When Should Governments Buy Assets?[*]

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Abstract

We study a policymaker buying and selling securities to manage financial market liquidity in "normal times," defined as periods where asset prices are not distorted by financial constraints. In contrast to crisis interventions, we find that asset purchases during normal times induce inefficient risk taking, while asset sales improve risk sharing but may distort intertemporal smoothing. Optimal quantity-based policy rules align averages of private and public price impact. A calibration using institutional portfolio data suggests that the 2014-2017 Eurozone quantitative easing programs modestly reduced risk-sharing efficiency. Our findings have implications for the optimal management of public portfolios.

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1 Introduction

Central banks around the world now commonly buy and sell securities in financial markets to achieve a variety of policy objectives. While these tools were initially deployed to stem financial crises and to circumvent constraints on short-term policy rates, they have since become established parts of the policy toolkit even outside of crises. As result, many central banks now hold, and dynamically manage, large balance sheets even during normal times.[1] What is the effect of these policies on market liquidity and efficiency?

A common view is that central bank purchases can foster liquidity by propping up asset prices during bad times. Is general, this view is based on the premise that financial constraints may force some market participants to sell assets at fire sale prices, so that public interventions can restore efficiency by bringing market prices closer to "fair" value. It is unclear, however, whether this argument remains valid during normal times when asset prices are not depressed by constrained sales. It may instead be the case that public asset purchases are distortionary outside of crises.

This concern has been discussed by policymakers (Bernanke, 2012; Coeurè, 2015), and has found growing empirical support. Wallen and Stein (2023) provide evidence that price elasticities in Treasury markets are high when Treasury bills are scarce. Pelizzon, Subrahmanyam, and Tomio (2022) measure spreads between highly similar assets to argue that central bank policies distort private portfolio choices and liquidity provision. Pinter and Walker (2023) show that financial institutions reduce their hedging of interest rate risk when central banks engage in expansionary polices, and this occurs prior to any concerns about fire sales. Contrary to models of fire sales, the main theme of these findings is that central bank purchases may ration scarce private liquidity provision.

To assess theoretical foundations of these concerns, we develop a model of risk sharing in endogenously illiquid financial markets in which there are no forced sales. The only impediment to trade is the empirically relevant concern of price impact in inelastic markets (Gabaix and Koijen, 2020; Wallen and Stein, 2023).[2] This yields a framework in which central bank purchases can affect asset prices and market liquidity, but investors still choose privately optimal portfolios in an unconstrained manner. This allows us to derive policy lessons for liquidity management outside of acute fire sales.

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[1] The traditional motivation for quantitative easing and large-scale asset purchase programs of government bonds was to manage interest rates, while the Federal Reserve's Secondary Market Corporate Credit Facility was introduced to improve corporate bond liquidity. In Japan, the Bank of Japan uses stock purchases to manage the equity premium and has become the largest holder of Japanese equities in the world. In Europe, the ECB has extensively purchased both sovereign and corporate bonds, among other assets.

[2] Price pressure in concentrated markets is an acute concern in markets that are central for the transmission of monetary policy, such as interest rate swaps (Pinter and Walker, 2023). More generally, imperfect

The main mechanism in our setting is that price impact leads assets to be overvalued in normal times, reflecting endogenous scarcity and markups due to inelastic trading. Hence, public asset purchases that further inflate asset prices harm liquidity and risk sharing, while selling (or issuing) assets can raise liquidity and improve risk sharing, albeit at the cost of potentially distorting intertemporal trade. We characterize this mechanism in detail and derive implications for optimal public portfolio management, including a notion of "capacity constraints" beyond which the welfare impact of asset sales is negative. We further calibrate our equilibrium model to data on institutional portfolios and prices from Koijen, Koulischer, Nguyen, and Yogo (2021), and assess the efficiency costs of the Eurozone Large-scale Asset Purchase program from 2014-2017.

Trading volumes play the central role in our model. Because of imperfect liquidity, investors are wary of trading large quantities and ration trades to manage their terms of trade. As a result, financial demand shocks, such as changes in gains from trade between investors, are not fully accommodated. This leads to inefficient risk sharing and distortions in duration management between investors with different trading needs. Asset purchases by the government affect this margin both directly, by altering the level of prices and price impact, and indirectly, by changing investors' trading needs. For example, if the government trades against a particular investor, that investor may need to trade more with others to undo the effects of government asset purchases.

Our key positive result characterizes the link between government interventions and risk sharing. To isolate our novel channel, we remove all frictions other than imperfect liquidity: markets are complete and integrated, the government fully funds expenditures on assets with non-distortionary taxes, and redistributes payouts from its portfolio

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liquidity because of price impact is a widespread concern. Ben-David, Franzoni, Moussawi, and Sedunov (2021) show the largest institutional investor oversaw 6.3% of total U.S. equity assets in 2016, while the top 10 investors managed 26.5%. They find "large financial institutions have bigger price impact than a collection of smaller entities." Koijen and Yogo (2019) show many financial institutions have substantial price impact even in relatively liquid U.S. equity markets, with price elasticities around 3 on average. Koijen, Koulischer, Nguyen, and Yogo (2021) find that insurance companies and pension funds specifically had the least elastic demands among participants in the Eurozone quantitative easing program. Bretscher, Schmid, Sen, and Sharma (2022) report similarly inelastic behavior in corporate bond markets. Allen and Wittwer (2022) and Hau, Hoffmann, Langfield, and Timmer (2017) document imperfect competition in Treasury and derivatives markets, respectively.

to investors as lump-sum payments. We also focus mainly on *simple* policies, such as public trading of risk-free debt. Despite this, we find that government interventions that lower interest rates (e.g. buying risk-free debt from the market) generically *increase* price impact and lead investors to accumulate excess risk exposures. In contrast, interventions that raise interest rates (e.g. by selling debt to the market) improve private risk sharing arrangements. We measure this improvement using a model-derived metric that links the cross-sectional dispersion in (unobservable) state prices in a given asset market to (measurable) price impact and the cross-sectional dispersion their asset positions.

The link between risk sharing and public trading of *risk-free* debt is surprising. After all, markets are complete, and risk-free debt offers payoffs that are orthogonal to risk sharing. The mechanism relies on an intuitive property of optimal portfolios, which is that investors opt to sell claims on a given future state if and only if they expect to be relatively wealthy in that state. Under convex marginal utility (e.g. CRRA), this implies that sellers always face a relatively low marginal utility cost of portfolio distortions. Thus, sellers ration supply more than buyers ration demand, creating endogenous undersupply of state-contingent consumption. In equilibrium, asset prices are too high relative to the competitive benchmark and the government can improve risk sharing through asset sales.

The risk-taking channel we develop is distinct from and complements the canonical "reach for yield" channel of risk-taking under low interest rates. According to this view, financial institutions substitute toward riskier assets when interest rates are low to earn higher returns. In our setting, risk taking instead reflects inefficient diversification and may not lead to higher expected returns. The two mechanisms also have different causes. While reach for yield is driven by portfolio restrictions, high leverage or moral hazard (e.g. Martinez-Miera and Repullo (2017)), our mechanism operates when financial institutions are unconstrained and well-capitalized. Finally, our mechanism depends directly on *quantities* traded, not just on interest rates.

This proposed mechanism for the impact of interest rates on the portfolios of financial institutions is consistent with empirical evidence. Pinter and Walker (2023) shows that non-bank financial institutions, including pension funds and insurance companies, do not fully hedge interest rate risks in derivatives markets. More specifically, they document that these markets are concentrated, and that monetary expansions worsen non-bank financial institutions' management of interest rate risk. Additionally, Joyce, Liu, and

Tonks (2017) provides causal evidence that U.K. insurance companies and pension funds shifted from Gilts toward less liquid corporate bonds during the Global Financial Crisis because of the Bank of England's quantitative easing program.

Next, we extend our insight to the complementary case in which all gains from trade are intertemporal, for example because investors are initially endowed with portfolios of different duration. The main difference now is that public trading of risk-free debt aligns with the desired direction of trade of some investors, but not others. This leads to distributional concerns and asymmetric consequences of asset purchases. Our key result here is that large (or "fast") asset sales by the government can distort welfare even if they improve risk sharing. That is, the government faces a trade-off between improving risk sharing and distorting intertemporal smoothing, and managing this trade-off requires that the government trade cautiously.

Regarding normative implications, we derive simple rules for optimal government tax-and-trading schemes. We find that the government can minimize distortions to risk sharing and intertemporal smoothing by choosing its portfolio holdings to equalize weighted averages of government (or public) and private agent price impacts. When the government trades risk-free debt, these weights are marginal-utility averages of asset positions and position elasticities de-trended by per capita government demand. The government trades off improving terms-of-trade with the redistribution and erosion of rents that strategic agents garner by distorting their portfolios.

We consider several extensions of the optimal policy problem. First, we to allow for richer asset market interventions, such as how the U.S. government traded Agency Mortgage-Backed Securities during the Global Financial Crisis and corporate bonds during the COVID-19 crisis. We show the government in this case would aim to equate public and private price impact, appropriately weighted, within each asset market. Second, we consider unfunded interventions, whereby the government can purchase assets using funds raised "outside" of the model. Such wealth injections can have a direct effect on liquidity, and can rationalize why unfunded interventions may have a positive impact on liquidity during periods of financial stress.

To assess the empirical relevance and practical implications of our framework, we calibrate our model to demand elasticities estimated by Koijen, Koulischer, Nguyen, and Yogo (2021) for the Eurozone 2014-2017 Quantitative Easing program. Recent evidence across asset markets suggests that financial institutions' price elasticities of demand are much lower than implied in classical asset pricing models. Our model can rationalize these low elasticities when calibrated to portfolio characteristics, such as duration and demand elasticities, and price effects, such as the risk-free rate and yield response to the large-scale asset purchase program. Interpreting the effects of the program facts through the lens of our fully micro-founded model, we find that the program lowered sovereign debt yields with only a modest loss in trading efficiency. This is because low demand elasticities imply that portfolio positions changed only modestly for the less elastic participants in response to the program.

Finally, Section 7 contains extensions and discussion of our framework. First, we explore the implications of our model for the yield curve. We demonstrate that demand shocks at one tenor primarily affect interest rates at that tenor with limited transmission to other maturities even in the absence of market incompleteness or segmentation. Second, we show how government bond sales can *crowd-in* capital investment by improving market liquidity. Third, we discuss our theory's implications for dynamic liquidity management, such as when the government must sometimes intervene to stem fire sales.

Related literature. We contribute to the literature studying the implications of public asset purchases for market liquidity. Often, quantitative easing and large-scale asset purchases are studied in the context of financial crises with firesales (e.g., Davila and Korinek (2018)). A recurring theme in this literature is that governments can alleviate the downward spiral in asset prices by buying assets when their prices are depressed because of forced sales by market participants (e.g., Shleifer and Vishny (2011)). More recently, there is growing interest in the potential risks and unintended consequences of public liquidity provision. For example, Schmid, Liu, and Yaron (2021) shows that although government debt issuance can improve market liquidity, it can increase risk in the economy and raise firms' cost of capital. Li (forthcoming) examines how quantitative easing can mitigate bank liquidity crises but makes treated banks vulnerable to fluctuations in the real economy. Wallen and Stein (2023) show that heterogeneity in the demand elasticities among money market funds can amplify how Treasury yields and reverse repurchase rates respond to supply shocks when Treasury bills are scarce. Diamond, Jiang, and Ma (2022) show empirically that quantitative easing crowded out bank lending to firms. We provide a conceptual framework for understanding how government asset purchases can have unintended consequences outside of crises by inducing inefficient risk-taking among large institutional investors.

In its methodology, our approach is related to the burgeoning literature that emphasizes the role of asset *quantities* for understanding asset prices and the transmission of government policy. Gabaix and Koijen (2020) shows that large investor portfolios matter for asset prices when financial markets are inelastic, and Koijen and Yogo (2019) and Bretscher, Schmid, Sen, and Sharma (2022) provide evidence of this in U.S. equity and corporate bond markets, respectively. Jansen (2021) studies how changes in demand from pension funds and insurance company for Dutch sovereign bonds at different maturities directly impacts the shape of the yield curve. Du, Hebert, and Li (forthcoming) shows that the increase in the supply of U.S. treasuries can explain why interest rate swap-treasury spreads turned negative post-global financial crisis.

Our paper relates to a literature that studies market power in financial markets, and bond markets in particular. Eisenschmidt, Ma, and Zhang (2022) examines how large dealers' market power impacts the transmission of monetary policy in European repo markets. Wang (2018) studies how monetary policy transmission is affected by the market power of financial intermediaries, while Huber (forthcoming) and Wallen (2020) study dealer market power in tri-party repo and foreign exchange derivatives markets, respectively. Choi, Kirpalani, and Perez (2022) argues that the U.S. government has market power in safe assets. We ask how government trading can affect private trading efficiency by ameliorating the undersupply of risk sharing. Kacperczyk, Nosal, and Sundaresan (2021) investigates the impact of large institutional investors on asset price informativeness. We study how the supply of public liquidity interacts with private risk sharing arrangements, and how it can attenuate strategic distortions from market power.

More broadly, previous literature has emphasized that barriers to trade, such as limits to arbitrage (e.g., Gertler and Karadi (2013)) and market segmentation (e.g., Droste, Gorodnichenko, and Ray (2021)), are necessary for large-scale asset purchase programs to affect asset prices and real outcomes. For instance, Vayanos and Vila (2021) illustrates how such purchases can impact different parts of the yield curve when investor demand is segmented across maturities. Our analysis demonstrates how government trading can have real effects outside of crisis times even when there are no exogenous barriers to trade or forced asset sales.

2 Model

There are two dates, $t = \{1, 2\}$. Uncertainty is represented by a set of states of the world $\mathcal{Z} \equiv \{1, 2, \ldots Z\}$, one of which realizes at date 2. The probability of generic state $z \in \mathcal{Z}$ is $\pi(z) \in (0, 1)$, and all agents share common beliefs.

Demographics. There are two classes of agents: a continuum of competitive agents with mass m_f called the *competitive fringe* who takes prices as given, and a discrete number of *strategic agents* who are large relative to the economy and internalize their impact on prices in financial markets. The presence of a competitive fringe can represent, for instance, retail investors and smaller institutional investors. There is also a government that can buy or sell risk-free debt, but is constrained to balance its budget at each date.

There are N *types* of strategic agents, indexed by $i \in \{1, 2 \ldots, N\}$, where an agent's type determines her income process. Within each type, there exist $1/\mu$ *symmetric* agents who each has mass μ. For an individual strategic agent j of type i, μ determines how much she internalizes her *market power* because it affects her size relative to the economy. In the aggregate, average μ proxies for *market concentration*, or the extent to which the same wealth and income is concentrated in the hands of a few investors. In what follows, we focus on equilibria in which strategic agents within each type follow symmetric strategies.

Preferences. Strategic agents share common preferences over consumption at both dates. These are represented by the utility index $u(c)$ that is $\mathcal{C}^2$, strictly increasing, strictly concave, homothetic, and satisfies the Inada condition. Marginal utility $u'(c)$ is further assumed to be strictly convex. Risk aversion captures the notion that even large financial institutions can exhibit limited risk-bearing capacity under a variety of frictions, such as capital and risk management constraints. The fringe has quasi-linear preferences: linear in consumption at date 1 and risk-averse at date 2. Its date-2 utility function, $u_f(c)$, satisfies the same properties as that of strategic agents. Although a price-taking fringe is essential for our results, quasi-linearity of its preferences is not.

Income and Consumption. The fringe receives initial wealth w_f and state-contingent endowment $y_f(z) > 0$. A strategic agent j of type i receives initial endowment μw_i at date 1, and state-contingent endowment $\mu y_i(z) > 0$ in state z. The total initial endowment and state-contingent income of agents of type i are consequently also w_i and $y_i(z)$, respectively, and the *aggregate endowment* of all strategic agents is $Y(z) = \sum_i y_i(z)$. These income processes can be interpreted in multiple ways. One interpretation is that they represent the operational cash flow exposures of institutional investors. Another is that they represent the payoffs of asset portfolios that were in place before the government intervenes. Risk sharing needs then could represent the outcome of shocks to the expected payoffs of these portfolios. In the context of insurance companies and pension funds, these could reflect not only differences in existing asset exposures, but also in net cash flows from premiums less payouts to insurees or defined benefit pensioners.

Aggregate resource constraints are as follows. Let $c_{1,j,i}$ and $c_{2,j,i}(z)$ denote consumption of agent j of type i at date 1 and in state z, respectively, and similarly with c_{1f} and c_{2f} for the fringe. Aggregating within types gives $c_{1,i} = \sum_{j=1}^{1/\mu} \mu c_{1,j,i}$ and $c_{2,i}(z) = \sum_{j=1}^{1/\mu} \mu c_{2,j,i}(z)$. The aggregate resource constraints are

$$\begin{aligned} \sum_{i=1}^{N} c_{1,i} + m_f c_{2f} &= \sum_{i=1}^{N} w_i + w_f, \\ \sum_{i=1}^{N} c_{2,i}(z) + m_f c_{2f}(z) &= Y(z) + m_f y_f(z). \end{aligned}$$

Financial Markets. Financial markets are complete and open at date 1. The traded assets are the full set of Arrow securities; i.e., there are Z securities such that security z pays one unit of the numeraire in state z and zero otherwise. We show below that equilibrium allocations are invariant to the precise security menu, holding fixed the asset span. As such, it is without loss of generality to focus on trading in Arrow securities only.

Let $a_{j,i}(z) \in \mathbb{R}$ denote the position of agent j of type i in claim z, where $a_{j,i}(z) < 0$ denotes a sale. Aggregating within and across types yields $a_i(z) \equiv \sum_{j=1}^{1/\mu} \mu a_{j,i}(z)$ and $A(z) \equiv \sum_{i=1}^{N} a_i(z)$. The fringe's and the government's positions in security z are $a_f(z)$ and $a_G(z)$, respectively. Market clearing in the market for claim z requires:

$$A(z) + a_G(z) + m_f a_f(z) = 0. \tag{1}$$

So that the government trades only risk-free debt, we impose $a_G(z) = a_g$, i.e., it can only hold a portfolio that has the same position in all Arrow securities. That is, if the government demands a_g units of risk-free debt, then it buys a_g units of each Arrow security $z \in \mathcal{Z}$, and similarly if it sells a_g units. Because markets are complete, this is equivalent to the government instead trading only risk-free debt in a risk-free debt market. In Section 7, we allow the government to trade other assets.

Finally, define $\mathbf{A}$ to be the $(N+2) \times Z$ matrix summarizing portfolios choices of all agents and the government. The equilibrium price function of asset z is denoted $Q(\mathbf{A}, z)$. In contrast, the *perceived pricing functional* used by agent j of type i to forecast her influence on the price of security z is $\tilde{Q}_{i,j}(\mathbf{A}, z)$.

Government. The government can either buy or sell Arrow assets at date 1 subject to budget balance at each date. It maintains budget balance through uniform lump sum transfers τ_1 and $\tau_2(z)$ at dates 1 and 2 to all agents that can be positive or negative. This imposes the budget constraints

$$\left(N + m_f\right) \tau_1 + \sum_{z \in \mathcal{Z}} \tilde{Q}_G(\mathbf{A}, z) a_G(z) = 0, \tag{2}$$

$$\left(N + m_f\right) \tau_2(z) + a_G(z) = 0. \tag{3}$$

Decision Problems and Equilibrium Concept. The government is a Stackelberg leader and sets its tax and trading policies first. Conditional on these policies, we search for a *Cournot-Walras* equilibrium in which the competitive fringe takes asset prices as given and strategic agents place limit orders while taking into account their price impact.[3]. A *strategy* $\sigma_{j,i}$ for strategic agent j of type i consists of asset positions and consumption, $\sigma_{j,i} = \{\{a_{j,i}(z)\}_{z \in \mathcal{Z}}, c_{1,j,i}, c_{2,j,i}\}$. The decision problem is

$$\begin{aligned} U_{j,i} = \max_{\sigma_{j,i}} \quad & u\left(c_{1,j,i}\right) + \sum_{z \in \mathcal{Z}} \pi(z) u\left(c_{2,j,i}(z)\right) \\ \text{s.t.} \quad & \mu c_{1,j,i} = \mu w_i - \mu \tau_1 - \sum_{z \in \mathcal{Z}} \tilde{Q}_{i,j}(\mathbf{A}, z) \mu a_{j,i}(z), \\ & \mu c_{2,j,i}(z) = \mu y_i(z) + \mu a_{j,i}(z) - \mu \tau_2(z). \end{aligned} \tag{4}$$

We define preferences and controls in this manner recognizing that the consumption of strategic agent j of type i is actually $\mu c_{1,j,i}$ and $\mu c_{2,j,i}(z)$ at dates 1 and 2, respectively, and similarly with optimal asset holdings, $\mu a_{j,i}(z)$. Given homothetic utility, however, optimal policies are invariant to defining a strategic agent's preferences over $\mu c_{t,j,i}$.

A *strategy* σ_f for the competitive fringe consists of asset positions and consumption, $\sigma_f = \{\{a_f(z)\}_{z\in\mathcal{Z}}, c_{1,f}, c_{2,f}\}$. Because it takes prices as given, its perceived pricing function satisfies $\tilde{Q}_f(\mathbf{A}, z) = \tilde{Q}_f(z)$. The fringe's decision problem is

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[3] Malamud and Rostek (2017) and Rostek and Yoon (2020) instead study risk sharing among traders in CARA-normal settings using the alternative Equilibrium-in-demand-schedules approach of Kyle (1989). An advantage of our Cournot-Walras equilibrium concept is the mapping between asset prices and strategic agents' demands is unique, and consequently so is the equilibrium pricing function. This allows us to incorporate rich hetereogeneity across investors and trading needs, which is critical for calibrating our model to real-world policies. See Neuhann and Sockin (2021) for a comparison of the two concepts.

$$\begin{aligned} U_f = \max_{\sigma_f} \quad & c_{1f} + \sum_z \pi(z) u(c_{2f}(z)) \\ \text{s.t.} \quad & c_{1f} = w_f - \tau_1 - \sum_z \tilde{Q}_f(z) a_f(z), \\ & c_{2f}(z) = y_f(z) - \tau_2(z) + a_f(z). \end{aligned} \tag{5}$$

This allows us to define our equilibrium concept as follows.

Definition 1 (Cournot-Walras Equilibrium) *Fixing a strategy of the government, a Cournot-Walras equilibrium consists of a strategy $\sigma_{j,i}$ for each strategic agent, a strategy σ_f for the competitive fringe, and pricing functions $Q(\mathbf{A}, z)$ for all $z \in \mathcal{Z}$ such that:*

1. *Fringe optimization: σ_f solves decision problem (5) given $\{\tilde{Q}_f(z)\}_{z\in\mathcal{Z}}$*
2. *Strategic agent optimization: For each agent j of type i, $\sigma_{j,i}$ solves decision problem (4) given (i) other agents' strategies $\{\sigma_{-j,i}, \sigma_f\}$ and perceived pricing functions $\{\tilde{Q}_{j,i}(\mathbf{A}, z)\}_{z\in\mathcal{Z}}$.*
3. *Market-clearing: Each market clears with zero excess demand according to (1).*
4. *Consistency: all agents have rational expectations, which requires for strategic agents that $\tilde{Q}_{j,i}(\mathbf{A}, z) = Q(\mathbf{A}, z)$ for all i, j and z.*

The competitive fringe intermediates strategic interaction in our model. Although a strategic agent takes the asset positions of other strategic agents as given, he does internalize how his own demand impacts equilibrium asset prices by altering the marginal utility of the fringe. Through this channel, how one strategic agent type trades *indirectly* affects how another strategic agent type trades by altering the prices (and price impact) that agent type faces.

3 Equilibrium Characterization

We now characterize fundamental properties of equilibrium. In models of strategic trading, a crucial step is characterizing the pricing functional that determines an investors' equilibrium influence on prices. In our setting, this is simplified by the fact that the price-taking competitive fringe optimally aligns its marginal utility in a given state with the associated Arrow security price. Asset prices are consequently pinned down by the fringe's consumption process. Holding other large agents' portfolios fixed, each large agent can then infer her price impact from how much the fringe's marginal utility will move when she demands more or less of a given security. Because a strategic agent's influence scales with her mass, μ, her individual price impact does as well. This is shown in Lemma 1.

Lemma 1 (Prices and Price Impact) *The price of the Arrow security referencing state z is*

$$Q(\mathbf{A}, z) = q(z) \equiv \pi(z) u_f' \left(c_{2f}(z) \right). \tag{6}$$

The price impact of strategic agent i satisfies

$$\frac{\partial Q_{j,i}(\mathbf{A}, z)}{\partial a_i(z)} = \frac{\mu}{m_f} q'(z) \qquad \text{where} \qquad q'(z) \equiv \frac{\partial q(z)}{\partial A(z)} = -\pi(z) u_f'' \left(c_{2f}(z) \right) > 0, \tag{7}$$

and price impact $q'(z)$ is increasing and convex in strategic agent demand. The law of one price holds, and equilibrium consumption allocations are invariant to the presence of a risk-free or redundant assets. With increasing, concave utility, price impact is increasing in the price level.

In addition to determining asset prices and price impact, Proposition 1 has two additional important implications. The first is that the law of one price holds, so that consumption allocations are invariant to the asset span. As such, we do not need to explicitly model a risk-free asset. The second is that price impact vanishes in the limit as $\mu \to 0$. As such, our model nests perfect competition as a benchmark. Since the equilibrium is efficient under perfect competition, deviations from the perfect-competition limit allow us to distill welfare consequences.

3.1 Government Policies as "Asset Endowments"

One way to interpret the effects of government trading is that they endow private agents with an inventory of assets that is either in line with privately desired trading or must be undone through financial markets. In particular, if the government's portfolio is $\{a_g(z)\}$, budget balance requires that

$$\tau_1 = \frac{1}{N+m_f}\sum_{z\in\mathcal{Z}} q\left(z\right) a_G(z) \qquad \text{and} \qquad \tau_2\left(z\right) = -\frac{1}{N+m_f} a_g(z).$$

If markets were perfectly competitive, such changes in“asset endowments” would be entirely neutral as private investors could always undo them by taking the reverse position in asset markets. The main role of inelastic trading is to ensure that it is never optimal to fully undo the effects of government trading, thereby allowing the government to influence asset prices and private portfolios. To capture the net effects of public and private trading on an investor’s state-contingent income, we define *normalized* asset positions as follows:

$$\hat{a}_{j,i}\left(z\right) = a_{j,i}\left(z\right) + \frac{a_G(z)}{N+m_f}. \tag{8}$$

Consumption can then be described purely as a function of normalized asset positions,

$$c_{1i} = w_i - \sum_{z\in\mathcal{Z}} q\left(z\right)\hat{a}_{j,i}\left(z\right) \qquad \text{and} \qquad c_{2i}(z) = y_i\left(z\right) + \hat{a}_i\left(z\right).$$

as can the equilibrium asset price,

$$q\left(z\right) = \pi\left(z\right) u_f'\left(y_f\left(z\right) - \frac{1}{m_f}\sum_{i=1}^{N}\hat{a}_{j,i}\left(z\right)\right).$$

3.2 Optimal Portfolios and Inelastic Trading

We can now formally state the conditions for the optimal portfolio choice of each strategic agent. Marginal valuations can be summarized by the *state price* of strategic agent j of type i, $\Lambda_{j,i}(z) \equiv \frac{\pi(z)u'\left(c_{2,j,i}(z)\right)}{u'\left(c_{1,j,i}\right)}$, and depends only on normalized asset positions.

Lemma 2 (Optimal Portfolio) *At an optimum, asset positions $\{a_i(z)\}$ satisfy*

$$\Lambda_{j,i}\left(z\right) - q\left(z\right) = \frac{\mu}{m_f} q'(z)\underbrace{\left(\hat{a}_i(z) - a_G(z)/(N+m_f)\right)}_{=a_i(z)}, \tag{9}$$

The left-hand side of equation (9) shows marginal valuations and asset prices, which are optimally equal to each other in competitive markets. The right-hand side captures the distortions induced by price impact, which is the motive to ration quantities to preserve favorable terms of trade. The distortion scales with position size $a_i(z)$ because this affects the infra-marginal importance of the terms of trade. It is also defined over the *gross position* $a_i(z)$, and not the normalized asset positions $\hat{a}_i(z)$. This is because price changes affect the terms of trade on all assets that need to be reallocated, irrespective of the final normalized position.

We next show that investors with price impact trade inelastically, in the sense that asset quantities respond sluggishly to shocks that need to be accommodated through financial markets. For concreteness, we do so in context of wealth-neutral demand shocks, which are shocks that increase diversifiable income risk but do not alter total resources available in the economy (state-by-state). While such shocks are neutral under perfect competition, they are not when investors face price impact.

Definition 2 *A wealth-neutral demand shock is a reshuffling of large investor endowments* $y_i\left(z\right)$ *that leaves unchanged the total endowment in each state* $\sum_{i=1}^{N} y_i\left(z\right)$ *and the present-value of each endowment under prevailing market prices* $\sum_{z\in\mathcal{Z}} q\left(z\right) y_i\left(z\right)$.

Lemma 3 (Inelastic Trading) *In the Cournot-Walras equilibrium where large investors have price impact, a wealth-neutral demand shock:*

(i) alters asset prices and consumption allocations even though it would have been entirely neutral in the perfect competition limit where $\mu \to 0$.

(ii) worsens the efficiency of risk sharing, as measured by state price dispersion, if it raises the endowment of sellers and lowers that of buyers in all asset markets.

In concentrated financial markets, investors do not efficiently absorb demand shocks because price impact induces them to trade too little. As such, consumption allocations and asset prices are more responsive to demand shocks than under perfect competition, asset quantities are less responsive, and the efficiency of risk sharing falls when it increases gains from trade (i.e., distributes more resources to sellers and less to buyers). The response of asset prices to demand shocks in this setting is driven by both a direct and an indirect effect. The direct effect is that the imperfect reallocation of consumption in a specific state of the world because of market power alters the state price for that state of large investors, and consequently how they trade with the fringe. This alters the fringe's consumption in that state and consequently the associated Arrow price. The indirect effect is that a change in the price of one asset alters the wealth of all large investors, which alters all their state prices and consequently their demands for other assets. This spillover effect gives rise to cross-asset elasticities from demand shocks.

3.3 A Model-implied Measure of Risk Sharing

Since our theory is focused on risk management, we want to measure the efficiency of risk sharing in a model-consistent manner. Inefficient risk sharing leads to lost gains from trade, which are differences in state prices across investors. Hence, we can measure the inefficiency of risk sharing in a state z as the cross-investor *dispersion* in state prices:

$$\omega(z) = \frac{1}{N}\sum_{i=1}^{N}\left(\Lambda_i(z) - \frac{1}{N}\sum_{j=1}^{N}\Lambda_j(z)\right)^2. \tag{10}$$

While state prices are unobservable, portfolio positions and prices are not. Using the first-order condition (9), we can substitute out state prices using observable measures to arrive at the risk-sharing wedge,

$$\omega(z) = \left(\frac{\mu}{m_f}q'(z)\right)^2 \frac{1}{N}\sum_{i=1}^{N}\left(a_i(z) - \frac{1}{N}\sum_{j}a_j(z)\right)^2 \tag{11}$$

As such, trading efficiency is directly linked to two channels: price impact, which deters the realization of gains from trade, and dispersion in gross quantities, which are linked to the underlying gains from trade.

4 Positive Effects of Government Trading

In the previous section, we characterized the basic properties of strategic agents' portfolios and the extent to which they share risks in equilibrium under price impact. In this section, we derive positive and normative implications of government trading in securities markets. For realism and starkness, we restrict the government to trading risk-free debt. This is a natural benchmark intervention because it is not only simple, but practically relevant. It also means that government interventions are *neutral* across investors: the government does not take a distorted position in any single security, and all taxes and transfers are symmetric across all investors. Despite this neutrality, we show that government trading can still affect the degree of risk sharing that occurs in equilibrium.

4.1 Benchmark: Ricardian Equivalence under Perfect Competition

We first show that, absent price impact, government trading is completely neutral with respect to consumption allocations and asset prices.

Benchmark 1 (Ricardian Equivalence) *In the limit with perfect competition ($\mu \to 0$), government purchases and sales do not affect consumption allocations or asset prices. Moreover, risk sharing is perfect and there are no lost gains from trade for any government policy.*

The reason for this neutrality is that all costs and profits of government trading are passed on and rebated to investors, respectively. Since trade is frictionless absent price impacts, investors can therefore always undo any undesirable effects of government policies by adjusting their gross positions in financial markets, leaving net positions unchanged.

4.2 Prices and Liquidity

We now turn to the model with price impact. As the Ricardian benchmark in the previous section illustrates, it is not obvious that the government can affect equilibrium asset prices, and therefore price impact. We now show that it does. To build intuition, we use the portfolio condition from Proposition 2 to derive a simple asset pricing equation. Summing the optimality condition across strategic agents and imposing market-clearing shows that prices are satisfy a distorted consumption-based equation,

$$q(z) = \frac{1}{N} \sum_i \Lambda_i(z) + \frac{\mu}{m_f} q'(z) m_f a_f(z) + \frac{\mu}{m_f} q'(z) a_g. \tag{12}$$

The first term on the right-hand side of equation (12) is familiar from consumption-based asset pricing in which asset prices reflect the average marginal valuation of investors, as determined by the marginal rate of substitution. The remaining two terms reflect that the average wedge between asset prices and state prices is related to the net demand absorbed by the remaining investors in the market, i.e., the competitive fringe and the government. The third term specifically suggests that the direct effect of government trading is standard: prices fall when the government sells, and rise when it buys. We show this is indeed the case in the following proposition, and derive implications for liquidity as well. The risk-free rate is defined as the inverse sum of asset prices,

$$r_f = \left(\sum_{z \in \mathcal{Z}} q(z) \right)^{-1}. \tag{13}$$

Proposition 1 (Price and Liquidity Effects of Government Trading) *In the model with price impact ($\mu > 0$), budget-balanced tax and trading schemes affect asset prices as follows:*

(i) public purchases of risk-free bonds raise prices and price impact of all assets, and lower the risk-free rate.

(ii) public sales of risk-free bonds lower prices and price impact of all assets, and raise the risk-free rate.

Proposition 1 represents our first main result, which is that the government can improve market liquidity by reducing price impact by using tax-and-trading schemes that would be entirely neutral under perfect competition. This is because price impact deters private investors from fully undoing government purchases via financial markets. Interestingly, liquidity is low when prices are high (and risk-free rates are low). This is because high prices indicate high average marginal values of consumption, which makes it costly to reallocate consumption on the margin.

4.3 Effects on Risk Sharing

Given that the government can affect liquidity, it is natural investigate whether it can improve trading efficiency. Liquidity improvements alone are not enough to guarantee this because risk sharing distortions are the product of price impact and trading quantities. In particular, (11) shows that lost gains from trade are given by

$$\omega(z) = \left(\frac{\mu}{m_f} q'(z) \right)^2 \frac{1}{N} \sum_{i=1}^{N} \left(a_i(z) - \frac{1}{N} \sum_j a_j(z) \right)^2$$

As such, asset sales that lower price impact may still worsen risk sharing if they sufficiently raise the cross-sectional variance of trading quantities.

To assess the effects of government interventions on risk sharing, we consider income processes under which the only gains from trade because of risk sharing. In such economies, all gains from trade are orthogonal to the payoffs of risk-free assets. This ensures that government trades in risk-free debt do not directly affect risk sharing.

We also restrict attention to an economy populated almost exclusively by strategic agents, which we call the *strategic limit*. This allows us to isolate the effects of government purchases on the portfolios of large institutional investors. Although this is not necessary for our results, it provides the cleanest theoretical laboratory in which to derive them. To ensure that price impact remains well-defined in the limit, we consider the joint limit where $\mu \to 0$ and μ / m_f converges to a nontrivial constant.[4]

Definition 3 (Pure Risk Sharing Economy) *A pure risk sharing economy is one where all strategic agents are ex-ante symmetric but ex-post heterogeneous. As such, they face identical decision problems up to a relabeling of the states.*

Definition 4 (Strategic Limit) *The strategic limit is the limit of a sequence of economies in which* $\mu, m_f \to 0$ *and* $\mu / m_f \to \kappa$ *for some constant* $\kappa > 0$.

To understand the role of government interventions, it is to first the consider the benchmark without government trading. We have the following benchmark.

Benchmark 2 (Asset Prices without the Government) *In the strategic limit of the pure risk sharing economy, all Arrow asset prices are inflated above their competitive equilibrium counterparts (i.e, prices are higher than when* $\mu = 0$.*)*

The proof can be adapted from Neuhann and Sockin (2022), who focus on the dynamics of financial market power without a government. The underlying mechanism is that, for any asset, supply curves for are always more elastic than demand curves. The reason is that sellers *choose* to sell precisely because they are rich when the asset pays off, and thus have relatively flat marginal utility. Hence sellers always ration supply more than buyers ration demand, and all asset prices are too high relative to the efficient benchmark.

We then have our second main result, which is that public purchases of risk-free debt reduce the efficiency of risk sharing. Since purchases of risk-free debt also lower interest rates, our model predicts that expansionary policies brought about by quantitative interventions are associated with inefficient risk taking by financial institutions.

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[4] Neuhann and Sockin (2021) provides a formal analysis of this particular limit economy.

Proposition 2 (Effects of Government Trading on Risk Sharing) *In the strategic limit of a pure risk sharing economy, risk sharing distortions (as measured by the dispersion of state prices* $\omega(z)$*) are increasing in government asset purchases* a_g *for all states* z*. Conversely, government asset sales lead to a decline in risk sharing distortions.*

This result is striking because the payoffs of risk-free assets are orthogonal to gains from trade due to risk sharing, and all wealth effects are neutralized by taxes and subsidies. As such, our mechanism operates *only* through improved liquidity and by crowding in efficient asset supply. In particular, because sellers are the "marginal distorters" in the economy, selling risk-free debt can improve risk sharing allowing rationed buyers to obtain more insurance. As a result, state price dispersion declines and risk sharing improves. The converse argument can be made for government asset purchases.

While Proposition 2 relied on the strategic limited to obtain analytical results for relatively general income processes, this is not necessary. In particular, the following example considers an analytically tractable pure risk sharing economy with two strategic types, two states, and a "large" competitive fringe. This setting allows us to derive clear analytical expressions that highlight private distortions to risk sharing and the government's role in ameliorating them.

Example 1 *There are two types of strategic agents,* $i \in \{1, 2\}$ *and two states* $z \in \{1, 2\}$ *that are equally likely. Endowments satisfy* $y_1(1) = 2\bar{y}$ *and* $y_1(2) = 0$*, and* $y_2(1) = 0$ *and* $y_2(2) = 2\bar{y}$*. All agents have an initial wealth* w*. The fringe receives* $\frac{\bar{y}}{w}$ *in every state. There are Arrow assets for states 1 and 2, both of which will have equilibrium price* q^* *by symmetry. Preferences are of the CRRA type and taxes and transfers are such that* $\tau_1 = \frac{2}{2+m_f} q^* a_g$ *and* $\tau_2(z) = -\frac{1}{2+m_f} a_g$*.*

Since the two strategic agent types are symmetric, we can search for an equilibrium where Type 1 agents sell $a_s < 0$ *units of the claim to state 1 and buy* $a_b > 0$ *units of the claim to state* 2*. Type 2 agents take the reverse positions. Define* $\hat{a}_s = a_s + \frac{a_g}{2+m_f}$ *and* $\hat{a}_b = a_b + \frac{a_g}{2+m_f}$*. The claim in each state has a price* q^* *based on the fringe's marginal utility,* $q^* = u'\left(\frac{\bar{y}}{w} - \frac{1}{m_f}\left(\hat{a}_b + \hat{a}_s\right)\right)$*. and price impact is* q'^**. Moreover, strategic agents net expenditures on assets at date 1 are* $q^*\left(\hat{a}_b + \hat{a}_s\right)$*.*

As such, asset positions satisfy

$$\text{Seller optimality:} \qquad \frac{\frac{1}{2}u'(2\bar{y}+\hat{a}_s)}{u'(w-q^*(\hat{a}_b+\hat{a}_s))} = q^* + \frac{\mu}{m_f}q'^*\left(\hat{a}_s - \frac{a_g}{2+m_f}\right), \tag{14}$$

$$\text{Buyer optimality:} \qquad \frac{\frac{1}{2}u'(\hat{a}_b)}{u'(w-q^*(\hat{a}_b+\hat{a}_s))} = q^* + \frac{\mu}{m_f}q'^*\left(\hat{a}_b - \frac{a_g}{2+m_f}\right). \tag{15}$$

With perfect competition (i.e., $\mu = 0$), $a_b = -a_s = \bar{y}$, and government intervention in financial markets has no real effects. With market concentration, in contrast, sellers sell fewer claims ($a_s > -\bar{y}$) while buyers buy fewer claims ($a_b < \bar{y}$) because of price impact. As a result, Type 1 agents are over-exposed to state 2 risk, while Type 2 agents are over-exposed to state 1 risk.

If the government sells a small amount of claims, or $a_g = \epsilon < 0$, then government sales reduce the market power wedge for the seller type, $\frac{\mu}{m_f}q'^*\left(\hat{a}_s - \frac{\epsilon}{2+m_f}\right)$, and increase it for the buyer type, $\frac{\mu}{m_f}q'^*\left(\hat{a}_b - \frac{\epsilon}{2+m_f}\right)$. The direct effect from the first-order conditions is to induce the seller type to sell more claims and the buyer type to buy fewer claims. The direct effect decreases the demand that the fringe must absorb $\hat{a}_b + \hat{a}_s$, reducing the claim price q^* and consequently price impact q'^* (the indirect effect). The indirect effect that lowers the claim price and price impact, in turn, mitigates the reduction in purchases by the buyer type. On net, this improves risk sharing according to $\omega(z)$ by raising $\frac{\frac{1}{2}u'(2\bar{y}+\hat{a}_s)}{u'(w-q^*(\hat{a}_b+\hat{a}_s))}$ more than it lowers (or raises) $\frac{\frac{1}{2}u'(\hat{a}_b)}{u'(w-q^*(\hat{a}_b+\hat{a}_s))}$ because the seller's supply curve is more elastic in each market.

4.4 Effects on Intertemporal Trade

Beyond risk sharing, the other motive for trade is intertemporal smoothing across investors with different income duration. We now ask how government trading affects this margin. As under pure risk sharing, price impact and inelastic trading lead to inefficient intertemporal smoothing whenever market participants face income streams of different duration. The key difference to the case of risk sharing is that government purchases of risk-free debt are in *same* direction as at least some agents in the economy. This leads to distributional effects that differ from those under risk sharing, and is particularly relevant when analyzing duration mismatch among pension funds and insurance companies.

For simplicity, we focus on the following transparent setting. There two types of strategic agents, $i \in \{1, 2\}$. Endowments satisfy $y_1 = 2y$, and $y_2 = 0$. Type 1 agents have initial wealth 0 while Type 2 agents have initial wealth $2y$. The fringe receives 1 at date 2.

There is a risk-free bond with equilibrium price q^*. Preferences are of the CRRA type and taxes and transfers are such that $\tau_1 = \frac{1}{2+m_f} q^* a_g$ and $\tau_2(z) = -\frac{1}{2+m_f} a_g$.

We search for an equilibrium where Type 1 agent sells $a_s < 0$ units of risk-free debt and Type 2 agents buy $a_b > 0$ units of risk-free debt. Define $\hat{a}_s = a_s + \frac{a_g}{2+m_f}$ and $\hat{a}_b = a_b + \frac{a_g}{2+m_f}$. Risk-free debt has a price q^* based on the fringe's marginal utility, $q^* = u'\left(1 - \frac{1}{m_f}(\hat{a}_b + \hat{a}_s)\right)$. and price impact is q'^*. Moreover, strategic agents net expenditures on assets at date 1 are $q^* \hat{a}_s$ and $q^* \hat{a}_b$, respectively. As such, asset positions satisfy

$$\text{Seller optimality:} \quad \frac{u'(2y + \hat{a}_s)}{u'(-q^* \hat{a}_s)} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{a_g}{2+m_f}\right), \tag{16}$$

$$\text{Buyer optimality:} \quad \frac{u'(\hat{a}_b)}{u'(2y - q^* \hat{a}_b)} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{a_g}{2+m_f}\right). \tag{17}$$

With perfect competition (i.e., $\mu = 0$), $a_b = -a_s = y$, $q^* = u'(1) = 1$ with CRRA preferences, and government intervention in financial markets has no real effects. With market concentration, in contrast, Type 1 agents sell fewer claims ($a_s > -y$) while Type 2 agents buy fewer claims ($a_b < y$) because of price impact. As a result, the assets of Type 1 agents have too long a duration, and the assets of Type 2 agents have too short a duration.

If the government sells a small amount of risk-free debt, or $a_g = \epsilon < 0$, then the direct effect reduce the market power wedge for the seller type (Type 1 agents), $\frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{\epsilon}{2+m_f}\right)$, and increase it for the buyer type (Type 2 agents), $\frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{\epsilon}{2+m_f}\right)$. As a result, Type 1 agents sell more claims while Type 2 agents buy fewer claims. This reduces the demand the fringe must absorb $\hat{a}_b + \hat{a}_s$, reducing the risk-free debt price q^* and consequently price impact q'^* (the indirect effect), which mitigates the reduction in purchases by the buyer type. This improves inter-temporal smoothing by raising Type 1's state price $\frac{u'(2y+\hat{a}_s)}{u'(-q^* \hat{a}_s)}$ more than it lowers (or raises) Type 2's $\frac{u'(\hat{a}_b)}{u'(2y - q^* \hat{a}_b)}$.

4.5 Endogenous Capacity Constraints

The previous sections showed that the government can improve the efficiency of trade by crowding in the supply of rationed assets, no matter the underlying gains from trade. We now argue that the model also gives rise to a notion of endogenous "capacity" constraints that may determine limits on the appropriate size of interventions. To see this, recall that the effect of price impact on optimal portfolios is determined by the product of trade volumes and price impact. Absent the government, market clearing forces some investors to take short positions if other take long positions. For sufficiently large government interventions, however, *all* private investors may take a long or short position, sharply raising gross private trading volumes. Observe that equations (14)-(17) imply that gross trading volumes in the pure risk sharing or the intertemporal smoothing economy are

$$Vol = |\hat{a}_s - \frac{a_g}{2+m_f}| + |\hat{a}_b - \frac{a_g}{2+m_f}|$$

where $\hat{a}_s < 0 < \hat{a}_s$ for a_g sufficiently close to zero. This leads to the following observation.

Corollary 1 (Capacity Constraints) *Government sales reduce gross trade volumes if* $a_g < (2 + m_f)\hat{a}_b$ *but raise gross trade volumes if* $a_g \geq (2+m_f)\hat{a}_b$.

Since trading efficiency is related to the product of price impact and gross volumes, large interventions may therefore lower trading efficiency even if they reduce price impact.

4.6 Integrating Risk Sharing and Intertemporal Smoothing

We now integrate our results on risk sharing and intertemporal smoothing. In particular, we consider an economy in which there are gains from trade stemming from both risk sharing and intertemporal smoothing needs. The next result establishes that government asset sales, on the margin, can improve risk sharing in the economy, while asset purchases worsen it if the fringe has sufficiently limited risk-bearing capacity.[5] However, the the government still faces a trade-off even when it can improve risk sharing using asset sales. In particular, the cost of distorting intertemporal smoothing may locally reduce welfare even if risk sharing improves. As such, the government may face "capacity constraints" beyond which financial markets cannot appropriately accommodate government interventions, even if the government trades in the appropriate direction.

Since endowments can be interpreted as pre-determined asset holdings, this result has the practical implication that sufficiently "fast" asset sales may be sub-optimal relative to smaller interventions. This allows our model to speak to events such as the Gilt market crash in the fall of 2022, where relatively sudden quantitative tightening by the Bank of England *revealed* that large institutional investors were poorly hedged ex-ante (for example, because central banks had previously maintained large positive balance sheets), and then distorted intertemporal trade among investors with different duration.

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[5] While this condition is sufficient and intuitive, it is not necessary: we find that the same mechanism holds numerically under quite general conditions.

Proposition 3 (General Gains from Trade) *Let $\underline{\gamma}$ be the competitive fringe's minimum coefficient of absolute risk aversion across all asset markets z in the absence of government intervention. Government asset purchases cannot achieve the competitive outcome. In addition, for each z:*

(i) if the government buys a small amount risk-free bonds and $\frac{\gamma}{m_f}$ is sufficiently large, this lowers risk sharing efficiency in the economy by raising $\omega(z)$. Large asset purchases worsen risk sharing further by driving $\omega(z)$ to its autarky value.

(ii) if the government sells a small amount risk-free bonds and $\frac{\gamma}{m_f}$ is sufficiently large, this improves risk sharing efficiency in the economy by lowering $\omega(z)$. Large asset sales have diminishing returns by driving $\omega(z)$ for each z to an asymptotic, but positive lower bound.

(iii) Since asset sales may distort intertemporal trade, it may not be welfare-optimal to drive the risk sharing wedge to its lower bound. In particular, smaller interventions may deliver higher welfare than large interventions.

Figure 1 illustrates our results by plotting equilibrium outcomes as a function of the government's position a_G. We study the strategic limit of the simple two-state, two-type from Example 1, enriched to allow for intertemporal gains from trade stemming from differences in initial wealth. The top left panel shows that the risk sharing wedge, which measures the inefficiency of risk sharing, falls as the government sells more risk-free debt. The bottom left panel shows that the underlying mechanism is in part driven by improving liquidity (falling price impact) for all assets. The top right panel shows that utilitarian welfare is hump-shaped: increasing for relatively small interventions, but falling for sufficiently large interventions. The bottom right panel reveals the source of this non-linearity: for sufficiently large interventions, all investors begin to trade "against" the government to partially undo the effects of policy. Since this sharply raises gross volumes, the price impact friction grows in importance, reducing the efficiency of the allocation. This shows the importance of endogenous "capacity constraints" (Corollary 1).

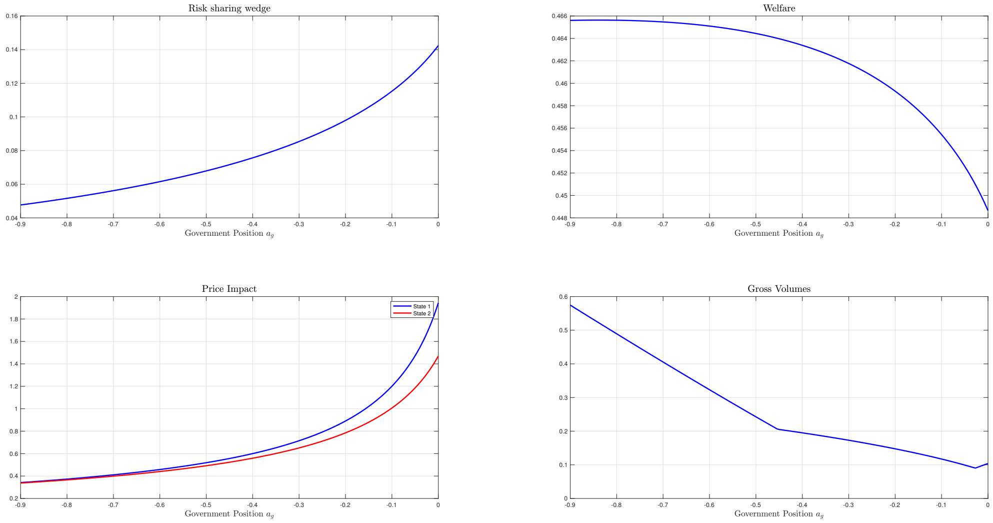

Figure 1: Effects of government trading given intertemporal and risk sharing gains from trade. *Remarks.* We compute equilibrium allocations in the strategic limit of the two-state, two-type economy discussed in Example 1, with the modification that types are also allowed to differ in their initial wealth. The ratio of market concentration to fringe mass is $\frac{\mu}{m_f} = 1$. Average income in every state is $\bar{y} = 1$. The within-state income dispersion that determines risk sharing needs is $\Delta = 0.3$. Type 1's initial wealth is $w_1 = 1$. There are intertemporal gains from trade because Type 2's initial income is higher, $w_2 = 2.5$. The risk sharing wedge is the average of the state-contingent wedges, $\frac{1}{2}\sum_{z=1}^{2}\omega(z)$. Liquidity as measured by price impact is plotted separately for each Arrow security. Gross volumes are the sum of all gross Arrow security positions. Welfare is utilitarian welfare as in Equation (18).

5 Optimal Interventions

The previous sections showed that there are endogenous limits to the optimal size of interventions, even taking as given the ability to improve risk sharing. To evaluate the net benefit of interventions, we now formally examine how a government would optimally buy or issue risk-free debt to maximize utilitarian welfare, defined in the usual way as

$$W = \sum_{i=1}^{N} u\left(c_{1,j,i}\right) + \sum_{z\in\mathcal{Z}} \pi\left(z\right) u\left(c_{2,j,i}\right) + m_f\left(\sum_{z\in\mathcal{Z}} \pi\left(z\right) u\left(c_{2,f}\right) - c_{1,f}\right), \tag{18}$$

The government is a Stackelberg leader in that it declares a budget-balanced tax and trading policy and internalizes all investors' reactions to this policy. We then have the following analytical characterization of the optimal policy rule for risk-free debt.

Proposition 4 (Optimal Government Debt) *The government's optimal holding of risk-free debt* *satisfies the necessary condition:*

$$\sum_{z\in\mathcal{Z}} w_p(z)\frac{\mu}{m_f}q'(z) - w_g(z)\frac{dq(z)}{da_g} = 0\;(< if a_g = -\infty, > if a_g = \infty), \tag{19}$$

where $w_p(z) = \sum_{i=1}^{N} u'(c_{1,i})\,a_i(z)\frac{d\hat{a}_i(z)}{da_g}$ *and* $w_g(z) = \sum_{i=1}^{N}(u'(c_{1,i}) - 1)\,\hat{a}_i(z)$ *are the weights on private and public price impact, respectively. This condition is trivially satisfied at the competitive equilibrium where price impact is zero.*

Proposition 4 reveals that the government chooses its risk-free debt position based on a weighted average across states, with weights $w_p(z)$ and $w_g(z)$, of the difference between private $\frac{\mu}{m_f}q'(z)$ and public (or the government's) price impact $\frac{dq(z)}{da_g}$, respectively. A strategic investor's price impact multiplied by her position $\frac{\mu}{m_f}q'(z)\,a_i(z)$ represents the wedge between her marginal valuation and marginal cost of consumption in a given state. This wedge is zero when there is perfect risk sharing in the competitive benchmark. The government's price impact is the total derivative of the price with respect to its trading position because it internalizes that it shifts private agents' demand in equilibrium. It is the government's ability to impose taxes and its internalization of its impact on prices (i.e., the total vs partial derivative) that introduces a role for policy. The government, as the Stackelberg leader, recognizes it can influence other large agents' asset demands, and not just the fringe's.

The government chooses its risk-free debt position until, on average across states, it counteracts large investors' private price impact. In markets in which $w_p(z)\frac{\mu}{m_f}q'(z) - w_g(z)\frac{dq(z)}{da_g}$ is positive, the government wants to sell debt if it has a higher weighted price impact than private agents to provide buyers relief with lower prices and price impact. In contrast, when $w_p(z)\frac{\mu}{m_f}q'(z) - w_g(z)\frac{dq(z)}{da_g}$ is negative, the government wants to buy debt to help sellers if it has higher weighted price impact.

From the optimality condition (19), it is apparent that social welfare can be non-monotonic in the government's holdings of risk-free debt. This is because there may be an interior choice of risk-free debt that balances the redistribution and erosion of rents among strategic agents with the improvement in risk sharing when the government sells debt, and the reverse when the government buys risk-free debt. The erosion of rents is more pronounced the more concentrated are financial markets (i.e., higher μ), which can lead to an interior choice of optimal risk-free debt.

In the following two subsections, we consider the government's choice of large-scale asset purchases in two alternative cases. First, we discuss optimal government asset market interventions when they are targeted to specific markets. Such interventions are relevant because in addition to sovereign bonds, many governments, including the US and EU, intervened in corporate bond markets in 2020 during the COVID crisis, and other governments, including Japan, have purchased equity Exchange Traded Funds (ETFs). Second, we consider the role of different taxation schemes (unfunded interventions) for our results. This second extension clarifies that typical analyses of the effects of large-scale asset purchases jointly study not only their impact on liquidity and risk-taking, which is our focus, but also on the portfolio rebalancing channel because the government crowds-out the market for risk-free debt.

5.1 Targeted Asset Purchases

The role of public liquidity in concentrated financial markets is not specific to risk-free debt. If the government could trade a richer set of linearly-independent securities, then, for a security with payoffs $X(z)$, its position $a_{\tilde{g}x}$ should satisfy

$$\sum_{z \in \mathcal{Z}} X\left(z\right) \left(w_p\left(z\right) \frac{\mu}{m_f} q'\left(z\right) - w_g\left(z\right) \frac{dq\left(z\right)}{da_{gx}} \right) = 0, \tag{20}$$

where $w_p\left(z\right)$ and $w_p\left(z\right)$ are given in Proposition 4. The government continues to align weighted public and private price impact, but achieves more targeted interventions based on distortions in individual asset markets.

5.2 Unfunded Asset Purchases

We conducted our analysis assuming that the government fully funds its large-scale asset purchases or sales using non-distortionary lump-sum taxation. This was to ensure a clear benchmark of Ricardian Equivalence under perfect competition, and to isolate the impact of government trading on market liquidity from that on market risk-bearing capacity and investment opportunities. Here, we examine how unfunded purchases of risk-free debt have the additional effect of shifting aggregate resources across dates.

Suppose the government has an endowment y_{1g} at date 1 and y_{2g} at date 2, and its objective is to trade until it equates its consumption at both dates

$$y_{1g} - \sum_{z \in \mathcal{Z}} q(z)\, a_g = y_{2g} + a_g, \tag{21}$$

from which follows, defining $\Delta y_g = y_{1g} - y_{2g}$ that

$$a_g = \frac{\Delta y_g}{1 + \sum_{z \in \mathcal{Z}} q(z)}. \tag{22}$$

This has the interpretation of the government wanting to inelastically smooth its expenditures at each date by issuing or purchasing risk-free debt. By varying Δy_g (the endowment mismatch), this fixed rule will provide an exogenous source of demand (or supply) to financial markets at date 1. We focus on the strategic limit from Definition 4 in which $\mu, m_f \to 0$ but $\frac{\mu}{m_f} \to \kappa$ to clarify how unfunded purchases impact risk sharing and inter-temporal smoothing among strategic agents.

To examine the role of large-scale asset purchase programs like quantitative easing, which were prevalent during the 2010s, we assume $y_{1g} < y_{2g}$ so that $a_g = \bar{a}_g > 0$ and the government buys risk-free debt. The complementary case when $y_{1g} < y_{2g}$, which captures the more quotidian behavior of governments to issue debt to finance expenditures, has analogous results. We then have the following proposition.

Proposition 5 (Unfunded Large-Scale Asset Purchases) *Suppose Δy_g increases and the government demands more risk-free debt. Then, under both perfect and imperfect competition, all asset prices $q(z)$ fall and all agents consume more at date* 1 *and less at date* 2*.*

Proposition 5 shows that unfunded government large-scale asset purchases break Ricardian Equivalence. They effectively reallocate resources among strategic agents from date 2 to date 1 by crowding-out investment in risk-free bonds. This induces agents to consume more at date 1. In a more general setting, such a crowding-out of investment in risk-free bonds can crowd-in investment into alternative investment opportunities, such as riskier equities or corporate debt if we modeled firms (as in Neuhann and Sockin (2021)). For instance, Joyce, Liu, and Tonks (2017) show that the Bank of England's quantitative easing program during the Global Financial Crisis shifted the investments of U.K. insurance companies and pension funds from Gilts toward corporate bonds.

6 Calibration to the Eurozone

Our theoretical results show that government trading can have rich and nuanced effects on market outcomes. To study how government asset purchases might impact equilibrium trading arrangements, asset prices, and welfare in practice, we calibrate our model to Eurozone data from 2014-2017 based on Koijen, Koulischer, Nguyen, and Yogo (2021). Such a setting is ideal because the European Central Bank conducted large-scale asset purchases outside of a crisis period, which allows us to examine how such purchases impact market liquidity absent forced asset sales. A novelty of our calibration is that we directly target cross-sectional characteristics of the portfolio holdings and demand elasticities of different institutional investors. In what follows, we consider date 1 to represent one year and date 2 to represent ten years; as such, risk-free debt in our model is akin to a ten-year government bond.

To calibrate our model to Koijen, Koulischer, Nguyen, and Yogo (2021), we assume that there are three groups of strategic agents: 1) insurance companies and pension funds (ICPFs) that have long duration portfolios; 2) banks and corporations that have short duration portfolios; and 3) mutual funds and hedge funds that have portfolios of intermediate duration. All strategic agents have constant relative risk aversion (CRRA) preferences with risk aversion γ, as does the competitive fringe at date 2. To focus on risk sharing among these institutional investors, we examine the strategic limit from Definition 4.

To flexibly capture the distinct portfolios and trading motives of these different agent types, we specify initial wealth and endowments as follows. At date 2, there are two possible states $z \in \{1, 2\}$ with $\pi(z) = \frac{1}{2}$. Strategic agents of type $i \in \{1, 2\}$ that represent pension funds and insurance companies receive initial wealth $(1 - k_1)\,\bar{y}$ and an endowment at date 2, $y_i(i) = k_1\bar{y}\,(1 + \Delta)$ and $y_i(-i) = k_1\bar{y}\,(1 - \Delta)$. That is, in every state one of the two types has high income and the other has low income. Similarly, strategic agents of type $i \in \{3, 4\}$ that represent banks and corporations receive initial wealth $(1 - k_2)\,\bar{y}$ and an endowment at date 2, $y_i(i - 2) = k_2\bar{y}\,(1 + \Delta)$ and $y_i(5 - i) = k_2\bar{y}\,(1 - \Delta)$. Strategic agents of type $i = 5$ that represent hedge funds and mutual funds receive initial wealth $(1 - k_3)\,\bar{y}$ and a certain endowment at date 2 of $k_3\bar{y}$. The competitive fringe receives $\bar{y}$ at both dates.

We set $\bar{y} = 10$, $\kappa = 1$, and calibrate the remaining parameters $\{\gamma, k_1, k_2, k_3, \Delta, a_g\}$ as follows. We target γ to match the risk-free rate r_f with the Euro area 10 Years Government

Bencnhmark Bond Yield in March 2014 of 2.8%.[6]. We target k_1 , k_2, and k_3 to match the durations of government bond holdings of IPCFs, banks and corporations, and mutual funds from Table 14 of Koijen, Koulischer, Nguyen, and Yogo (2021). We weight the durations of each group across vulnerable and non-vulnerable countries by the size of their holdings to arrive at values of 8.94 years, 4.62 years, and 6.92 years, respectively.[7] We measure duration D in our model using Macaulay's Duration for strategic agent i D_i based on the fraction of present-value consumption derived at each date

$$D_i = \frac{c_{1i}}{w_i + \sum_{z=1}^{2} q\left(z\right) y_i\left(z\right) - \tau_1 - \frac{1}{r_f}\tau_2} + 10 \frac{\sum_{z=1}^{2} q\left(z\right) c_{2i}\left(z\right)}{w_i + \sum_{z=1}^{2} q\left(z\right) y_i\left(z\right) - \tau_1 - \frac{1}{r_f}\tau_2}. \tag{23}$$

We target Δ to match the mean demand elasticity for risk-free bonds of ICPFs from Table 13 of Koijen, Koulischer, Nguyen, and Yogo (2021) of -4.04. Defining $\hat{a}_1(rf) = \min_{\{z \in \{1,2\}} a_i\left(z\right)$ to be agent i's holding of risk-free bonds, we can calculate this demand elasticity as $-\frac{d \log\left|\hat{a}_1(rf)\right|}{d \log\left(1/r_f\right)}$. Finally, we target the initial size of government trading a_g such that a 26% purchase of outstanding government bonds (the effective size of the Eurozone's asset purchases from 2014-2017) reduces the ten-year yield by 65 bp based on the calculations of Koijen, Koulischer, Nguyen, and Yogo (2021).

Table 1 reports the parameters that we recover from estimating our model using the simulated method-of-moments approach, and Table 2 compares the calibrated moments that we target in our model with their empirical counterparts. It is worth emphasizing the ambition of our exercise in that it is very difficult to match not only asset pricing moments (i.e., the risk-free rate and yield changes), but also portfolio characteristics (i.e., duration and demand elasticities) that are typically ignored in models of strategic trading. Of note is that our calibrated model estimates very realistic demand elasticities for strategic agents compared to the perfect competition benchmark in which elasticities are (locally) infinite. Although we target only the demand elasticity for risk-free debt of ICPFs, those of banks/corporates and mutual/hedge funds in our model are also relatively low at 6.80 and 23.83 (compared to 2.08 and 2.93 in Koijen, Koulischer, Nguyen, and Yogo (2021)), respectively, with mutual and hedge funds sensibly being the most elastic.

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[6] See `https://data.ecb.europa.eu/data/datasets/FM/FM.M.U2.EUR.4F.BB.U2_10Y.YLD`.

[7] Specifically, ICPFs have an average duration of (1284 * 9.8 + 493 * 6.7) / (1284 + 493) = 8.94 years. Banks have an average duration of (1346 * 5.0 + 963 * 4.1)/ (1346 + 963) = 4.62 years. Mutual funds, which include hedge funds, have an average duration of (895 * 7.6 + 333 * 5.1) / (895 + 333) = 6.92 years.

In addition, price impact in risk-free bonds, i.e., $\kappa\frac{q'}{q}$ for bond price q, is 0.30.

Parameter	Interpretation	Value
$\bar{y}$	Average Endowment	10.000
κ	Relative Size of Strategic Agents	1.000
k_1	Fraction of Bank/Corporate Endowment at Date 2	0.2973
k_2	Fraction of ICPF Endowment at Date 2	0.6198
k_3	Fraction of Mutual/Hedge Fund Endowment at Date 2	0.6703
Δ	Distributional Endowment Risk (% of $\bar{y}$)	0.0109
γ, γ_f	Agent Risk Aversion	0.3117
a_g	Government Initial Asset Position	-0.4258

Table 1: Parameter choices for the baseline calibration.

Moment	Data	Model
Ten-year Risk-free Rate	1.29%	1.43%
Bank / Corporate Duration	4.62	4.21
ICPF Duration	8.94	6.21
Mutual/Hedge Fund Duration	6.92	6.54
ICPF Demand Elasticity	-4.04	-3.86
Asset Purchase Yield Response	65bp	64bp

Table 2: Model vs empirical moments for the parameters given in Table 1.

Figure 2 illustrates our “strategic rent-seeking” channel according to which strategic agents retain too much diversifiable risk when asset prices are high and interest rates are low. We simulate the equilibrium for different values of the government's position a_g. The vertical line plots the calibrated “initial position” of the government. Starting from this point, asset purchases thus represent a rightward move along the x-axis. An advantage of our no-arbitrage framework with complete markets is that we can rewrite Arrow security positions in terms of more interpretable assets. In the left panel, we plot the normalized positions of insurance companies and pension funds (ICPFs) in terms of a risk-free bond $\hat{a}_1(rf)$ and a swap that pays 1 in state 2 and -1 in state 1, $\hat{a}_1(swap)$. In the middle panel, we plot the risk-free rate, and in the right panel, we plot our risk-sharing efficiency measure $\omega(z)$ (which is the same across both states in this exercise).

When interest rates are low, ICPFs trade too little of their diversifiable risk. In the competitive equilibrium, they should trade 2.005 shares of risk-free debt and 0.0648 shares of the swap. Instead, they trade only 0.6232 shares of risk-free debt and 0.0169 shares of

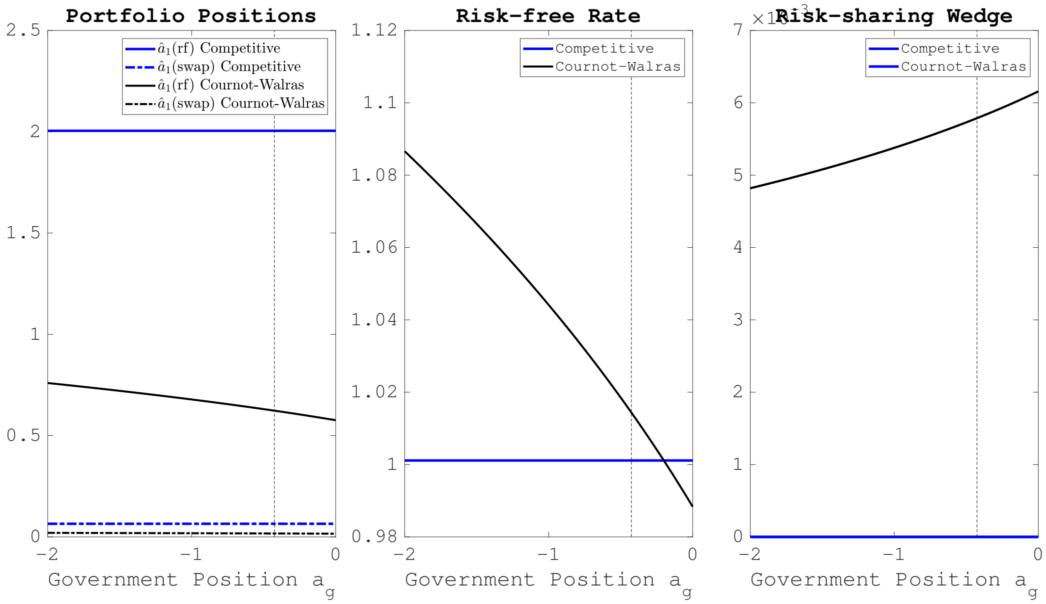

Figure 2: Agent 1's Holdings (Left Panel), Risk-free Rate (Middle Panel), and Risk-sharing Efficiency $\omega(z)$ (Right Panel) Across Government Positions for the parameters in Table 1

the swap, voluntarily over-exposing themselves to their own state and mismatched duration to extract surplus. As the government sells more assets and interest rates rise, their positions in risk-free debt and the swap increase to 0.7210 and 0.0191 shares, respectively when $a_g = -1.25$, leading agents to realize more gains from trade in financial markets. However, because the demand elasticities of agents are very low, government trading has only limited impact in improving risk sharing. The right panel reveals that risk sharing improves as the government sells more assets and reduces price impact, although it cannot achieve the level of risk sharing in the competitive equilibrium.

We plot welfare according to our utilitarian objective (18) in Figure 3. Although welfare is always below the competitive benchmark (0.84% lower consumption-equivalent welfare in the baseline specification), it is decreasing in public debt purchases. As the government buys debt, the risk-free rate falls and price impact rises. This induces ICPFs to buy less bonds and to take a smaller position in the swap that shares risk. However, because ICPFs, banks, and mutual funds all have very inelastic demands, the 26% large-scale asset purchase by the European Central Banks government bonds over 2014-2017 mostly impacted prices rather than allocations. As a result, it modestly reduced consumption-equivalent welfare by -0.012%.[8] Our analysis consequently cautions that examining only (even large) changes in yields is insufficient for evaluating the transmission of large-scale asset purchases to investor portfolios and their implications for welfare.

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[8] In principle, there could be an interior solution to optimal government debt issuance. When markets

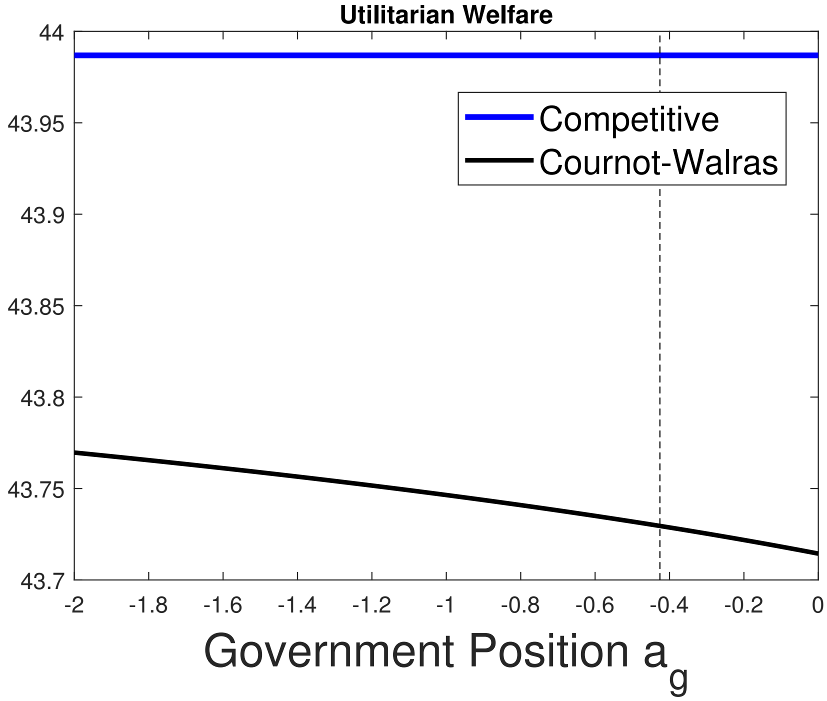

Figure 3: Welfare Across Government Positions for the parameters in Table 1

7 Extensions and Applications

In this section, we discuss several extensions of our model. First, we consider an application to the yield curve to demonstrate how demand shocks impact interest rates at different tenors in a manner consistent with preferred habitat-like behavior. Second, we explore how government asset sales can crowd-in real investment by improving market liquidity. Finally, we discuss the implications of financial market concentration for a government that engages in dynamic liquidity management.

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are very illiquid, marginally reducing prices and price impact at some point harms sellers by compressing their inframarginal gains from reducing prices more than it incrementally aids in risk sharing. The special case of a monopolist trading against the competitive fringe is another example of where reducing price impact need not always improve welfare.

7.1 An Application to the Yield Curve

Previous research has demonstrated the importance of institutional investor demand for understanding the pass-through of shocks along the yield curve. Theoretically, this research has relied on the premise that markets are starkly segmented, whereby certain traders can only trade in certain asset (their *habitats*). Our theory provides a complementary view, which is that markets may be inelastic because of concentration and price impact. This will allow us to demonstrate how quantity-based interventions, such as large-scale asset purchase programs, can impact interest rates at a specific tenor of the yield curve even though there is no market incompleteness or segmentation. For instance, D'Amico, English, Lôpez-Salido, and Nelson (2012) and D'Amico and King (2013) provide evidence that targeted interventions altered the U.S. yield curve by changing the supply of bonds, and Krishnamurthy and Vissing-jorgensen (2011) of segmentation in the response of asset prices to quantitative easing. In contrast to Vayanos and Vila (2021), our theory of the term-structure does not require that we assume market segmentation for quantitative easing / tightening to have real effects.[9]

Example 2 (A Strategic Term-structure Model) *Suppose now there are three dates* $t \in \{1, 2, 3\}$ *and* Z_2 *states of the world at* $t = 2$*, indexed by* $z_2 \in \mathcal{Z}_2$*, and* Z_3 *states of the world at* $t = 3$*, indexed by* $z_3 \in \mathcal{Z}_3$*. An agent of type* i *has initial wealth* w_i *and receives random endowment* $y_i(z_t)$ *for* $t \in \{2, 3\}$*, while the fringe has initial wealth* w_f *and receives random endowments* $y_f(z_t)$ *for* $t \in \{2, 3\}$*.*

All agents have time-separable preferences over consumption at all three dates and do not discount the future. Agents of type i *have concave utility index* $u(c)$ *over consumption at each date, while the competitive fringe has quasi-linear preferences: linear at date* 1 *and concave preferences* $u_f(c)$ *at dates* 2 *and* 3*. All agents trade consumption claims in complete financial markets that open at date* 0 *and close at date* 1*, and settle their asset positions at dates* 2 *and* 3*.*

We allow the government to trade a short-term and a long-term risk-free asset referencing dates 2 *and* 3 *with risk-free rates* $r_{ft} = \left(\sum_{z_{tk} \in \mathcal{Z}_t} q\left(z_{tk}\right)\right)^{-1}$*. These two interest rates form the yield curve in the economy with* r_{f2} *the short-term rate and* r_{f3} *the long-term rate. In complete markets, this is equivalent to government taking the same position* $a_G\left(z_{tk}\right) = a_{gt}$ *for each* k*. Budget balance at each date consequently requires transfers* $\tau_t = -\frac{1}{N+m_f} a_{gt}$ *for* $t \in \{2, 3\}$ *and*

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[9] Our strategic model also differs from equilibrium term structure models in the literature, which focus on competitive investors, such as the popular affine term structure models (e.g., Piazzesi (2010)).

$\tau_1 = \frac{1}{N+m_f}\left(\sum_{z_{2k}\in\mathcal{Z}_2} q\left(z_{2k}\right)a_{g2} + \sum_{z_{3k}\in\mathcal{Z}_3} q\left(z_{3k}\right)a_{g3}\right)$.

Analogous to equation 9, we can derive the optimal normalized asset positions of strategic agents

$$\frac{\pi\left(z_{tk}\right)u'\left(y_i\left(z_{tk}\right)+\hat{a}_{j,i}\left(z_{tk}\right)\right)}{u_1'\left(w_i-\sum_{t,z_{tk}\in\mathcal{Z}_t} q\left(z_{tk}\right)\hat{a}_{j,i}\left(z_{tk}\right)\right)} - q\left(z_{tk}\right) = \frac{\mu}{m_f}q'(z_{tk})\left(\hat{a}_{j,i}\left(z_{tk}\right)-\frac{a_{gt}}{N+m_f}\right), \qquad (24)$$

for $\hat{a}_{j,i}\left(z_{tk}\right) = a_{j,i}\left(z_{tk}\right)+\frac{a_{gt}}{N+m_f}$*, where Arrow asset prices are given by*

$$q\left(z_{tk}\right) = u_f'\left(y_f\left(z_{tk}\right)-\frac{1}{m_f}\hat{A}\left(z_{tk}\right)\right), \qquad (25)$$

and $\hat{A}\left(z_{tk}\right)$ *is the total normalized demand of strategic agents.*

With perfect competition, there is Ricardian equivalence (Proposition 1), and government asset purchases have no impact on equilibrium allocations or interest rates. In contrast, with market power, the government can target the long- or short-end of the yield curve. For instance, by selling more risk-free debt against date 2 *(i.e., smaller* a_{g2}*), the government induces sellers of assets referencing states in date* 2 *to sell more, and buyers of assets referencing states in date* 2 *to buy less. This reduces the prices of assets referencing date* 2*, which raises the risk-free rate* r_{f2}*. Similarly, targeted interventions with* a_{g3} *primarily affect the long-term risk-free rate,* r_{f3}*. Wealth effects, however, imply some transmission of targeted policies to the other end of the yield curve.*

A key implication of our term-structure example is that bond prices with strategic trading will exhibit behavior that resembles a "preferred habitat" interpretation of the term-structure. Because risk is imperfectly shared in financial markets, shocks to large investor demand at a given tenor will impact bond prices at that tenor while having a weaker response at other maturities because of spillovers from wealth effects.

We now provide a numerical exercise based on example 2 to illustrate how quantities impact the term structure of interest rates. Suppose there are two types of strategic agents, $i \in \{1,2\}$. At dates 2 and 3, there are two possible states $z \in \{1,2\}$ with $\pi(z) = \frac{1}{2}$. Strategic agents face pure idiosyncratic risk: $y_i(it) = \bar{y} + \Delta_t$ and $y_i(-it) = \bar{y} - \Delta_t$. That is, in every state one type has high income and the other has low income. The fringe receives $\bar{y}$ in every state. All agents have log preferences and initial endowments of w_i and w_f for strategic agents and the fringe, respectively.

Figure 4 depicts the term structure of interest rates at tenors 2 and 3 for different

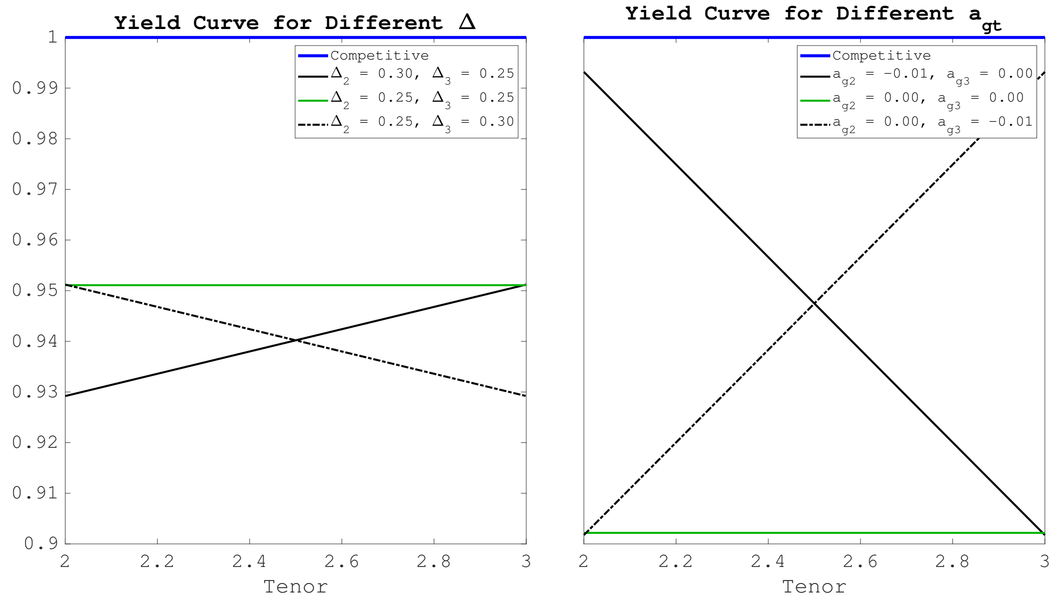

Figure 4: Term Structure Under Different Dispersions in Endowments Δ_t (Left Panel) and Levels of Government Asset Sales a_G (Right Panel). The parameters are $m_f = 0.01, \mu = 0.10, \bar{y} = 2, w_f = 0$, and $w_1 = w_2 = 1$. The baseline values are $a_{g2} = a_{g3} = 0$ for the Left Panel and $\Delta = 0.25$ for the Right Panel

dispersions in endowments Δ_t (Left Panel) and different levels of government asset sales a_{gt} (Right Panel). In the absence of shocks to the dispersion in endowments or government asset trading, market concentration reduces risk-free rates at all tenors. Interestingly, a shock to the short rate through higher-dispersion intermediate endowments (i.e., higher Δ_2) steepens the yield curve by lowering the date 2 risk-free rate and raising the date 3 risk-free rate, while a shock to the long rate through a higher Δ_3 has the opposite effect of inverting the yield curve. Similarly, government demand shocks through asset sales at date 2 (i.e., $a_{g2} < 0$) raise the date 2 risk-free rate and modestly lower the date 3, while asset sales at date 3 (i.e., $a_{g3} < 0$) steepen the yield curve by raising the date 3 risk-free rate and modestly lowering that at date 2. Such shocks and asset sales are, in contrast, neutral with perfect competition.

This numerical example illustrates how demand shocks can impact one part of the yield curve while having a more attenuated impact at other tenors, similar to preferred habitat models. It also illustrates that it is difficult to relate the slope of the yield curve to forecasts of aggregate macroeconomic conditions because demand shocks (which would have no effect with perfect competition) distort the slope of the yield curve.

7.2 Capital Investment

We can extend our model to include real investment in physical capital. Kubitza (2023), for instance, provides evidence that shocks to insurance companies affects the investment of non-financial firms through their price impact in corporate bond markets. We show how government trading in risk-free debt can improve the efficiency of how institutional investors allocate capital in the real economy.

Suppose now large agent i, in lieu of endowments, can invest k in physical capital, and his future income in state z is $y_i(z)k$. If there is heterogeneity in technological productivity and financial markets are illiquid, there is capital misallocation and agents may scale down their risky investment because risk sharing is impaired (e.g., Neuhann and Sockin (2021)). In this case, our results suggest that government debt issuance can crowd *in* real private investment.

To illustrate this point, we consider the following example based on Example 2 of Neuhann and Sockin (2021). In lieu of endowments at date 2, the future income for large agent i in state z is $y_i(z)k_i$ if she invests k_i in risky capital. Example 3 shows that by attenuating the incentives of more productive firms to reduce their sales of financial claims when raising capital, government debt issuance improves capital allocation.

Example 3 (Asymmetry with dominated technologies) *There are two types of strategic agents, $i \in \{1, 2\}$. Production technologies satisfy $y_1(z) = y^h$ and $y_2(z) = y^l \in (R_f, y_h)$ so that Type 2's production technology is strictly dominated. All agents have an initial wealth w. The fringe receives $\bar{y}$ in every state. Preferences are of the CRRA type and taxes and transfers are such that $\tau_1 = \frac{2}{2+m_f}q^* a_g$ and $\tau_2(z) = -\frac{1}{2+m_f}a_g$.*

Since there is no risk, we can search for an equilibrium where agent 1 sells a_1 units of the claim to its production and agent 2 buys a_2 units. Define $\hat{a}_1 = a_1 + \frac{a_g}{2+m_f}$ and $\hat{a}_2 = a_2 + \frac{a_g}{2+m_f}$. The claim has a price q^* based on the fringe's marginal utility, $q^* = u'\left(\bar{y} - \frac{1}{m_f}(\hat{a}_1 + \hat{a}_2)\right)$. and price impact is q'^*. Moreover, strategic agents net expenditures on assets at date 1 are $k_1 - q^* a_1$ and $k_2 + q^* a_2$, respectively. As such, asset positions satisfy

$$\text{Seller optimality:} \qquad \frac{u'(y^h k_1 + \hat{a}_1)}{u'(w - q^*\hat{a}_1 - k_1)} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_1 - \frac{a_g}{2+m_f}\right).$$

$$\text{Buyer optimality:} \qquad \frac{u'(y^l k_2 + \hat{a}_2)}{u'(w - q^*\hat{a}_2 - k_2)} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_2 - \frac{a_g}{2+m_f}\right).$$

With perfect competition (i.e., $\mu = 0$), $k_2 = 0$ and government intervention in financial markets has no real effects. With market concentration, in contrast, Type 1 agents sell fewer claims ($a_1 < 0$) while Type 2 agents buy fewer claims ($a_2 > 0$) because of price impact. Type 2 agents with the dominated technology consequently over-invest relative to the competitive benchmark.

If the government sells claims, or $a_g < 0$, then government sales reduce the market power wedge for Type 1 agents, $\frac{\mu}{m_f} q'^* \left(\hat{a}_1 - \frac{a_g}{2+m_f} \right)$, and increase it for Type 2 agents, $\frac{\mu}{m_f} q'^* \left(\hat{a}_2 - \frac{a_g}{2+m_f} \right)$. As a result, Type 1 agents sell more claims while Type 2 agents buy less. This increases the supply the fringe must absorb $\hat{a}_1 + \hat{a}_2$, reducing the claim price q^* and consequently price impact q'^*. This increased sale of claims raises risky investment by Type 1 agents k_1, and mitigates the reduction in purchases by Type 2 agents.

Our example, however, demonstrates that government debt issuance alone cannot achieve constrained efficiency. This is because although government debt sales can induce productive agents that sell financial claims to sell more, and even achieve their optimal scale of risky investment, they cannot always induce less productive agents to buy enough claims compared to the competitive equilibrium.

7.3 Dynamic Liquidity Management

To illustrate how government asset market interventions can impact market liquidity, holding fixed the aggregate resources in the economy, we imposed budget balance on the government. In a dynamic setting, we can relax this to allow the government to dynamically optimize its management of financial market liquidity. This would also allow the government more flexibility in choosing its taxation policies.

Our analysis suggests that mitigating financial market concentration will make government debt sales more countercyclical. A government managing financial market liquidity will sell more debt when financial markets are relatively more illiquid, such as during business cycle troughs, compared to when they are relatively more liquid, such as during business cycle booms. Because governments tax less and spend more during recessions to support the economy, a countercyclical debt policy to mitigate financial market illiquidity would be complementary to such countercyclical spending policies. This role for government asset market interventions to smooth cyclical fluctuations in financial market liquidity is relevant because of the presence of large institutional investors.

Integrating our model with explicit fire sales also suggests that it would be optimal for a government to buy assets when a fire sale occurs, and then to gradually sell its holdings to reduce its public balance sheet afterward. This is to minimize the distortionary impact on market liquidity that government asset purchases have in normal times.

8 Conclusion

We provide a novel perspective of how the public provision of liquidity impacts risk sharing among investors with price impact. In otherwise frictionless financial markets, government asset sales can improve liquidity by attenuating financial market power. In contrast, when the government buys assets and lowers interest rates, investors instead intensify rent-seeking at the expense of risk management. Our results can help explain why, after a long period of government asset purchases and low interest rates, many large institutional investors are now highly exposed to diversifiable risks. Our results on optimal policy deliver insights into the optimal management of public portfolios and central bank balance sheets.

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A Proofs of Propositions

A.1 Proof of Proposition 1:

As a preliminary, suppose we have some arbitrary asset span indexed by the $|\mathcal{Z}| \times |\mathcal{Z}|$ matrix X that is of full rank. In the special case of Arrow-Debreu assets, $X = I_{|\mathcal{Z}|}$, i.e., the identity matrix of rank $|\mathcal{Z}|$. Let x_k index the kth row vector of X, and $x_k(z)$ be the dividend asset k pays in state z.

If the competitive fringe trades assets with asset span X, it is immediate from the first-order conditions of the competitive fringe's optimization problem that the vector of asset prices $\vec{q}_X$ satisfies:

$$\vec{q}_X = X\vec{\Lambda}_f, \tag{26}$$

where $\vec{\Lambda}_f$ is vector of the fringe's state prices. Since the quasi-linear competitive fringe now maximizes $u_f\left(y_f(z) - \sum_{k=1}^{|\mathcal{Z}|} x(z)\, x_k(z)\, A_{x_k}(z)\right) + \sum_{k=1}^{|\mathcal{Z}|} x(z)\, q_{x_k} A_{x_k}(z)$, where $A_{x_k}(z)$ is the total demand for asset k of the strategic agents, the price impact function can be summarized by the matrix Γ:

$$\Gamma = XUX', \tag{27}$$

where U is the diagonal matrix with diagonal entries $-\frac{\mu}{m_f}\pi(z)u_f''\left(c_{2f}(z)\right)$.

Step 1: The Problem of the Fringe:

From the first-order condition for $a_f(z)$ from the competitive fringe's problem (5), we can recover the pricing equation of the Arrow-Debreu claim to security z:

$$\tilde{q}(z) = \pi(z)u_f'\left(c_{2f}(z)\right) = \Lambda_f(z),$$

where $\Lambda_f(z)$ is the competitive fringe's state price. Since $c_{2f}(z) = y_f(z) - \tau_2(z) + a_f(z)$, imposing the market-clearing condition, (1), reveals:

$$\tilde{q}(z) = \pi(z)u_f'\left(y_f(z) - \tau_2(z) - \frac{1}{m_f}\left(A(z) + a_G(z)\right)\right).$$

In equilibrium, this must be the realized price of the claim, $Q(\mathbf{A}, z)$. As this price is a function of state variables from the perspective of the fringe, we designate the realized price more concisely as:

$$q\left(z\right)=Q(\mathbf{A},z).$$

Let $\vec{q}$ be the vector of Arrow asset prices.

Step 2: Equilibrium Price Impact:

Because agents of type i take the demands of other agents (even within their type) as given, $u_f\left(z\right)$ is twice continuously differentiable, and each agent's position size scales by its mass μ, we can derive each agent's perceived price impact:

$$\frac{\partial\tilde{Q}_{j,i}(\mathbf{A},z)}{\partial a_i\left(z\right)}=-\frac{\mu}{m_f}\pi(z)u_f''\left(c_{2f}(z)\right)=-\frac{\mu}{m_f}\frac{\partial q(z)}{\partial A\left(z\right)},$$

which also implies that price impact is symmetric across all strategic agents. Defining $q'\left(z\right)=\frac{\partial q(z)}{\partial A(z)}$ yields the expression in the statement of the proposition.

Finally, we recognize price impact $q'\left(z\right)$ is convex because of the convex marginal utility of the fringe. It is straightforward to see:

$$q''\left(z\right)=\frac{\mu}{m_f}\pi(z)u_f'''\left(c_{2f}(z)\right)>0,$$

$$q'''\left(z\right)=-\left(\frac{\mu}{m_f}\right)^2\pi(z)u_f''''\left(c_{2f}(z)\right)>0.$$

Price impact is consequently convex in the net demand of strategic agents. In addition, because $-u_f''\left(c_{2f}(z)\right)$ is (weakly) decreasing in $c_{2f}(z)$ with increasing, concave utility, it follows prices and price impact are both increasing in the net demand of strategic agents and the government $A\left(z\right)$. Because $c_{2f}(z)$ is a sufficient statistic for both price impact and the price level, it follows price impact is increasing in the price level.

Step 3: Strategic Agent Demand:

Consider the optimization problem of strategic agent j of type i, (4). We attach the Lagrange multiplier φ_i to the budget constraint. The first-order necessary conditions for

$c_{i,j,1}$ and $\{a_{i,j}(z)\}_{z\in\mathcal{Z}}$ are given by:

$$\begin{aligned} c_{1,j,i} \quad &: \quad u'\left(c_{1,j,i}\right) - \varphi_{j,i} \leq 0 \ \left(= \ if \ c_{1,j,i} > 0\right), & (28) \\ a_{j,i}(z) \quad &: \quad -\pi(z)\, u_2^{i\prime}\left(c_{2,j,i}(z)\right) + \varphi_{j,i}\left(\tilde{Q}_{j,i}(\mathbf{A},z) + \frac{\partial \tilde{Q}_{j,i}(\mathbf{A},z)}{\partial a_{j,i}(z)} a_{i,j}(z)\right) = 0. & (29) \end{aligned}$$

The above represents the first-order necessary conditions for agent $i's$ problem. Because $u(\cdot)$ satisfies the Inada condition, $c_{1,j,i} > 0$ and (28) binds with equality.

Because strategic agent i has rational expectations, her perceived price impact must coincide with her actual price impact from (7). Consequently, equation (29) reduces to:

$$a_{j,i}(z) : \Lambda_{j,i}(z) = q(z) + \frac{\mu}{m_f} q'(z) a_{j,i}(z) \,\forall\, z \in \mathcal{Z}. \tag{30}$$

Step 4: The Law of One Price:

It is immediate from equation (26) because $\vec{q} = \vec{\Lambda}_f$:

$$\vec{q}_X = X\vec{q}. \tag{31}$$

The Law of One Price consequently holds if we introduce redundant assets into the complete markets economy.

Step 5: Invariance:

Let us conjecture that consumption allocations are unchanged if strategic agents and the fringe instead trade with asset span X. Because the fringe's consumption is unchanged, its state prices $\vec{\Lambda}_f$ and consequently Arrow prices $\vec{q}$ are unchanged.

Notice next we can stack the first-order conditions for strategic agent i with asset span $I_{|\mathcal{Z}|}$ from equation (30) as:

$$\vec{\Lambda}_i = \vec{\Lambda}_f + U\vec{a}_i, \tag{32}$$

where $\vec{\Lambda}_i$ are the stacked state prices of agent i, $\vec{a}_i$ is the vector of her asset positions, and we have substituted for Arrow-Debreu prices $\vec{q}$ with $\vec{\Lambda}_f$.

Let $\vec{a_{i,x}}$ be the vector of asset positions of agent i when she instead trades with asset span X. Imposing invariance of the consumption allocations of strategic agent i requires:

$$\vec{a_i} = X'\vec{a_{i,x}}. \tag{33}$$

Substituting with equation (33), we can manipulate equation (32) to arrive at:

$$X\vec{\Lambda}_i = X\vec{\Lambda}_f + XUX'\vec{a_{i,x}} = X\vec{\Lambda}_f + \Gamma\vec{a_{i,x}}, \tag{34}$$

where we have also substituted with equation (27). This is the identical stacked first-order conditions if strategic agent instead traded asset span X.

Both strategic agents and the competitive fringe therefore choose the same state-specific asset exposures under both asset spans. Finally, because the Law of One Price holds, the cost of each agent's portfolio is the same under both asset spans. We conclude that consumption allocations are invariant to the complete markets asset span.[10]

A.2 Proof of Proposition 2:

The first-order necessary condition equation (30) is derived in Step 3 of Proposition 1. Substituting for $a_{j,i}(z)$ with equation (8), we arrive at the equation in the statement of the proposition.

A.3 Proof of Proposition 3:

Step 1: Wealth-neutral Demand Shock with Perfect Competition:

Let us conjecture that asset prices are unaffected by the demand shock.

With perfect competition (i..e, $\mu = 0$) and complete markets, large agents of type i maximizes their utility from consumption subject to the intertemporal budget constraint

$$\sum_{z\in\mathcal{Z}} q(z)\, y_i(z) + w_i = c_{1,j,i} + \sum_{z\in\mathcal{Z}} q(z)\, c_{2,j,i}(z). \tag{35}$$

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[10] This result is true for any full-rank asset span X. See, for instance, Carvajal (2018) and Neuhann and Sockin (2022).

Since the demand shock is *wealth-neutral*, $\sum_{z\in\mathcal{Z}} q\left(z\right) y_i\left(z\right)$ is the same as before the demand shock if asset prices are unchanged. Because the present-value of their endowments is unchanged under a wealth-neutral demand shock, as are the aggregate resources available in each state $\sum_{i=1}^{N} y_i\left(z\right)$, their problem is unchanged. Consequently, they choose the same consumption allocations that they did before the demand shock.

If all large agents' consumption allocations are unchanged, then so is the competitive fringe's by market clearing. If the fringe's consumption is unchanged, then so are asset prices, which are equal to the fringe's marginal utility in each state. This confirms the conjecture and the neutrality of *wealth-neutral* demand shocks for asset prices and consumption allocations.

Step 2: Wealth-neutral Demand Shock with Imperfectly Competition:

With market power, large agents of type *i* instead maximize their utility from consumption subject to the intertemporal budget constraint

$$\sum_{z\in\mathcal{Z}} \Lambda_{j,i}\left(z\right) y_i\left(z\right) + w_i = c_{1,j,i} + \sum_{z\in\mathcal{Z}} \Lambda_{j,i}\left(z\right) c_{2,j,i}\left(z\right). \tag{36}$$

Importantly, although the demand shocks is *wealth-neutral* under market prices, it is not under type *i*'s state prices. Consequently, the demand shock is not neutral for their consumption allocation decision, and type *i* agents choose a different consumption bundle.

If all agents choose different consumption bundles after the demand shock, then generically so will the competitive fringe. If the fringe's consumption changes, then so do asset prices. A *wealth-neutral* demand shock is therefore not neutral with market power.

Step 3: Elasticity of Consumption and Asset Prices:

It is immediate because consumption allocations are altered by the demand shock, but are invariant in the perfect competition benchmark, that the elasticity of consumption with respect to demand shocks is lower with imperfect competition. This is because market power induces both sellers and buyers of Arrow securities to trade less, which deters reallocation of the demand shocks through financial markets.

Similarly, because asset prices change with market power but are invariant in the perfect competition limit, the elasticity of asset prices with respect to demand shocks is higher with imperfect competition.

Step 4: Dispersion in State Prices:

Suppose there is a wealth-neutral demand shock that increases the endowments of sellers in a given state and reduces the endowment of buyers. Then, because this raises the state prices of buyers and lowers it of sellers, and is only imperfectly reallocated, the dispersion of state prices, as measured according to $\omega(z)$, rises.

A.4 Proof of Proposition 1:

Step 1: The Case of the Competitive Equilibrium:

When all agents behave competitively, and $\mu = 0$, equation (**??**) reduces to:

$$q(z) = \frac{\pi(z)\, u'(c_{2,i}(z))}{u'(c_{1,i})} = \pi(z)\, u'_f\left(c_f(z)\right) = \Lambda^{CE}(z), \tag{37}$$

and all agents align their state prices state-by-state. There is therefore perfect risk sharing in the competitive equilibrium. This implies for the N types of agents with homothetic preferences:

$$\frac{c_{2,i}(z)}{c_{1,i}} = \frac{c_{2,j}(z)}{c_{1,j}} = \eta(z), \tag{38}$$

and for the competitive fringe:

$$c_f(z) = \eta_f(z) = u_f^{-1}\left(u'(\eta(z))\right). \tag{39}$$

Substituting for date 2 consumption into the budget constraint at date 1, the intertemporal budget constraint for agents of type i is:

$$c_{1,i} + \sum_{z\in\mathcal{Z}} q(z)\, c_{2i}(z) = w - \tau_1 + \sum_{z\in\mathcal{Z}} q(z)\left(y_i(z) - \tau_2(z)\right). \tag{40}$$

Recognizing that $\tau_1 = -\sum_{z\in\mathcal{Z}} q(z)\,\tau_2(z)$, it follows that equation (41) reduces to:

$$c_{1,i} + \sum_{z\in Z} q(z)\, c_{2i}(z) = w + \sum_{z\in\mathcal{Z}} q(z)\, y_i(z)\,. \tag{41}$$

Finally, substituting with equation (56), we arrive at:

$$c_{1,i} = \frac{w + \sum_{z\in\mathcal{Z}} q(z)\, y_i(z)}{1 + \sum_{z\in\mathcal{Z}} q(z)\, \eta(z)}. \tag{42}$$

Suppose the consumption of the fringe is invariant to the government's policies, then so is $q(z)$ because it is equal to the state prices of the fringe. If prices are unchanged, then from equation (42) $c_{1,i}$ is also unchanged, and so are $c_{2,i}(z) = \eta(z)\, c_{1,i}$. By market-clearing then, so is the consumption of the fringe, confirming the conjecture.

The government's trading activity is therefore irrelevant for all agents and there is Ricardian Equivalence. This is because agents can frictionlessly transfer wealth across both states and dates.

Step 2: The Case of the Cournot-Walras Equilibrium:

First, we recognize that equation (41) remains valid when large agents are strategic. As such, there are no direct effects from government trading in the Cournot-Walras equilibrium. Notice from equation (30) that $q(z) = \Lambda_i(z) - \frac{\mu}{m_f} q(z)\, a_i(z)$. Substituting this into equation (41), we arrive at:

$$c_{1,i} + \sum_{z\in\mathcal{Z}} \Lambda_i(z)\, c_{2i}(z) = w + \sum_{z\in\mathcal{Z}} \Lambda_i(z)\, y_i(z) + \frac{\mu}{m_f} \sum_{z\in\mathcal{Z}} q(z)\, a_i(z)^2\,. \tag{43}$$

The last term on the right-hand side of equation (43) is zero in the competitive equilibrium. This reveals that in the Cournot-Walras equilibrium, asset positions $a_i(z)$ are not irrelevant for equilibrium allocations. As such, trading away from the effective endowment of $-\frac{1}{N+m_f} a_g$ in assets from the government transfers is not frictionless.

Notice now the Cournot-Walras equilibrium features too little trading relative to the competitive benchmark by all agents, based on the implied wedges between state prices and asset prices in equation (30). It then follows that the competitive fringe absorbs at least part of the government's trades. As a result, if the government sells risk-free debt

$a_g < 0$, then the fringe has more consumption at date 2, all else equal. This reduces its marginal utility in all states and all Arrow asset prices. In contrast, if the government buys risk-free debt $a_g > 0$, then the fringe has less consumption. This raises marginal utility in all states and all Arrow asset prices.

A.5 Proof of Proposition 2:

We first recognize that in the type-symmetric case, all agents consume the same initial consumption $c_{1,j,i}$ at date 1. Further, in the limit $m_f \to 0$ and $\frac{\mu}{m_f} = \kappa$,

$$\sum_{i=1}^{N} \hat{a}_i (z) = 0, \tag{44}$$

and it must be the case that $c_{1,j,i} = w$, i.e., agents consume their initial wealth. Consequently, the state price of agent j of type i is $\Lambda_{j,i}(z) = \frac{u'\left(y_i(z)+\hat{a}_{j,i}(z)\right)}{u'(w)}$. Consequently, the state prices of strategic agents move inversely with their asset positions $\hat{a}_{j,i}(z)$ state-by-state. In addition, because agents are type-symmetric, we need only focus on characterizing one asset market because they each behave identically for all z.

We next consider the seller side of asset market z. It is immediate from equations (9) and Proposition 1 that when $a_g < 0$ that sellers in asset market z sell more (i.e., $\hat{a}_{j,i}(z)$ becomes more negative. This is because a negative a_g effectively endows each agent with a long position that they must undo in financial markets and because a negative a_g lowers asset prices and consequently price impact. Both forces reduce the market power wedge between state prices $\Lambda_{j,i}(z)$ and asset prices $q(z)$.

Because an increase in normalized asset sales by seller j of type i raises her state price in state z, all sellers' state prices in asset market z rise. Since sellers have lower state prices than buyers (by definition of how agents sort into both sides of financial markets), government asset sales $a_g < 0$ decreases the dispersion in state prices referencing state z $\omega(z)$. By the converse argument, government asset purchases $a_g > 0$ instead raise this dispersion.

Finally, we consider the buyer side of asset market z. Although lower prices and price impact increase a buyer's normalized asset purchases from equation (9), the endowment of buyers with a long position $-\frac{a_g}{N+m_f}$ instead reduces her position. However, we recognize by market-clearing in the limit $m_f \to 0$ and $\frac{\mu}{m_f} = \kappa$ that

$$\sum_{i=1}^{N} \hat{a}_i\left(z\right) = 0. \tag{45}$$

Because each seller increases her sales in market z, it must be the case that buyers, on net, buy more assets based on their normalized demand. This lowers the state prices $\Lambda_{j,i'}\left(z\right)$ for those buyers who buy more, which further reduces the dispersion in state prices referencing state z, $\omega\left(z\right)$.

If all buyers increase their normalized demands, then $\omega\left(z\right)$ falls for all z. Otherwise, notice in the cross-section, strategic agents who buy the least (i.e., smallest $\hat{a}_{j,i}\left(z\right)$) have the lowest state prices among buyers, while those that buy the most (i.e., largest $\hat{a}_{j,i}\left(z\right)$) have the highest state prices. Consequently, a decline in price impact because of government sales must increase the normalized asset demand of the high state price buyers, and lower their state prices, and decrease the normalized asset demand of low state price buyers, and raise their state prices. Because lower state price buyers have more elastic demand than high state price buyers, it follows the overall effect is to decrease the dispersion in state prices ω_z.

An analogous argument establishes that asset purchases $a_g > 0$ raise the state-price dispersion ω_z. Consequently, ω_z is increasing in a_g.

A.6 Proof of Proposition 3:

We begin with the Euler Equations from equation (9) expressed in terms of risk sharing wedges $\Lambda_{j,i}\left(z\right) - q\left(z\right)$ and the normalized asset demands $\hat{a}_{j,i}$, which we write as

$$\Lambda_{j,i}\left(z\right) - q\left(z\right) = \frac{\mu}{m_f} q'(z) \hat{a}_{j,i}\left(z\right) - \frac{\mu}{m_f} q'(z) \frac{a_g}{N + m_f}, \tag{46}$$

and depends on $\frac{a_g}{N+m_f}$ only through the last term on the right-hand side. If $a_g > 0$, then the last term is negative, while if $a_g < 0$, then the last term is positive.

Notice government asset purchases cannot generically achieve the competitive equilibrium. This is because a_g (which is one degree of freedom) cannot be chosen such that the wedges $\frac{\mu}{m_f} q'(z)\left(\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f}\right)$ ($N \times |\mathcal{Z}|$ equations) are all zero.[11]

In what follows, we measure the aggregate efficiency of risk sharing in an asset market using $Var_i(\Lambda_i(z))$ defined in equation (11). The change in efficiency in risk sharing in state z for a change in government policy a_g is

$$\Delta \log Var_i(\Lambda_i(z)) = 2\Delta \log q'(z) + \Delta \log Var_i(\hat{a}_i(z)). \tag{47}$$

From Proposition 1, $\frac{\partial q'(z)}{\partial a_g} > 0$ and the first-term in equation (48) is positive if $a_g > 0$ and negative if $a_g < 0$ for all z. Because sellers' supply curves are more elastic than buyers' demand curves, it follows $\frac{\partial}{\partial a_g} Var_i(\hat{a}_i(z)) < 0$ because sellers reduce their selling positions more than buyers increase their buying positions.

We can further approximate equation (47) to first-order for a change in government policy Δa_g as

$$\Delta \log Var_i(\Lambda_i(z)) \approx 2\frac{\gamma_f(z)}{m_f}\sum_{i=1}^{N} \Delta \hat{a}_i(z) + 2\sum_{i=1}^{N} w_i \frac{\Delta \hat{a}_i(z)}{\hat{a}_i(z)}, \tag{48}$$

where $\gamma_f(z)$ is the competitive fringe's coefficient of absolute risk aversion in state z and $w_i = \frac{1}{N}\frac{\hat{a}_i(z)^2 - \hat{a}_i(z)\frac{1}{N}\sum_{j=1}^{N}\hat{a}_j(z)}{Var_i(\hat{a}_i(z))}$ are bounded weights that sum to 1 with the convention that $w_i\hat{a}_i(z) = -\frac{1}{N}\frac{\frac{1}{N}\sum_{j=1}^{N}\hat{a}_j(z)}{Var_i(\hat{a}_i(z))}$ when $\hat{a}_i(z) = 0$.

To first-order, the change in the efficiency of risk sharing is driven by how the change in each agent's net asset position $\hat{a}_i(z)$ impacts not only market liquidity but also their position relative to the mean exposure $\frac{1}{N}\sum_{j=1}^{N}\hat{a}_j(z)$. The first term is increasing in $\frac{\gamma(z)}{m_f}$, which is the inverse of the effective risk-bearing capacity of the fringe. Let $\underline{\gamma} = \min_{z\in\mathcal{Z}} \gamma(z)$, i.e., the minimum coefficient of risk aversion across all asset markets.

Step 1: Asymptotic Absorption Capacity:

Consider first large government asset purchases $a_g >> 0$. Such a large purchase induces sellers in each market to ration severely their supply in each market (even becoming buyers), and buyers to increase their demand. This demand is absorbed by the fringe until $\sum_{i=1}^{N} \hat{a}_i(z) = m_f y_f(z)$, in which case the asset price in market z $q(z)$ becomes infinite, as does price impact. This immiseration pushes $Var_i(\Lambda_i(z))$ to its autarky value. Consequently, sufficiently large asset purchases severely worsen the efficiency of risk sharing.

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[11] This is also true even if the government can trade each Arrow asset separately because there are still more Euler Equations than asset markets. Because prices (and consequently price impact) cannot be zero by no arbitrage, these wedge will not vanish if government asset sales are arbitrarily large.

Consider next large government asset sales $a_g << 0$. In this case, the market is saturated with supply of each asset and prices and price impact are very low from Proposition 1. Because the wedge is bounded from below by 0, and from above because state prices cannot become infinite since strategic agent utility satisfies the Inada condition, the efficiency measure eventually asymptotes. Notice now from equation (46) that for the measure to asymptote, it must be the case that $\lim_{a_g \to -\infty} q'(z) a_g = 0$. For this to be the case, $q'(z)$ must fall faster than $|a_g|$ rises (in fact, $q'(z)$ is convex in a_g). As such, $Var_i(\Lambda_i(z))$ asymptotically decreases to its limit. Because from above, government asset purchases cannot achieve the competitive equilibrium, it follows this limit is bounded away from 0.

Consequently, when a_g is arbitrarily negative, $Var_i(\Lambda_i(z))$ converges to its lower limit above 0, while when it is positive and too large, $Var_i(\Lambda_i(z))$ converges to its autarky value. Because $Var_i(\Lambda_i(z))$ is continuous in a_g, it follows $Var_i(\Lambda_i(z))$ has an even number of turning points in a_g. Given $Var_i(\Lambda_i(z))$ is driven by two monotonic forces that move in opposite directions, price impact and asset position variance, there are either zero or two turning points.

Step 2: Small Government Asset Purchases $a_g > 0$:

Suppose the government purchases a small amount of assets, i.e., $a_g > 0$. Because $a_g > 0$, the overall wedge for asset sellers $\frac{\mu}{m_f} q'(z) \left(\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f} \right)$ increases, and consequently they trade less. As a result, asset buyers buy more from the fringe (which is why prices increase). In contrast, the wedge may increase or decrease (or even become negative) for buyers depending on whether the increase in price impact is offset by the decrease in $\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f}$. Consequently, the risk sharing wedge unambiguously worsens for sellers, but may improve or worsen for buyers.

Consider now the first-order change in risk sharing efficiency, given by equation (48), of a small increase in a_g from 0 to $\Delta a_g > 0$. From our above discussion, the first piece is positive for all z while the second piece is negative for all z. If $\frac{\gamma(z)}{m_f}$ is sufficiently large, then the price impact effect dominates and position variance effect and the efficiency measure rises.

Step 3: Small Government Asset Sales $a_g < 0$:

Suppose the government sells a small amount of assets, i.e., $a_g < 0$. Because $a_g < 0$, the overall wedge for asset sellers $\frac{\mu}{m_f} q'(z) \left(\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f} \right)$ decreases, and consequently they sell more. In contrast, the wedge may increase or decrease (or even become positive) for buyers depending on whether the decrease in price impact is offset by the increase in $\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f}$. Consequently, risk sharing unambiguously improves for sellers, but may improve or worsen for buyers.

Consider now the first-order change in risk sharing efficiency, given by equation (48), of a small increase in a_g from 0 to $\Delta a_g > 0$. From our above discussion, the first piece is positive for all z while the second piece is negative for all z. If $\frac{\gamma(z)}{m_f}$ is sufficiently large, then the price impact effect dominates and position variance effect and the efficiency measure falls.

A.7 Proof of Proposition 4:

Consider a perturbation to the welfare objective (18) from increasing the government's debt position, a_G. This has two effects. First, mechanically, it marginally shifts all agents' resources from date 1 to date 2. This is because taxes increase at date 1 and are more negative at date 2 to finance the purchase of the debt and lump-sum rebate of its proceeds.

Second, it marginally increases the price of every Arrow asset from the government's increased demand. To first-order, the change in each agent's asset position on her own utility is zero by the Envelope Theorem applied to their respective optimization problems from Proposition (1). There is, however, a wealth effect on each agent's utility from having to buy a more expensive asset portfolio, and a strategic indirect effect that a change in large agents' asset positions affects the price impact of other large agents.

Perturbing in utilitarian welfare by altering Δa_g reveals:

$$\Delta W = \sum_{i=1}^{N} u'\left(c_{1,i}\right)\left(\Delta c_{1,i} + \sum_{z\in\mathcal{Z}} \Lambda_i\left(z\right)\Delta c_{2,i}\left(z\right)\right) + m_f\left(\sum_{z\in\mathcal{Z}} u_f'\left(c_{2,f}\left(z\right)\right)\Delta c_{2,f}\left(z\right) - \Delta c_{1,f}\right), \tag{49}$$

which we can expand with equations (4) (5), and (8)

$$\begin{aligned}\frac{\Delta W}{\Delta a_g} \quad &= \quad \sum_{i=1}^{N} u'\left(c_{1,i}\right)\sum_{z\in\mathcal{Z}}\left(\left(\Lambda_i\left(z\right) - q\left(z\right)\right)\frac{\Delta\hat{a}_i\left(z\right)}{\Delta a_g} - \frac{\Delta q\left(z\right)}{\Delta a_g}\hat{a}_i\left(z\right)\right) \\ &\quad + m_f\left(\sum_{z\in\mathcal{Z}}\left(u_f'\left(c_{2,f}\left(z\right)\right) - q\left(z\right)\right)\frac{\Delta\hat{a}_f\left(z\right)}{\Delta a_g} - \frac{\Delta q\left(z\right)}{\Delta a_g}\hat{a}_f\left(z\right)\right),\end{aligned} \tag{50}$$

where $\frac{\Delta q(z)}{\Delta a_g}$ indicates the total change in $q\left(z\right)$ with respect to a_g. Notice that the first m_f term in equation (50) is zero by the definition of the Arrow price $q\left(z\right)$ and $m_f\hat{a}_f = -\sum_{i=1}^{N}\hat{a}_i\left(z\right)$. Equations (50) and 30 consequently reduces to:

$$\frac{\Delta W}{\Delta a_g} = \sum_{z\in\mathcal{Z}}\frac{\mu}{m_f}q'\left(z\right)\sum_{i=1}^{N} u'\left(c_{1,i}\right)a_i\left(z\right)\frac{\Delta\hat{a}_i\left(z\right)}{\Delta a_g} - \frac{\Delta q\left(z\right)}{\Delta a_g}\sum_{i=1}^{N}\left(u'\left(c_{1,i}\right) - 1\right)\hat{a}_i\left(z\right). \tag{51}$$

Taking the limit of equation (51) as $\Delta a_g \to da_g$, defining $w_p\left(z\right) = \sum_{i=1}^{N} u'\left(c_{1,i}\right)a_i\left(z\right)\frac{d\hat{a}_i(z)}{da_g}$ and $w_g\left(z\right) = \sum_{i=1}^{N}\left(u'\left(c_{1,i}\right) - 1\right)\hat{a}_i\left(z\right)$, and recognizing that a necessary condition for optimality is $\frac{\partial W}{\partial a_g} = 0$, at the optimal a_g it must be the case from (51):

$$\sum_{z\in\mathcal{Z}} w_p\left(z\right)\frac{\mu}{m_f}q'\left(z\right) - w_g\left(z\right)\frac{dq\left(z\right)}{da_g} = 0\ (< if a_g = -\infty, > if a_g = \infty) \tag{52}$$

If all large agents behaved competitively, then $\frac{\mu}{m_f}q'\left(z\right) = 0$. Further, because asset prices are invariant to the government's debt position at the competitive equilibrium from Proposition 1, $\frac{dq(z)}{da_g} = 0$. Therefore, the government's first-order condition is satisfied at the competitive equilibrium, and its debt choice is irrelevant.

A.8 Proof of Proposition 5:

We first consider how unfunded government purchases $\bar{a}_g > 0$ impact equilibrium allocations in the case of perfect competition. We then consider the complementary case with concentrated financial markets.

Step 1: Perfect Competition:

With perfect competition, it is immediate that the First Welfare Theorem holds and optimal risk-sharing arrangements solve the appropriate social planner's problem. In this case, with symmetric, homothetic preferences, perfect risk sharing calls for $\frac{c_{2i}(z)}{c_{1i}} = \eta(z) = u'^{-1}(q(z))$ for all i. This is Wilson's optimal risk sharing rule. In this case, $\sum_{z\in\mathcal{Z}} q(z) = \sum_{z\in\mathcal{Z}} u'(\eta(z))$

By the inter-temporal budget constraint for agent i at date 1

$$c_{1i} + \sum_{z\in\mathcal{Z}} q(z)\, c_{2i}(z) = \left(1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)\right) c_{1i} = w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z), \tag{53}$$

from which follows

$$c_{1i} = \frac{w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z)}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)}. \tag{54}$$

Further, by market clearing in the consumption market at date 2 in the strategic limit in which $m_f \to 0$

$$\sum_{i=1}^{N} c_{2i}(z) + \bar{a}_g = \eta(z) \sum_{i=1}^{N} c_{1i} + \bar{a}_g = \sum_{i=1}^{N} y_i(z), \tag{55}$$

from which follows from equation (54) and $a_g = \frac{y_{1g} - y_{2g}}{1 + \sum_{z\in\mathcal{Z}} q(z)}$ that $\eta(z)$ solve

$$\eta(z) \sum_{i=1}^{N} \frac{w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z)}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)} + \frac{\Delta y_g}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))} = \sum_{i=1}^{N} y_i(z). \tag{56}$$

Recovering $\eta(z)$ from equation (56) is consequently sufficient to solve for the competitive equilibrium.

Notice because $\Delta y_g = y_{1g} - y_{2g} > 0$ (i.e., $\bar{a}_g > 0$) and $u'(\eta(z))$ is decreasing in

$\eta(z)$ from applying the Implicit Function Theorem to equation (56) that $\frac{\partial \eta(z)}{\partial \Delta y_g} < 0$. In addition, by the Chain Rule, $\frac{\partial q(z)}{\partial \Delta y_g} = \frac{\partial q(z)}{\partial \eta(z)} \frac{\partial \eta(z)}{\partial \Delta y_g} > 0$ for all $z \in \mathcal{Z}$ because $\frac{\partial q(z)}{\partial \eta(z)} < 0$ with convex marginal utility.

Consequently, as $\Delta y_g > 0$ increases, all agents consume more at date 1 and less at date 2, and asset prices rise state-by-state. Because the government crowds out investment in risk-free bonds, agents are forced to consume more resources at date 1.

Step 2: Imperfect Competition:

Notice that in the limit $m_f \to 0$ that asset prices are given by the average of strategic agents' state prices, $q(z) = \frac{1}{N} \sum_{i=1}^{N} \Lambda_i(z)$.

Suppose that Δy_g increases to $\Delta y_g + \varepsilon_g$. We conjecture that $\bar{a}_g$ increases and that, based on Step 1, all asset prices fall and strategic agents consume more at date 1 and less at date 2, state-by-state. In this case, all strategic agents' state prices (weakly) rise state-by-state, $\Lambda_i(z)$, which raises asset prices and lowers the risk-free rate. Because the risk-free rate falls, the government has to (weakly) buy more assets to achieve its pre-existing date 2 position based on Δy_g, as well as more to cover its incremental position based on ε_g. Consequently, $\bar{a}_g$ increases, confirming the conjecture.

In addition to the direct effect of government purchases, the increase in asset prices raises price impact $q'(z)$ because the competitive fringe's marginal utility is convex. This further attenuates how much risk-free debt is traded among strategic agents. As a result, buyers reduce their purchases of risk-free debt more than with perfect competition, while sellers must sell to fulfill the government's incremental orders, exacerbating the rise in asset prices.

Consequently, as with perfect competition, asset prices rise and strategic agents consume more at date 1 and less at date 2.