Collateral Quality and Intervention Traps*

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Abstract

What determines the supply of good collateral? We study a dynamic model in which borrowers must exert effort to maintain collateral quality and markets become illiquid when average quality is too low. Average quality grows quickly when it is high initially, but deteriorates or grows slowly otherwise. As such, even long-run market conditions are sensitive to a wide array of fundamental and non-fundamental shocks. Recoveries from illiquidity can occur, but only if funding is inefficiently rationed for some time. Policymakers without commitment may fall into *intervention traps* in which ex-post efficient liquidity injections cause permanent declines in collateral quality.

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1 Introduction

In many markets, borrowers must pledge collateral to obtain funding. For example, corporate bonds are secured by firm assets, and securitized debt instruments are backed by pools of loans. Such arrangements are often plagued by asymmetric information about underlying asset quality. This renders collateralized lending markets vulnerable to negative shocks. In particular, the ability to borrow against a certain asset (its collateral liquidity) may disappear during bad times, and in extreme cases, may not recover for extended periods of time. A striking example is private-label mortgage backed securities, which were commonly used as collateral prior to the global financial crisis, but not thereafter. Motivated by such market freezes, policy interventions to restore liquidity have become an important part of the central bank toolkit around the world.

What determines long-run liquidity in collateralized lending markets? What are the short and long-run effects of policy interventions designed to restore liquidity? We study these questions in a dynamic model of collateralized lending under asymmetric information with two key features: borrowers must exert effort to maintain the quality of long-lived collateral assets, and markets are liquid only if average collateral quality is sufficiently high. Our answers hinge on the link between current collateral quality in the market and the scope to sustain growth in collateral quality over time. We find that this link is pro-cyclical: average quality grows quickly when it is high to begin with, but deteriorates or grows slowly otherwise. This allows us to establish two main results.

The first main result is that even very long-run market liquidity is vulnerable to a wide array of shocks. With respect to fundamental shocks, we show that transitory shocks to asset quality can lead to a permanent decline in growth rates because the scope to maintain collateral quality depends on current market conditions. This provides an explanation for why certain asset classes may *permanently* fail to be considered good collateral after adverse shocks. More generally, because borrowers are forward-looking, current and future market conditions are sensitive to any shock that affects *expectations* about future payoffs, including changes in discount rates, the persistence of asset quality, or beliefs regarding others' actions even if they have no direct effect on collateral values. Our framework thus offers a broad taxonomy of risks, both fundamental and non-fundamental in nature, affecting collateralized lending markets.

The second main result is that recoveries from periods of illiquidity can occur even absent policy interventions, but only if there is a period in which lending is inefficiently rationed. A striking implication is that benevolent policymakers without commitment power can fall into *intervention traps* in which ex-post efficient lending subsidies hamper long-run growth by destroying private incentives to maintain collateral quality. In such a trap, well-intended policies to overcome inefficient market freezes in the short run induce *permanent* declines in asset quality in the long run, forcing the policymaker to intervene time and again at ever greater cost. This offers a stark contrast with important findings in the literature, such as Tirole (2012), in which the equivalent intervention would improve welfare because asset quality is assumed to be fixed. As such, our analysis offers an explanation for why some real-world interventions, such as the Targeted Long-Term Refinancing Operations of the European Central Bank, have had to be extended multiple times.

Formally, we consider a model in which a continuum of infinitely-lived borrowers each own a durable productive asset with privately known quality that is either good or bad. Types are persistent but evolve stochastically, with transition probabilities determined by hidden effort. Assets can represent physical assets or pools of securities; effort can represent the maintenance, monitoring, or screening required to sustain cash flows over time. In every period, assets offer a collateralizable cash flow and an additional investment opportunity that requires a unit of new investment. Borrowers can fund this investment by pledging their collateral to borrow from short-lived competitive lenders. Good assets offer more collateral and possibly higher returns on investment than bad assets.

Because lenders unable to distinguish between good and bad assets, they provide funding only if aggregate asset quality (the share of good assets) is above a *liquidity threshold*. To compensate for adverse selection, the market-wide interest rate includes a mark-up that is decreasing in aggregate asset quality. Because bad borrowers prefer to default ex-post, the mark-up is paid only by good borrowers. This creates a two-way feedback mechanism whereby the additional benefit of owning a good asset (the good assets' *premium*) depends on the expected path of liquidity and interest rates, and liquidity and interest rates depend on the expected path of the premium.

We decompose this feedback mechanism into two forms of strategic interactions. The first is a *strategic complementarity* that operates if borrowers expect to be able to borrow. Since the adverse selection mark-up in interest rates is paid by good borrowers, the good asset's premium is increasing in current and expected future asset quality. A higher premium raises incentives to exert effort, which in turn supports a higher premium. This positive feedback loop links future growth to current asset quality. Accordingly, a *decline* in current asset quality, or any shock that lowers the expected future value of good assets, can lead to a permanent decline in the growth rate. Contrary to canonical models of adverse selection, these effects are not due to contemporaneous breakdowns in trade; rather they obtain because poor expected market conditions discourage investment in collateral quality today. Indeed, we find that pessimistic self-fulfilling beliefs alone may be enough to trigger a decline in long-run liquidity. Since such poor outcomes can occur even if all borrowers would have been better off if everyone had exerted effort, our results indicate the presence of *dynamic coordination failures* in collateralized lending markets.

The second form of strategic interaction is a *local strategic substitutability* that operates when asset quality is close to the liquidity threshold. Below the liquidity threshold, borrowers retain all collateral because they cannot borrow. At the liquidity threshold, borrowers must instead pledge *all* of their collateral. Since good borrowers have more collateral to pledge, the transition to liquidity leads to a sharp decrease in the *relative* (but not absolute) value of good assets. This reduces individual borrowers' willingness to contribute to an increase in market-wide asset quality. For intermediate costs of effort, markets either remain frozen indefinitely or resume growth only after a period of *inefficient rationing* in which some borrowers are not funded even though asset quality is high enough to sustain lending to all borrowers. Rationing improves incentives because unfunded borrowers remain fully exposed to the value of their collateral. Whenever investment is efficient, this leads to a strict trade-off between efficient investment in the short run and sustained growth in the long run.

This trade-off presents an acute dilemma for policymakers with the ability to restore liquidity in frozen lending markets. Such liquidity interventions have become important tools for central banks around the world. Tirole (2012) and Philippon and Skreta (2012) provide theoretical foundations for this type of policy in an environment with fixed asset quality, and find that optimal intervention generally entails restoring market liquidity at minimum cost. In line with these findings, we show that a policymaker without commitment power finds it strictly optimal to restore ex-post efficient lending when funding is rationed. With endogenous asset quality, however, this policy can reduce rather than improve welfare because it destroys private incentives to maintain collateral quality. Under some conditions, average quality falls *zero* under the intervention but would have grown indefinitely otherwise. The decline in asset quality induces further interventions, leading to *intervention traps* in which lending markets become permanently reliant on subsidies.

1.1 Related Literature

The general notion that scarcity of collateral hampers investment and output is well-established. Our specific focus is on markets where the quality of collateral is endogenously determined and subject to asymmetric information. Fluctuations in information about collateral quality has been cited as a key catalyst of the recent financial crisis (e.g. Gorton and Ordoñez (2014)). Our contribution is to analyze the endogenous dynamics of collateral quality and the feedback between liquidity and incentives. In this respect, our setting provides sharp predictions on how asset quality responds to adverse shocks. This differentiates our work from Bigio (2015), Kurlat (2013), and Eisfeldt (2004) who show that exogenous quality shocks can depress investment under asymmetric information.

Previous work has argued that increasing asset prices and market liquidity can *decrease* incentives to produce high-quality assets. Examples include Chemla and Hennessy (2014), Vanasco (2017) Neuhann (2018), Caramp (2020) Fukui (2018), Daley et al. (2020), and Asriyan et al. (2019). They consider settings where agents expect to sell more of their assets when asset prices rise. The resulting lack of exposure to asset returns then reduces incentives. We study markets in which agents can borrow the same amount using less collateral when interest rates fall. This *increases* exposure to asset quality, and generates *positive* co-movement between asset quality and output.

Similar to Asriyan et al. (2019), incentives in our model are determined by expectations over future market conditions. They focus on sunspot fluctuations in a model with transient types, while we study persistent types and endogenous fluctuations in fundamentals. Zryumov (2015) and Hu (2022) study dynamic models in which asset quality fluctuates due to the entry of bad types.

Our policy analysis relates to the theoretical literature on interventions in adversely-selected asset markets, including the previously discussed Tirole (2012) and Philippon and Skreta (2012). Fuchs and Skrzypacz (2015) consider a model with dynamic trading but fixed asset quality. Camargo and Lester (2014) study decentralized markets in which trading dynamics are decided by selective exit of seller types. Camargo et al. (2016) study the design of interventions in the presence of an information externality from trade. We do not consider information externalities or dynamic trading, but focus on the endogenous determination of effort incentives in a dynamic environment. Li and Li (2021) show that mispriced capital subsidies to firms hamper the cleansing effect of recessions.

2 Framework

There is a unit mass of risk-neutral long-lived *borrowers* with discount factor $\beta \in (0,1)$, who each own a long-lived *asset*. In any given period, assets are defined by their current quality θ, which can be either good or bad, $\theta \in \{b, g\}$, and may evolve over time. The share of good assets in period t is denoted by λ_t, and λ_0 is exogenously given. The initial condition λ_0 should be interpreted as the result of a single un-modeled shock to aggregate asset quality; our analysis then corresponds to the dynamics of asset quality in response to such a shock. Online Appendix B.1 considers the effects of anticipated aggregate shocks.

Assets offer two types of cash flows in every period: a fixed payoff L_θ that does not require further investment (similar to assets in place), and an additional payoff $R_\theta > L_\theta$ that accrues only if the borrower invests one unit of capital within the period (e.g., a growth option). We summarize the payoff differences across types by the difference in fixed cash flow $\Delta L = L_g - L_b$ and the difference in growth options $\Delta R = R_g - R_B$. Online Appendix B.2 shows that our results are robust to allowing variable investment scale.

In order to invest, borrowers must obtain the unit of capital from a mass $m > 1$ of risk-neutral *lenders*. Lenders are short-lived. A new generation is born with one unit of capital in the beginning of every period and exits at the end of the period. We introduce financial frictions by assuming that R_θ cannot be seized by lenders if the borrower defaults. Hence only L_θ can be pledged as collateral. Since $R_\theta > L_\theta$, borrowers are always willing to pledge collateral to invest.

Lending is subject to asymmetric information. Period-t lenders know the share of good assets λ_t, but individual asset quality is the borrower's private information. That lenders observe λ_t is convenient but not important; equivalent results obtain if lenders have rational expectations about average quality. What is important is that interest rates and liquidity depend on λ_t.

We make two main assumptions on parameter values. The first is that investment is efficient, which implies that lending market freezes are generally inefficient.

Assumption 1 (Investment is Efficient)**.** *$R_\theta > 1$ for all types θ.*

The second is that only good assets offer enough collateral to sustain borrowing. This ensures that the availability of funding depends on the share of good assets.

Assumption 2 (Collateral Supply)**.** *Only good types can fully collateralize one unit of capital, $L_g > 1 > L_b$.*

Given our focus on collateral quality and market liquidity, we also make the auxiliary assumption that collateral heterogeneity is the dominant source of heterogeneity[1], and we follow the literature in assuming $R_g \geq R_b$. These assumption are not central to our results, but they allow us to greatly simplify the exposition. Section 2.5 provides a more detailed discussion.

Assumption 3. *Collateral heterogeneity is greater than growth option heterogeneity:* $\Delta L > \Delta R \geq 0$.

2.1 Separating Static and Dynamic Components

A useful feature of the model is that it can be studied in two separate blocks: a static lending market game described in Section 2.2 in which asset quality is taken as given, and a dynamic game described in Section 2.3 in which asset quality is determined taking as given the payoff functions of the static game. This allows us to separately characterize the economic mechanisms shaping market liquidity and asset quality. Figure 1 shows the timeline in a generic period.

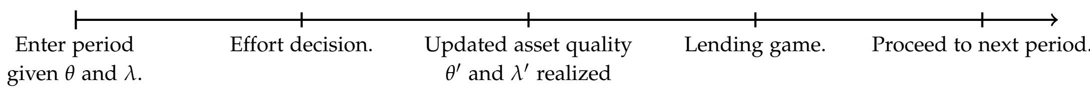

Figure 1: Timing of events in period t.

2.2 The Lending Game

We begin by describing the static lending market game, taking the distribution of asset quality and the share of good assets λ as given. We study the Bayes-Nash equilibrium of a game where borrowers offer collateralized contracts consisting of a promised interest payment B and an endogenous amount of collateral in exchange for 1 unit of capital. If lenders accept the contract, borrowers invest and earn the returns of their projects within the period. They must then decide whether to make

[1]This assumption is in line with Tirole (2012), who assumes $\Delta R = 0$. This is important because our policy analysis is directly related to that paper.

the promised interest payment. If they do not, lenders seize the pledged collateral. Since lenders are short-lived, borrowing is short-term.

We first describe two benchmarks. With full pledgeability, all borrowers obtain funding at gross interest rate of 1. When types are observable, only good borrowers obtain funding. Hence, asymmetric information matters only if pledgeability is limited.

Benchmark 1 (Full Pledgeability)**.** *Suppose R_θ and L_θ can both be pledged. Since $R_\theta > 1$, all borrowers obtain funding at gross interest rate* 1*.*

Benchmark 2 (Symmetric Information and Limited Pledgeability)**.** *Suppose information is symmetric but R_θ is not pledgeable. Since $L_b < 1 < L_g$, only good types obtain funding, and the gross interest rate is* 1*.*

Now turn to the case with frictions. Since investment is efficient and $L_b < 1 < R_b$, any equilibrium in which good borrowers obtain funding must involve pooling. Since the interest payment B must exceed L_b, good borrowers repay and bad borrowers default. Lenders' participation constraint is $\lambda B + (1-\lambda)L_b \geq 1$, and can be satisfied only if λ exceeds *liquidity threshold* $\bar{\lambda}$ given by

$$\bar{\lambda} \equiv \frac{1-L_b}{L_g - L_b}. \tag{1}$$

We select the equilibrium in which the participation constraint holds with equality if it can be satisfied.[2] Then borrowers cannot borrow if $\lambda < \bar{\lambda}$, and if $\lambda > \bar{\lambda}$ they obtain funding at interest payment

$$B^*(\lambda) = \frac{1-(1-\lambda)L_b}{\lambda}, \tag{2}$$

which is strictly decreasing in aggregate quality λ. If aggregate quality is at the liquidity threshold, $\lambda = \bar{\lambda}$, the participation constraints holds with equality if borrowers offer the maximum feasible

[2]Choosing other off-equilibrium beliefs can support equilibria in which fewer borrowers obtain funding. The selected equilibrium thus minimizes inefficient market breakdowns arising from lending market frictions.

payment L_g. This permits multiple equilibria indexed by borrowers' probability of obtaining funds $\phi \in [0,1]$. In the full model, this probability is pinned down by an optimality condition for effort.

We summarize equilibrium using the *payoff function* $u(\theta, \lambda)$ for type θ and aggregate quality λ,

$$u(\theta,\lambda) = \begin{cases} R_\theta + L_\theta - \min\{B^*(\lambda), L_\theta\} & \text{if } \lambda > \bar{\lambda} \\ \phi R_\theta + (1-\phi) L_\theta & \text{if } \lambda = \bar{\lambda} \text{ for some } \phi \in [0,1] \\ L_\theta & \text{if } \lambda < \bar{\lambda}. \end{cases} \tag{3}$$

The payoff function is strictly increasing in aggregate asset quality λ when markets are liquid, and independent of λ when markets are illiquid. This is the source of strategic interactions in the model.

2.3 The Dynamic Asset Quality Game

The evolution of asset quality over time is endogenously determined by hidden effort. Effort $e \in [0,1]$ is a choice with cost $c \cdot e$ that is made by the onwer of the asset at the beginning of the period. It affects the probability $p(\theta'|\theta, e)$ of obtaining an asset of quality θ' at the end of the period given current quality θ. We say there is *full effort* if $e = 1$ and *partial effort* if $e \in (0,1)$. We specify the technology

$$p(g|\theta, e) = \rho \cdot \mathbb{1}(\theta = g) + \pi e, \tag{4}$$

where $\rho \in [0,1]$ and $\pi \in (0, 1-\rho]$ are the persistence of asset quality and marginal efficacy of effort, respectively. Persistent quality is an important feature of our model; we discuss it in Section 2.5.

The state variable for a borrower is (θ, λ), and a *strategy* for borrower i is a decision rule $e^i(\theta, \lambda) \in [0,1]$. *Aggregate effort* (or equivalently, average effort) is

$$E(\lambda) = \int_i e^i(\theta^i, \lambda). \tag{5}$$

By the law of large numbers, the updated share of good assets is $\lambda' = \rho\lambda + \pi E(\lambda)$ and depends only on current quality and aggregate effort. Hence the law of motion associated with effort rule E is

$$m(\lambda, E) \equiv \rho\lambda + \pi E(\lambda). \tag{6}$$

Then aggregate asset quality after $\tau \geq 1$ periods given aggregate effort rule E is

$$m^\tau(\lambda, E) \equiv \rho^\tau \lambda + \sum_{k=0}^{\tau-1} \rho^k \pi E\left(m^{\tau-1-k}(\lambda, E)\right) \tag{7}$$

where $m^0(\lambda, E) \equiv \lambda$. Quality grows if $E(\lambda) \geq \frac{1-\rho}{\pi}\lambda$. The maximum sustainable asset quality is

$$\lambda^{\max} = \lim_{\tau \to \infty} m^\tau(\lambda, 1) = \frac{\pi}{1-\rho}.$$

We assume that it is feasible to sustain liquidity in the long-run.

Assumption 4. $\lambda^{\max} > \bar{\lambda}$. *This inequality holds if* $\Delta L > \frac{(1-L_b)(1-\rho)}{\pi}$.

We also assume that effort is privately optimal in the benchmarks without frictions. This requires that the expected discounted sum of incremental cash flows exceeds the cost of effort.

Assumption 5. *Effort is optimal when there are no frictions,* $c < \pi\frac{\Delta L + \Delta R}{1-\rho\beta}$.

Individual agents take the aggregate effort rule as given. Let $\hat{E}$ denote an agent's belief about the aggregate effort rule. Then the value function for an agent in state (θ, λ) satisfies

$$\begin{aligned} V(\theta, \lambda | \hat{E}) = \max_e \quad & \sum_{\theta} p(\theta' | \theta, e)\Big(u(\theta', \lambda') + \beta V(\theta', \lambda' | \hat{E})\Big) - ce \\ \text{s.t.} \quad & \lambda' = m(\lambda, \hat{E}). \end{aligned} \tag{8}$$

The max operator on the right-hand of the value function defines the *effort choice problem*. We are now ready to define an equilibrium of the dynamic game.

Definition 1 (Competitive Equilibrium)**.** *A competitive equilibrium of the dynamic game consists of strategies e^i for all borrowers i such that*

1. *Individual effort choices e_t^i solves the optimal effort problem defined by value function* (8) *for all borrowers i and all time periods t given beliefs $\hat{E}^i$.*

2. *Beliefs are correct in equilibrium, $\hat{E}^i(\lambda) = E(\lambda)$ for all borrowers i.*

2.4 Utilitarian Benchmark

We evaluate the efficiency of competitive equilibrium by comparing it with the allocation that maximizes utilitarian welfare, taking as given the lending market frictions. The utilitarian payoff function is the sum of type-specific payoffs, $w(\lambda) = \lambda u_g(\lambda) + (1-\lambda)u_b(\lambda)$. Using the lender's participation constraint yields the strictly increasing function

$$w(\lambda) = \begin{cases} L_b + R_b + \lambda(\Delta L + \Delta R) - 1 & \text{if } \lambda > \bar{\lambda} \\ L_b + \lambda \Delta L + \phi\Big(R_b + \lambda \Delta R - 1\Big) & \text{if } \lambda = \bar{\lambda} \text{ for some } \phi \in [0,1] \\ L_b + \lambda \Delta L & \text{if } \lambda < \bar{\lambda}. \end{cases} \tag{9}$$

Given an aggregate effort rule E, welfare is $W(\lambda, E) = w(\lambda) + \beta W(\lambda') - cE(\lambda)$, where $\lambda' = m(\lambda, E)$. This is the same welfare criterion as the λ-weighted sum of type-specific value functions.

The socially optimal effort rule is $E^W = \arg\max_E W(\lambda_0, E)$. The key difference to private payoffs is that welfare is independent of the interest rate. This is because adverse selection redistributes across types without affecting aggregate cash flows. Hence a social planner prefers effort if the marginal value of good assets is sufficiently high and/or markets become liquid sufficiently quickly. This leads to a cut-off rule whereby effort is efficient when initial asset quality is sufficiently high.

Lemma 1 (Socially Efficient Effort Rule)**.** *Socially efficient effort follows a cut-off rule: there exists a unique λ^W such that full effort is optimal in all periods if and only if $\lambda_0 \geq \lambda^W$. Full effort is always socially optimal when markets are expected to be liquid within one period, i.e. $\lambda^W < (\bar{\lambda} - \pi)/\rho$.*

2.5 Model Discussion

Our model considers long-lived assets whose quality is persistent and evolves endogenously over time. There are a number of ways to map this setting to the real world. The underlying asset can represent physical capital, such as a plant, machinery, or real estate, which depreciates over time in a manner that is sensitive to owner effort. The asset could also represent a pool of relatively long-term financial securities, such as corporate loans or residential mortgages, where total payoffs of the pool determine the overall pledgeable and private income accruing to its owner. Even if the quality of an individual security is fixed, the pool's overall quality may evolve over time if there is prepayment, default, or maturation of some securities, and the borrower may further influence portfolio quality through monitoring of loans or by screening securities that replace expired ones.

We also assume that effort positively affects both R_θ and L_θ (that is, L and R are positively correlated). This is standard in the literature, but it is not essential. All of the key results would obtain if effort mainly affects collateral quality while non-pledgeable cash flows R_θ are negatively correlated with L_θ. This is because market liquidity is determined by the availability of collateral L_θ.

3 Optimal Effort: The Value of Good Assets

We begin our analysis by providing conditions under which borrowers find it optimal to improve asset quality through effort. Since quality is persistent, exerting effort today raises expected asset quality in all future periods. Thus the key determinant of borrower effort is the difference in asset *values* between good and bad assets, $\Delta(\lambda, \hat{E}) \equiv V(g, \lambda|\hat{E}) - V(b, \lambda|\hat{E})$. We refer to $\Delta(\lambda, \hat{E})$ given beliefs $\hat{E}$ as the good asset's *premium*. We can then rewrite value function (8) in the intuitive form

$$V(\theta,\lambda|\hat{E}) = \max_{e} \quad u(b,\lambda') + \beta V(b,\lambda'|\hat{E}) + p(g|\theta,e)\Delta(\lambda',\hat{E}) - ce \tag{10}$$
$$\text{s.t.} \quad \lambda' = m(\hat{E},\lambda).$$

The first two terms represent the baseline current and future value obtained by all borrowers. The third term is the premium multiplied by the probability of obtaining a good asset. The only term that depends on current type θ is $p(g|\theta,e)$. That is, the sole advantage of having a good asset at the beginning of the period is that it affords a higher probability of owning a good asset at the end of the period for any given level of effort. Since $p(g|\theta,e)$ is linear in effort, the value of effort is equal to the premium evaluated at updated aggregate quality λ'.

Lemma 2 (Optimal Effort Rule)**.** *For any θ, it is privately optimal to exert effort if and only if*

$$\pi\Delta\left(\lambda',\hat{E}\right) \geq c \quad \textit{where} \quad \lambda' = m(\lambda,\hat{E}). \tag{11}$$

This optimality condition is the same for both types. Hence it is without loss of generality with respect to aggregate outcomes to focus on symmetric effort rules $e(g,\lambda) = e(b,\lambda) = e(\lambda)$.[3]

Since asset values are discounted sums of future payoffs, we can write the premium recursively using the difference in per-period payoffs (i.e. per-period utility) earned by owners of good assets given asset quality λ. Given payoff function $u(\theta,\lambda)$ from the lending game, the payoff difference is $\Delta u(\lambda) \equiv u(g,\lambda) - u(b,\lambda)$. The premium then satisfies the recursion

$$\Delta(\lambda,\hat{E}) = \Delta u(\lambda) + \beta\rho\Delta(\lambda',\hat{E}), \tag{12}$$

[3]The choice of effort still determines the path of an *individual* borrower; as in Kaniel and Kondor (2013), individual borrowers might therefore opt for asymmetric strategies. However, in our setting this would not change the dynamics of *aggregate* asset quality because the marginal efficacy of effort is the same for all borrowers. Given our focus on market liquidity and borrowing costs, we thus focus on symmetric strategies.

where the discount rate is adjusted by ρ because, even absent future effort, an asset of good quality today has a higher probability of being a good asset in all future periods than an asset that is currently of bad quality.

Much of the intuition regarding the properties of the premium can be gleaned from analyzing the per-period payoff difference. Using the payoff function derived in the static game yields

$$\Delta u(\lambda) = \begin{cases} \Delta R + \Delta L - \frac{1-L_b}{\lambda} & \text{if } \lambda > \bar{\lambda} \\ \phi \Delta R + (1-\phi)\Delta L & \text{if } \lambda = \bar{\lambda} \text{ for some } \phi \in [0,1] \\ \Delta L & \text{if } \lambda < \bar{\lambda}. \end{cases}$$

While per-period payoffs monotonically increase in aggregate asset quality, the payoff difference increases in asset quality when markets are liquid, but is locally non-monotonic around the liquidity threshold. This feature leads to rich state-contingent asset quality dynamics.

Lemma 3. *The per-period payoff $u(\theta, \lambda)$ increases in aggregate asset quality λ for all types. However, the payoff difference $\Delta u(\lambda)$ is non-monotonic: it is constant when markets are illiquid, drops discontinuously at the liquidity threshold, and increases in asset quality when markets are liquid.*

When markets are liquid, the payoff difference is increasing in λ because there is less redistribution from good to bad borrowers as average quality increases. The non-monotonicity obtains because the redistribution due to adverse selection occurs only when markets are liquid. Hence there is a discrete change in relative payoffs as markets become liquid.

Proposition 1 uses these observations to characterize strategic interactions among borrowers and how they vary with market conditions. To isolate the effects of adverse selection, we decompose the interest payment into $B(\lambda) = 1 + b(\lambda)$, where 1 is the gross interest rate absent frictions and $b(\lambda) = (1 - L_b)\left(\frac{1}{\lambda} - 1\right)$ is the markup for adverse selection.

Proposition 1 (Strategic Interactions)**.** *The private value of effort is determined by the premium* Δ*, which depends on the expected path of asset quality as follows:*

(i) If markets are expected to be liquid forever, effort is a strategic complement in that the premium is increasing in expected future asset quality. In particular, the premium under liquidity is equal to

$$\Delta(\lambda, \hat{E}) = \frac{\Delta R + L_g - 1}{1 - \rho\beta} - \sum_{k=0}^{\infty} (\beta\rho)^k \underbrace{b(m^{k+1}(\lambda, \hat{E}))}_{\text{adverse selection markups}}, \tag{13}$$

where the adverse selection markups are decreasing in future asset quality.

(ii) If markets are expected to be illiquid forever, the premium is constant and independent of asset quality, $\Delta(\lambda, \hat{E}) = \frac{\Delta L}{1-\beta\rho}$*. Hence the premium is also independent of others' effort choices.*

(iii) If markets are initially illiquid but all borrowers always exert effort, the premium drops discontinuously when asset quality crosses the liquidity threshold. Hence effort is a strategic substitute during the transition to liquidity.

The economic intuition is as follows. When markets are expected to be liquid, an improvement in asset quality lowers interest rates. Since borrowers pledge their assets as collateral rather than sell them, lower interest rates increase owners' exposure to asset quality and raise the relative value of owning a good asset. This *strategic complementarity* implies that growth is easier to sustain when initial conditions are favorable. When markets are expected to remain illiquid, there are no strategic interaction because the premium is independent of aggregate quality. However, there is a local *strategic substitutability* during a transition to liquidity. Since there is adverse selection only when borrowing takes place, there is more redistribution just above the liquidity threshold than just below. This induces individual borrowers to free-ride on others' attempts to restore liquidity, making it particularly difficult to recover from periods of illiquidity. We formally show the dynamic implications of these effects next.

4 Equilibrium Asset Quality

In the previous section, we derived a simple condition for the private optimality of effort that depended on the expected path of aggregate quality. Since the path of aggregate quality is determined by effort, there is a feedback mechanism between individual beliefs and aggregate outcomes. To distinguish fundamental forces from belief-driven fluctuations, we first focus on the Pareto-dominant competitive equilibrium (PCE) in which beliefs are "most favorable" for effort and growth. Even this equilibrium may feature coordination failures that prevent efficient asset quality growth. Based on this result, we later show that there generically exist multiple equilibria in which self-fulfilling beliefs amplify such coordination failures. We use the following definition of Pareto efficiency.

Definition 2. *A competitive equilibrium with effort rule E is a* Pareto-dominant competitive equilibrium (PCE) *given initial condition λ_0 if this equilibrium leads to higher utility for all borrower types than any other equilibrium. That is, $V(\theta, \lambda_0|E) \geq V(\theta, \lambda_0|E')$ for all θ and all other equilibrium effort rules $E' \neq E$.*

Since the marginal value of effort is identical across types and investment is efficient, the PCE is the competitive equilibrium in which beliefs are most conducive to effort. Because effort is privately optimal when the expected value of the premium exceeds the cost of effort, the first step is to compute the premium under the conjecture that all borrowers exert effort , $\hat{E} = 1$. If effort is optimal given this conjecture, it can be sustained in equilibrium.

Given $\hat{E} = 1$, asset quality is expected to immediately exceed the liquidity threshold if current quality exceeds the lower threshold $\underline{\lambda}$ defined by $m(\underline{\lambda}, 1) = \bar{\lambda}$. For all such quality levels, the premium can be solved for in closed form as

$$\Delta(\lambda, 1) = \frac{\Delta R + L_g - 1}{1 - \beta\rho} - (1 - L_b) \sum_{k=0}^{\infty} (\beta\rho)^k \left(\frac{1}{\rho^{k+1}\lambda + \pi \left(\frac{1-\rho^{k+1}}{1-\rho} \right)} - 1 \right). \tag{14}$$

Given that there is maximal adverse selection at the liquidity threshold $\bar{\lambda}$, the premium under $\hat{E} = 1$

attains its minimum at the liquidity threshold $\bar{\lambda}$ and increases in λ otherwise. To allow for interesting dynamics, we assume that effort can be sustained for some quality levels but not for all.

Assumption 6 (Intermediate Cost of Effort)**.** $\pi\Delta(\bar{\lambda}, 1) < c < \pi\Delta(\lambda^{\max}, 1)$.

Given this assumption, there exists a unique quality level $\lambda^E \in [\underline{\lambda}, \lambda^{\max})$ such that the expected premium given $\hat{E} = 1$ when starting at λ^E is equal to the cost of effort. This *effort threshold* satisfies $\pi\Delta(m(\lambda^E, 1), 1) = c$. Its existence implies that equilibrium effort satisfies a cut-off rule.

Lemma 4 (Existence, Uniqueness, and Structure of PCE)**.** *There exists a unique Pareto-dominant competitive equilibrium in which borrowers exert full effort in all periods if and only if initial condition λ_0 is above the effort threshold λ^E. Below the effort threshold, there is less than full effort either temporarily or permanently.*

Lemma 4 establishes a strong link between initial conditions and subsequent asset quality. In particular, permanent effort can be sustained only if asset quality is sufficiently high. The next result shows that, below this threshold, asset quality either permanently falls, stagnates, or grows slowly, with periods of inefficient illiquidity along the way.

Proposition 2 (Quality Dynamics Given Poor Initial Asset Quality)**.** *If initial asset quality λ_0 is strictly below the effort threshold λ^E, there are two possible paths for asset quality depending on the cost of effort:*

(i) If $c > \pi\Delta L/(1 - \beta\rho)$, aggregate quality converges to zero and markets remain illiquid forever.

(ii) If $c \leq \pi\Delta L/(1 - \beta\rho)$, aggregate quality either (a) grows to $\bar{\lambda}$, in which case some borrowers cannot borrow in every period, or (b) grows to $\lambda^{\max}$, in which case growth is slowed down by partial effort in some periods and liquidity is rationed (not all borrowers obtain funding even though quality is high enough to fund all borrowers) in some periods. Rationing is integral to the recovery: without it, quality cannot reach $\lambda^{\max}$.

The first statement of this proposition highlights the fragility of long-run outcomes to small and/or temporary shocks to various model primitives. In particular, effort falls to zero if asset quality falls just below the effort threshold λ^E. Since the effort threshold is a function of the discount factor β, the persistence of quality ρ, and the efficacy of effort π, a small shock to any of these variables can drive current asset quality below the effort threshold. This leads to fragility of long-run asset quality with respect to shocks that are entirely irrelevant for cash flows in the current period (in particular, shocks that do not directly affect collateral availability in the current period).

This fragility allows us to speak to some of the observations that motivate this study. Specifically, the collateral liquidity of a particular asset class may be *fragile in the long-run*, in the sense that small exogenous shocks may change the long-run behavior of market liquidity and borrowing costs. Such permanent market freezes are akin to the sudden and persistent decline in the usefulness of mortgage-backed securities as collateral after the 2008 financial crisis.

The second statement considers parameters where the cost of effort is relatively low. We find that asset quality may recover in the long-run, but that this involves a period of slow growth and inefficient liquidity rationing. When the cost of effort is such that effort can be sustained when starting from the liquidity threshold, this rationing is temporary and liquidity recovers to $\lambda^{\max}$ in the long run. If effort cannot be sustained starting at $\bar{\lambda}$, asset quality instead remains at $\bar{\lambda}$ indefinitely.

In either case, the temporary inefficiency induced by liquidity rationing are crucial for the recovery because they raise incentives to exert effort. That is, there is a trade-off between short-term costs of illiquidity and the long-run benefits of sustained asset quality. Online Appendix C provides a graphical illustration of the PCE given parameters such that asset quality remains at $\bar{\lambda}$ indefinitely.

To see why rationing improves incentives, observe that the private value of effort is sensitive to the redistribution induced by adverse selection: an individual borrowers is less willing to exert effort because he knows that good types cross-subsidize bad types when markets are liquid. If aggregate quality is at the liquidity threshold $\bar{\lambda}$, incentives are thus higher under rationing (which induces a premium that is *as if* markets were illiquid) than if all borrowers were funded. Because rationing can only be sustained at the liquidity threshold, asset quality must grow slowly to this point. This leads to a striking implication: even though quality is sufficient to sustain borrowing, the PCE may sometimes involve a stasis at the liquidity threshold and a *permanent* partial market shutdown. Even in the case where asset quality eventually grows, temporary slowdowns have permanent effects: since quality is persistent, aggregate quality is lower in all periods after a period with low effort than in a counterfactual with full effort.

4.1 Inefficiency and Coordination Failures

The preceding discussion highlights that shortfalls in effort arise because the redistribution induced by adverse selection lowers the *private* value of effort. Since redistribution is immaterial from the perspective of utilitarian welfare, we show that PCE is socially inefficient unless effort is too costly even from the perspective of the social planner.

Corollary 1 (Inefficiency of the PCE)**.** *Consider parameters such that effort is socially optimal. If initial asset quality is below the effort threshold λ^E, then*

(i) Asset quality in the PCE is strictly lower than that in the utilitarian benchmark in all periods after an initial phase.

(ii) In at least one period, fewer borrowers are funded in the PCE than in the utilitarian benchmark.

(iii) Any PCE in which asset quality converges to $\bar{\lambda}$ or $\lambda^{\max}$ is Pareto-inefficient.

The fact that the competitive equilibrium can be Pareto-inefficient reveals a dynamic coordination failure. When there is effort only by some borrowers, all borrowers must be indifferent between effort and shirking. But individuals prefer to free-ride if everybody else exerts effort. In equilibrium, all borrowers obtain lower utility than if they coordinated on full effort.

These coordination failures are even stronger when beliefs are less favorable. In fact, *any* other competitive equilibrium necessarily worsens the dynamic coordination problem. This is because pessimistic beliefs regarding others' effort choices depress effort, and thus amplify inefficiencies stemming from expectations about the trajectory of aggregate asset quality. Hence, changes in beliefs alone may be enough to precipitate a breakdown in effort and a decline in liquidity and asset quality.

Corollary 2 (Self-fulfilling Beliefs)**.** *For an intermediate range of λ in the liquidity region, there always exist multiple equilibria in which differences in the path of asset quality are driven by self-fulfilling beliefs about other borrowers' effort – aggregate asset quality can increase, stagnate, or decrease over time.*

5 Intervention Trap

Market freezes in collateralized lending markets have frequently motivated policy interventions to restore liquidity. These policies are grounded in the view that public liquidity provision can raise welfare if shutdowns are inefficient. This view has theoretical backing from studies, such as Tirole (2012) and Philippon and Skreta (2012), in which asset quality is fixed. We now evaluate the welfare consequences of liquidity interventions in our model with endogenous quality.

We consider a benevolent policymaker who aims to maximize utilitarian welfare. The policymaker has no informational advantage or disadvantage over lenders: he observes λ but not θ. Based on this information, he can inject liquidity by providing a *subsidy* $s(\lambda)$ to any lender whose borrower defaults when the realized asset quality within a given period is λ.[4] Such a subsidy lowers borrowing costs. The required interest payment given s is $B(\lambda, s) = (1-(1-\lambda)(L_b + s(\lambda)))/\lambda$. The smallest subsidy that allows borrowers to obtain funding if $\lambda \leq \bar{\lambda}$ is the one that ensures that $B(\lambda, s) = L_g$. We call this the *minimal subsidy* $\underline{s}(\lambda)$ and observe that $\underline{s}(\lambda) \equiv \frac{1-\lambda L_g-(1-\lambda)L_b}{1-\lambda}$.

Deficits have a social deadweight cost $\delta > 0$. We focus on the limit $\delta \to 0$, where the deadweight loss serves to select the "smallest possible" intervention that delivers a particular allocation.

To maintain a close analogy with the optimal policy in Tirole (2012), we assume that the policymaker has limited commitment. Under limited commitment, any intervention in the lending game

[4] The particular policy instrument is not important. Alternative policy implementations with the same aggregate consequences include outright asset purchase collateral exchanges.

must be ex-post optimal *taking as given* realized asset quality. Since continuation values depend only on asset quality, ex-post optimality is equivalent to the following definition.

Definition 3 (Ex-post Optimal Subsidy)**.** *Subsidy $s(\lambda)$ is ex-post optimal if and only if*

$$s(\lambda) = \arg\max_{s'} \qquad \lambda u_g(\lambda, s') + (1-\lambda)u_b(\lambda, s') - (1+\delta)(1-\lambda)s'.$$

The ex-post optimal policy is the precise analogue of Tirole (2012): when the deadweight cost of interventions is small, the policymaker always unfreezes the market using the cheapest intervention. The motive is clear: since investment is efficient for all types, market freezes are inefficient. This is particularly evident when quality is at the liquidity threshold: since any subsidy is enough to induce borrowing, the policymaker always ensures that all borrowers receive funding at $\bar{\lambda}$.

Lemma 5 (Ex-post Optimal Policy)**.** *Let $\delta \to 0$. The policymaker injects the minimal subsidy if and only if not all borrowers would have been funded in the competitive equilibrium. At the liquidity threshold, the policymaker can ensure that all borrowers are funded by providing an infinitesimal subsidy.*

Importantly, the ex-post optimal subsidy rule ignores incentives because effort is sunk during the lending game. While the policy unambiguously increases welfare *given* λ, under mild conditions it lowers asset quality and reduces welfare once the feedback to effort is taken into account. Since it is again optimal to intervene when asset quality has fallen, optimal policy can lead to *intervention traps* in which asset quality permanently shrinks and the costs of subsidies rises.

Proposition 3 (Intervention Trap)**.** *Let $\lambda_0 < \lambda^E$ and consider a PCE in which borrowers exert effort at some date. Then asset quality converges to 0 under the ex-post optimal subsidy, but to $\bar{\lambda}$ or $\lambda^{\max}$ in the PCE. Despite the fall in asset quality, the policymaker finds it optimal to provide subsidies in every period, and the subsidy is strictly increasing over time. Moreover, the ex-post optimal policy can strictly reduce welfare.*

The intuition is that the intervention creates a region of the state space where incentives are "as if" all borrowers obtain the funding at the competitive equilibrium interest rate associated with

$\lambda = \bar{\lambda}$. However, Proposition 2 showed that, when $\lambda_0 < \lambda^E$, equilibrium asset quality can grow or remain at the liquidity threshold only if some borrowers are rationed at the liquidity threshold. Since this rationing is ex-post inefficient, the policymaker has a strict preference to induce borrowing by all agents, which destroys private incentives to exert effort. Hence asset quality falls.

In the next period, the policymaker faces the same problem. While asset quality has shrunk, investment is still efficient for all types. Hence the policymaker again provides a subsidy that creates interest rates *as if* $\lambda = \bar{\lambda}$ and borrowers respond by not improving the quality of their assets. What is more, the required subsidy is now larger because there are fewer good assets. This is the intervention trap: expectations of ex-post efficient interventions lead to a steady decline in asset quality that induces repeated interventions at ever greater cost, and the market grows more reliant on subsidies because quality can only recover slowly once support is withdrawn.

Figure 2 illustrates this mechanism. We plot equilibrium outcomes for various starting values λ_0 below the liquidity threshold $\bar{\lambda}$. The first panel shows the evolution of asset quality over time. In the PCE, asset quality converges to $\lambda^{\max} = 1$ in the long run but there is a period of low effort near the liquidity threshold that temporarily slows down growth. Under the intervention, asset quality instead converges to zero. The second panel shows total net cash flows generated. The intervention initially delivers higher total cash flows than in the PCE because the subsidy induces efficient investment. After some periods, however, the decline in asset quality induced by the subsidy leads to *lower* cash flows than in the PCE. The third panel shows that subsidy payments grow over time because the share of good assets falls. The last panel shows that welfare is higher in the PCE than under the policy for any initial condition.

The dynamics of asset quality and liquidity induced by the ex-post efficient intervention are reminiscent of observed credit market interventions by the European Central Bank in the aftermath of the Great Financial Crisis and the Eurozone Sovereign Default Crisis. Programs such as the Long-term Refinancing Operations and various collateral upgrading facilities had to be renewed

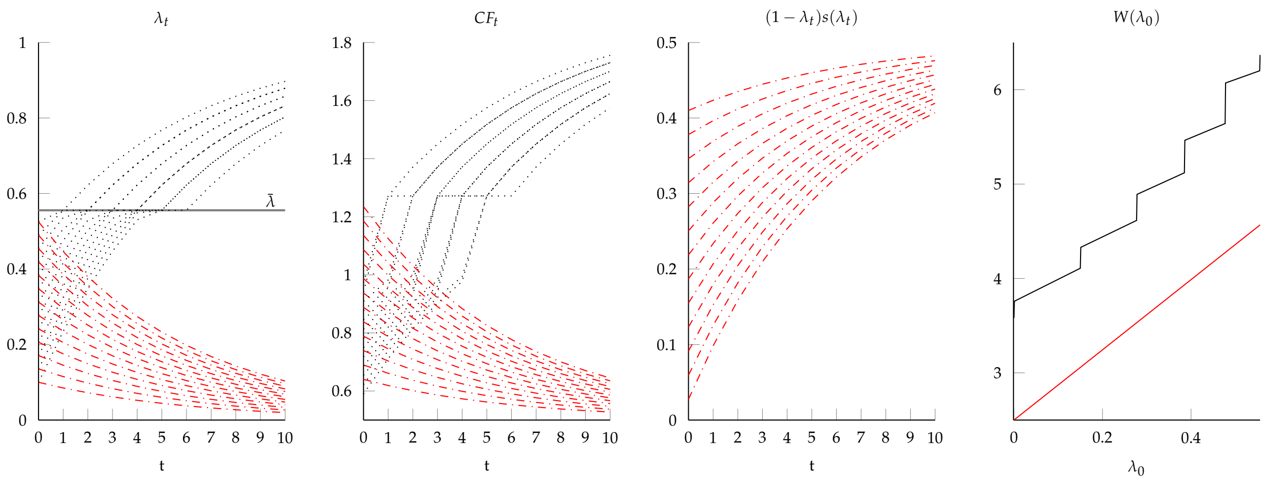

Figure 2: Welfare comparison between PCE and the ex-post optimal intervention. PCE outcomes are in black, equilibrium under the ex-post optimal subsidy are in red. In the second panel, total cash flows CF_t are $CF_t = \phi(\lambda)\Big(\lambda_t R_g + (1-\lambda_t)R_b - 1\Big) + \lambda_t L_g + (1-\lambda_t)L_b$. Parameters: $R_g = 1.5$, $R_b = 1$, $L_g = 1.4$, $L_b = 0.5$, $\beta = 0.8$, $\pi = 1-\rho = 0.15$, $\frac{c}{\pi} = 1.975$.

multiple times once it became clear that capital markets would not remain liquid without additional support. In this context, an important implication of our results is that it may be optimal to provide large subsidies that induce interest rates *as if* $\lambda \gg \bar{\lambda}$. However, such interventions would require commitment. A full analysis of optimal policy under commitment is beyond the scope of this paper.

6 Conclusion

We propose a tractable framework to study the dynamic evolution of liquidity and interest rates in collateralized lending markets with uninformed lenders and endogenous collateral quality. Our first main result is that the evolution of asset quality is pro-cyclical, so that even long-run market conditions are vulnerable to a wide array of fundamental and non-fundamental shocks. These shocks may be small, and may have significant consequences even without a discernible impact on current cash flows. Our second main result is that recoveries from periods of illiquidity may require inefficient rationing of borrowers for some time, and that benevolent policymakers without commitment may therefore harm welfare. Specifically, policymakers may fall into intervention traps, in which ex-post efficient liquidity injections lead to a permanent decline in fundamental asset quality. These results point to the need for future work to consider the optimal design and implementation of policy that takes into account the dynamics of endogenous asset quality.

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A Proofs

A.1 Proof of Lemma 1

For $\lambda > \bar{\lambda}$, the social return to effort is $\pi\frac{\Delta R+\Delta L}{1-\rho\beta}$, which exceeds c by Assumption 5. The result then follows because $m(\lambda, E)$ is strictly increasing in both arguments. □

A.2 Proof of Proposition 1

Statements (i) and (ii) follow directly from forward substitution of the per-period payoff difference in the recursive formulation of the premium in (12). Given $\lambda > \bar{\lambda}$, the premium is strictly increasing in λ because the adverse selection markup $b(\lambda)$ is strictly decreasing. For statement (iii), consider some λ' strictly above but arbitrarily close to liquidity threshold $\bar{\lambda}$. Then the per-period payoff difference at λ' is arbitrarily close to ΔR. Next, consider some λ'' strictly below but arbitrarily close to $\bar{\lambda}$. Then the per-period payoff difference at λ'' is $\Delta L > \Delta R$. Given that all borrowers are expected to exert effort in all periods, the premium is continuous and strictly increasing in $\lambda > \bar{\lambda}$. Hence the continuation values starting from λ' and λ'' can be made arbitrarily close. Since $\Delta L > \Delta R$, it follows that the premium is strictly higher at λ'' than at λ'. □

A.3 Proof of Lemma 4

We will argue that the PCE must involve full effort when full effort is sustainable (i.e. when $e = 1$ solves the effort choice problem for all borrowers and all λ on the path of play). We use of the following intermediate result showing that value functions are strictly increasing in λ given $E = 1$. Let $\underline{\Delta} \equiv \frac{\Delta L}{1-\rho\beta}$ and $\bar{\Delta} \equiv \Delta(\bar{\lambda}\rho + \pi, 1)$ for shorthand notation.

Lemma 6 (Increasing Value Functions)**.** *Suppose $E = 1$. Then $V(\theta, \lambda|1)$ is strictly increasing in λ on $[0, 1]$, and continuous and strictly increasing in λ on $[\underline{\lambda}, 1]$.*

Proof. For $\lambda \geq \bar{\lambda}$, $u(b, \lambda)$ is constant and $u(g, \lambda)$ is strictly increasing and continuous. Given that investment is efficient and $R_\theta > L_\theta \forall\theta$, the result follows from Theorem 4.7 in Stokey et al. (1989). □

We can then prove the main result by construction.

Step (i): We show that full effort is optimal when $\lambda \geq \lambda^E$. For any $\lambda \geq \lambda^E$, effort is privately optimal when all other borrowers are expected to exert effort. Moreover, $E = 1$ induces the maximum feasible asset quality period by period. By Lemma 6, utility must therefore be strictly higher than in any other equilibrium. Hence the PCE has $E(\lambda) = 1$ for all $\lambda \geq \lambda^E$.

Step (ii): Now consider the remainder of the state space, $\lambda < \lambda^E$. There two are possible cases.

Case (a). By Proposition 1, the premium under permanent illiquidity is $\underline{\Delta} = \frac{\Delta L}{1-\rho\beta}$. Hence if $c > \pi\underline{\Delta}$, effort is not optimal under permanent illiquidity. Proposition 1 also shows that the premium is strictly increasing in E when markets are expected to be liquid. Hence there does not exist any $\lambda < \lambda^E$ for which effort is privately optimal if asset quality remained at λ forever. This implies that the unique equilibrium is such that $E(\lambda) = 0$ for all $\lambda < \lambda^E$.

Case (b). If instead $c \leq \pi\underline{\Delta}$, then effort can be sustained if markets are expected to be illiquid forever. Hence the PCE must involve effort for some λ sufficiently small. To see that full effort cannot be sustained for all λ, observe that $\pi\Delta(\bar{\lambda}, 1) < c$ by Assumption 6. Since the premium is continuous in $\lambda \geq \bar{\lambda}$, there exists some λ^* such that $\pi\Delta(\lambda, 1) \leq c$ for all $\lambda \in [\bar{\lambda}, \lambda^*]$. This implies that full effort cannot be sustained for any λ such that $m(\lambda, 1) \in [\bar{\lambda}, \lambda^*]$.

It remains to be shown that PCE exists and is unique. There are again two cases to consider, depending on whether the cost exceeds $\bar{\Delta} = \Delta(\bar{\lambda}\rho + \pi, 1)$. This is the premium that obtains when all borrowers exert full effort starting at the liquidity threshold.

Subcase (b)1. If $c > \pi\bar{\Delta}$, full effort cannot be sustained at the liquidity threshold. However since $c < \pi\underline{\Delta}$, there cannot be zero effort, and this implies that the equilibrium must feature partial effort. Note also that since there cannot be an equilibrium in which asset quality is below $\bar{\lambda}$ forever, the unique equilibrium requires asset quality to remain at the liquidity threshold forever. Asset quality can remain fixed at $\bar{\lambda}$ only if some borrowers exert effort, which implies that borrowers must be indifferent to effort. This can be achieved by rationing, where the probability ϕ^* that a given borrower obtains funding is such that $c = \pi\frac{\rho^*\Delta R + (1-\phi^*)\Delta L}{1-\rho\beta}$.

Subcase (b)2. If $c \leq \pi\bar{\Delta}$, full effort can be sustained once liquidity has reached the liquidity threshold (i.e. $\lambda^E \leq \bar{\lambda}$). However, if all borrowers expect to be funded when asset quality is at $\bar{\lambda}$, following our previous arguments, full effort cannot be sustained for any λ such that $m(\lambda, 1) \in \lambda \in [\bar{\lambda}, \lambda^*]$. Hence the PCE must feature a single period of rationing and partial effort to reach the liquidity threshold, and then full effort from then on. □

A.4 Proof of Proposition 2

We maintain $\underline{\Delta} \equiv \frac{\Delta L}{1-\rho\beta}$ and $\bar{\Delta} \equiv \Delta(\bar{\lambda}\rho + \pi, 1)$ for shorthand notation. Asset quality dynamics follow from the proof of Lemma 4. In particular, if $c > \pi\underline{\Delta}$, asset quality converges to zero whenever $\lambda < \lambda^E$. If instead $c \leq \pi\underline{\Delta}$, then asset quality converges to $\lambda^{\max}$ if $c < \pi\bar{\Delta}$, and to $\lambda^{\max}$ otherwise.

To see why rationing is required for a recovery, recall that that $\pi\Delta(\bar{\lambda}, 1) < c$ by Assumption 6. Since the premium is continuous in $\lambda \leq \bar{\lambda}$, there thus exists some λ^* such that $\pi\Delta(\lambda, 1) \leq c$ for all $\lambda \in [\bar{\lambda}, \lambda^*]$. This implies that full effort cannot be sustained for any λ such that $m(\lambda, 1) \in [\bar{\lambda}, \lambda^*]$. Now consider a single period of rationing at the liquidity threshold, and then proceeding with some effort rule E. Letting ϕ denote the probability of funding at $\bar{\lambda}$, the premium for any such λ is

$$\Delta(\lambda) = \phi\Delta R + (1-\phi)\Delta L + \rho\beta\Delta(\bar{\lambda}, E), \tag{15}$$

which is decreasing in ϕ. Hence rationing improves incentives relative to full funding. If full effort can be sustained starting at $\bar{\lambda}$, a single period of rationing is sufficient. If not, there must be rationing in all future periods, which requires that $\lambda = \bar{\lambda}$ forever. □

A.5 Proof of Corollary 1

We maintain $\underline{\Delta} \equiv \frac{\Delta L}{1-\rho\beta}$ and $\bar{\Delta} \equiv \Delta(\bar{\lambda}\rho + \pi, 1)$ for shorthand notation.

(i) Let $\lambda_0 \in (\lambda^W, \lambda^E)$. Following Proposition 4, the PCE effort rule entails partial or no effort for at least one period, but the socially optimal effort rule entails full effort. Then asset quality is lower in the PCE than in the efficient allocation for all periods after the first period in which the PCE effort rules entails less than full effort.

(ii) Assume first that $c > \pi\underline{\Delta}$. Then, partial effort occurs along the path of play, which requires that a positive share $1 - \phi$ of borrowers are rationed. Since rationing can occur only if asset quality is at the liquidity threshold, all borrowers would obtain funding in the efficient benchmark. If $c \leq \pi\underline{\Delta}$, then no effort occurs in the PCE and all borrowers are shut out of lending markets in perpetuity. Since effort would have taken place in the efficient benchmark, there is at least one period in which markets are liquid in the efficient benchmark but not in the PCE.

(iii) By Proposition 2, in any PCE in which asset quality converges to $\lambda^{\max}$ requires partial effort on the path of play. This is consistent with individual optimality if and only if borrowers are indifferent between effort and shirking. Since value functions are strictly increasing in λ for all types, it follows that all borrowers would be strictly better off by coordinating on full effort.

□

A.6 Proof of Corollary 2

We maintain $\underline{\Delta} \equiv \frac{\Delta L}{1-\rho\beta}$ and $\bar{\Delta} \equiv \Delta(\bar{\lambda}\rho + \pi, 1)$ for shorthand notation. Following Lemma 4, there exists a unique equilibrium with maximal effort (PCE). Let $\tilde{\lambda}$ be the lowest initial condition such that asset quality converges to $\lambda^{\max}$ in a PCE.

Let initial condition $\lambda_0 > \tilde{\lambda}$, and first consider $\tilde{\lambda} < \bar{\lambda}$. If $\underline{\Delta} > \frac{c}{\pi}$, then effort incentives exist in illiquidity. There exists a steady state at $\bar{\lambda}$, where aggregate effort is $E = \frac{(1-\rho)\bar{\lambda}}{\pi}$ and funding occurs with probability ϕ pinned down by $\frac{(1-\phi)\Delta L + \phi\Delta R}{1-\rho\beta} = \frac{c}{\pi}$. This implies there exists an alternative equilibrium path in which, once asset quality reaches the interval $[\bar{\lambda} - \pi, \bar{\lambda}]$, beliefs are such that asset quality remains at $\bar{\lambda}$ indefinitely, and all borrowers exert partial effort $e = \frac{(1-\rho)\bar{\lambda}}{\pi} \in (0,1)$. If $\underline{\Delta} < \frac{c}{\pi}$, there always exists an alternative equilibrium path where asset quality depreciates to 0 given beliefs that $E = 0$. This is because effort is not optimal conditional on permanent illiquidity.

Next, consider the case $\tilde{\lambda} > \bar{\lambda}$. Let $\underline{\Delta} > \frac{c}{\pi}$. Consider an candidate equilibrium path in which quality deteriorates to $\bar{\lambda}$, whereby all borrowers shirk until $\bar{\lambda}$ is reached and exert partial effort as prescribed above. Since $\bar{\lambda}$ is reached within a finite number of periods, we can rewrite this condition as $\sum_{k=0}^{N}(\rho\beta)^k b(\rho^k\lambda') > \sum_{k=0}^{\infty}(\rho\beta)^k b(\lambda^E)$, where N such that $\rho^N\lambda' \geq \bar{\lambda} > \rho^{N+1}\lambda'$. Note that $\sum_{k=0}^{N}(\rho\beta)^k b(\rho^k\lambda)$ is a continuous and increasing function in λ for $\lambda \geq \bar{\lambda}$. Hence, there exists some $\lambda' \leq 1$ be such that either $\sum_{k=0}^{N}(\rho\beta)^k b(\rho^k\lambda') = \sum_{k=0}^{N}(\rho\beta)^k b(\lambda^E)$, or $\sum_{k=0}^{N}(\beta\rho)^k b(\rho^k \cdot 1) \geq \sum_{k=0}^{N}(\rho\beta)^k b(\lambda^E)$. This implies that for λ_0 less than some threshold above $\tilde{\lambda}$, there exists an alternative equilibrium with decreasing quality $\bar{\lambda}$. The remaining case is when $\underline{\Delta} < \frac{c}{\pi}$. Consider an alternative equilibrium path in which quality deteriorates at rate ρ in each period, until it reaches 0. Using a similar argument as before, note that there exists some threshold, bounded below by $\frac{\lambda^e}{\rho}$, such that for any initial condition λ_0 less than this threshold, there exists a competitive equilibrium where $E = 0$ and asset quality deteriorates to 0. □

A.7 Proof of Lemma 5

Given that cash flows are constant in the region of illiquidity, the policymaker either does not offer a subsidy or offers a subsidy that induces liquidity. If markets are illiquid, the per-period social payoff to borrowers and lenders is

$$\underline{w} = \lambda L_g + (1-\lambda)L_b.$$

If the policymaker intervenes and markets are liquid, the per-period payoff is

$$\bar{w}(s) = \lambda(R_g + L_g) + (1-\lambda)(R_b + L_b) - 1 - \delta(1-\lambda)s(\lambda).$$

Since $\bar{w}(s)$ is strictly decreasing in s, the policymaker never intervenes if markets are liquid in competitive equilibrium, and chooses a subsidy no higher than $\underline{s}(\lambda) = \frac{1-\lambda L_g-(1-\lambda)L_b}{1-\lambda}$. . Intervening minimally is optimal if and only if $\bar{w}(\underline{s}(\lambda)) > \underline{w}$, which is equivalent to

$$\lambda R_g + (1-\lambda)R_b > 1 + \delta(1-\lambda)\underline{s}(\lambda).$$

Since investment is efficient for all types, in the limit as $\delta \to 0$, this condition is satisfied for any λ. Moreover, there cannot be rationing at $\lambda = \bar{\lambda}$ because the policymaker could induce funding for all borrowers by offering an arbitrarily small subsidy. □

A.8 Proof of Proposition 3

We maintain $\underline{\Delta} \equiv \frac{\Delta L}{1-\rho\beta}$ and $\bar{\Delta} \equiv \Delta(\bar{\lambda}\rho + \pi, 1)$ for shorthand notation. From Lemma 4, given initial quality $\lambda < \lambda^E$ asset quality converges to $\bar{\lambda}$ or $\lambda^{\max}$ if $c < \pi\underline{\Delta}$. It suffices to show that,given $\lambda < \lambda^E$ and $c < \pi\underline{\Delta}$, quality converges to 0 under the ex-post optimal subsidy. Consider the equilibrium given the subsidy as characterized in Lemma 5. It follows directly from Proposition 2 that effort is not optimal if all borrowers are sure to be funded and the borrowing cost is $B(\bar{\lambda})$. Since the subsidy is optimal for $\lambda \leq \bar{\lambda}$, under $\tilde{E} = 0$, the government intervenes indefinitely, and the per-period cash flow is $u_\theta = R_\theta$ for all future periods. Hence value functions under subsidy are constants and satisfy $\tilde{V}_b = \frac{R_b}{1-\beta}$ and $\tilde{V}_g = \frac{\rho R_g + (1-\rho)\tilde{V}_b}{1-\rho\beta}$. The law of motion satisfies $\lambda' = \rho\lambda$ and the per-period deficit (i.e. the resources injected by the policy maker) at the updated asset quality is $d(\lambda') = (1-\lambda')\underline{s}(\lambda') = 1 - L_b - \lambda'\Delta L$. The deficit satisfies the recursion $D(\lambda) = 1 - L_b - \rho\lambda\Delta L + \beta D(\rho\lambda)$. Hence $D(\lambda_0) = \frac{1-L_b}{1-\beta} - \lambda_0 \frac{\rho\Delta L}{1-\rho\beta} > 0$. Since lender profits are unchanged, welfare under the optimal subsidy given $\delta \to 0$ is

$$W(\lambda_0) = \lambda_0 \tilde{V}_g + (1-\lambda_0)\tilde{V}_b - D(\lambda_0).$$

We now prove by construction that a PCE can yield strictly higher welfare than the intervention. Consider an initial condition $\lambda_0 \in (\underline{\lambda}, \bar{\lambda})$ and a cost of effort such that asset quality reaches the liquidity threshold using one period of partial effort and then grows to $\lambda^{\max}$ using full effort thereafter. In such a PCE, we can bound from below the utilitarian per-period payoff obtained in the first period by

$$\underline{w}^{PCE} = \lambda_0(\rho L_g + (1-\rho)L_b) + (1-\lambda_0)L_b.$$

Under the subsidy, the utilitarian payoff in the first period is instead given by

$$w = \lambda_0(\rho R_g + (1-\rho)R_b) + (1-\lambda_0)R_b.$$

Hence the difference in first period payoffs between the subsidy and the PCE is bounded above by the finite constant

$$\xi \equiv \lambda_0(\rho(R_g - L_g) + (1-\rho)(R_b - L_b)) + (1-\lambda_0)(R_b - L_b).$$

In all subsequent periods, the subsidy induces payoffs that are "as if" $\lambda = \bar{\lambda}$ period by period, but in the PCE effort is strictly optimal and asset quality grows in every period. Hence, PCE value functions in every subsequent period are strictly higher than those induced by the subsidy. We can then bound the welfare difference between the subsidy and the PCE by

$$W(\lambda_0) - W^{PCE}(\lambda_0) \leq \xi - D(\lambda_0).$$

There exist parameters s.t. the right-hand side is strictly negative. If $\beta \to 1$, then $D(\lambda_0) \to \infty$. $\square$

Online Appendix

B Extensions

We provide extensions to our basic framework. This shows the robustness of our results and points to avenues for future research.

B.1 Anticipated Aggregate Shocks

In our baseline model, we characterized the path of asset quality given an arbitrary initial condition λ_0, which we assumed to be the result of an *unanticipated* shock to asset quality. Here we consider the effects of *anticipated* aggregate shocks. We show this can reduce the efficacy of effort and the relative value of owning a good asset.

As before, borrowers make effort decisions at the beginning of every period. Once the outcome of their effort is determined, there is an aggregate shock that turns a fraction χ_g good assets into a bad assets and share χ_b of bad assets into good assets, where χ_θ is a random variable distributed over $[0, \bar{\chi}_\theta]$. All borrowers are equally likely to be affected by the aggregate shock. Let $\hat{\chi}_\theta$ denote expected values. Expected transition probabilities are $p(g|g,e) = (1-\hat{\chi}_g)(\rho + \pi e) + \hat{\chi}_b(1-\rho-\pi e)$ and $p(g|b,e) = (1-\hat{\chi}_g)\pi e + \hat{\chi}_b(1-\pi e)$. Hence the marginal effect of an increase in effort is $\frac{\partial p(g|\theta,a)}{\partial e} = \pi\left(1-\hat{\chi}_g-\hat{\chi}_b\right)$, which unequivocally falls with aggregate risk. This is because previous effort is either wasted (a bad shock conditional on being good), or because its benefits would have obtained for free (a good shock conditional on being bad). Moreover, since the premium is a concave function of λ when markets are liquid, mean-zero aggregate shocks also reduce the *value* of obtaining a good asset. Aggregate risk thus reinforces the key mechanisms.

B.2 Variable Investment Scale

In the baseline model, borrowers were required to raise an entire unit of capital in order to obtain R_θ. We now allow borrowers to select their investment scale. We find that illiquidity manifests as a borrowing constraint on the scale of investment rather than a binary outcome.

Consider the baseline model with the following modification. In lending markets, each borrower chooses an investment level $k \leq 1$, and investors offer funding conditional on k. Is there a separating equilibrium where good and bad borrowers borrow k_g, k_b, respectively, and $k_g \neq k_b$? If separation were to occur, bad borrowers could borrow at most $k_b = L_b$, their maximum pledgeable cash flow. In any candidate equilibrium with separation, bad borrowers thus obtain at most $L_b R_b$. In contrast, deviating to the level of investment made by good borrowers k_g yields strictly larger payoff as long as $k_g > k_b$. This implies that separation cannot occur unless $k_g < k_b$, which would be strictly dominated to pooling with bad borrowers at $k_b = L_b$. Thus, we consider the pooling equilibrium in which the equilibrium investment level k^* maximizes good borrowers' payoffs.

Conditional on λ, the per-period payoff of a good borrower is given by $kR_g + L_g - \frac{k-(1-\lambda)L_b}{\lambda}$. It follows that equilibrium investment is $k^* = 1$ if $\lambda > \frac{1}{R_g}$, and for $\lambda \leq \frac{1}{R_g}$, a lower scale $k^* = L_g\lambda + L_b(1-\lambda)$. Thus, changes in aggregate quality are reflected in the scale of investment. Consistent with the baseline model, a lack of good collateral thus leads to inefficient investment.

A borrower exerts effort only if the premium exceeds the cost of effort. With variable investment scale, this condition is $\pi\Delta(m(k(\lambda), E)) \geq c$. As before, the the premium is increasing in λ. Hence the modified model delivers the same link between incentives and current quality.

C Graphical Illustration of PCE

We now provide a graphical illustration of the PCE. The first panel shows the value functions of good types (in blue) and bad types (in red). Hypothetical value functions conditional on $E = 1$ are depicted in thin lines, while equilibrium value functions are shown using thick lines. The green shaded region depicts the liquidity region. The dashed vertical lines depict the number of periods required to reach the liquidity region if all borrowers exert effort.

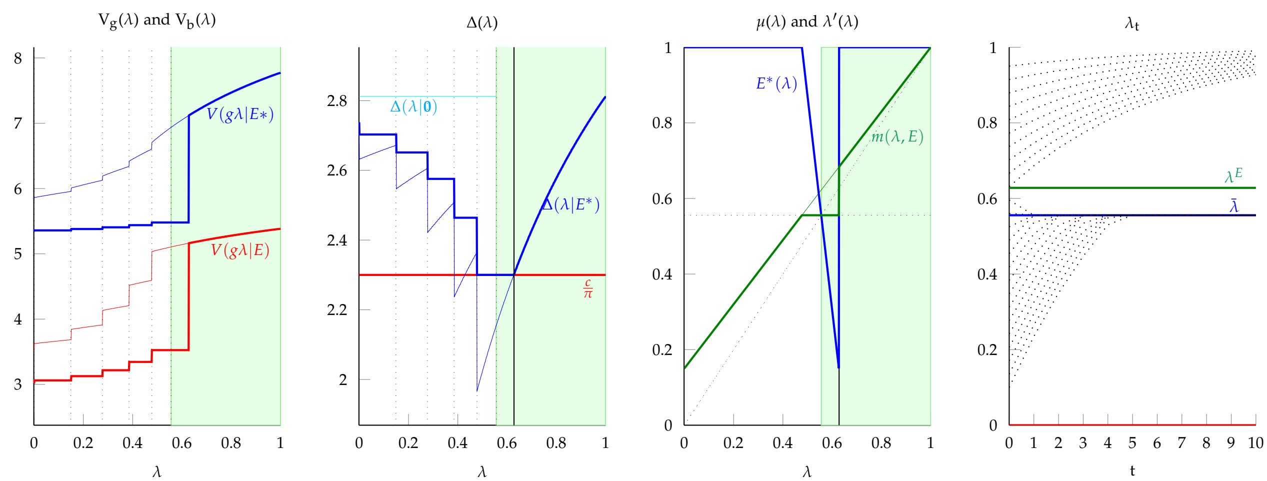

Figure 3: Parameters: $R_g = 1.5$, $R_b = 1$, $L_g = 1.4$, $L_b = 0.5$, $\beta = 0.8$, $\pi = 0.15$, $\rho = 0.85$, $\frac{c}{\pi} = 2.3$.

The second panel shows equilibrium incentives in the PCE (in thick blue) as well as in the counterfactuals where $E = 1$ (in thin blue) and $E = 0$ (in cyan). The cost of effort is shown in red. The solid vertical line depicts λ^E. The third panel illustrates the equilibrium effort strategy E^* in thick blue, the law of motion for asset quality given E^* in thick green, and the hypothetical law of motion given $E = 1$ in thin green. The upward-sloping dashed line is the 45-degree line. The horizontal dashed line is $\bar{\lambda}$. The fourth panel shows simulated paths for asset quality for various initial conditions λ_0 (in dashed lines). Incentives are such that average asset quality is able to *reach*, but not "jump" beyond, the liquidity threshold $\bar{\lambda}$. As a result, asset quality remains stuck at the liquidity threshold forever, and a fraction of borrowers fail to obtain funding in every period.