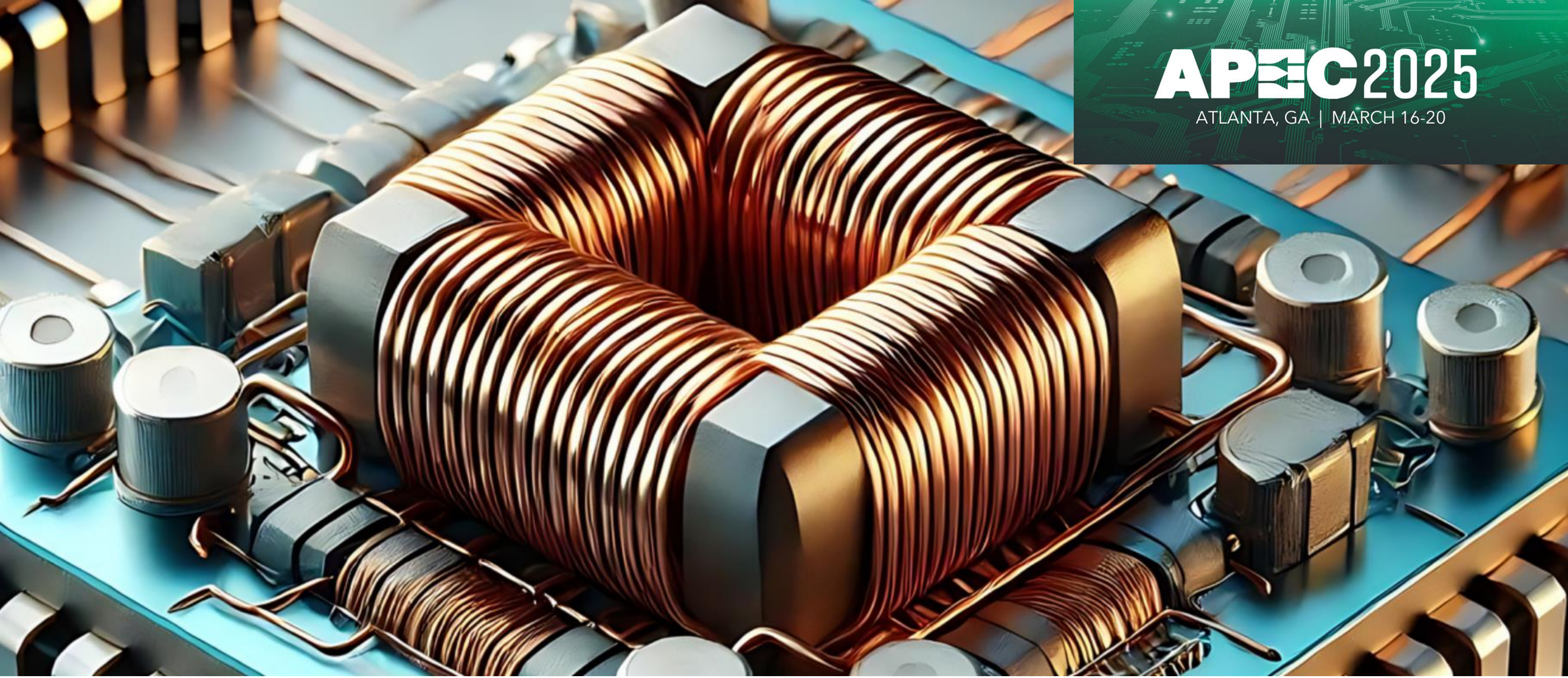

Magnetic Scaling and Constraints

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University of Texas at Austin

Our Goal: Escape the Tyranny of the Case Study

- Much of published magnetics design is for specific applications
- Often much doubt as to whether the conclusions will apply to different frequencies and power levels
- (Often no “conclusions” at all – just a good design)

- We’re looking for the opposite: generalizable conclusions that can easily be applied across size/power/frequency
- We’re willing to accept coarse precision

Ampere's Law

$$\oint H \cdot dl = \int J \cdot dA$$

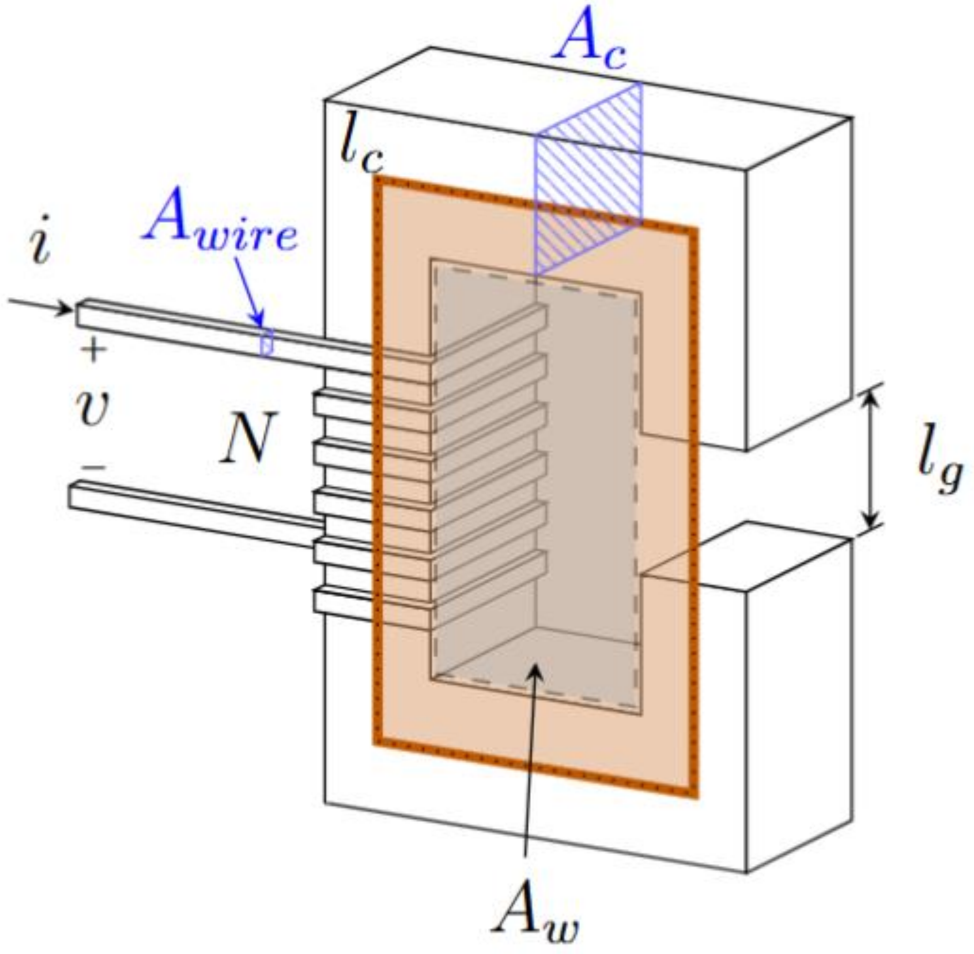
$$H_c l_c + H_g l_g = Ni$$

Constitutive Equation: $B = \mu H = \mu_r \mu_0 H$

Boundary Condition on B: $B_\perp$ is conserved

$$\Rightarrow \frac{B}{\mu_c} l_c + \frac{B}{\mu_0} l_g = Ni$$

$$\Rightarrow \boxed{B = \frac{Ni}{\frac{l_c}{\mu_c} + \frac{l_g}{\mu_g}}}$$



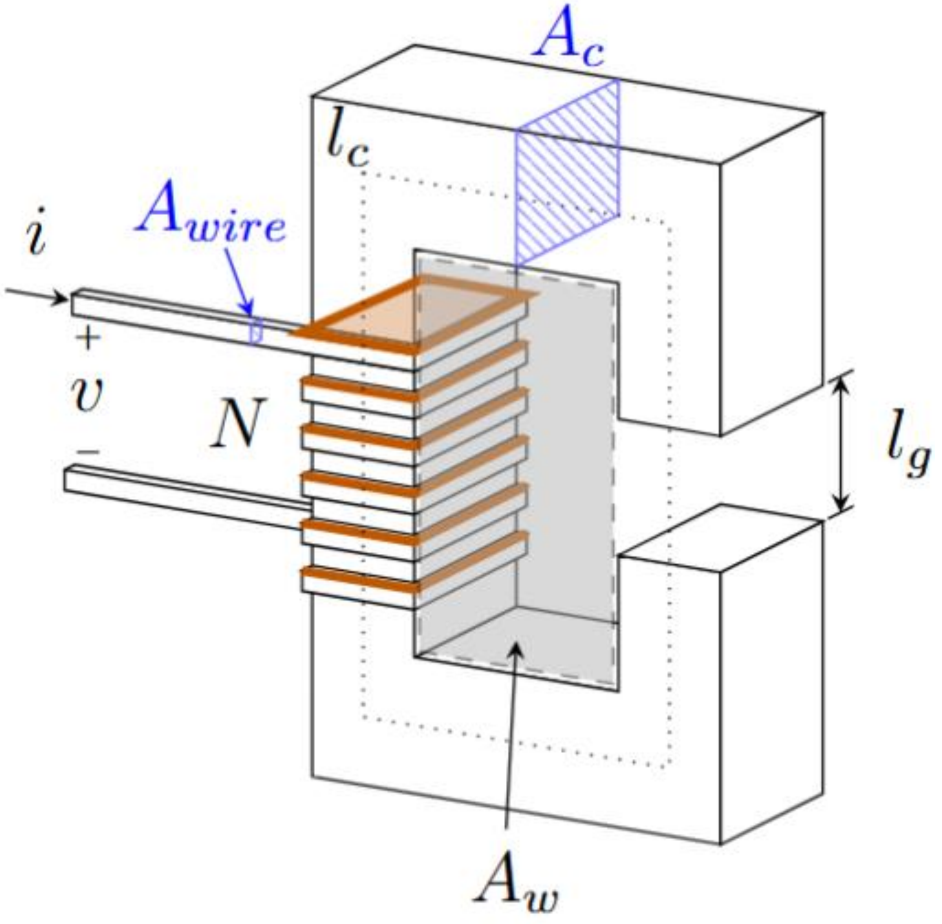
l_t: mean length of turn

Faraday's Law

$$\oint E \cdot \underline{dl} = -\frac{d}{dt}\int B \cdot \boxed{dA}$$

$$-V = -\frac{d}{dt}\left(\frac{Ni}{\frac{l_c}{\mu_c}+\frac{l_g}{\mu_0}}\right) \times (NA_c)$$

$$V = \underbrace{\frac{N^2}{\frac{l_c}{\mu_c A_c}+\frac{l_g}{\mu_0 A_c}}}_{L} \times \frac{di}{dt}$$



l_t: mean length of turn

A simplifying observation?

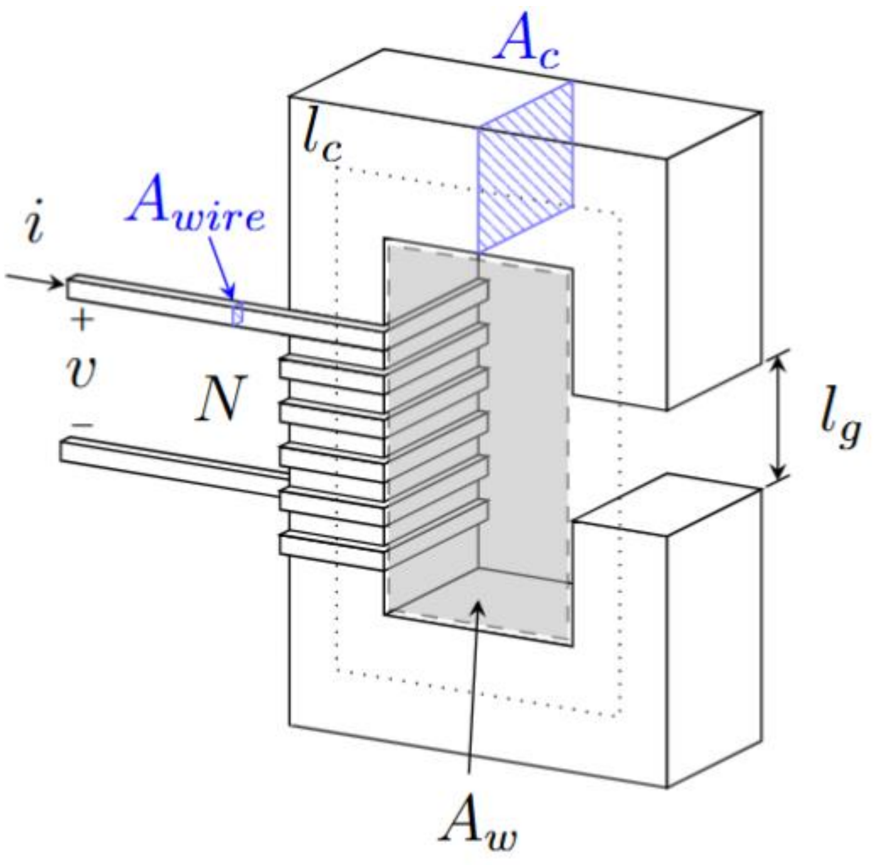
$B \times A_c$ has a suspicious form

$$BA_c = \frac{Ni}{\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}} = \phi \text{ (flux)}$$

This looks suspiciously like $I = \frac{V}{\frac{l_1}{\sigma_1 A_1} + \frac{l_2}{\sigma_2 A_2}} = \frac{V}{R_1 + R_2}$

Is it possible that magnetic flux "flows" following a sort of "Magnetic Ohm's Law"?

Yes! -- and we can use this observation to repurpose all the intuition and powerful analysis techniques we inherit from circuits



l_t: mean length of turn

Magnetic Circuits

$$\phi = \frac{Ni}{\mathcal{R}_1 + \mathcal{R}_2}$$

Where

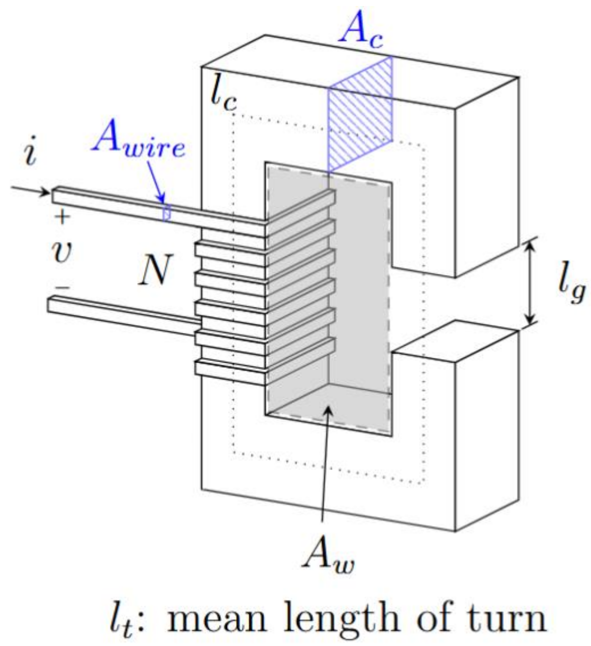
NI plays the role of voltage ⇒ MMF

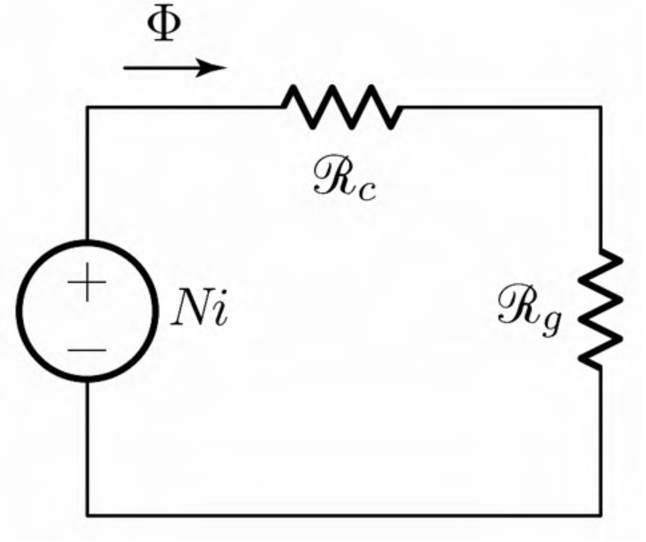
$\mathcal{R} = l/\mu A$ plays the role of resistance ⇒ Reluctance

ϕ plays the role of current ⇒ Flux

$$V = L\frac{di}{dt} = N\frac{d\phi}{dt} \quad \Rightarrow \quad \int V dt = Li = N\phi = \lambda$$

Inductance can be found from the flux flowing through an MMF source and the number of turns in that source, $L = \frac{N\phi}{i} = \frac{\lambda}{i}$





What constrains a magnetic component design?

- We **must** obtain L and be able to sustain an excitation $i(t)$
- We get to **choose** A_c, l_c, N, l_g, μ_c
- We **want** the component to be small and efficient

It's tempting to just say "everything affects everything" and fall into trial and error, scripting, computer-based optimization

We must avoid that temptation!

- Recast equations in terms of must-haves: L and $i(t)$

$$\lambda = N\phi = NA_cB = Li$$

$$\boxed{B = \frac{Li}{NA_c}} \quad \text{and} \quad \boxed{A_{wire} = \frac{A_w}{N}}$$

(l_g is whatever it has to be to achieve L for a given core geometry and number of turns ⇒ not an independent design choice)

The core's interests

Do not enter saturation

$$B_{pk} < B_{sat}$$

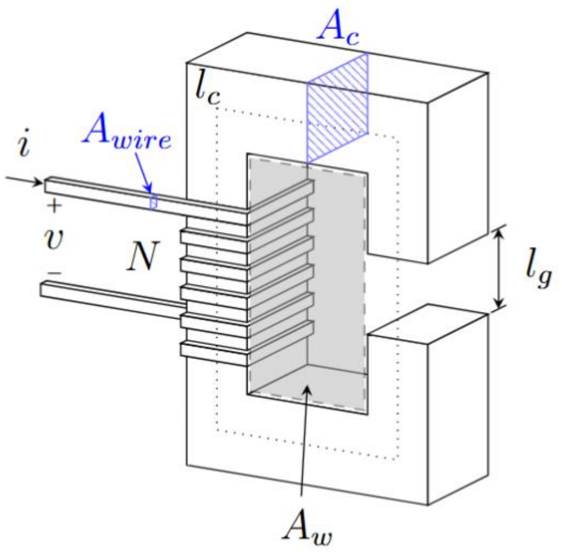
Avoid core loss

Keep $P_{core} = (VOL) \times kB_{ac}^{\beta}$ within acceptable bounds; otherwise, minimize along with P_{cu}

The winding's interests

Avoid copper loss

Keep $P_{cu} = I_{rms}^2 \left(\rho \dfrac{N l_t}{A_{wire}}\right) = I_{rms}^2 \left(\rho \dfrac{N^2 l_t}{A_w}\right)$ within acceptable bounds; otherwise, minimize along with P_{core}



l_t: mean length of turn

Key design intuition

- The design question: what prevents me from making the component infinitely small with one turn?

$$B = \frac{Li}{NA_c} \quad \text{and} \quad A_{wire} = \frac{A_w}{N}$$

1. Smaller size makes B bigger (bad for saturation/core loss) and A_{wire} smaller (bad for conduction loss)

⇒ There's a **minimum size** required to meet all the specifications

2. Larger N makes B smaller (good for saturation/core loss) and makes A_{wire} smaller (bad for conduction loss)

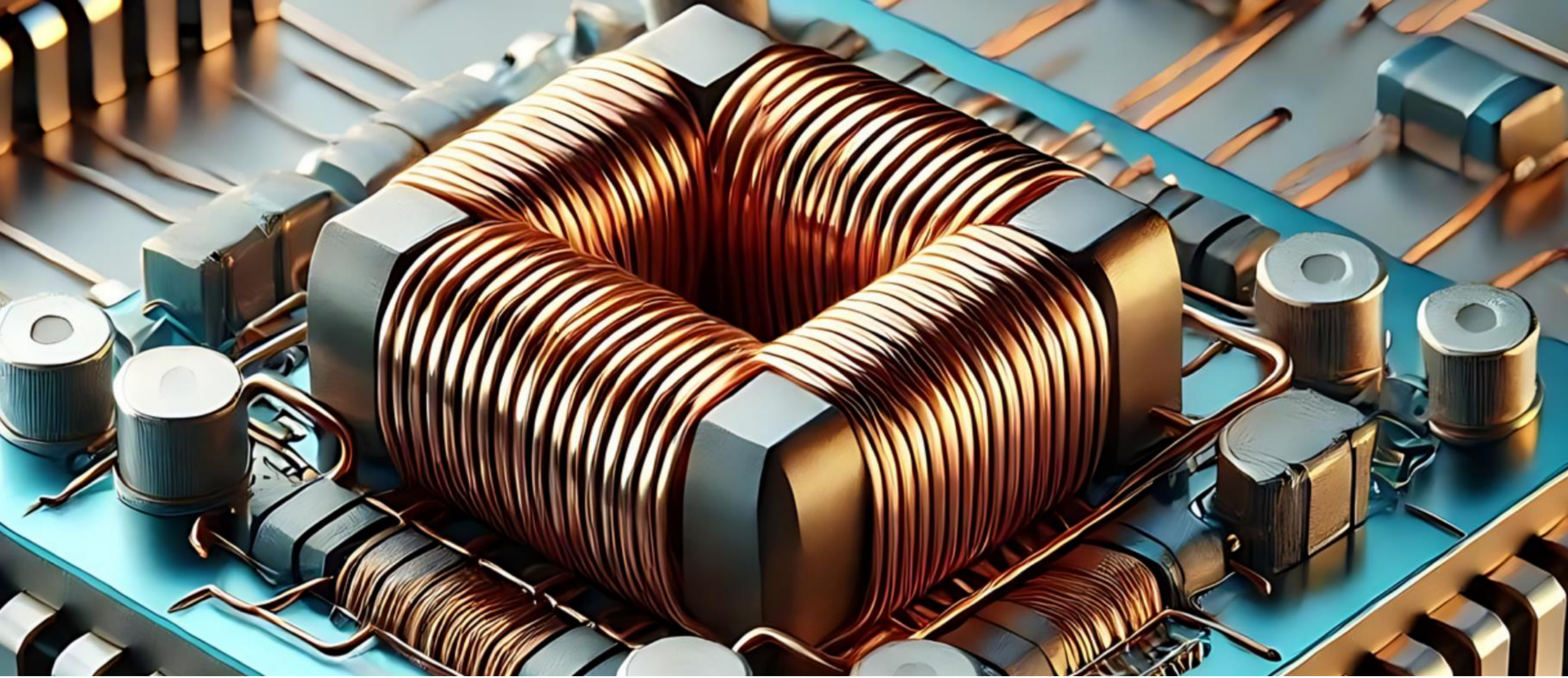
⇒ There's an **optimum N** that best balances the interests of the core and the winding

Key design intuition

- Very often, the goal is to minimize size, and inefficiency mainly matters from a thermal perspective – loss keeps you from making the component smaller
- To reach broadly useful conclusions, let us **assume from the beginning that a component is size-optimized, i.e., it has been shrunk to its absolute limit.**

- Copper loss **will** be *at its maximum tolerable amount*. If it weren't, then the window could be shrunk, contrary to our assumption
- The B field **will** be *at its maximum tolerable value*. If it weren't, then the core area could be shrunk, contrary to the assumption

But – **is the maximum B field set by saturation or by core loss?**



Will saturation or core loss limit a design?

Elaine Ng

B field limits – there can be only one!

Do not enter saturation

$B_{pk} < B_{sat}$

Avoid core loss

Keep $P_{core} = (VOL) \times kB_{ac}^{\beta}$ within acceptable bounds; otherwise, minimize along with P_{cu}

$B_{ac} < \hat{B}$ such that

$P_v = kB_{ac}^{\beta} < P_{v,max}$~200-500 mW/cm^3

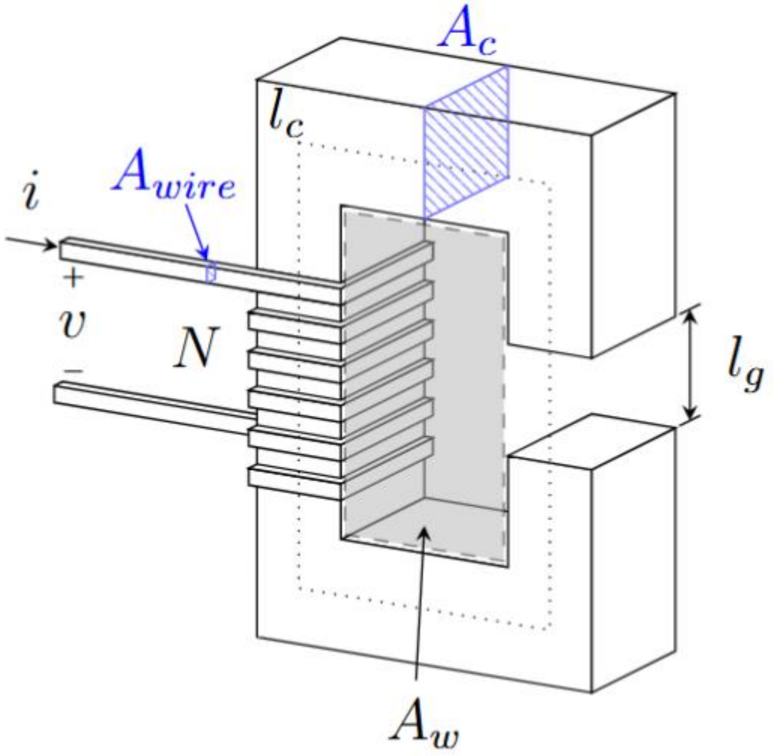
Design is limited by saturation or core loss (not both)

From flux linkage relationship:

$$B = \frac{LI}{NA_c}$$

B can be kept low by growing the core ($A_c \uparrow$) or by adding turns ($N \uparrow$)

We use the **same design parameters (N and A_c)** to avoid saturation or core loss (to limit B_{pk} or B_{ac})



l_t: mean length of turn

Design is limited by saturation or core loss (not both)

Either:

1. B_{pk} will reach B_{sat} first (core loss will still exist but $P_v \ll P_{v,max}$)

Or:

2. B_{ac} will reach $\hat{B}$ first (but $\hat{B} \ll B_{sat}$)

If (1), the design is saturation limited

If (2), the design is core loss limited

Which one will it be? Can we derive an easy inequality to determine this?

How do we quantify if a design is core loss or sat limited?

First, let's relate B_{pk} and B_{ac}

From flux linkage relationship, B is related to I:

$$B = \frac{LI}{NA_c}$$

An inductor will have a certain I_{pk} and I_{ac} which corresponds to B_{pk} and B_{ac}

The relationship between I_{pk} and I_{ac} is given by the ripple ratio $\mathcal{R} = I_{ac}/I_{dc}$

$$\frac{I_{pk}}{I_{ac}} = \frac{1+\mathcal{R}}{\mathcal{R}}$$

How do we quantify if a design is core loss or sat limited?

Is B_{sat}/I_{pk} (saturation limit) lower or $\hat{B}/I_{ac}$ (core loss limit) lower?

If core loss limited:

$$\frac{B_{sat}}{I_{pk}} > \frac{\hat{B}}{I_{ac}} = \frac{\hat{B}}{I_{pk}} \frac{1+\mathcal{R}}{\mathcal{R}}$$

$$\Rightarrow \boxed{B_{sat} > \hat{B}\,\frac{1+\mathcal{R}}{\mathcal{R}}}$$

How do we quantify if a design is core loss or sat limited?

$$B_{sat} > \hat{B}\,\frac{1+\mathcal{R}}{\mathcal{R}} \qquad \text{(core loss limited)}$$

$$B_{sat} < \hat{B}\,\frac{1+\mathcal{R}}{\mathcal{R}} \qquad \text{(saturation limited)}$$

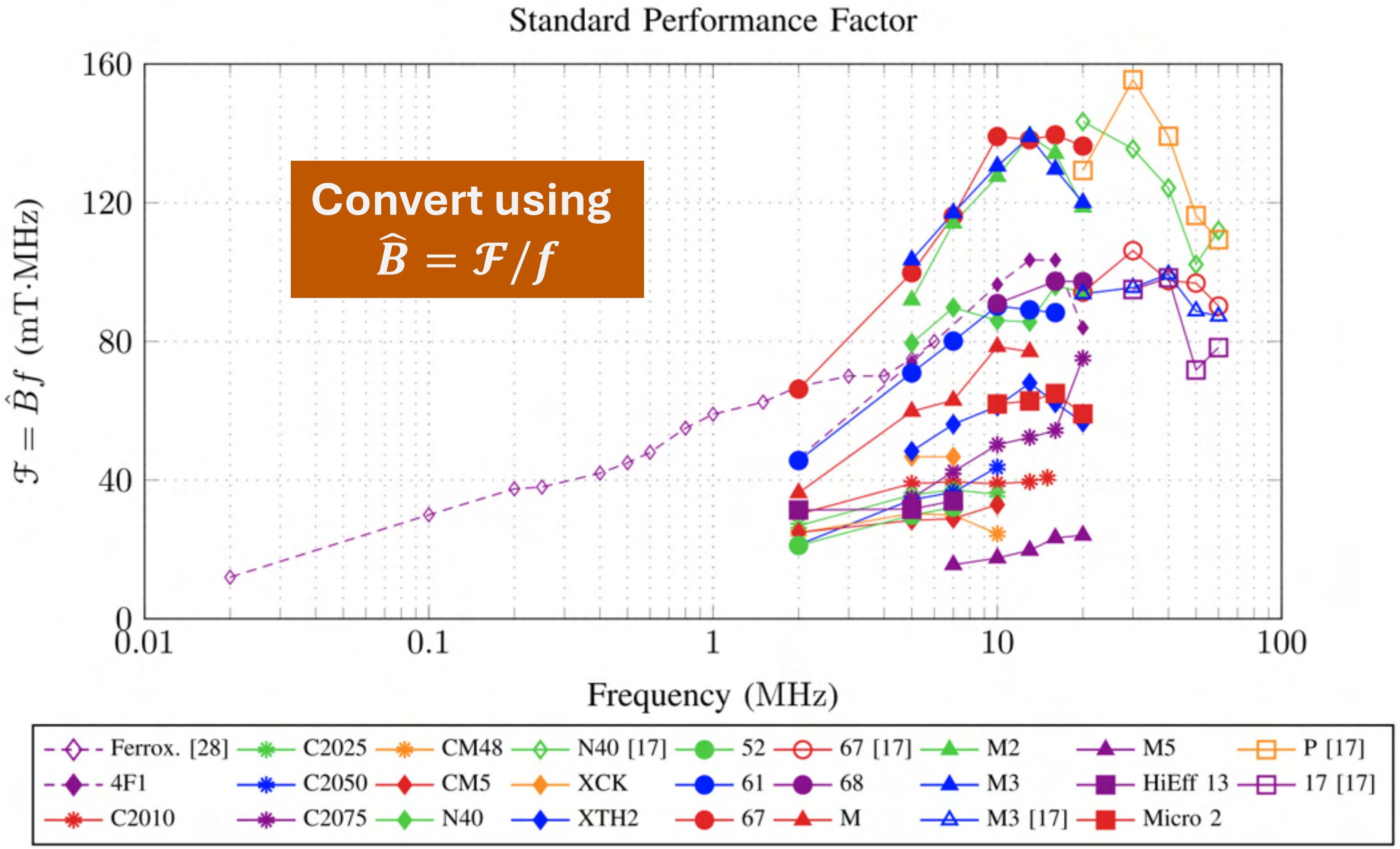
Before we get into numbers, what do we expect?

1) Low ripple should make a saturation limit more likely
2) High frequency should make a core loss limit more likely (at higher frequencies, materials can't sustain as much $\hat{B}$)

Core loss ($\hat{B}$) dataset

The reference below tabulates Steinmetz parameters ($k(f)$, $\beta(f)$) and the performance factor $\mathcal{F} = f\hat{B}$ across frequencies using industry data and original research

Frequency→		2 MHz		5 MHz		7 MHz	
Material	μ_r	k	β	k	β	k	β
Ceramic Magn. C2010 [39]	340	0.20	2.89	2.61	2.56	10.61	2.23
Ceramic Magn. C2025 [39]	175	0.49	2.67	3.14	2.58	11.33	2.27
Ceramic Magn. C2050 [39]	100	0.52	2.9	2.47	2.75	5.25	2.76
Ceramic Magn. C2075 [39]	50	–	–	2.31	2.77	3.42	2.77
Ceramic Magn. CM48 [39]	190	0.59	2.68	7.49	2.33	21.5	2.17
Ceramic Magn. CM5 [39]	290	0.61	2.66	9.42	2.29	22.55	2.19
Ceramic Magn. N40 [39]	15	–	–	1.52	2.09	3.04	2.00
Ceramic Magn. XCK [39]	210	–	–	1.07	2.75	4.86	2.44
Ceramic Magn. XTH2 [39]	80	–	–	0.83	2.82	1.72	2.72
Fair-Rite 52 [40]	250	0.46	2.97	5.44	2.53	14.44	2.32
Fair-Rite 61 [40]	125	0.08	2.79	0.42	2.67	0.83	2.62
Fair-Rite 67 [40]	40	0.10	2.44	0.69	2.20	1.11	2.18
Fair-Rite 68 [40]	16	–	–	–	–	–	–
Ferroxcube 4F1 [28]	80	0.15	2.57	1.11	2.27	–	–
Metamagnetics HiEff 13 [41]	425	0.11	3.06	10.44	2.10	12.69	2.32
Micrometals 2 [42]	10	–	–	–	–	–	–
National Magn. M [43]	125	0.03	3.36	0.45	2.83	1.35	2.69
National Magn. M2 [43]	40	–	–	0.41	2.44	0.69	2.36
National Magn. M3 [43]	20	–	–	0.85	2.10	1.66	2.03
National Magn. M5 [43]	7.5	–	–	–	–	90.34	2.14

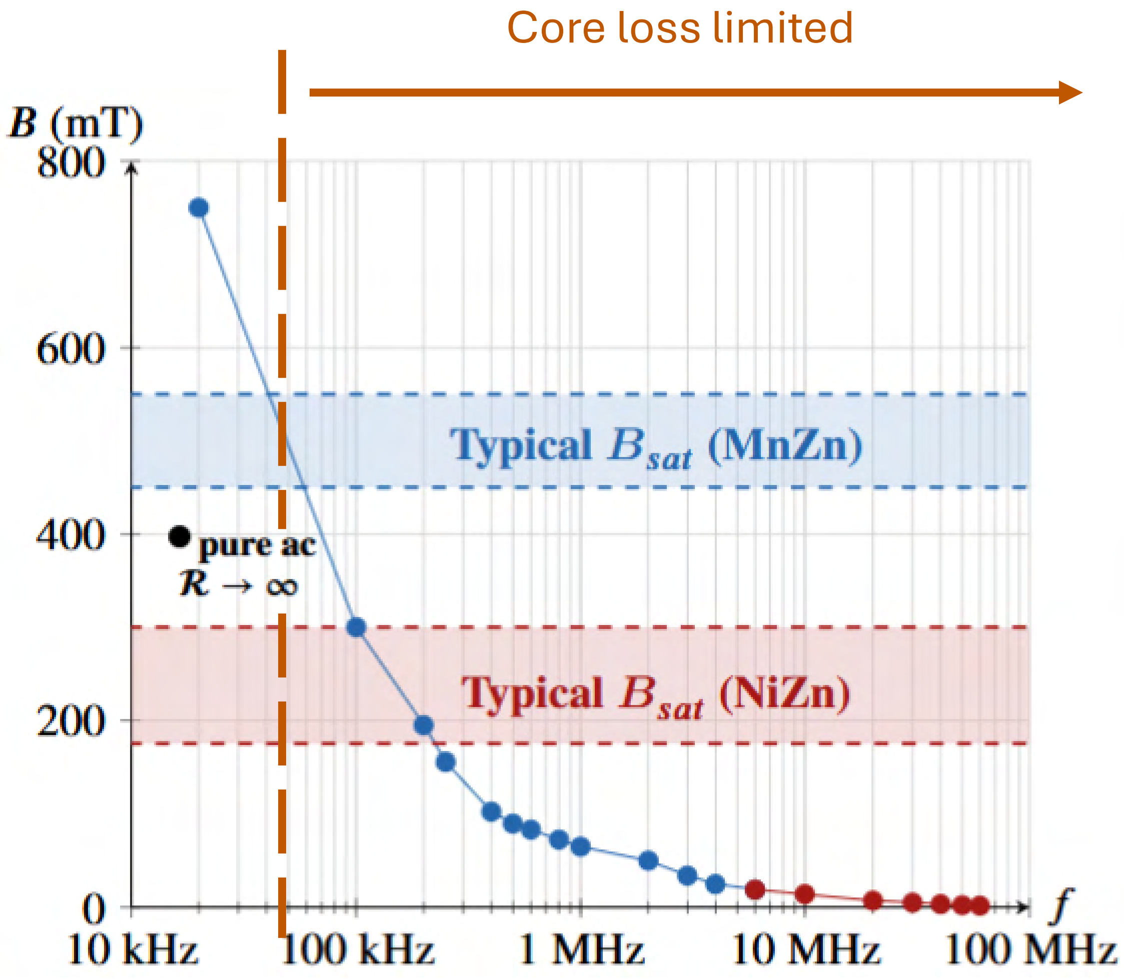


A. J. Hanson, J. A. Belk, S. Lim, C. R. Sullivan and D. J. Perreault, "Measurements and Performance Factor Comparisons of Magnetic Materials at High Frequency," in *IEEE Transactions on Power Electronics*, vol. 31, no. 11, pp. 7909-7925, Nov. 2016

$\frac{\hat{B}(1+\mathcal{R})}{\mathcal{R}}$ vs B_{sat} for **purely ac**

- Expect ac-dominated designs to more likely be core loss limited (only B_{ac} contributes to core loss, and there's no B_{dc} to push B_{pk} close to B_{sat})

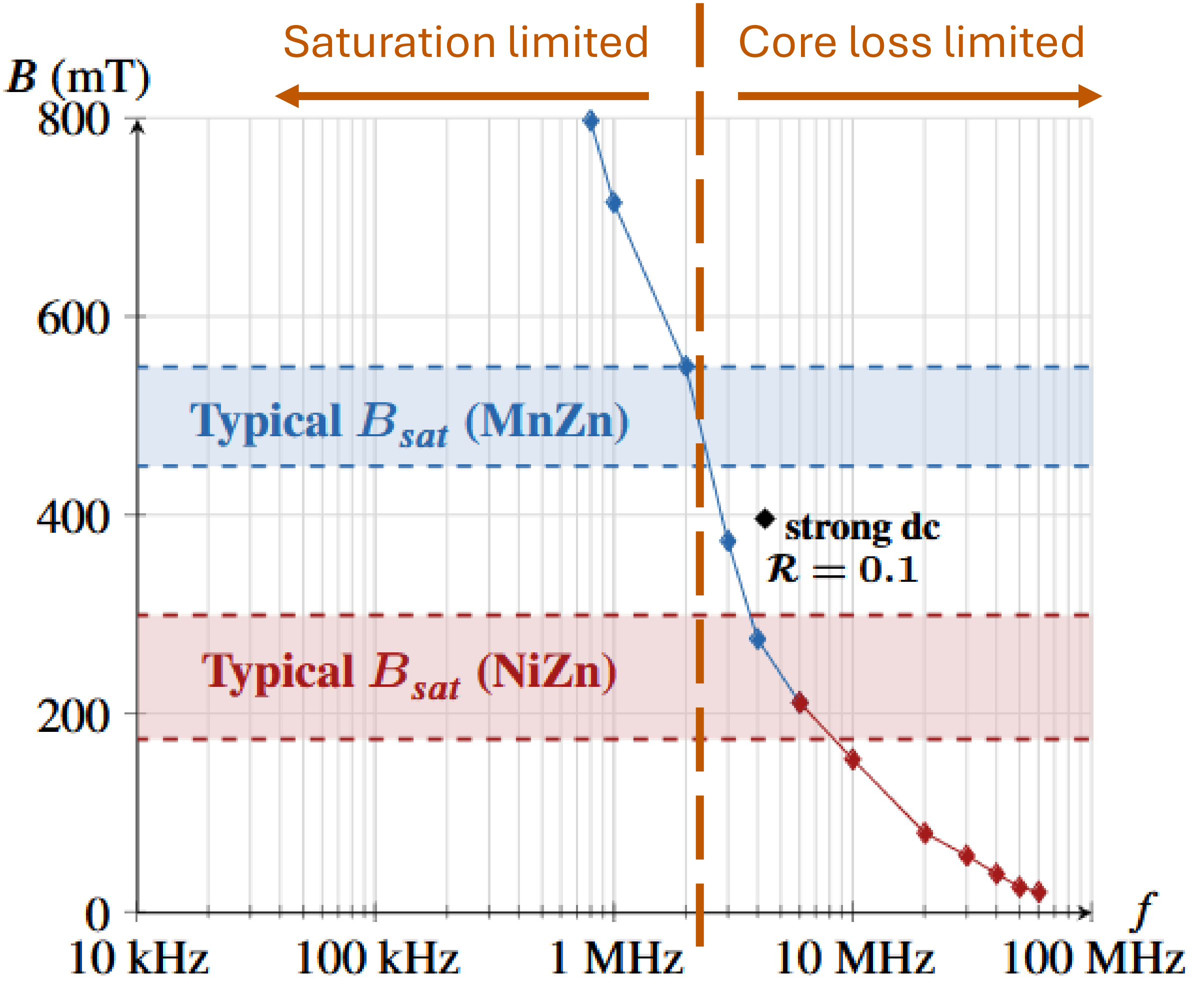
- Purely ac designs can be core loss limited as low as ~50 kHz



A. Brown, E. Ng, and A. J. Hanson, "Magnetic Material Selection for Power Inductors and Transformers," in *IET Handbook of Inductive Devices in Power Electronics*, (To appear 2025)

$\frac{\hat{B}(1+\mathcal{R})}{\mathcal{R}}$ vs B_{sat} for **low ripple (10%)**

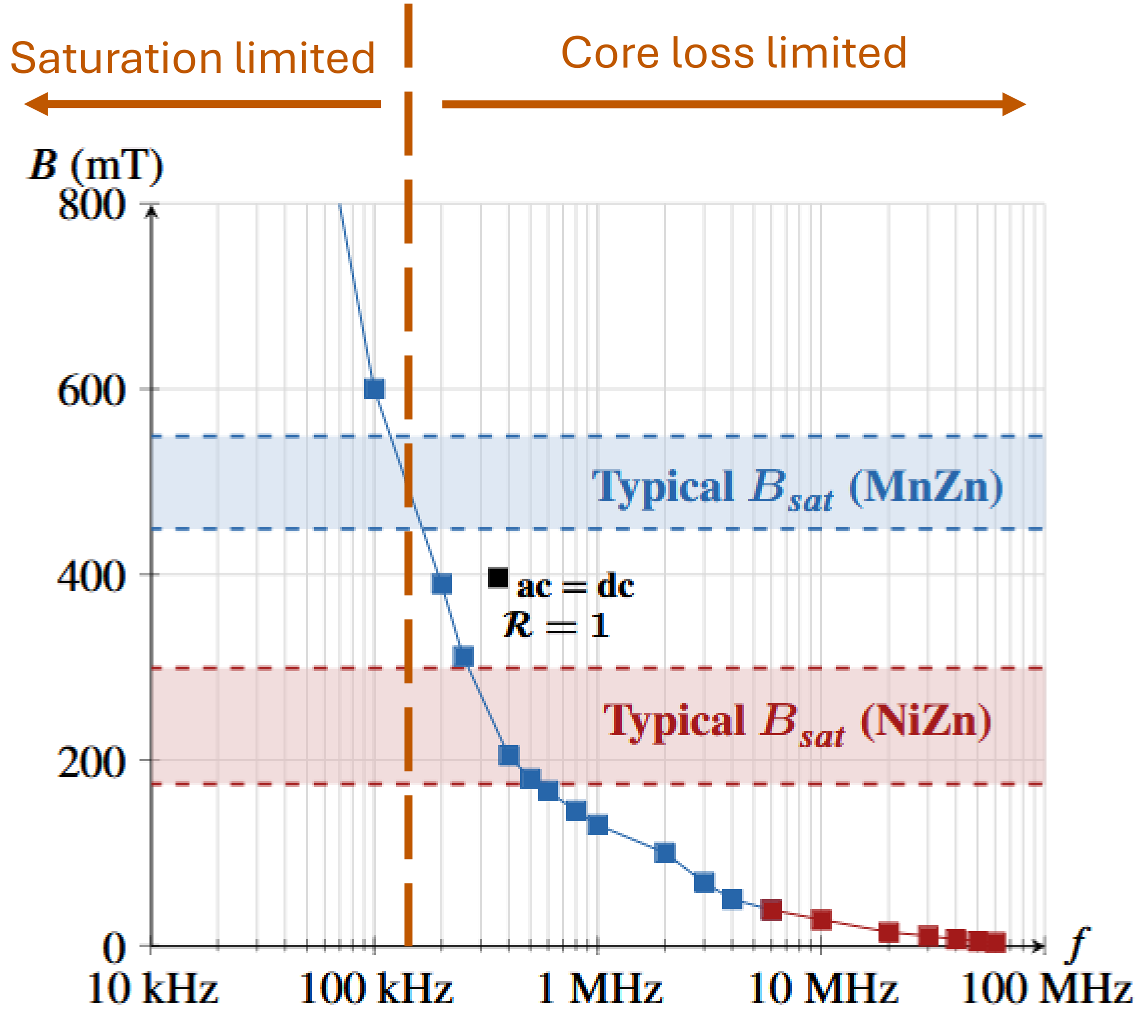
- Expect low ripple designs to more likely be saturation limited (only B_{ac} contributes to core loss, and large B_{dc} pushes B_{pk} close to B_{sat})
- Designs up to a few MHz are saturation limited as expected
- But high frequency designs start becoming core loss limited above ~2 MHz



A. Brown, E. Ng, and A. J. Hanson, "Magnetic Material Selection for Power Inductors and Transformers," in *IET Handbook of Inductive Devices in Power Electronics,* (To appear 2025)

$\frac{\hat{B}(1+\mathcal{R})}{\mathcal{R}}$ vs B_{sat} for **ac=dc** (e.g., BCM)

Designs start becoming core loss limited at frequencies as low as ~150 kHz!



A. Brown, E. Ng, and A. J. Hanson, "Magnetic Material Selection for Power Inductors and Transformers," in *IET Handbook of Inductive Devices in Power Electronics,* (To appear 2025)

How do we choose a material?

Material	Relative Permeability (μ_r)	$\hat{B}$ (mT)	B_{sat} (mT)
1	1000	150	700
2	100	300	400
3	500	100	800

Unless permeability is close to 1, ignore it (we'll discuss this in a later section!)

1. Use $B_{sat} > \hat{B}\,\frac{1+\mathcal{R}}{\mathcal{R}}$ to determine if each material is core loss or saturation limited
2. If material is core loss limited, $B_{max} = \hat{B}$
3. If material is saturation limited, $B_{max} = B_{sat}\,\frac{\mathcal{R}}{1+\mathcal{R}}$
4. Choose the material with the highest B_{max}

A. Brown, E. Ng, and A. J. Hanson, "Magnetic Material Selection for Power Inductors and Transformers," in *IET Handbook of Inductive Devices in Power Electronics,* (To appear 2025)

Which material should you choose for $\mathcal{R} = 0.4$?

Step 1: Determine if each material is core loss or saturation limited

If $B_{sat} > \hat{B}\frac{1+\mathcal{R}}{\mathcal{R}} = 3.5\hat{B}$, material is core loss limited

Otherwise, material is saturation limited

Material	Relative Permeability (μ_r)	$\hat{B}$ (mT)	B_{sat} (mT)	$\hat{B}\frac{1+\mathcal{R}}{\mathcal{R}}$ (mT)	Core or Sat Limited?
1	1000	150	700	525	Core Loss
2	100	300	400	1050	Saturation
3	500	100	800	350	Core Loss

A. Brown, E. Ng, and A. J. Hanson, "Magnetic Material Selection for Power Inductors and Transformers," in *IET Handbook of Inductive Devices in Power Electronics,* (To appear 2025)

Which material should you choose for $\mathcal{R} = 0.4$?

Step 2: Calculate B_{max} for each material

If material is core loss limited, $B_{max} = \hat{B}$

If material is saturation limited, $B_{max} = B_{sat}\frac{\mathcal{R}}{1+\mathcal{R}}$

Material	μ_r	$\hat{B}$ (mT)	B_{sat} (mT)	$B_{sat}\frac{\mathcal{R}}{1+\mathcal{R}}$ (mT)	Core or Sat Limited?	B_{max} (mT)
1	1000	**150**	700	200	Core Loss	**150**
2	100	300	400	**114**	Saturation	114
3	500	**100**	800	229	Core Loss	100

A. Brown, E. Ng, and A. J. Hanson, "Magnetic Material Selection for Power Inductors and Transformers," in *IET Handbook of Inductive Devices in Power Electronics*, (To appear 2025)

Which material should you choose for $\mathcal{R} = 0.4$?

Material	Relative Permeability (μ_r)	$\hat{B}$ (mT)	B_{sat} (mT)	Core or Sat Limited?	B_{max} (mT)
1	**1000**	**150**	**700**	**Core Loss**	**150**
2	100	300	400	Saturation	114
3	500	100	800	Core Loss	100

Step 3: Choose material with highest B_{max}

In this case, material 1 is the best choice for this application even though it has neither the highest $\hat{B}$ or highest B_{sat}

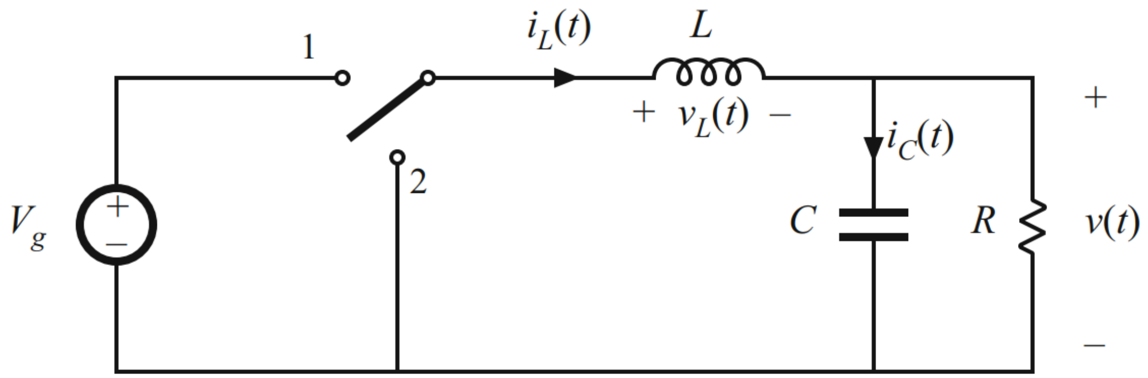
Material 2 has better $\hat{B}$ but is hamstrung by its very low B_{sat}
Material 3 has better B_{sat} but is hamstrung by its very low $\hat{B}$

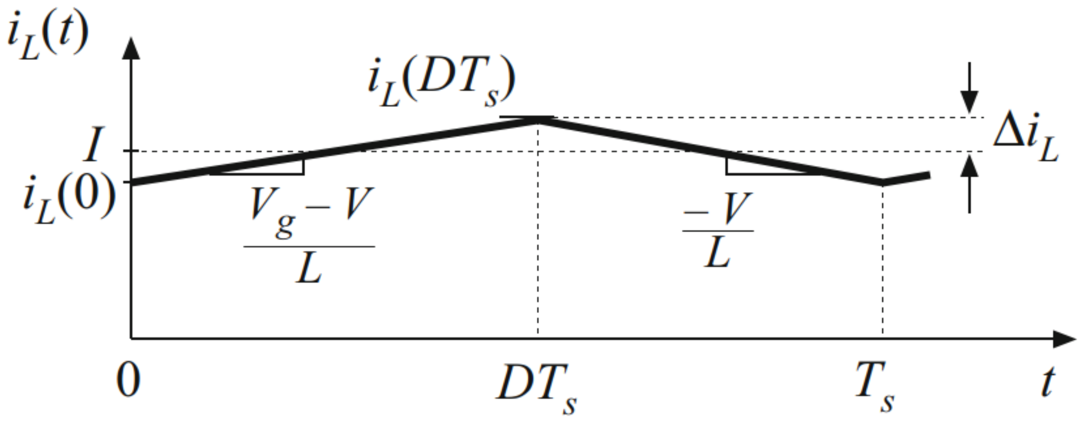
A. Brown, E. Ng, and A. J. Hanson, “Magnetic Material Selection for Power Inductors and Transformers,” in *IET Handbook of Inductive Devices in Power Electronics,* (To appear 2025)

Core material selection for buck converter inductor example

Consider selecting a core material for the inductor in a CCM buck converter with

- Switching frequency:
  - $f_s = 200$ kHz
- Inductor current ripple ratio:
  - $\mathcal{R} = \frac{\Delta i_L/2}{I_L} = 0.4$





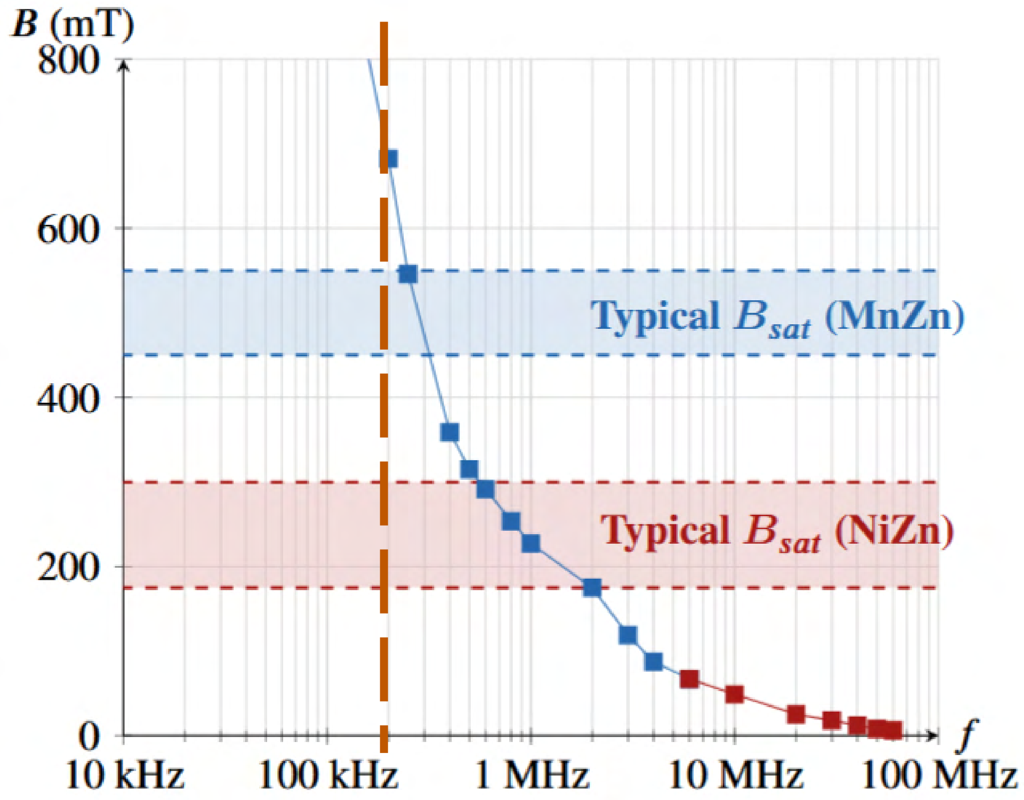
R. W. Erickson and D. Maksimović, *Fundamentals of Power Electronics*. Cham: Springer International Publishing, 2020.

Core material selection for buck converter inductor example

Let's plot saturation vs. core loss limit for ferrites for $\mathcal{R}$=0.4:

At 200 kHz, both MnZn and NiZn ferrites are **likely saturation limited (the threshold is fuzzier for MnZn)**

Does that mean we choose material with highest B_{sat}?

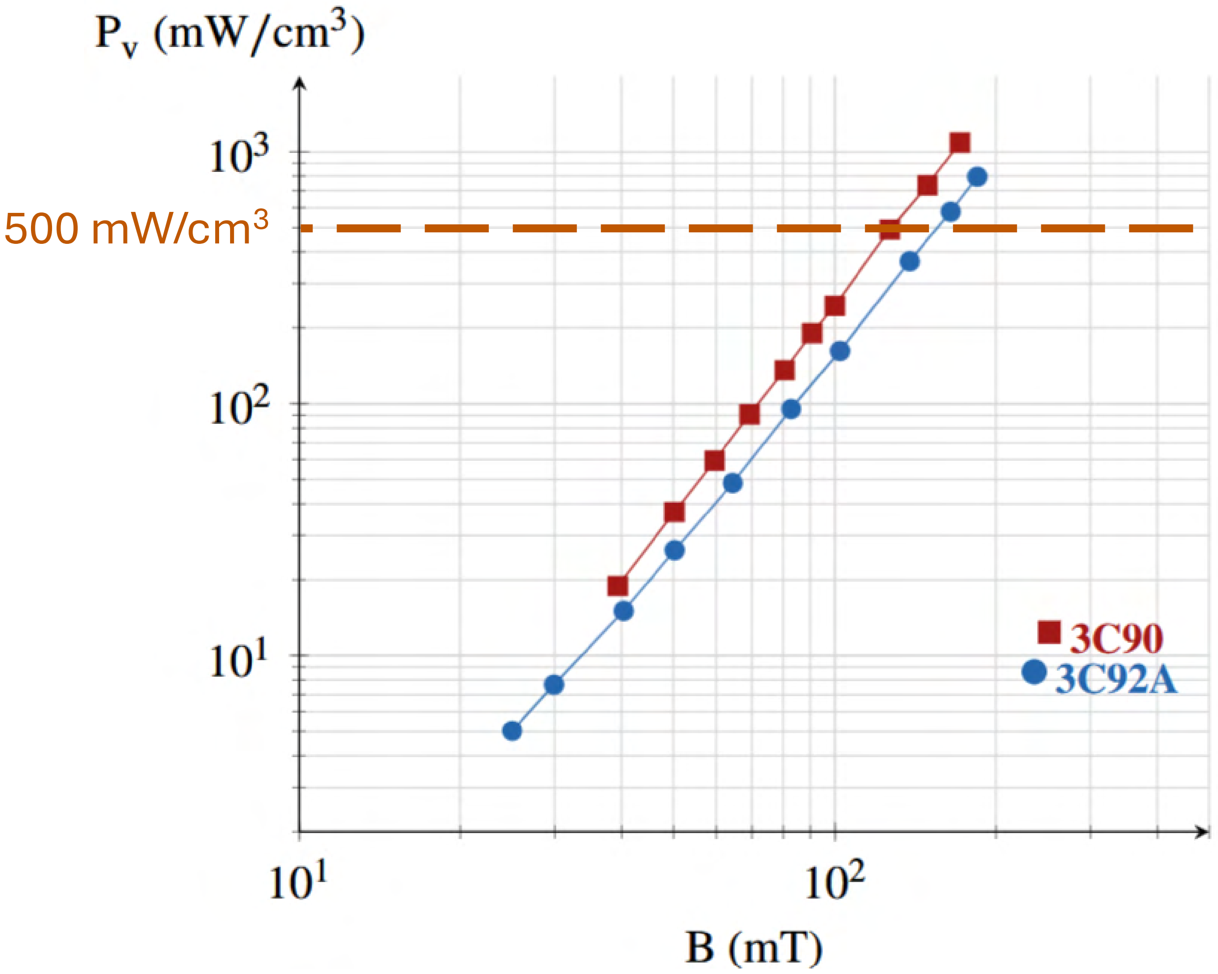


Core material selection for buck converter inductor example

Consider choosing between two MnZn materials from Ferroxcube (3C90 and 3C92A)

Get B_{sat} and $\hat{B}$ (which yields $P_v \leq 500$ mW/cm³) from datasheets

Ferroxcube Material	$\hat{B}$ (mT)	B_{sat} (mT)
3C90	140	470
3C92A	160	570



[1] 3C90 - ferroxcube, https://www.ferroxcube.com/upload/media/product/file/MDS/3c90.pdf
[2] 3C92a – ferroxcube, https://www.ferroxcube.com/upload/media/product/file/MDS/3c92a.pdf

Core material selection for buck converter inductor example

Determine saturation vs. core loss limits for 3C90 and 3C92A:

If $B_{sat} > \hat{B}\frac{1+\mathcal{R}}{\mathcal{R}} = 3.5\hat{B}$, (where $\mathcal{R} = 0.4$), material is core loss limited

3C90 is **just barely** saturation limited
3C92A is **just barely** core loss limited
(usually the answer is far more obvious)

From previous plots, we expected these materials to be near the threshold!

Ferroxcube Material	$\hat{B}$ (mT)	B_{sat} (mT)	$\hat{B}\frac{1+\mathcal{R}}{\mathcal{R}}$ (mT)	Sat vs. Core Limited?
3C90	140	470	490	Saturation
3C92A	160	570	560	Core Loss

[1] 3C90 - ferroxcube, https://www.ferroxcube.com/upload/media/product/file/MDS/3c90.pdf
[2] 3C92a – ferroxcube, https://www.ferroxcube.com/upload/media/product/file/MDS/3c92a.pdf

Core material selection for buck converter inductor example

Determine B_{max} for each material:

If material is core loss limited, $B_{max} = \hat{B}$

If material is saturation limited, $B_{max} = B_{sat} \frac{\mathcal{R}}{1+\mathcal{R}}$

Pick 3C92A b/c it has the highest B_{max} attributed to its core loss limit, NOT because it has higher B_{sat}

Ferroxcube Material	$\hat{B}$ (mT)	B_{sat} (mT)	Sat vs. Core Limited?	B_{max} (mT)
3C90	140	470	Saturation	131
3C92A	**160**	**570**	**Core Loss**	**160**

[1] 3C90 - ferroxcube, https://www.ferroxcube.com/upload/media/product/file/MDS/3c90.pdf
[2] 3C92a – ferroxcube, https://www.ferroxcube.com/upload/media/product/file/MDS/3c92a.pdf

Real designs are core loss limited at even lower frequencies

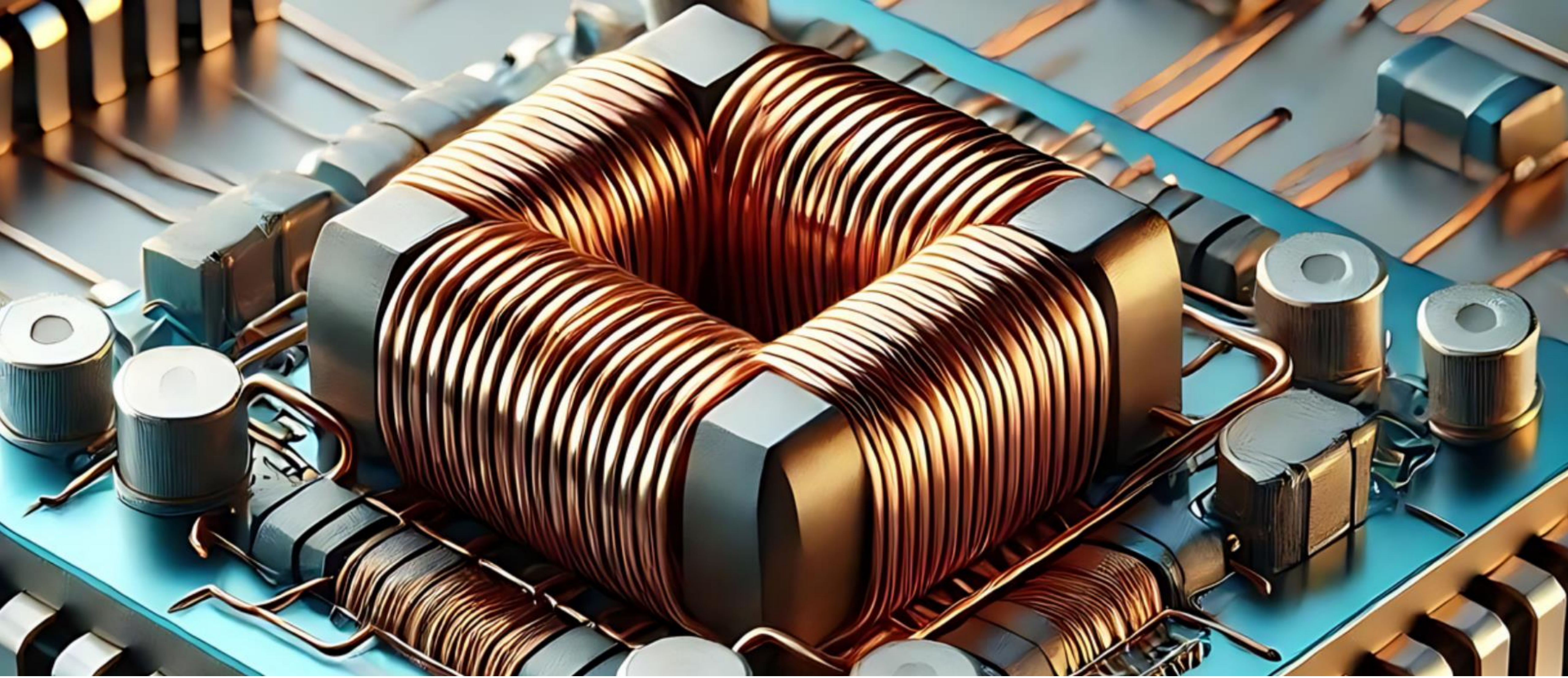
- Previous plots are based on $\hat{B}$
- $\hat{B}$ typically based on sinusoidal excitations with no dc bias
- ac+dc excitations cause more core loss than purely ac excitations
- Non-sinusoidal ac excitations have higher core loss than purely sinusoidal excitations for a given Δi_{pkpk} and a given fundamental frequency

$\Rightarrow$ Real applications will have higher core loss than $\hat{B}$ alone suggests and applications will become core loss limited at lower frequencies

Core loss is a fairly likely limit: so what?

- A lot of design approaches (like K_g) assume B_{sat} as the core's limit
- A lot of students only learn B_{sat} as a limit
- A lot of sales pitches for magnetic materials focus heavily on B_{sat}

…Yet for many designs, B_{sat} is irrelevant!



How does magnetic goodness scale with size?

Elaine Ng

Size scaling of magnetics through the lens of…

- Dc-dominated inductors
  - K_g method (limited by winding resistance)
  - Limited by current density $\hat{J}$
- Ac-dominated inductors
  - Core-area product

How big does an inductor need to be?

- Consider a **choke** or **dc-dominated** inductor
- The core is likely to be limited by **saturation**

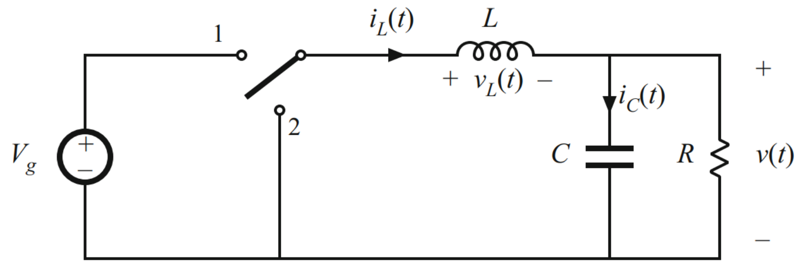
$$B_{pk} = \boldsymbol{B_{dc}} + B_{ac} = \frac{LI_{pk}}{NA_c} = B_{sat}$$

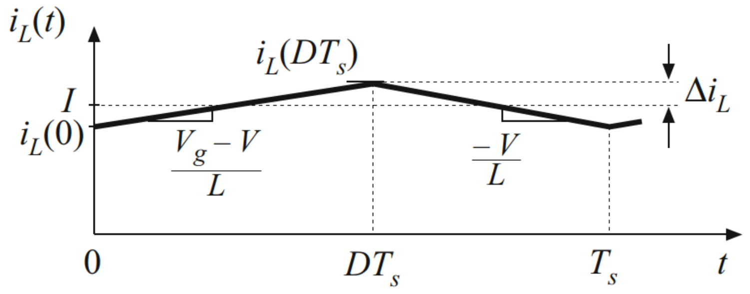
(equals, not less than!)

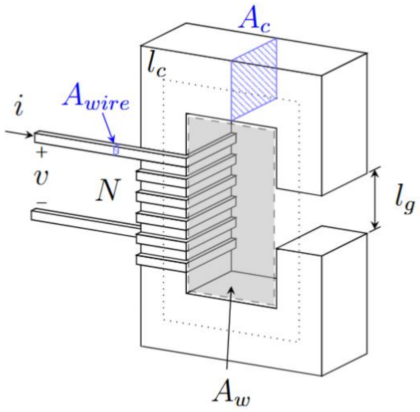
- Let's give ourselves a maximum tolerable winding resistance as well

$$R = \rho\frac{l_{total}}{A_{wire}} = \rho\frac{Nl_{turn}}{A_w/N} = \rho\frac{l_tN^2}{A_w} = R_{max}$$

(equals, not less than!)







l_t: mean length of turn

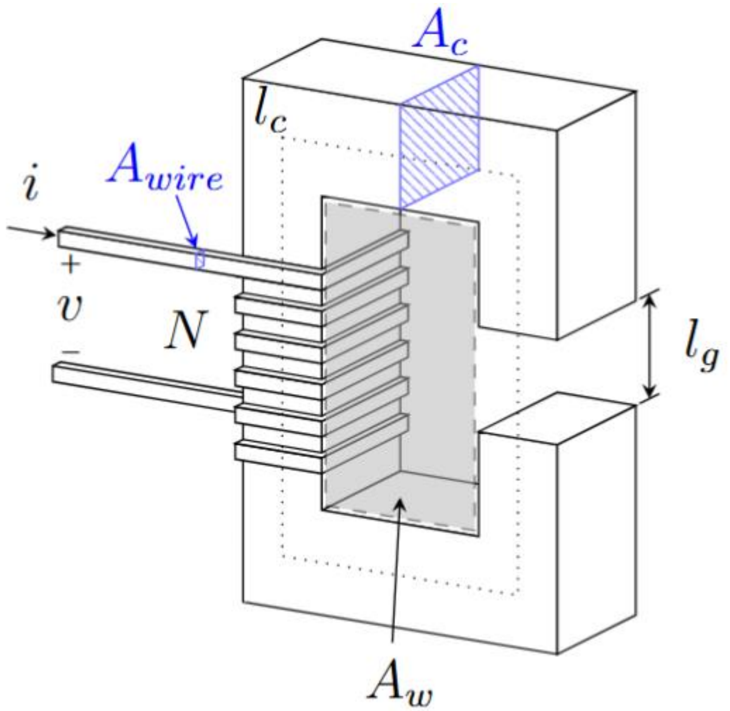
R. W. Erickson and D. Maksimović, *Fundamentals of Power Electronics*. Cham: Springer International Publishing, 2020.

How big does an inductor need to be? ⇒ K_g

To achieve both constraints $B_{pk} \leq B_{sat}$ **and** $R \leq R_{max}$:

$$K_g = \underbrace{\frac{A_w A_c^2}{l_t}}_{\text{Size}} \geq \underbrace{\frac{\rho L^2 I_{pk}^2}{R_{max} B_{sat}^2}}_{\text{Constraints given to us by the application and the material}}$$

Core geometric factor



l_t: mean length of turn

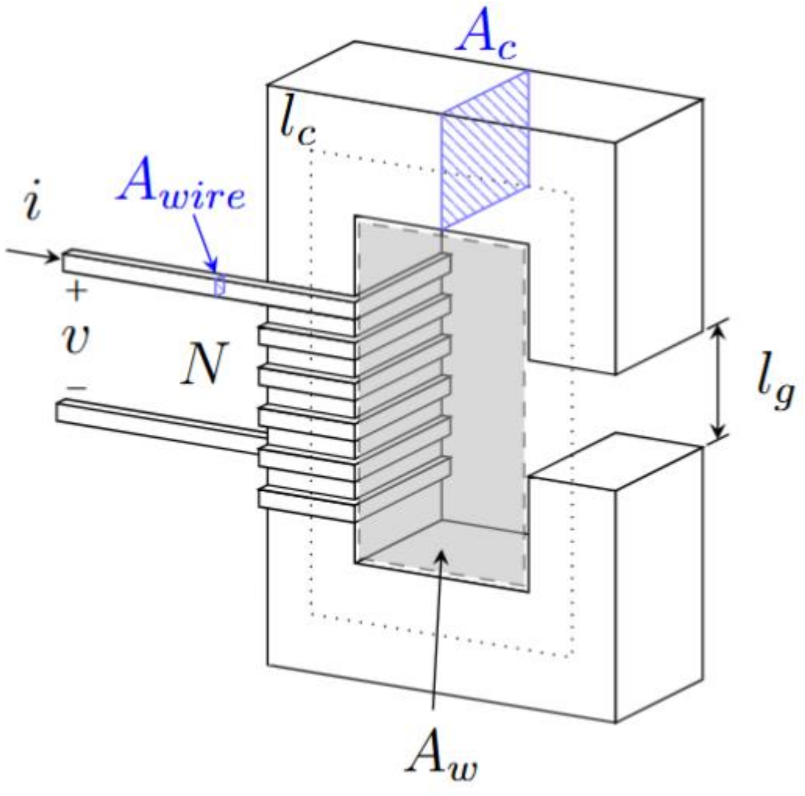
Let's dwell on the power of K_g

$$K_g = \underbrace{\frac{A_w A_c^2}{l_t}}_{\text{Size}} \geq \underbrace{\frac{\rho L^2 I_{pk}^2}{R_{max} B_{sat}^2}}_{\text{Constraints given to us by the application and the material}}$$

Take a moment to ponder how K_g allows us to cut through a lot of confusing equations. Given **only** application and material constraints, we can **immediately** calculate how big of a core we need to **guarantee** that we can meet the interests of the core and the winding

K_g method

1. Choose $B_{pk} = B_{sat}$
2. Obtain a core with $K_g = \frac{A_w A_c^2}{l_t} \geq \frac{\rho L^2 I_{pk}^2}{R_{max} B_{sat}^2}$
3. Choose $N = \frac{L I_{pk}}{B_{sat} A_c}$ (i.e., use the minimum number of turns to avoid B_{sat}).
4. Make the turns as big as possible to fill the window.
5. Choose gap length g to achieve L with given geometry and N



l_t: mean length of turn

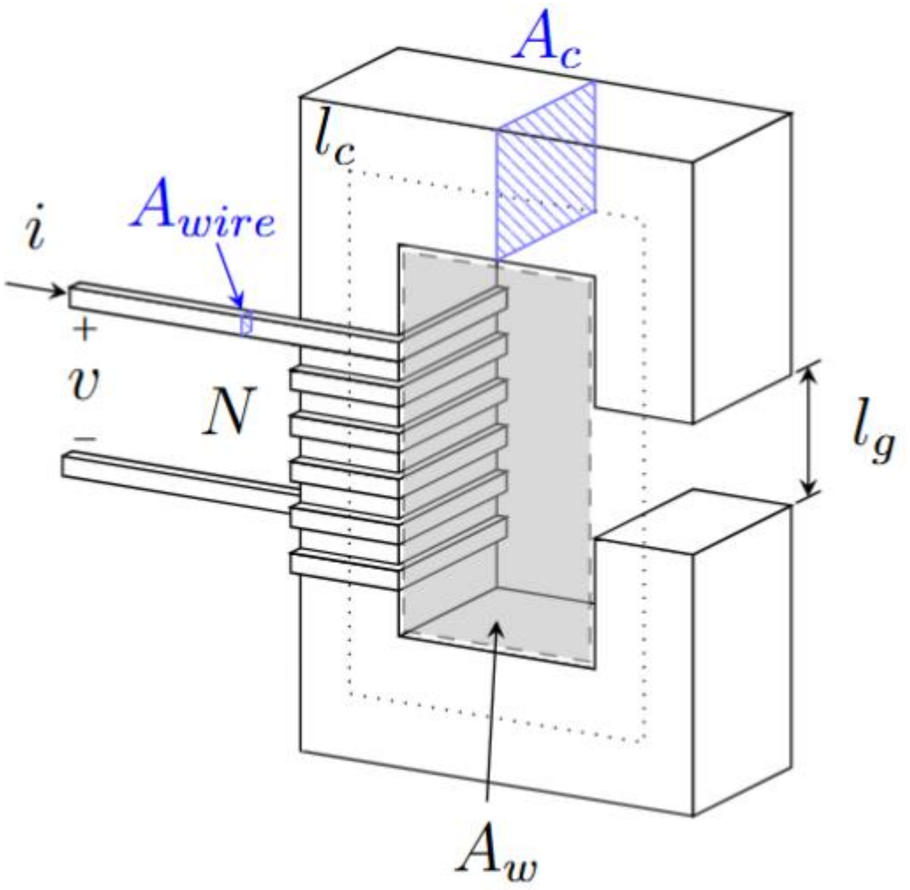
K_g method

If $K_g = \frac{\rho L^2 I_{pk}^2}{R_{max} B_{sat}^2}$

- yields the smallest possible component to meet R requirement

If $K_g > \frac{\rho L^2 I_{pk}^2}{R_{max} B_{sat}^2}$

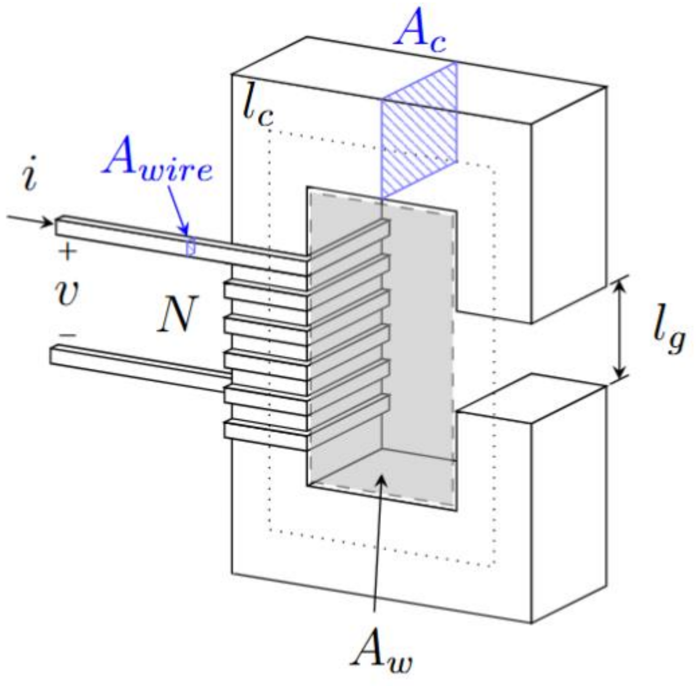
- Bigger core than minimum size
- Guaranteed to have $R < R_{max}$



l_t: mean length of turn

Based on K_g, how do inductors scale?

$$K_g = \underbrace{\frac{A_w A_c^2}{l_t}}_{(\text{length})^5} \geq \underbrace{\frac{\rho L^2 I_{pk}^2}{R_{max} B_{sat}^2}}_{\text{Inductor "goodness"}}$$



l_t: mean length of turn

"Inductor goodness" for R-limited, dc inductor

scales as (length)^5

Is resistance the right limit?

- K_g uses a limit on winding resistance R
- A huge component has more surface area to dissipate heat and can tolerate larger R.
- A small component can only tolerate smaller R.
- But if we don't know beforehand how big the component will be, can we really specify a tolerable R_{max}?
- What if we specified a tolerable winding loss *density* $P_{v,cu,max}$?

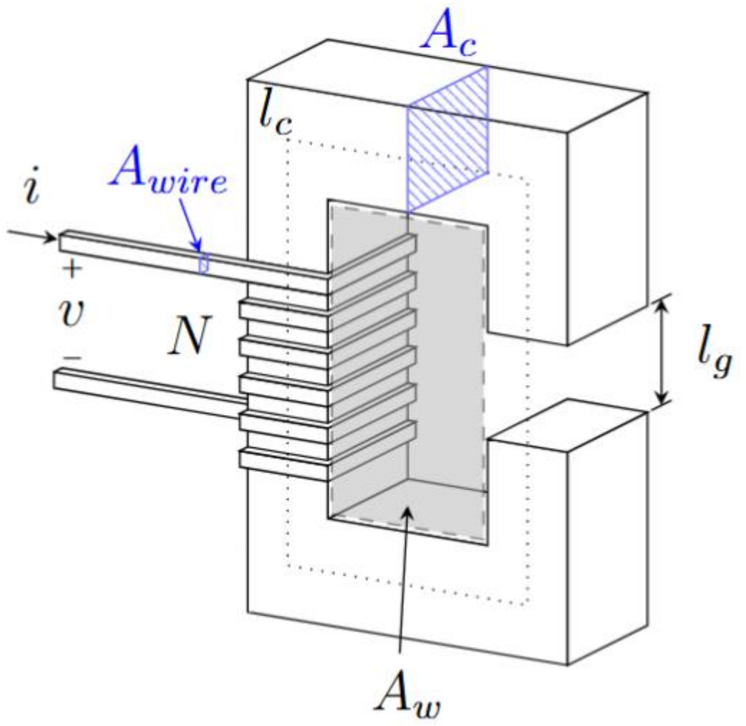
What if we are limited by a winding loss density?

- Winding loss density is $P_{v,cu}$

$$P_{v,cu} = \frac{I^2 R}{VOL_{cu}} = \frac{I^2 \rho \frac{N l_t}{A_w/N}}{l_t \times A_w} = \left(\frac{I}{A_w/N}\right)^2 \rho$$

$$\Longrightarrow \boxed{P_{v,cu} = \boldsymbol{J}^2 \rho}$$

- Holding $P_{v,cu} < P_{v,cu,max}$ is the same as $J < \hat{J}$
  - where $\hat{J}$ is the current density that results in $P_{v,cu,max}$.



Inductor goodness using $\hat{J}$ instead of R

Consider an inductor with mostly dc current and a *sinusoidal* voltage excitation $v(t) = V_{pk}\sin(\omega t)$.

The peak flux linkage is

$$\lambda_{pk} = NA_cB_{sat} = I_{pk}L \quad \text{where}$$

$$L = \frac{\int v dt}{\Delta i_{pkpk}} = \frac{2V_{pk}}{\omega \Delta i_{pkpk}} \;\text{ and }\; I_{pk} = I_{dc} + I_{ac} = I_{dc} + \frac{\Delta i_{pkpk}}{2}$$

Inductor goodness using $\hat{J}$ instead of R

Recall the ripple ratio $\mathcal{R} = \frac{I_{ac}}{I_{dc}} = \frac{\Delta i_{pkpk}/2}{I_{dc}}$

Rewrite flux linkage in terms of $\mathcal{R}$:

$$I_{pk} = I_{dc}(1 + \mathcal{R}) \text{ and } \Delta i_{pkpk} = 2I_{dc}\mathcal{R}$$

$$\Longrightarrow \lambda_{pk} = NA_cB_{sat} = \frac{V_{pk}}{\omega}\left(\frac{1 + \mathcal{R}}{\mathcal{R}}\right)$$

Applied voltage is constrained by B:

$$V_{pk} = \omega NA_cB_{sat}\left(\frac{\mathcal{R}}{1 + \mathcal{R}}\right)$$

Inductor goodness using $\hat{J}$ instead of R

The rms current can be set by the current density limit $\hat{J}$:

$$I_{rms} = \hat{J}_{rms}\frac{A_w}{N}$$

Inductor “goodness” can be defined as its power handling capability:

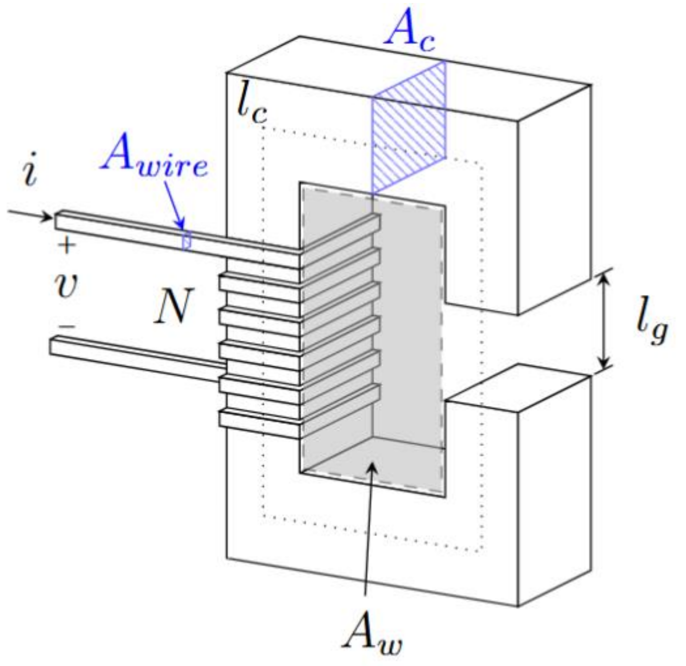
$$V_{pk}I_{rms} = \omega B_{sat} N A_c \frac{\mathcal{R}}{1+\mathcal{R}}\left(\frac{\hat{J}_{rms}A_w}{N}\right)$$

Based on $\hat{J}$, inductor goodness scales with (length)^4

To achieve a target power handling capability within a winding density limit:

$$\underbrace{A_w A_c}_{(\text{length})^4} \geq \frac{V_{pk} I_{rms}}{\omega B_{sat} \hat{J}_{rms} \frac{\mathcal{R}}{1+\mathcal{R}}}$$

Core Area Product



l_t: mean length of turn

Maximum power processing capability for $\hat{J}$-limited, dc inductor **scales as (length)^4**

Comparing K_g and $\hat{J}$ approaches

- For K_g method with fixed R,

$$\frac{A_w A_c^2}{l_{turn}} \geq \frac{\rho L^2 I_{pk}^2}{R_{max} B_{sat}^2}$$

- For fixed $\hat{J}_{rms}$,

$$A_w A_c \geq \frac{V_{pk} I_{rms}}{2\pi f B_{sat} \hat{J}_{rms} \frac{\mathcal{R}}{1+\mathcal{R}}}$$

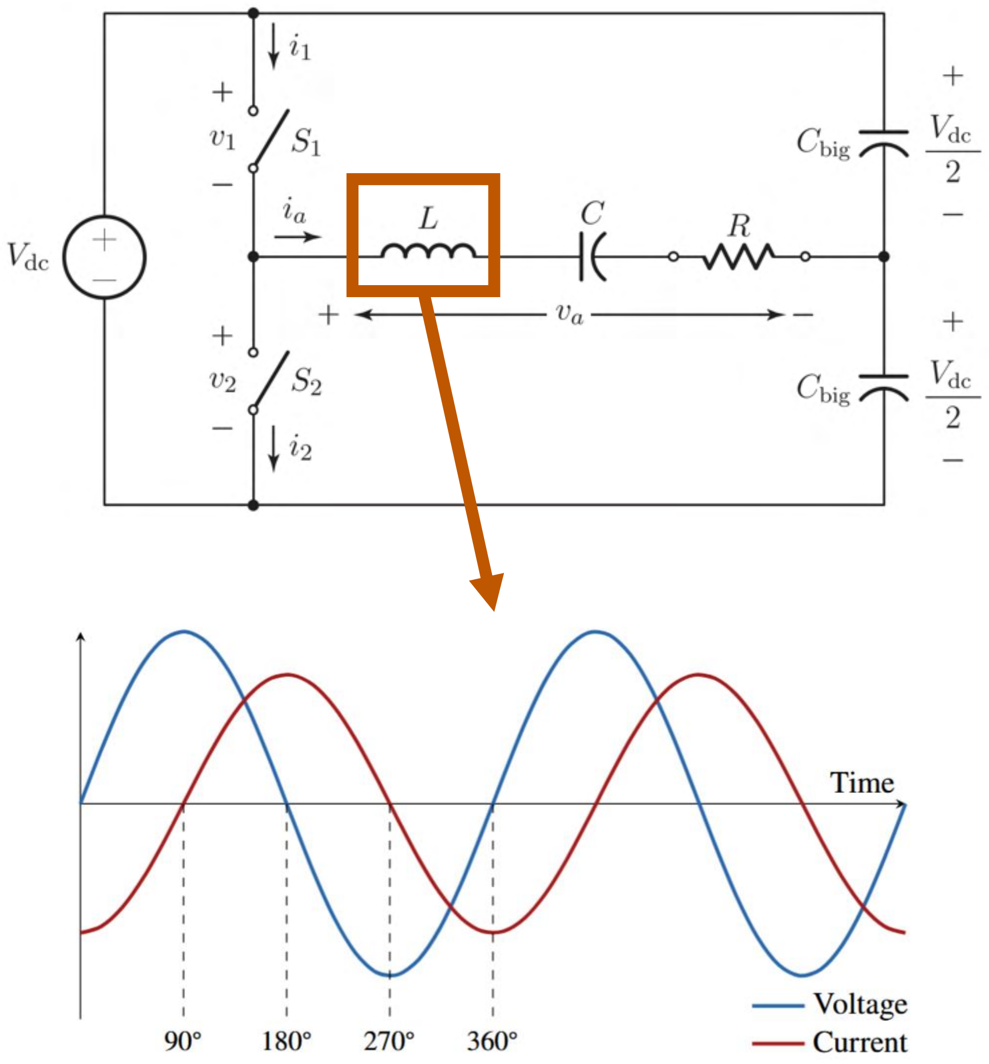
For the K_g method, inductor goodness scales as m^5

For fixed $\hat{J}_{rms}$, inductor goodness scales as m^4

1) Different assumptions yield different conclusions!
2) Inductor goodness does seem to scale faster than m^3 (volume)

How about for ac inductors?

- Choke inductors have mostly dc current
- AC inductors have mostly/entirely ac current
- (All inductors have only ac voltage in steady state)
- Consider an inductor with purely sinusoidal voltage and current
- Define power handling capability as $V_{pk}I_{pk}$



Sizing ac-inductors with core-area product

- V_{pk} is constrained by B_{max}:
- Ac B field limit: $B_{ac} = B_{max}$
  - We've already learned that B_{max} is often based on core loss
- Recall that B_{max} is related to flux linkage:

$$\lambda_{pk} = NA_c B_{max} = \frac{V_{pk}}{\omega}\left(\frac{1+\mathcal{R}}{\mathcal{R}}\right)$$

- Since we are considering ac inductors, $\mathcal{R} \longrightarrow \infty$

$$NA_c B_{max} = \frac{V_{pk}}{\omega}$$

$$\Rightarrow \quad V_{pk} = N\omega B_{max} A_c$$

Sizing ac-inductors with core-area product

- I_{pk} is constrained by current density limit J_{max} :

$$I_{pk} = J_{max} A_{wire} = \frac{J_{max} A_w}{N}$$

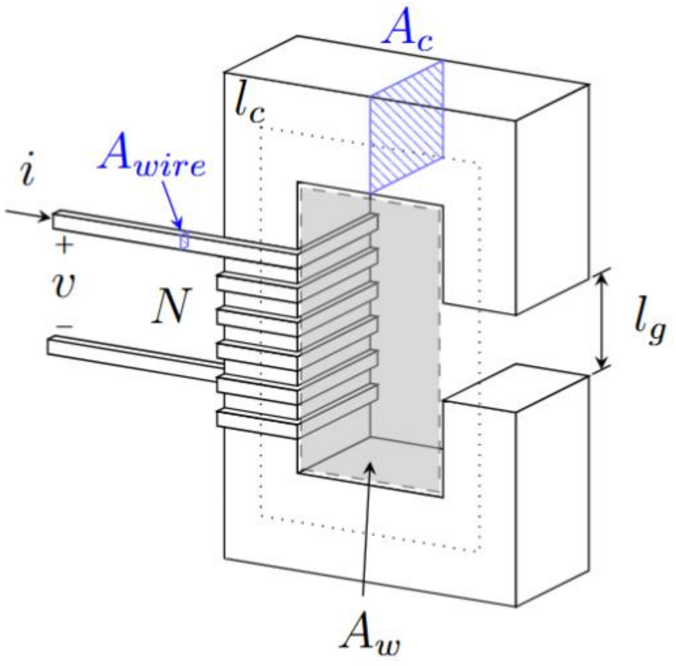
- Recall $V_{pk} = N\omega B_{max} A_c$ from previous slide

$$\Longrightarrow V_{pk}\, I_{pk} = \omega B_{max} J_{max} \underbrace{A_c A_w}_{\text{Core-area product}}$$

Core-area product ⇒ ac-inductor goodness scales with (length)^4

To achieve a target power handling capability for ac-inductors:

$$\underbrace{A_w A_c}_{(\text{length})^4} \geq \frac{V_{pk} I_{pk}}{\omega B_{max} \hat{J}_{max}}$$



l_t: mean length of turn

Maximum power processing capability for ac inductor

scales as (length)^4

Comparing methods for magnetics scaling

Design Method	Application	"Goodness" Metric	Condition to Achieve Good Metric	How Metric Scales with Length
K_g (Constant R)	Dc-dominated	$\frac{L^2 I_{pk}^2}{R_{max}} = E_{store}\frac{X_L/R}{\omega}$	$K_g = \frac{A_w A_c^2}{l_t} \geq \frac{\rho L^2 I_{pk}^2}{R_{max} B_{sat}^2}$	$(\text{length})^5$
Constant $\hat{J}$	Dc-dominated	Power processing capability = $V_{pk} I_{rms}$	$A_w A_c \geq \frac{V_{pk} I_{rms}}{\omega B_{sat} \hat{J}_{rms} \frac{\mathcal{R}}{1+\mathcal{R}}}$	$(\text{length})^4$
Core Area	Ac-dominated	Power processing capability = $V_{pk} I_{pk}$	$A_w A_c \geq \frac{V_{pk} I_{pk}}{\omega B_{max} \hat{J}_{max}}$	$(\text{length})^4$

For all methods, inductor goodness

scales faster than inductor volume = $(\text{length})^3$

Power density scaling for different forms of energy storage

- $$\text{Volumetric power density} = \frac{\text{Power processing capability}}{\text{volume}}$$
- If each dimension of energy storage component is scaled by a linear factor α: $\Longrightarrow$

Energy Storage Form	Power Density Scaling	Ideal for Miniaturization?
Dc-dominated inductor limited by constant $\hat{J}$	α	$\times$
Ac-dominated inductor	α	$\times$
Capacitor	1	✓
Piezoelectric Resonator [1]	α^{-1}	✓

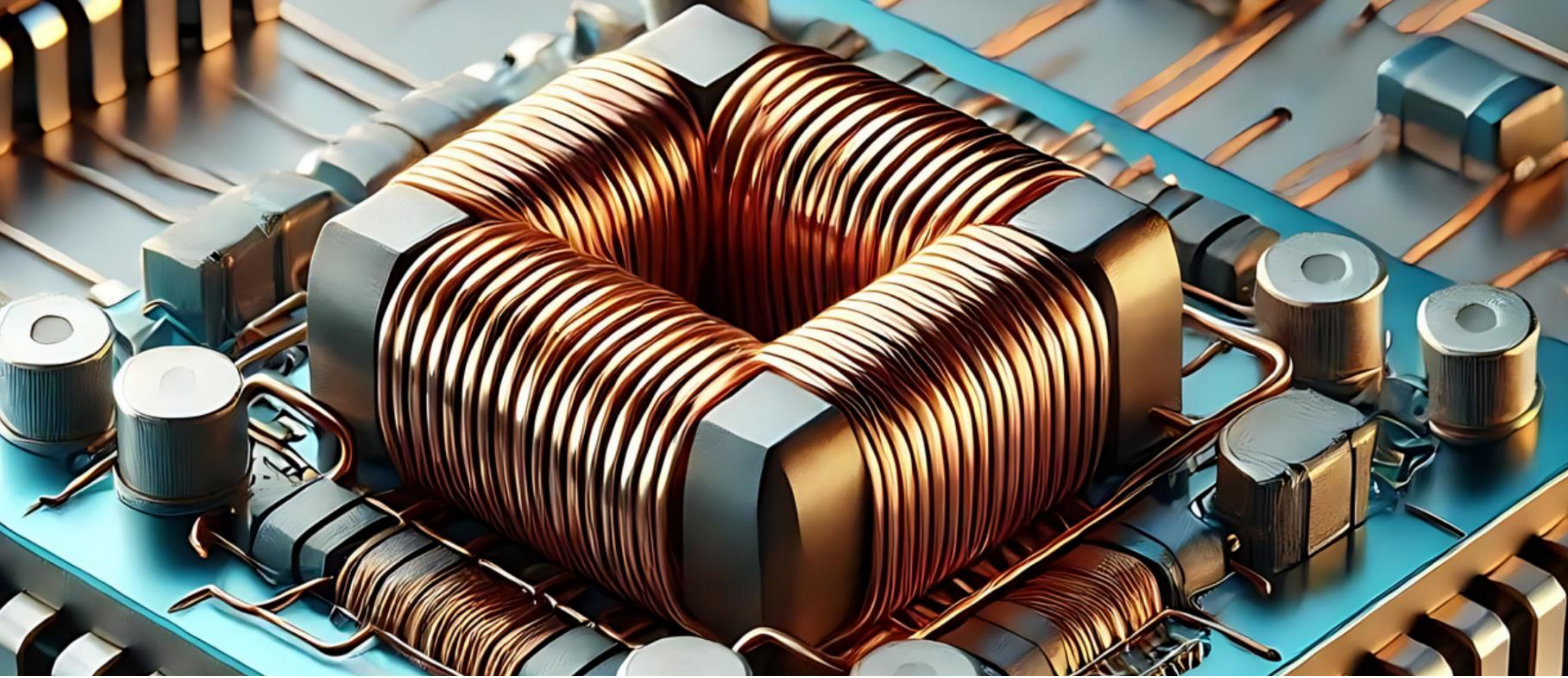
[1] J. D. Boles, J. E. Bonavia, P. L. Acosta, Y. K. Ramadass, J. H. Lang and D. J. Perreault, "Evaluating Piezoelectric Materials and Vibration Modes for Power Conversion," in *IEEE Transactions on Power Electronics*, vol. 37, no. 3, pp. 3374-3390, March 2022.

Broad conclusions about inductor scaling

For all presented methods, inductor goodness does seem to scale faster than volume

So what?

- Expect physics to resist miniaturization: half-power will not yield half size.
- Capacitors or piezoelectric solutions may be best at the smallest sizes.
- Splitting a big inductor into multiple smaller inductors is likely to lose out on density (but slowly).



How do magnetics scale with frequency?

Elaine Ng

Why do we always want higher frequency?

Power electronics has been moving to higher frequency for a long time

Why? – mainly because we expect the required L and C to get smaller

Example: buck ripple $= \frac{V_o}{L}(1-D)T = \frac{V_o\left(1-\frac{V_o}{V_i}\right)}{Lf}$

$\Rightarrow \quad L \propto 1/f$ to maintain constant ripple

But small *inductance* L does not necessarily imply a small component!

Volume scaling with frequency

Consider again expressions for power processing capability of inductors:

DC: $V_{pk} I_{rms} = 2\pi f \, B_{sat} \hat{J}_{rms} \frac{\mathcal{R}}{1+\mathcal{R}} A_c A_w$

AC: $V_{pk} I_{pk} = 2\pi f \, B_{max} \hat{J}_{max} A_c A_w$

- Increasing f *does* allow $A_c A_w$ to decrease
- Volume only scales as $1/\omega^{3/4}$ if B_{max} and J_{max} are unchanged
- But it's worse than that – **$\widehat{\boldsymbol{B}}$ in particular does reduce, strongly, with $\boldsymbol{f}$**

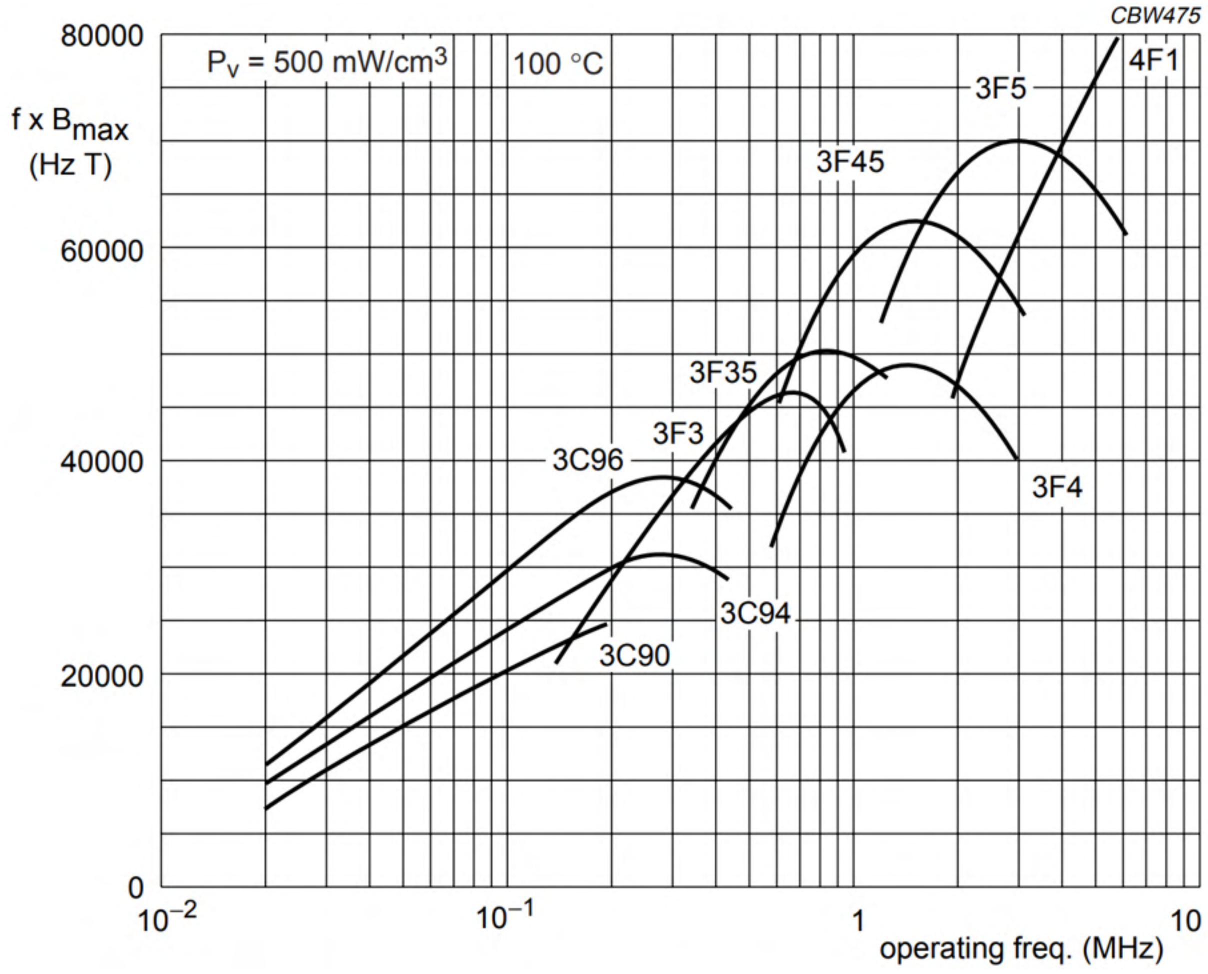
The Performance Factor

- What if, instead of tracking $\hat{B}$, we tracked the product $\hat{B} \times f$? This merged quantity being bigger or smaller *does* directly predict volume
- The Performance Factor $\mathcal{F} \equiv \hat{B} f$
- At low frequencies $\hat{B}$ decreases slowly and $\mathcal{F}$ increases with f
- At high frequencies, $\hat{B}$ decreases rapidly and $\mathcal{F}$ decreases
- Any given material has a frequency that maximizes $\mathcal{F}$

"Soft Ferrites and Accessories," Data Handbook, Ferroxcube, 2013

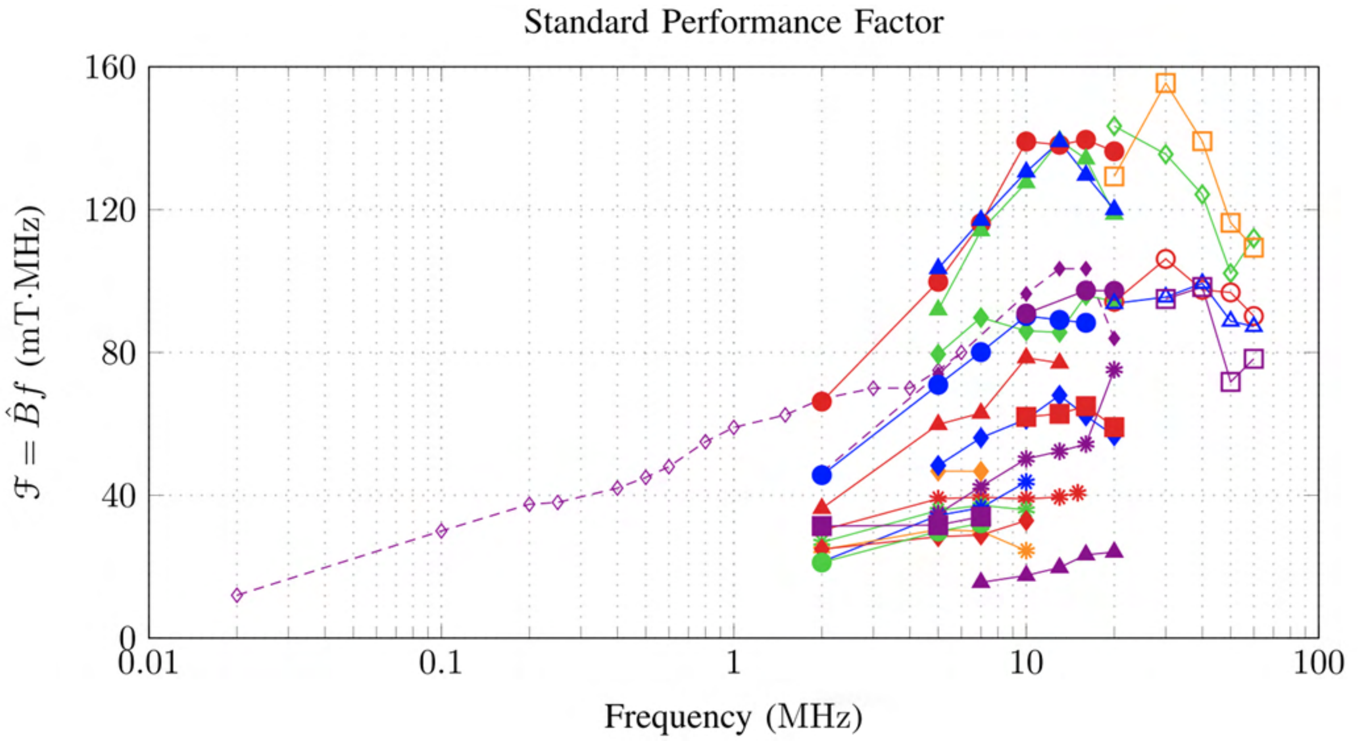
Performance Factor Trends

- Each material has an optimum f
- Different materials peak at different f and $\mathcal{F}$
- Higher frequency does tend to improve $\mathcal{F}$, but slowly
  - 2x from 100 kHz to 1 MHz
  - Another ~1.3x per decade above that



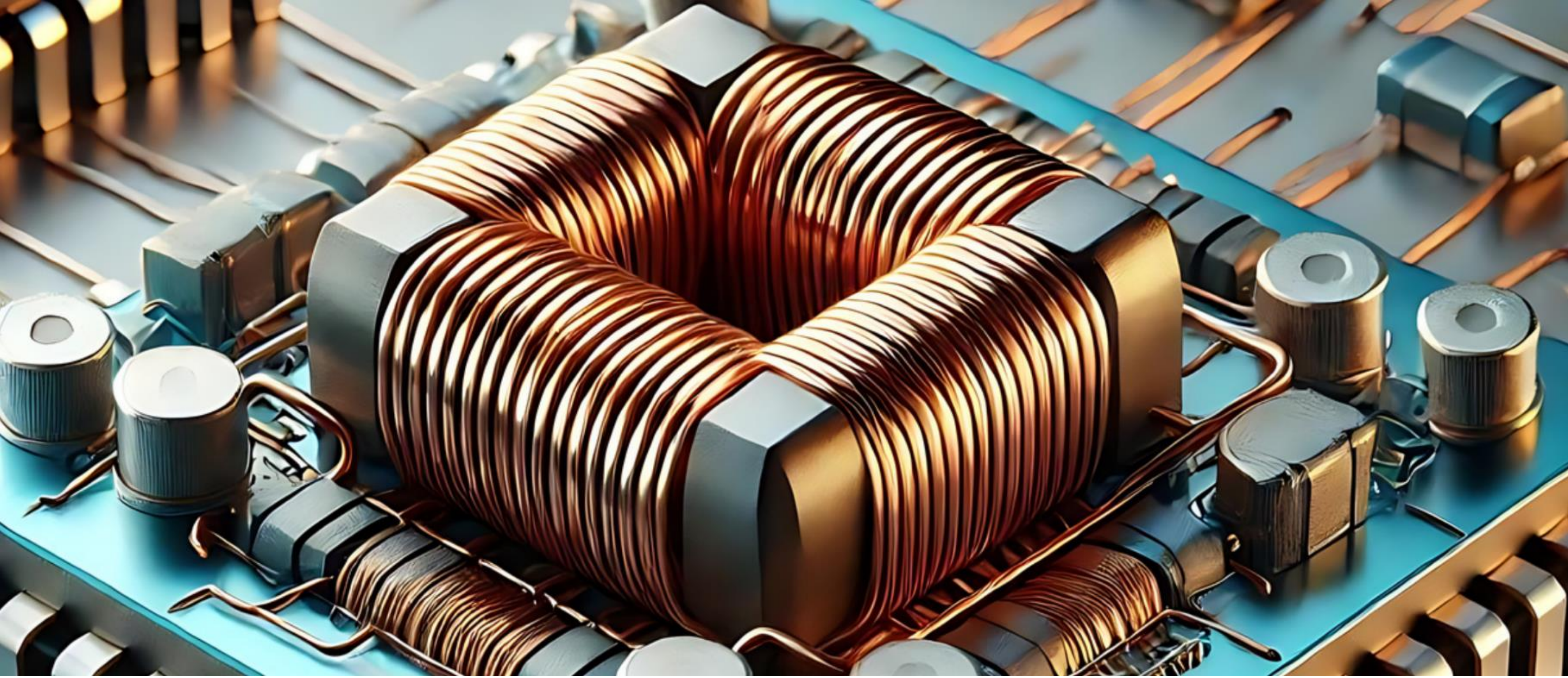
"Soft Ferrites and Accessories," Data Handbook, Ferroxcube, 2013

Looking to even higher frequencies



- Purple = envelope of data from previous slide
- Big jump in moving from MnZn to NiZn materials in the ~10 MHz range
- Recall $\mathcal{F}$ is predicting power density ⇒ expect continued improvements to ~30 MHz

A. J. Hanson, J. A. Belk, S. Lim, C. R. Sullivan and D. J. Perreault, "Measurements and Performance Factor Comparisons of Magnetic Materials at High Frequency," in IEEE Transactions on Power Electronics, vol. 31, no. 11, pp. 7909-7925, Nov. 2016



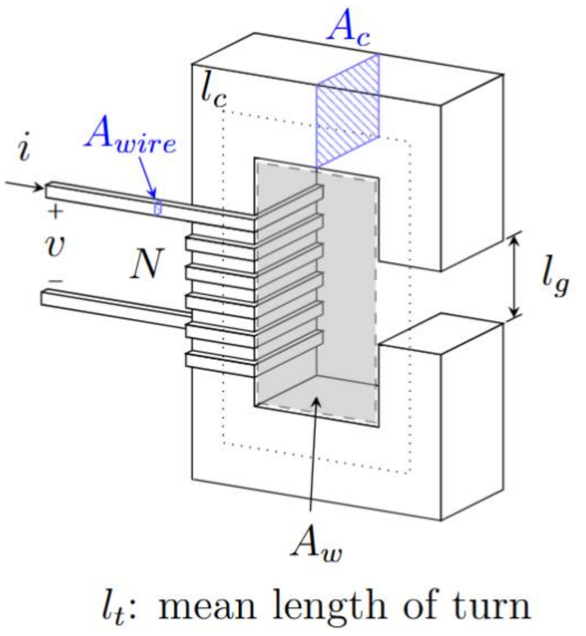
How much permeability is necessary?

Alyssa Brown

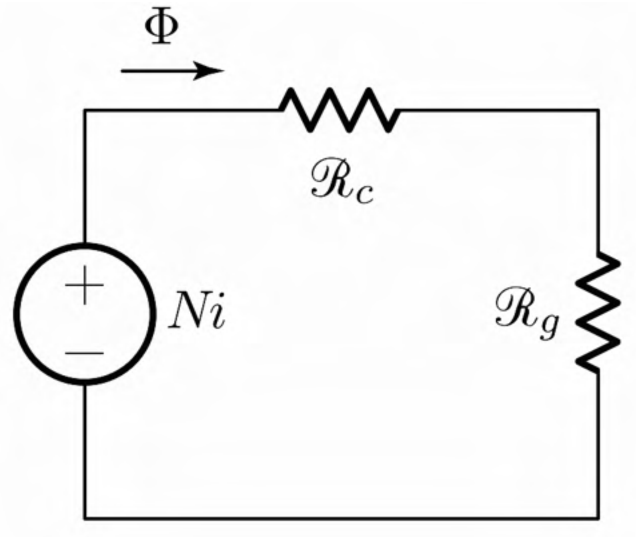
Where did permeability go?

Why can μ_c often be approximated out of key equations?

$$B = \frac{Ni}{\frac{l_c}{\mu_0\mu_r} + \frac{l_g}{\mu_0}} \approx \frac{\mu_0 Ni}{l_g} \qquad L = \frac{N^2}{\frac{l_c}{\mu_0\mu_r A_c} + \frac{l_g}{\mu_0 A_c}} \approx \frac{\mu_0 A_c N^2}{l_g}$$



- Electric: $R_{wire} = \frac{l}{\sigma A}$ is much lower than any other resistance in the circuit
- Magnetic: $\mathcal{R}_{core} = \frac{l}{\mu A}$ is much lower than any other reluctance in the circuit
- Then **when is $\boldsymbol{\mu_r}$ "large enough"?**



Permeability limits for inductors

- Consider a perfectly-designed inductor with inductance L and a peak B field B_{max}

$$L = \frac{N^2}{\frac{l_c}{\mu_0 \mu_r A_c} + \frac{l_g}{\mu_0 A_c}}$$

- To compensate for $\downarrow \mu_r$, we need $\downarrow l_g$, but this **doesn't affect the performance** because

$$B_{max} = \frac{Li}{NA_c}$$

- Same B field → same core loss, same buffer before saturation.
- Same window geometry, same N → same winding loss.

μ_r doesn't matter, until…

$$L = \frac{N^2}{\frac{l_c}{\mu_0 \mu_r A_c} + \frac{l_g}{\mu_0 A_c}} \qquad B_{max} = \frac{Li}{NA_c}$$

- If we keep dialing down μ_r and dialing down l_g to compensate, absolutely nothing else changes, ***until*** $\boldsymbol{l_g = 0}$**.**
- Once $l_g = 0$, it can't be used to compensate for lower μ_r. It becomes necessary to change the core geometry or increase N, both of which make the component worse

$\Rightarrow$ **Find the limit where** $\boldsymbol{l_g = 0}$

The Critical Permeability for Inductors

$$l_g = 0 \quad \Rightarrow \quad \mu_{r,critL} = \frac{Ll_c}{\mu_0 A_c N^2} = \frac{Ll_c}{\mu_0 A_c \dfrac{L^2 I^2}{A_c^2 B_{max}^2}} = \frac{A_c l_c \dfrac{1}{2\mu_0} B_{max}^2}{\dfrac{1}{2} L I^2}$$

$$\mu_{r,critL} = \begin{cases} \dfrac{A_c l_c \dfrac{1}{2\mu_0} B_{sat}^2}{\dfrac{1}{2} L I_{pk}^2} & \text{if saturation limited} \\[2em] \dfrac{A_c l_c \dfrac{1}{2\mu_0} \hat{B}^2}{\dfrac{1}{2} L I_{ac}^2} & \text{if core loss limited} \end{cases}$$

Deeper interpretation of $\mu_{r,critL}$

- When $B_{max} = B_{pk}$ (saturation limited or purely ac cases)

$$1 = \frac{A_c l_c \frac{1}{2} \frac{1}{\mu_0 \mu_{r,critL}} B_{pk}^2}{E_{store}} = \frac{E_{core,max}}{E_{store}}$$

As μ_r increases, less energy can be stored in the core for a given B limit. Only when the core itself becomes incapable of storing the necessary energy for the application, E_{store}, does a design include a gap and true optimization is possible.

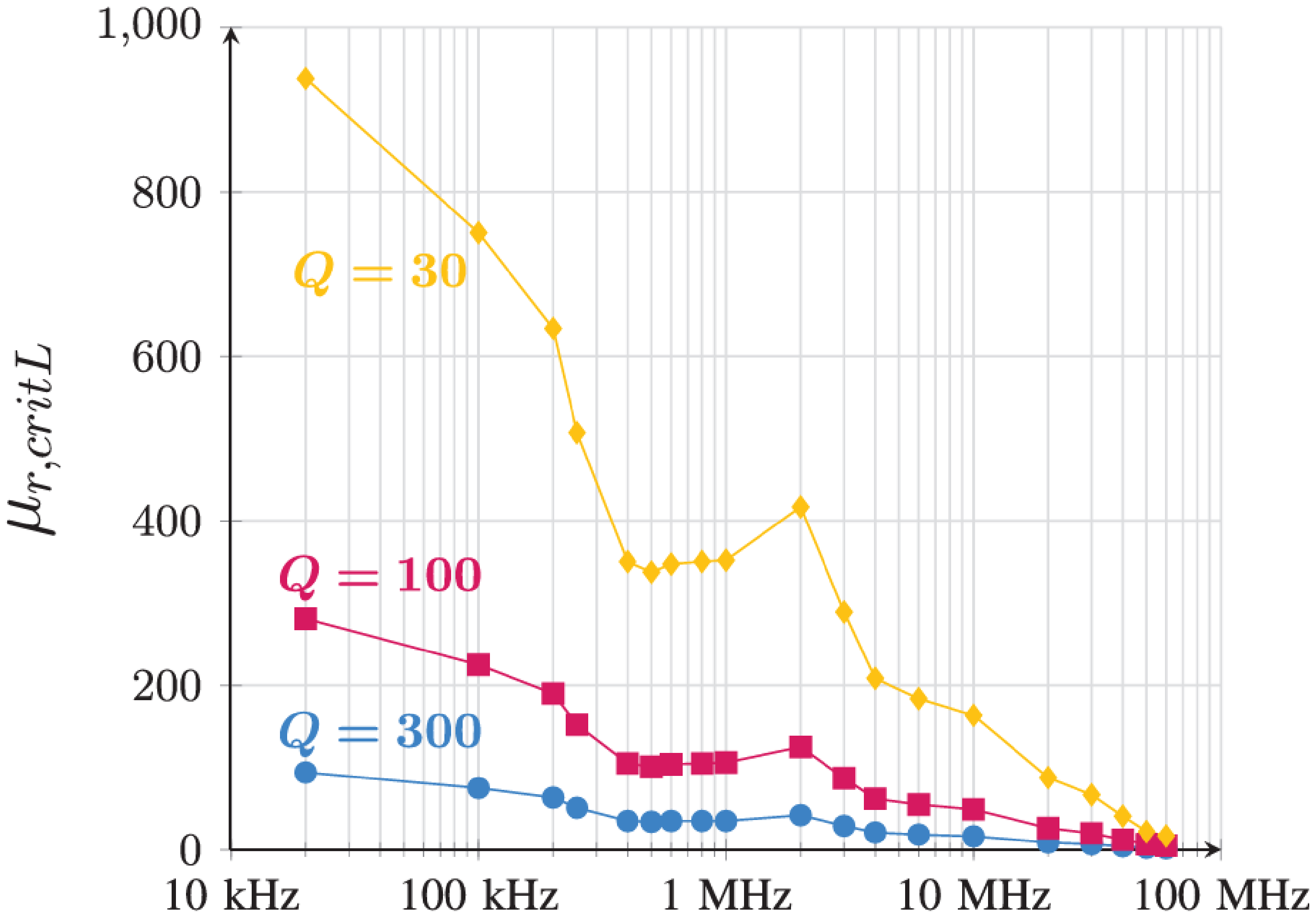
In terms of peak energy storage and Q

$$\mu_{r,critL} = \frac{A_c l_c \frac{1}{2\mu_0} B_{max}^2}{E_{store}}$$

- So long as the core has sufficient permeability that the core does not store the energy, a gap will be used, and more permeability is no longer beneficial.
- For size optimized designs: $P_{core} = P_{cu}$
- In terms of Quality Factor, $Q = \frac{2\pi E_{store}}{E_{Loss}}$, where $E_{Loss} \approx \frac{2P_{core}}{f} = \frac{2P_{v,core}}{f A_c l_c}$

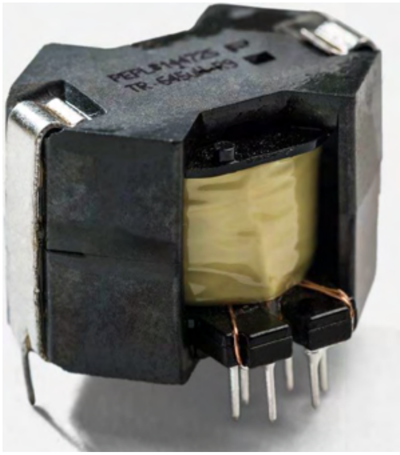
$$\mu_{r,critL} = \frac{\frac{1}{2\mu_0} 2\pi A_c l_c B_{max}^2}{Q E_{Loss}} = \frac{\frac{1}{\mu_0} \pi f B_{max}^2}{Q P_{v,core}}$$

$\mu_{r,critL}$ for best ferrites vs. frequency



- $P_{v,core} = 500\,\frac{mW}{cm^3}$
- For reasonable values of Q, the required permeability can be **well under 100 in MHz regime.**

Permeability Example



$$L = \frac{N^2}{\frac{l_c}{\mu_0 \mu_r A_c} + \frac{l_g}{\mu_0 A_c}} \qquad B_{max} = \frac{Li}{NA_c}$$

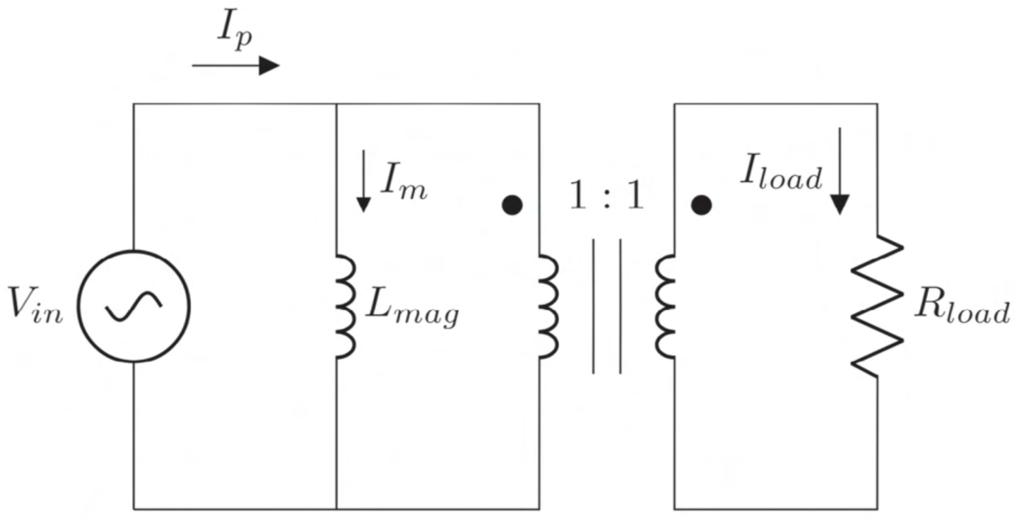
- We have a 500 kHz RM10 inductor designed to be size optimized for a DC-DC converter:
  - L = 24 μH, $B_{max} = 100 \text{ mT}$, $\boldsymbol{\mu_r = 1600}$, $A_c = 98 \text{ mm}^2$, $l_c = 44\, mm$, N = 11, $l_g = 0.6\, mm$
- How much permeability do we actually need?

$$\mu_{r,critL} = \frac{Ll_c}{\mu_0 A_c N^2} \approx \mathbf{74}$$

We have **21 times** the permeability we need for the same performance
⇒ **Permeability is not a limiting design factor**

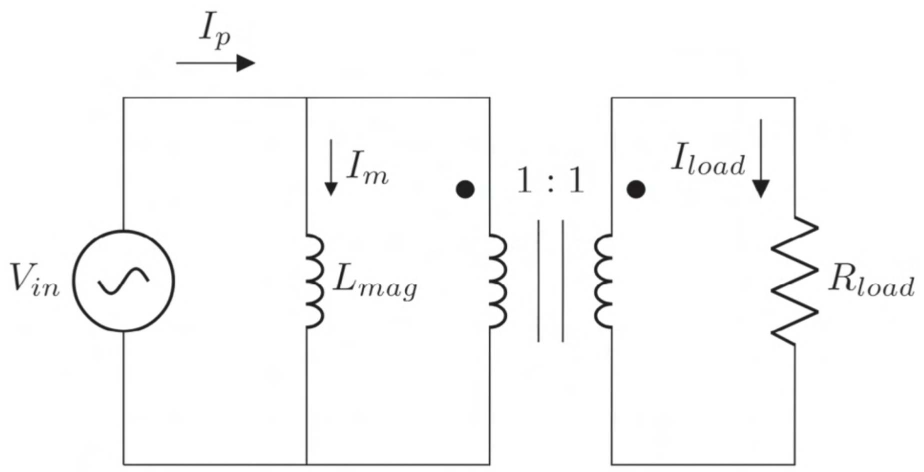
Critical permeability for transformers

- High μ_r increases L_{mag} for no gap and a given number of turns (N is fixed by loss constraints)
- As L_{mag} increases, the transformer can handle more I_{load} for a given limit on I_p (i.e., can deliver more power)
- Once $\omega L_{mag} \gg R_{load}$, further increasing L_{mag} has little further impact
- Identify $\omega L_{mag} = R_{load}$ as an important turning point



Critical permeability for transformers

- $P_{load} = \frac{1}{2} V_{load} I_{load} = \frac{1}{2} V_{in} I_{load} = \frac{1}{2} \omega N A_c B_{max} I_{load}$
- The primary current I_p is limited by current density in the wires: $I_p = \frac{\hat{J} A_w / 2}{N}$
- Since I_{load} and I_m are orthogonal: $I_{load} = \sqrt{I_p^2 - I_m^2}$
- $P_{load} = \frac{1}{2} V_{load} I_{load}$
- $= \frac{1}{2} \omega N A_c B_{max} \sqrt{\left(\frac{\hat{J} A_w}{2N}\right)^2 - \left(\frac{V_{in} l_c}{\omega N^2 \mu_0 \mu_r A_c}\right)^2}$



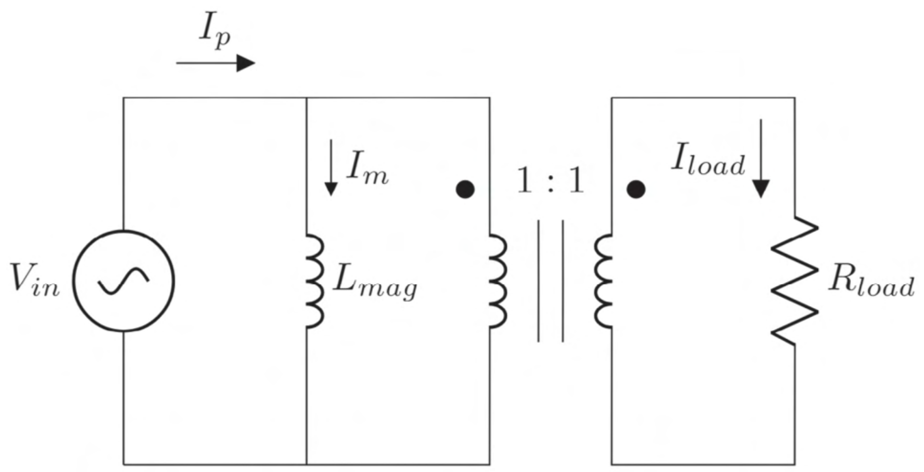
Critical permeability for transformers

$$= \frac{1}{4}\omega A_c A_w B_{max}\hat{J}\sqrt{1-\left(\frac{2B_{max}l_c}{\mu_0\mu_r\hat{J}A_w}\right)^2}$$

- Assume a core loss limited design and assume core loss is approximately equal to winding loss

$$P_{v,core}A_c l_c \approx \frac{1}{2}\hat{J}\rho A_w l_w$$

$$\Rightarrow \quad P_{load} \approx \frac{1}{4}\omega A_c A_w \hat{B}\hat{J}\sqrt{1-\frac{2B_{max}^2\rho}{\mu_0^2\mu_r^2 P_{v,core}}\frac{l_c l_w}{A_c A_w}}$$

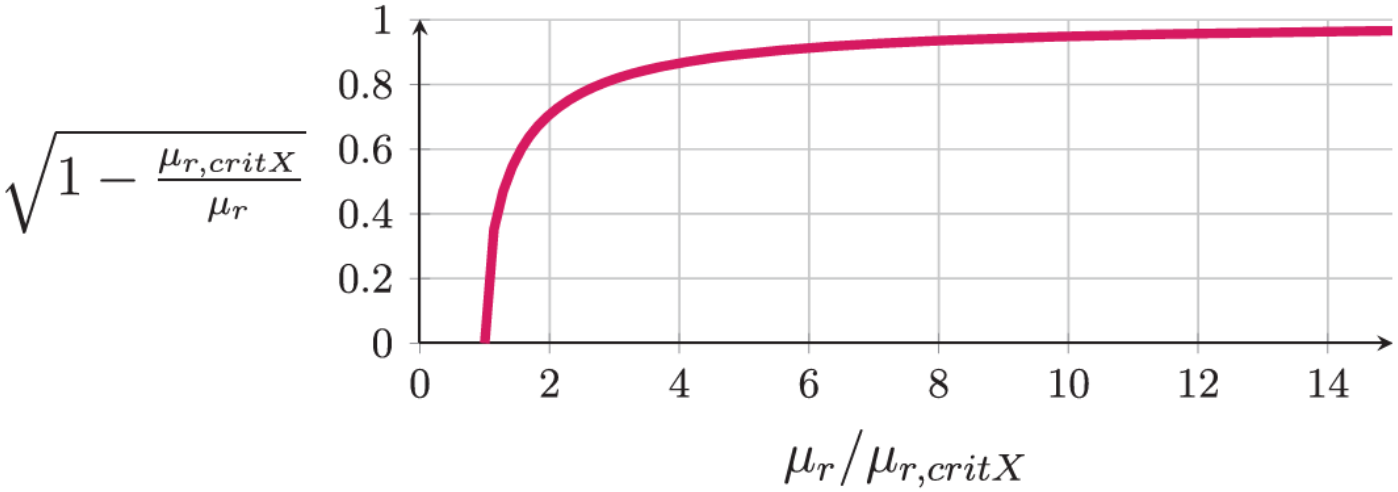


Critical permeability for transformers

$$P_{load} \approx \frac{1}{4}\omega A_c A_w \hat{B}\hat{J}\sqrt{1 - \frac{2B_{max}^2\rho}{\mu_0^2\mu_r^2 P_{v,core}}\frac{l_c l_w}{A_w A_c}}$$

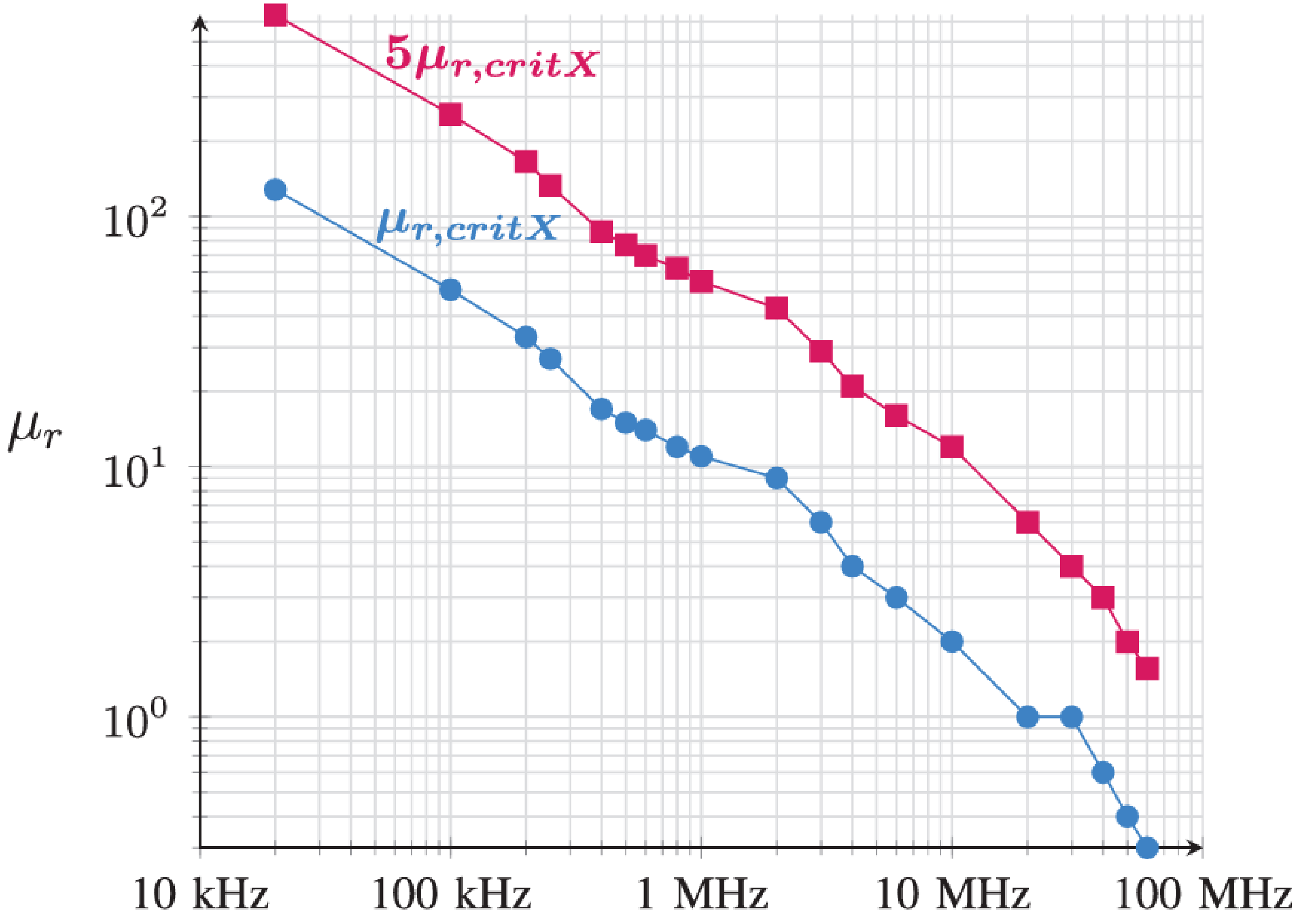
$$\mu_{r,critX} = \frac{B_{max}}{\mu_0}\sqrt{\frac{2\rho}{P_{v,core}}\frac{l_c l_w}{A_c A_w}}$$

So how much does μ_r need to be?

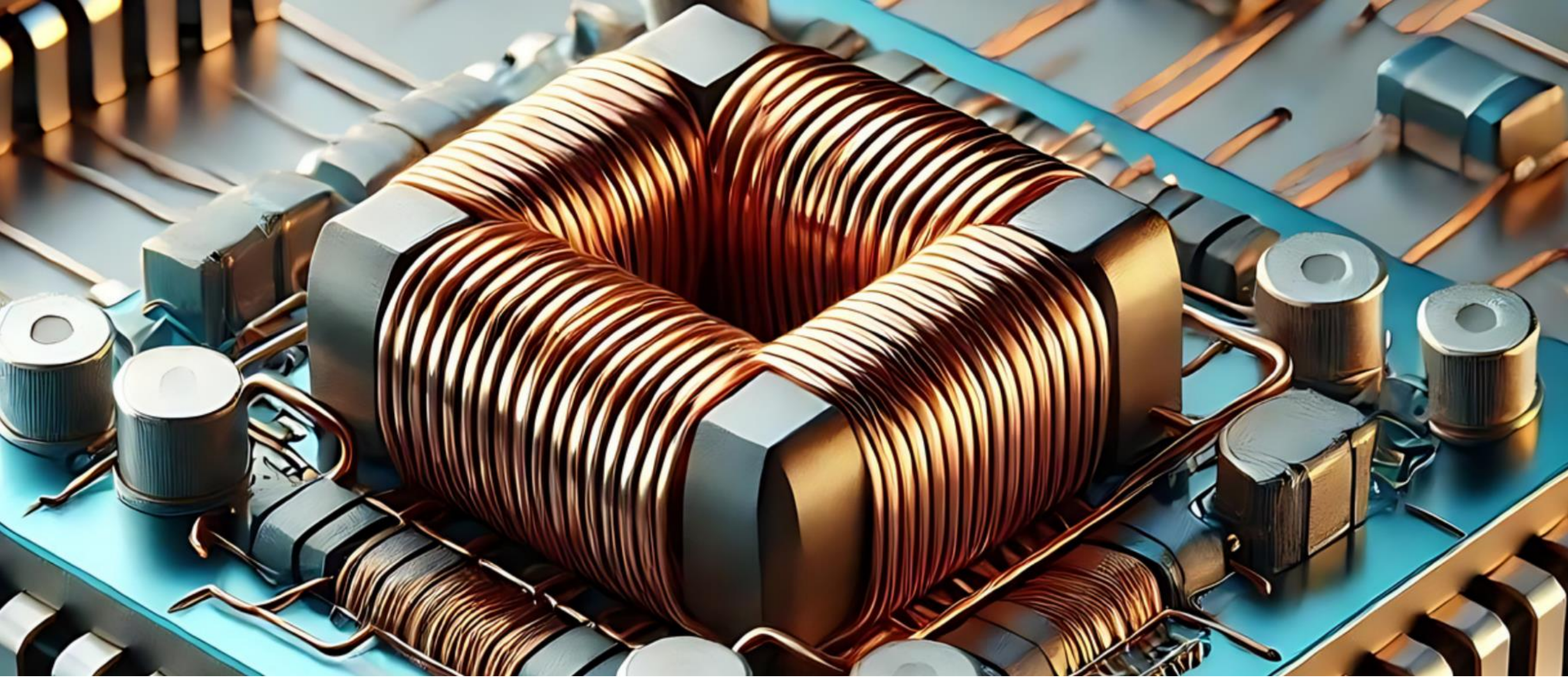


- $P_{load} \propto \sqrt{1 - \frac{\mu_{r,critX}}{\mu_r}}$
- As long as $\mu_r \gg \mu_{r,critX}$ then P_{load} is not affected by additional μ_r
  - Can define $\gg$ as $\times\, 5$

$\mu_{r,critX}$ for best ferrites vs. frequency



- $P_{v,core} = 500\,\frac{mW}{cm^3}$, RM7 core size
- Modest permeabilities(≤ 100) satisfy the demands of $5\mu_{r,critX}$ **above a few hundred kHz.**



When should air-core magnetics be used?

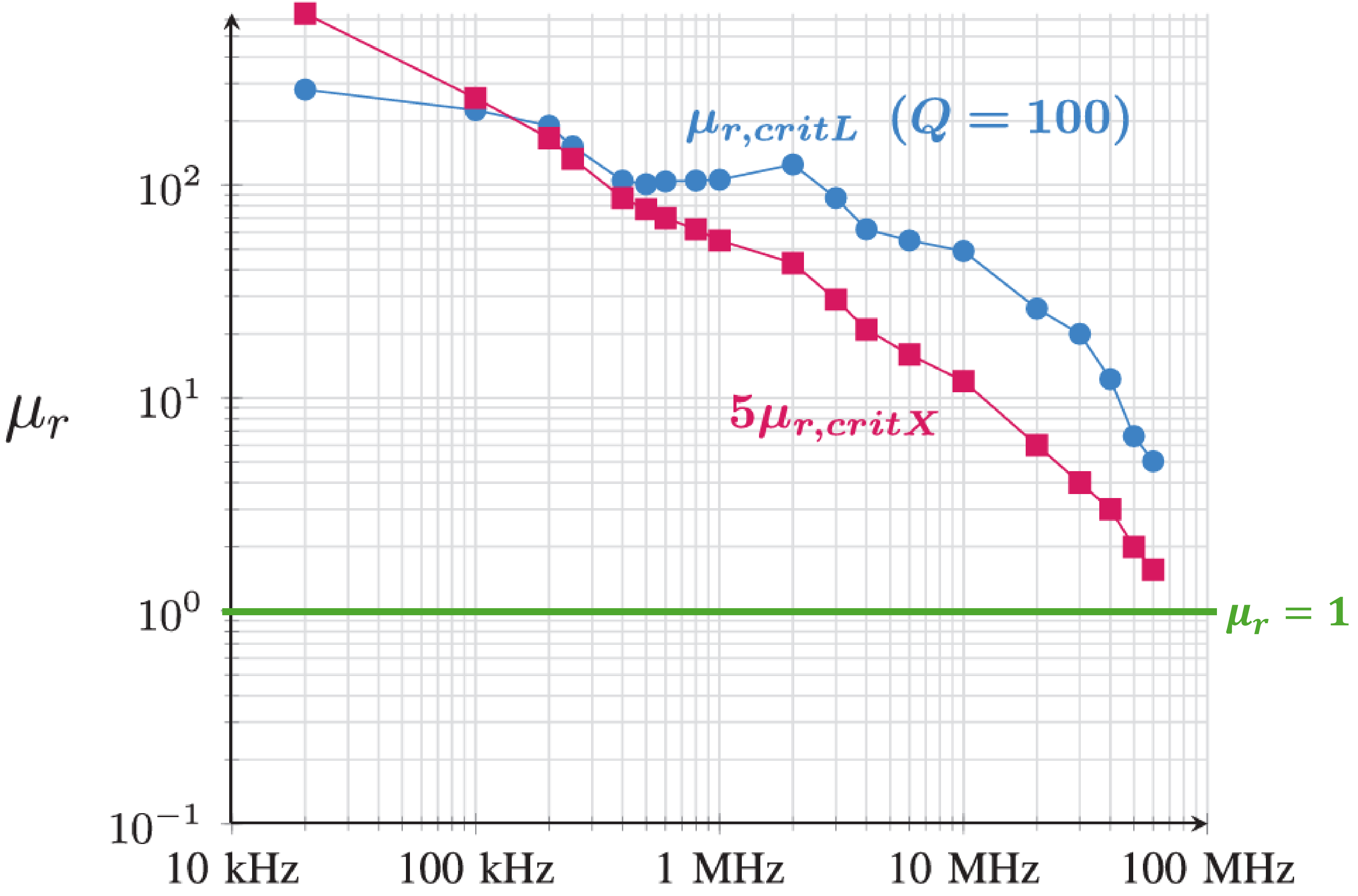
Alyssa Brown

Fundamental Tradeoff

- At typical operating frequencies, magnetic cores are used to reduce the number of turns needed to reach a desired inductance.
  - *Less copper loss* but now have *saturation limits and core loss*.
- Tradeoff leans towards use of core for sub-MHz applications.
- At many MHz, typical design intuition leads us to believe that the core incurs too much loss and needs to be omitted (“air-core” design).
- **At what frequency should we shift to air cores? 1 MHz? 10 MHz?**
- **The answer matters!** Many applications operate in this region, including plasma generation, rf heating, envelope tracking, on-chip power supplies, etc.

An air-core boundary estimate based on $\mu_{r,crit}$

- $\mu_{r,crit} = 1$ indicates that a core would no longer be beneficial
- For both $\mu_{r,critL}$ and $5\mu_{r,critX}$, this is predicted to happen in the 80-100 MHz range.



A more rigorous air-core boundary

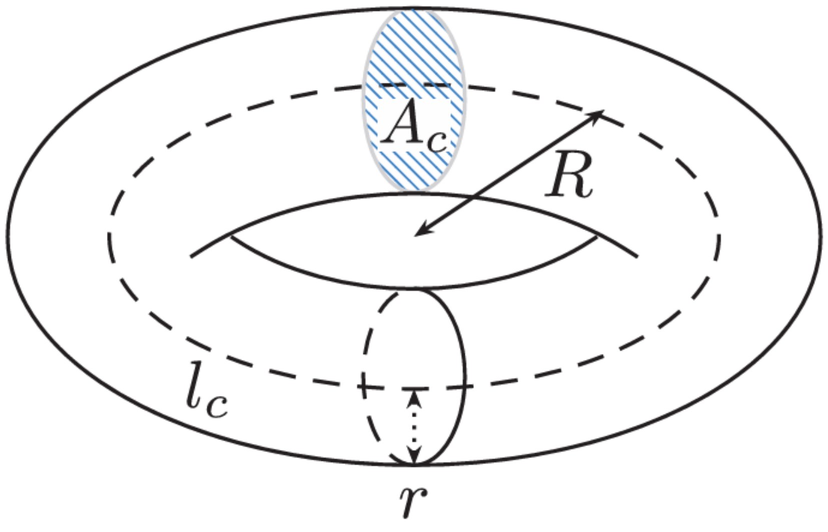
- At what frequency does the total power loss of an optimized magnetic-core inductor equal that of an optimized air-core inductor?

$$P_{diss} = P_{mag,diss} = P_{air,diss}$$

- Begin with an optimized magnetic-core toroidal inductor, which generally has copper loss ($P_{cu,mag}$) ≈ core loss ($P_{core,mag}$)

$$P_{mag,diss} = P_{cu,mag} + P_{core,mag}$$

$$= 2P_{cu,mag} = 2A_c l_c P_{v,core}$$



Constant P_{total} boundary: Magnetic-Core

- Current is restricted by the maximum copper loss

$$P_{cu,mag} = \frac{1}{2} I_{max}^2 R = \frac{I_{max}^2}{2} \frac{2\pi r N}{\delta\sigma\left(\frac{l_c}{N}\right)}$$

$$I_{max} = \sqrt{\frac{\delta\sigma l_c P_{cu,mag}}{\pi r N^2}} = \sqrt{\frac{\delta\sigma A_c l_c^2 P_{v,core}}{\pi r N^2}}$$

- The ac B field is restricted to $\hat{B}$, therefore $V_{max} = N A_c \hat{B} \omega$. Thus, maximum power for cored inductor is

$$P_{mag} = \frac{1}{2} I_{max} V_{max} = \frac{1}{2} A_c l_c \hat{B} \omega \sqrt{\frac{\delta\sigma A_c P_{v,core}}{\pi r}}$$

Constant P_{total} boundary: Air-Core

- For the same P_{diss}, we can set the copper loss of the air-core component equal to twice that of the magnetic-core component and follow the same procedure

$$P_{diss} = P_{cu,air} = 2P_{cu,mag}$$

$$I_{max} = \sqrt{\frac{2\delta\sigma A_c l_c^2 P_{v,core}}{\pi r N^2}} \text{ and } V_{max} = L I_{max} \omega$$

$$P_{air} = \frac{1}{2} L\omega I_{max}^2 = \frac{1}{2} \frac{N^2 A_c \mu_0 \omega}{l_c} \frac{2\delta\sigma A_c l_c^2 P_{v,core}}{\pi r N^2}$$

$$= \frac{1}{\pi r} A_c^2 l_c \delta\sigma\mu_0\omega P_{v,core}$$

Ratio of power processing capabilities

$$\frac{P_{mag}}{P_{air}} = \frac{\pi r \hat{B}}{2A_c \delta\sigma\mu_0 P_{v,core}} \sqrt{\frac{\delta\sigma A_c P_{v,core}}{\pi r}} = \frac{\hat{B} f^{\frac{1}{4}} \pi^{\frac{1}{4}}}{2\mu_0^{\frac{3}{4}} \sigma^{\frac{1}{4}} r^{\frac{1}{2}} P_{v,core}^{\frac{1}{2}}}$$

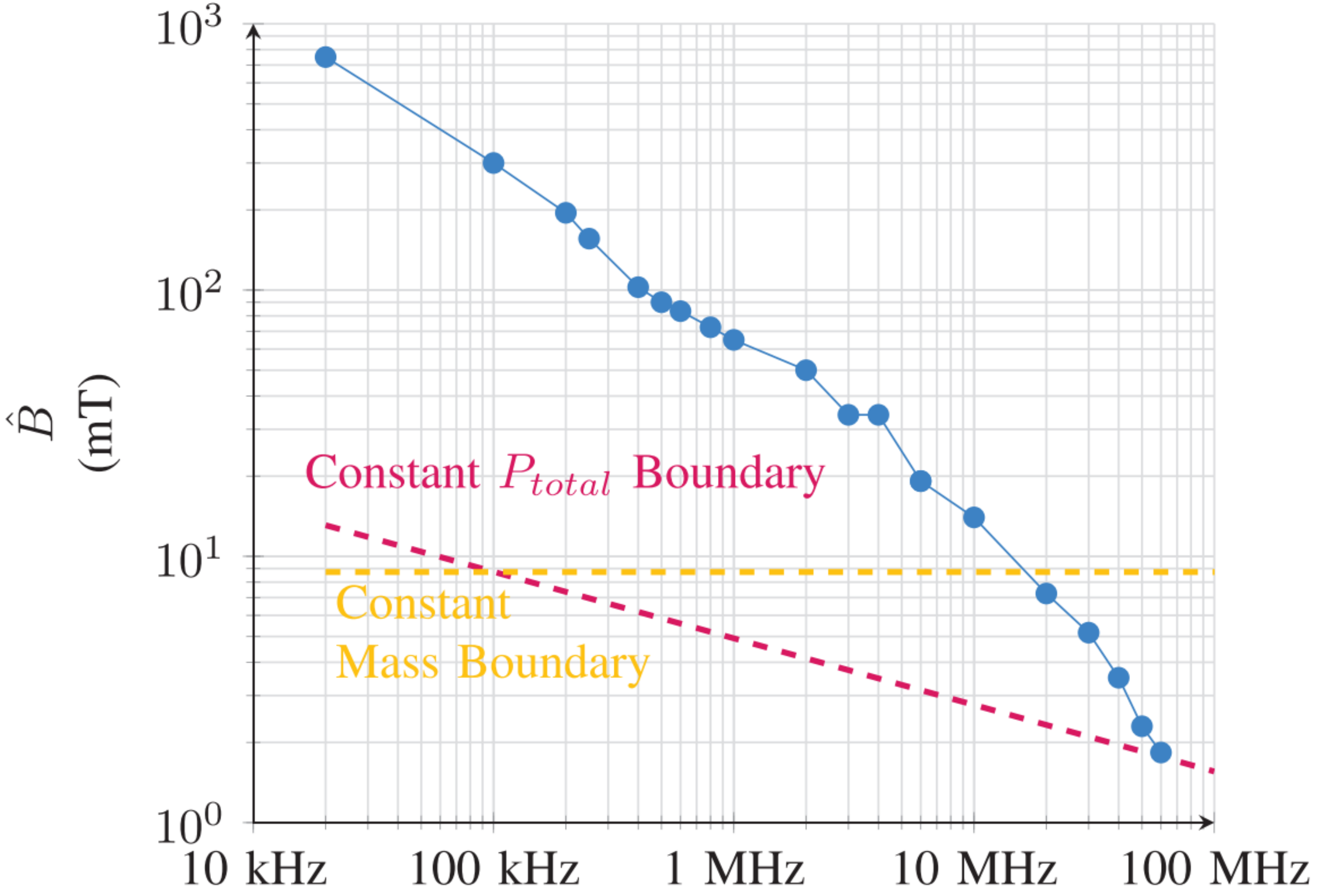
$$A_c = \pi r^2$$

Setting this ratio equal to 1 means that both devices have the same maximum power rating for a given P_{diss} and solving for the threshold $\hat{\mathrm{B}}$

$$1 \leq \frac{\hat{B} f^{\frac{1}{4}} \pi^{\frac{1}{4}}}{2\mu_0^{\frac{3}{4}} \sigma^{\frac{1}{4}} r^{\frac{1}{2}} P_{v,core}^{\frac{1}{2}}} \rightarrow \boxed{\hat{B} \geq 2 \sqrt{r P_{v,core} \sqrt{\frac{\mu_0^3 \sigma}{\pi f}}}}$$

Survey of maximum-$\hat{B}$ materials vs boundaries

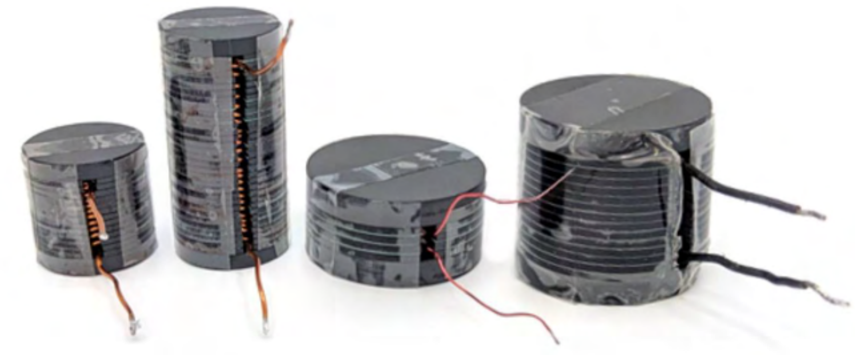
- $P_{v,core} = 200\,\frac{mW}{cm^3}, r = 5\,mm$
- A magnetic-core component would outperform an air-core counterpart with the same P_{diss} up to about 60 MHz
- For a mass constrained design (derived in [1]) , a magnetic-core component would be preferred over air-core up to about 15 MHz

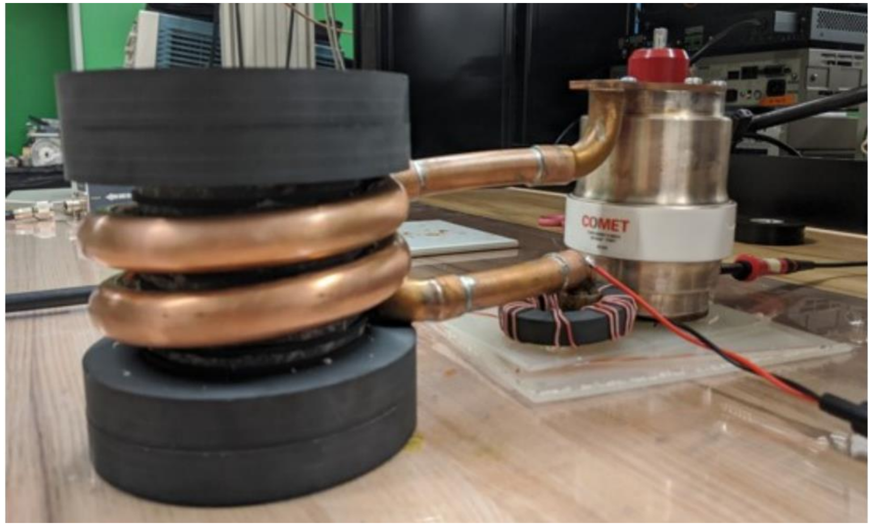


[1] M. Solomentsev and A. J. Hanson, "At What Frequencies Should Air-Core Magnetics Be Used?," in IEEE Transactions on Power Electronics, vol. 38, no. 3, pp. 3546-3558, March 2023

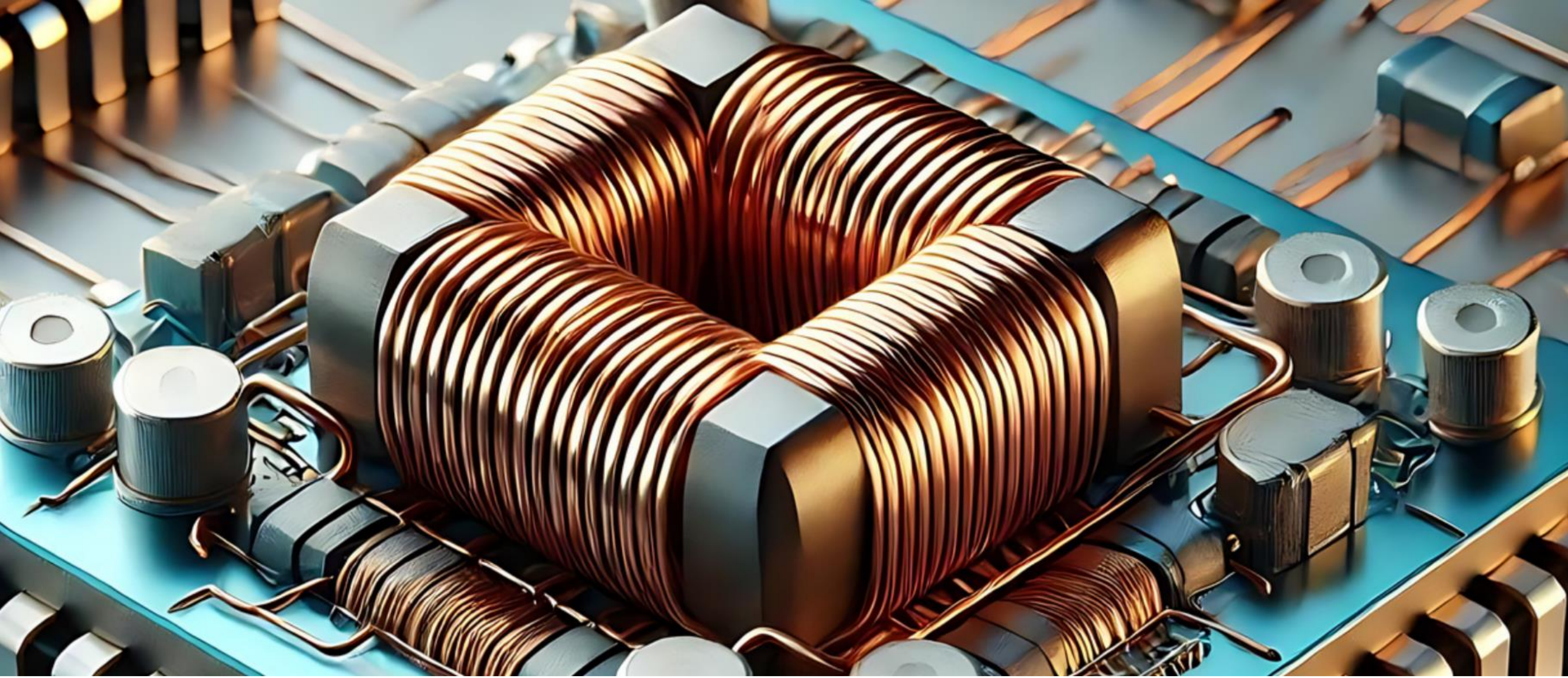
Real-world impact of magnetic/air choice

- Plasma generation at 13.56 MHz uses resonant matching networks to make the plasma impedance look like 50 Ω
- **It is assumed that the resonant inductors *must* be air core, but we have just shown that a magnetic-core inductor is very likely to win at 13.56 MHz**
- Rachel Yang (MIT) and Rod Bayliss (MIT, now Berkeley) have demonstrated magnetic-core inductors at this frequency with Q ~ 1000-1200
- Compare with air-core magnetics with similar effective volume, which have Q~200-300





R. S. Bayliss, R. S. Yang, A. J. Hanson, C. R. Sullivan and D. J. Perreault, "Design, Implementation, and Evaluation of High-Efficiency High-Power Radio-Frequency Inductors," *2021 IEEE Applied Power Electronics Conference and Exposition (APEC)*, Phoenix, AZ, USA, 2021



How much interstitial heat sinking can be included?

Alyssa Brown

Thermal Management in Magnetic Components

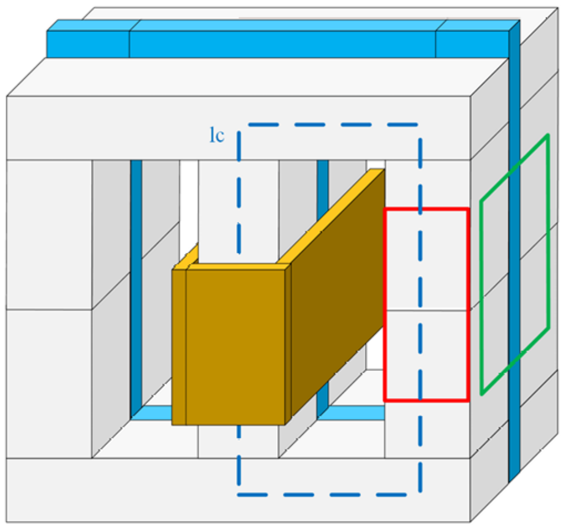
- Ferrite is a terrible thermal conductor
  - 1 to 5 $\frac{W}{mK}$ vs aluminum which has ~200 $\frac{W}{mK}$
- Many avenues taken to improve thermal performance
  - Optimization of insulation structures/bobbins
    - Low cost since both are typically necessary
  - Addition of materials/active cooling
    - Can add weight, cost, or volume but when optimized the benefits outweigh the cost
    - E.g. liquid/gas/air/hybrid cooling methods, immersion in oil, heat pipes, air channels, **cooling planes**, etc.
- We will focus on cooling planes/plates a.k.a. interstitial heat sinking

Wait, isn't metal a no-no?

Engineering intuition tells us that metal next to magnetic fields results in wasteful eddy currents.

However:

- If the metallic planes are kept in parallel with the flow of H fields, then little eddy currents will be induced in the plate.
- The reluctance model of the device is only slightly changed, so the impact on the component design is minimal.



A. Brown, D. T. Nguyen, and A. J. Hanson, "Heat Extraction from Ferrite Cores Using Metallic Laminations," 2025 IEEE Applied Power Electronics Conference and Exposition (APEC), Long Beach, CA, USA, 2025

Derivation of loss within cooling plane(s)

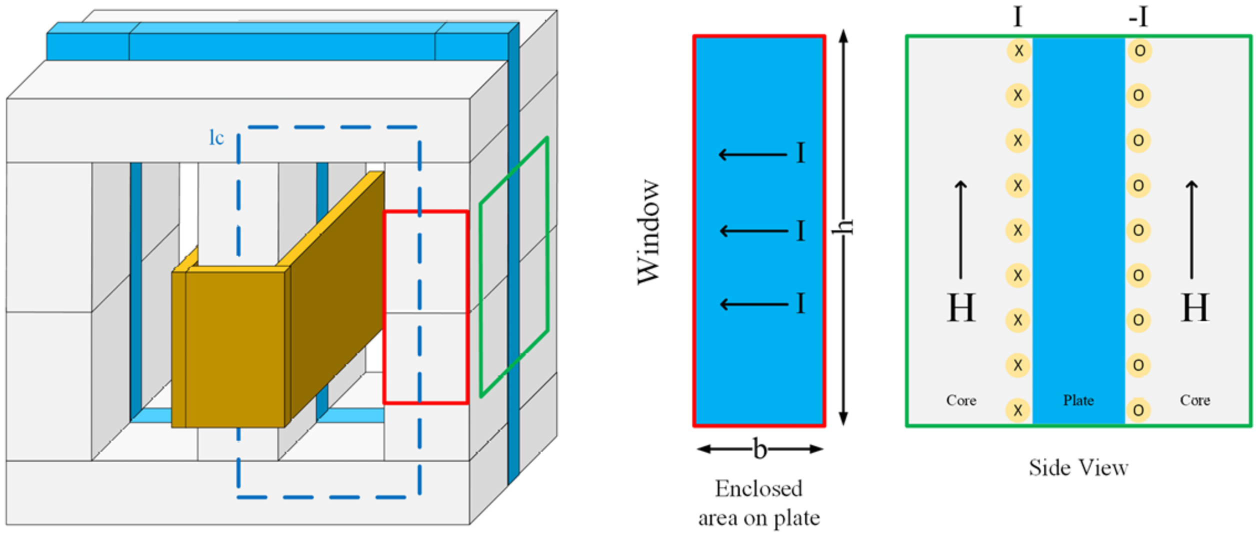
- The loss in the area enclosed in red is given by:

$$Loss\ for\ this\ section = 2(Kh)^2 \times \frac{\rho\sqrt{A_c/2}}{h\delta}$$

$$= \sqrt{2}(Kh)^2 \times \frac{\rho\sqrt{A_c}}{h\delta}$$

- For the full geometry $\Sigma h = 2l_c$ and $H = \frac{B}{\mu}$

$$Total\ loss = \frac{2\sqrt{2}B^2\sqrt{A_c}l_c\rho}{\mu^2\delta}$$



Loss ratio for comparison of materials

- Further generalize by finding the *loss in the plate* per unit volume of the *core* ($Vol_{core} \approx A_c l_c$) and plugging in $\delta = \sqrt{\frac{2\rho}{\omega\mu_0}}$

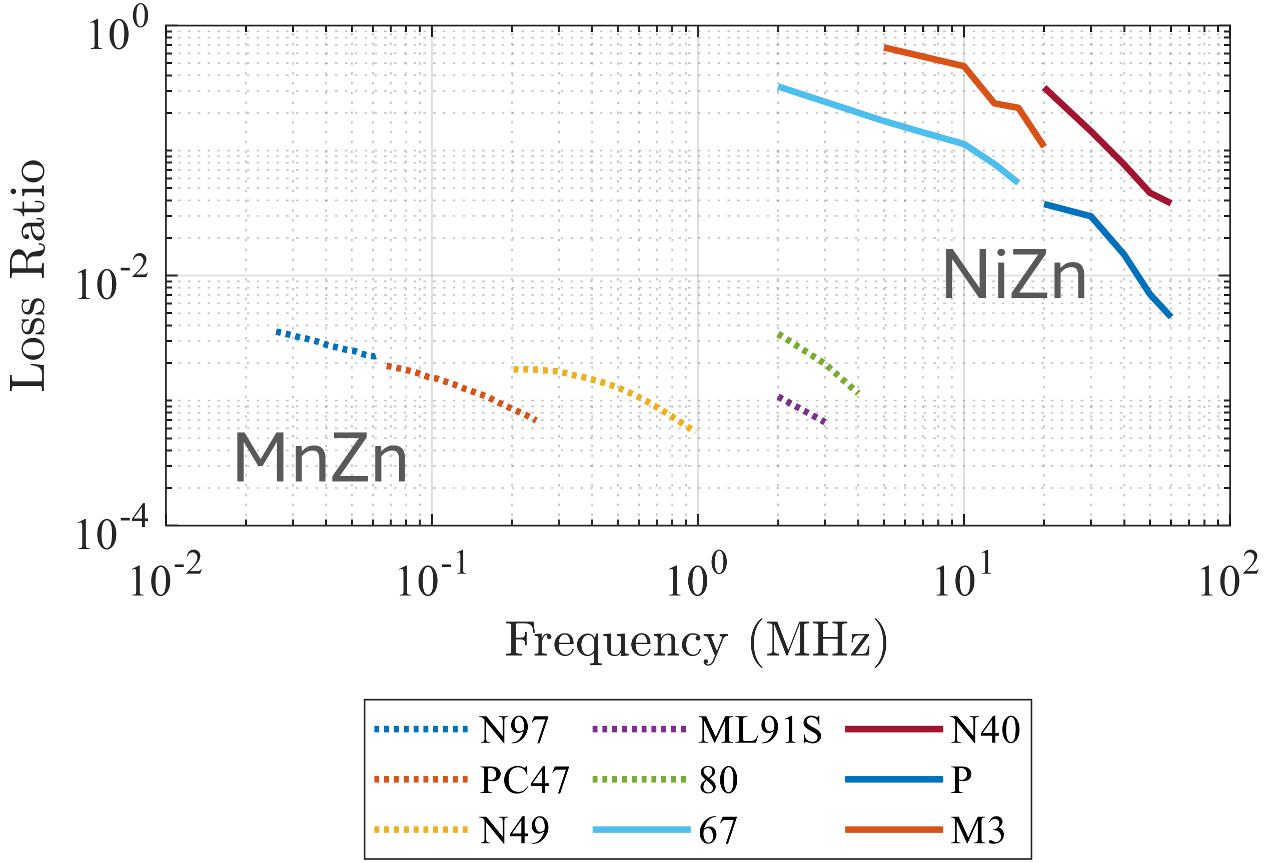
$$\frac{Loss\ in\ plate}{Vol_{core}} = \frac{2\sqrt{2}B^2\rho}{\mu^2\delta\sqrt{A_c}} = \sqrt{\frac{\omega\rho}{A_c\mu_0^3}}\frac{2B^2}{\mu_r^2}$$

- To compare between magnetic materials, we can set up a loss ratio

$$1 \gg \frac{Loss\ in\ Plate}{Loss\ in\ Core} = \sqrt{\frac{\omega\rho}{A_c\mu_0^3}}\frac{2B^2}{P_{v,core}\mu_r^2}$$

Loss for some ferrites

- For an aluminum plate with a core with $P_{v,core} = 200\,\frac{mW}{cm^3}$, $A_c = 0.2\ cm^2$
- All materials' loss ratios are < 1, but a cooling plate is better suited for lower frequency MnZn materials (<5 MHz) in terms of loss
- Works better with MnZn because permeability is higher (loss ratio $\propto 1/\mu_r^2$)



How many plates can be used?

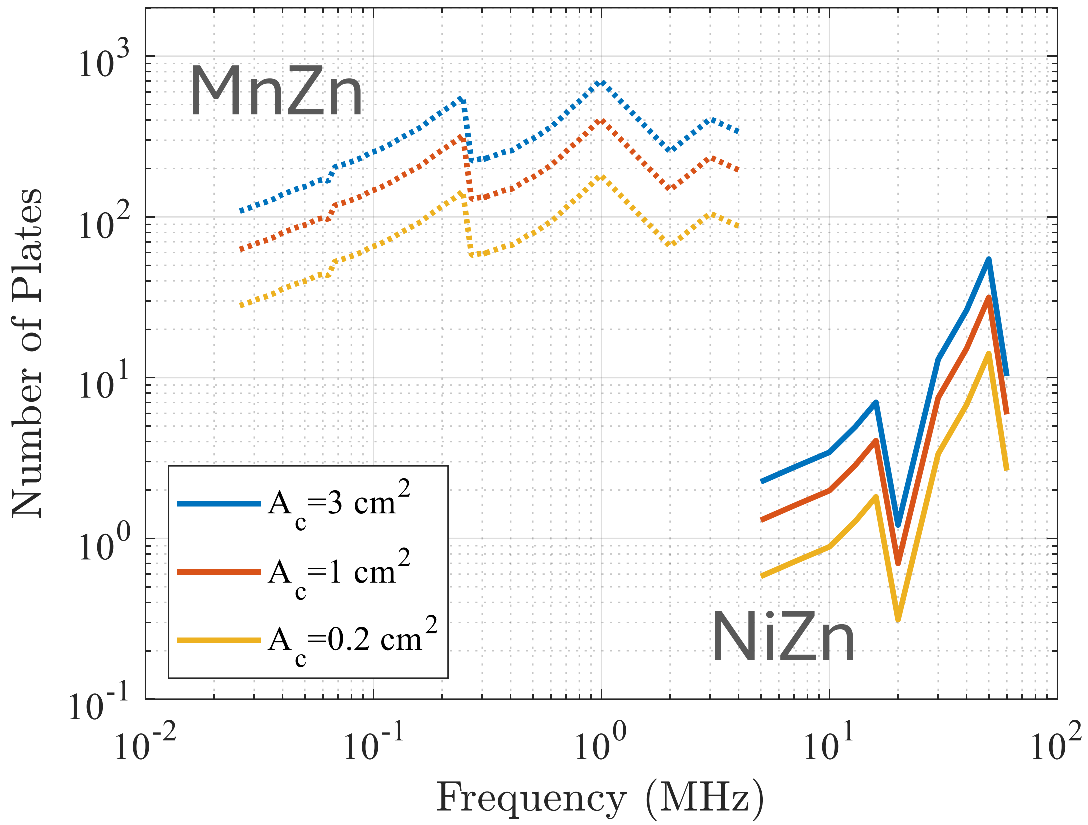
- We can multiply the loss ratio by N for multiple plates

$$\frac{Loss\ in\ N\ Plate}{Loss\ in\ Core} = \sqrt{\frac{\omega\rho}{A_c\mu_0^3}\frac{2NB^2}{P_{v,core}\mu_r^2}}$$

- If we set a limit on the increase in loss, then we can solve for N.

$$\frac{Loss\ in\ N\ Plate}{Loss\ in\ Core} = 0.1$$

$$N = \frac{0.1}{2}\sqrt{\frac{A_c\mu_0^3}{\omega\rho}\frac{P_{v,core}\mu_r^2}{B^2}}$$



For an Al plate with $P_{v,core} = 200\frac{mW}{cm^3}$ and 10% increase in loss

Example Cases

For an aluminum plate with a core with $P_{v,core} = 200\,\frac{mW}{cm^3}, A_c = 0.2\ cm^2$

$$1 \gg \frac{Loss\ in\ Plate}{Loss\ in\ Core} = \sqrt{\frac{\omega\rho}{A_c\mu_0^3}\frac{2B^2}{P_{v,core}\mu_r^2}}$$

MnZn: N97 at 40 kHz

$$Loss\ ratio = 0.003$$

NiZn: 67 at 10 MHz

$$Loss\ ratio = 0.14$$

For 10% increase in loss

N = 39

N ≈ 1

Learn more at the APEC technical session

Session: (T05) Magnetics Applications I (8:30 AM – 12:00 PM)

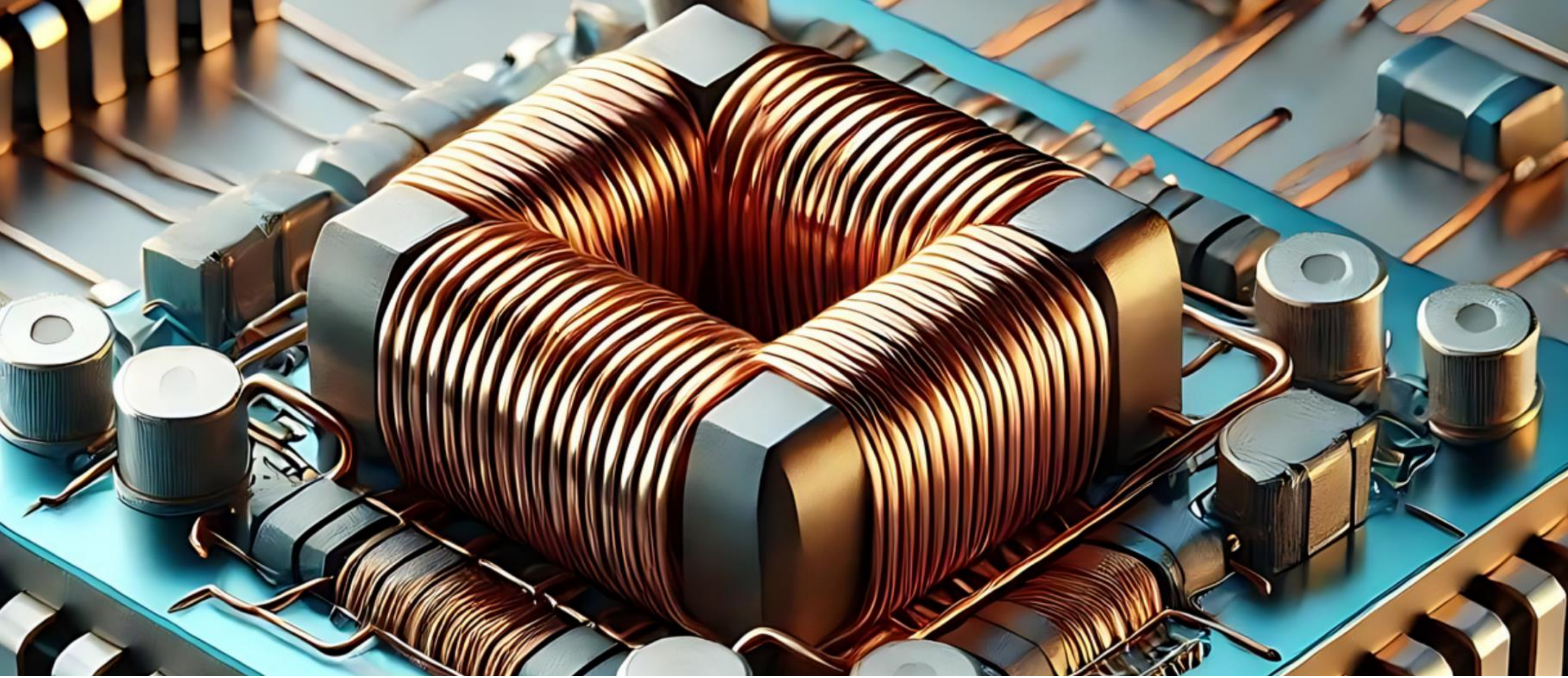
Tuesday, March 18, 2025

8:30 AM – 8:50 AM

(T05.1) Heat Extraction from Ferrite Cores Using Metallic Laminations

Alyssa Brown, Tan Duy Nguyen, Alex Hanson

The University of Texas at Austin



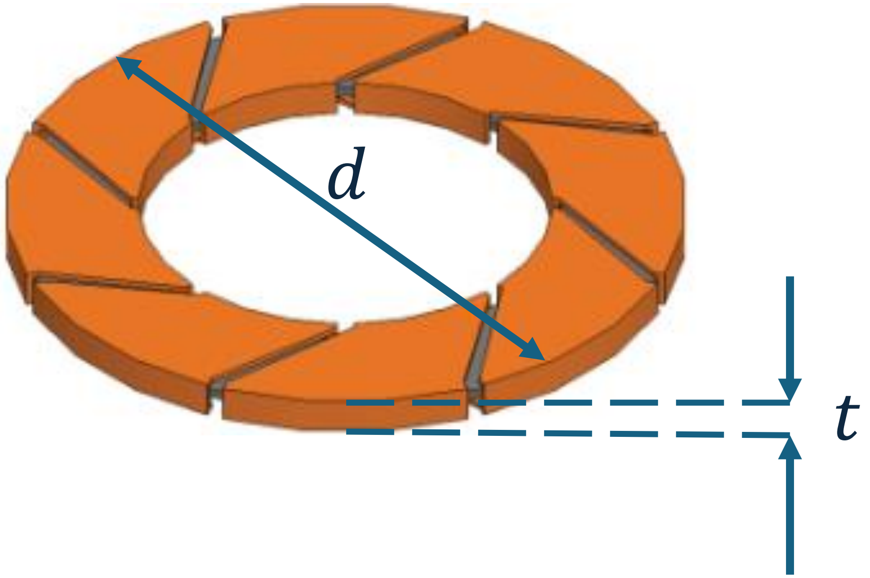
Is scaling different for hyper-planar magnetics?

Alex Hanson

On-chip magnetics are *extremely* planar

$$d \sim mm$$
$$t \sim 100\ um$$
$$\text{Aspect Ratio} \sim 10 \text{ or more}$$

- Surface Area/Volume ratio is huge compared to ordinary magnetics – maybe we can drive them harder?
- Planar magnetics aim to sit on top of CMOS, increasing substrate temp – maybe we can't drive them as hard?



J. Qiu, D. V. Harburg and C. R. Sullivan, "A toroidal power inductor using radial-anisotropy thin-film magnetic material based on a hybrid fabrication process," *2013 IEEE Applied Power Electronics Conference and Exposition (APEC)*, Long Beach, CA, USA, 2013

How much heat can hyper-planar magnetics handle?

Consider designing inductors to live on top of high-performance processors that can dissipate 100-1000 W/cm^2

- Most likely efficiency-limited (have sufficient thermal management solutions for the processor already)

Consider designing a hyper-planar inductor with

- $t = 100\ \mu m$, limited by on-chip manufacturing capability and cost
- A 1% loss budget per unit area P_A =1-10 W/cm^2

Yields a volumetric power density limit:

$$P_{v,planar} = \frac{P_A}{t} = 1 \times 10^5 - 1 \times 10^6\ \mathrm{mW/cm}^3$$

$P_{v,planar}$ is **200-5000 times larger** than typical $P_{v,bulk}$

(due to high aspect ratio and crazy aggressive heatsinking)

What should $\hat{B}$ be for hyper-planar magnetics?

Steinmetz equation at a given frequency:

$$P_v = kB_{ac}^{\beta} \text{ where } 2 < \beta < 3$$

AC B field is limited to:

$$B_{ac} = \left(\frac{P_v}{k}\right)^{1/\beta}$$

$\Longrightarrow \hat{B}_{planar}$ is $\left(\frac{P_{v,planar}}{P_{v,bulk}}\right)^{1/\beta}$ larger than $\hat{B}_{bulk}$

$\hat{B}_{planar}$ is **5.8 to 70.7 times larger** than typical $\hat{B}_{bulk}$ at a given frequency

How does sat vs core loss threshold change?

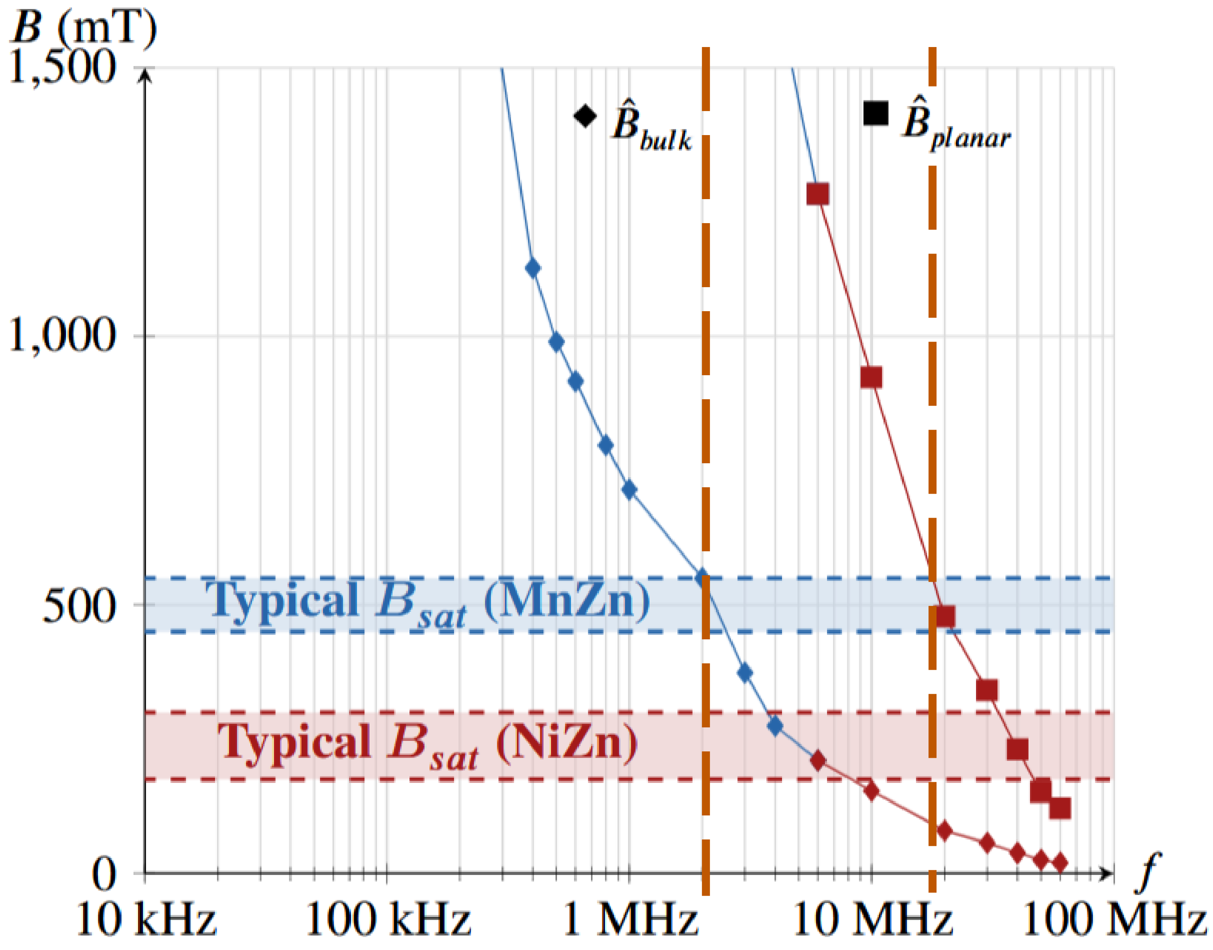
- Recall that saturation vs. core loss threshold is based on $\hat{B}$
- If $B_{sat} > \hat{B}\,\frac{1+\mathcal{R}}{\mathcal{R}}$, design is core loss limited, else saturation limited
- Hyper-planar magnetics can tolerate more core loss than bulk counterparts at a given frequency
- Hyper-planar designs can tolerate $\hat{B}$ that is 5x to 70x compared to what was shown previously
- Pushes threshold for core-loss-limited designs to higher frequencies

How does sat vs core loss threshold change?

- Assume

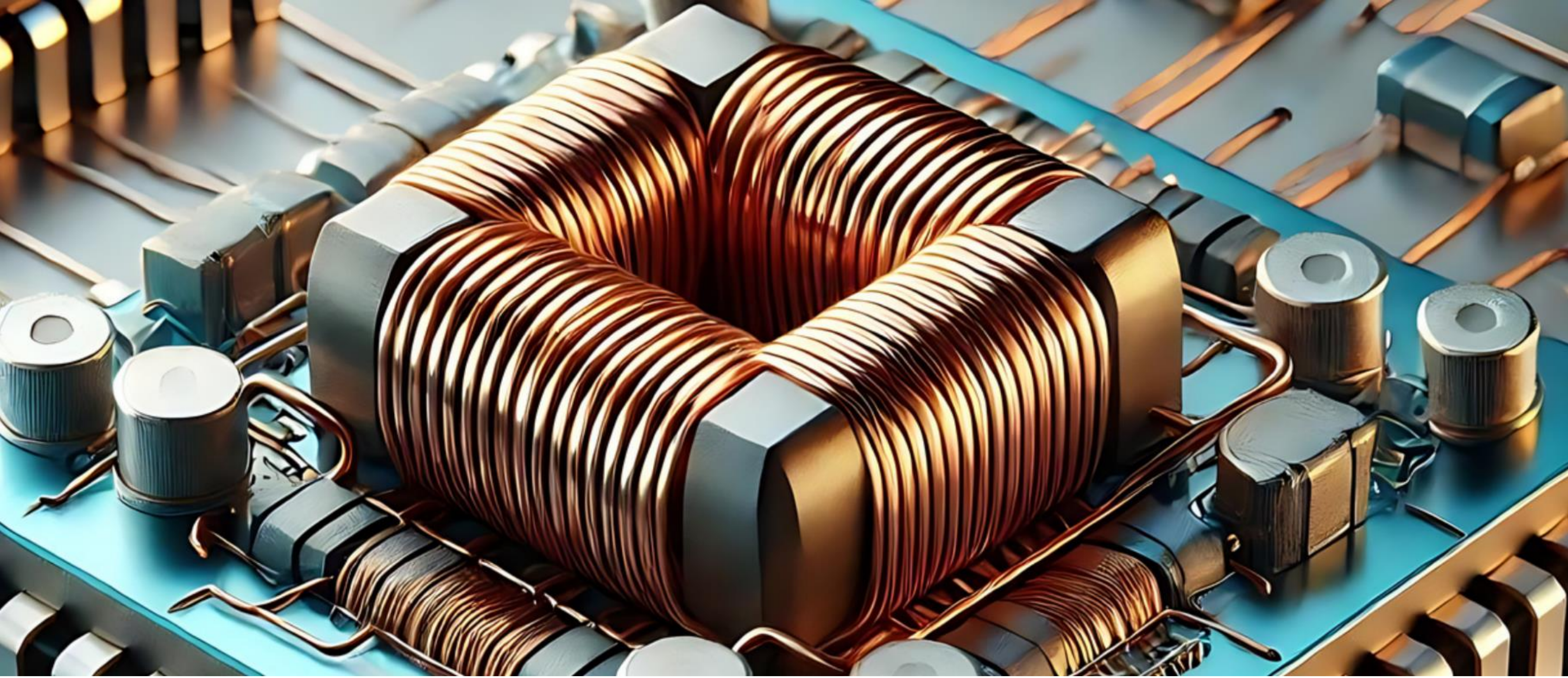
$$\hat{B}_{planar} = 6\,\hat{B}_{bulk}$$
$$\mathcal{R} = 0.1$$

- With $\hat{B}_{bulk}$, designs are core loss limited starting ~2 MHz
- With $\hat{B}_{planar}$, designs are core loss limited starting ~20 MHz
- Threshold between limits for on-chip magnetics (f>10 MHz) is fuzzier



Conclusions about hyper-planar magnetics

- Hyper-planar magnetics may potentially tolerate
  - $P_{v,planar}$ **200-5000 times larger** than typical $P_{v,bulk}$
  - $\hat{B}_{planar}$ is **5.8 to 70.7 times larger** than typical $\hat{B}_{bulk}$
- Hyper-planar magnetic designs may be saturation limited up to higher frequencies than bulk designs
- At the tens of MHz frequencies relevant for on-chip magnetics, hyper-planar designs may still be core loss limited



Take-Home Conclusions

Alex Hanson

Learn more:

“Magnetic Material Selection for Power Inductors and Transformers”

Chapter 6, IET Handbook on Inductive Devices in Power Electronics, 2025

Alyssa Brown, Elaine Ng, Alex Hanson

The University of Texas at Austin

Plus the references contained in the slides

Take-Home Conclusions

There's a lot we can learn by breaking free from the case study and seeking broadly applicable conclusions

⇒ Core loss *or* saturation will limit a design, but not both

⇒ Core-loss-limited designs more common at higher ripple ratio

⇒ Core loss limits kick in at surprisingly low frequencies

⇒ Many magnetic components' performance scales as l^4 or l^5, faster than volume (vs capacitance l^3 or piezoelectric l^2)

⇒ The performance factor predicts continued improvements in power density for frequencies increasing to the tens of MHz

Take-Home Conclusions

⇒ Additional μ_r has no effect on inductor performance above $\mu_{r,critL}$

⇒ $\mu_{r,critL}$ is surprisingly low (< 200) over most useful frequency range

⇒ Additional μ_r has diminishing returns on transformer performance above $\mu_{r,critX}$, which is likewise lower than usually thought

⇒ Air-core magnetics don't win volumetrically until > 50 MHz, leaving opportunity on the table for many applications in the 1-30 MHz range

⇒ Air-core magnetics start to win gravimetrically at ~15 MHz

⇒ Metallic cooling planes can be used over most frequencies, esp. with higher permeability MnZn materials – sometimes dozens of them

⇒ Planar magnetics can handle much higher $\hat{B}$ -- but still may be core loss limited