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A NOVEL APPROACH TO AUTONOMOUS LUNAR LOCALIZATION AND TIMING

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The ability of deep-space spacecraft to estimate its position, velocity, and clock errors without external clock error estimation is investigated. A novel approach to onboard estimation of clock errors is proposed. Uncertainty quantification on the effects of clock errors on translational states is performed. The dynamic correlation between timing errors and localization errors are included in the filter design and leveraged for time estimation. Simulations are performed to numerically validate the proposed approach.

INTRODUCTION

Deep space position, navigation, and timing (PNT) is dominated by ground tracking via the Deep Space Network (DSN). Space vehicles typically establish a two-way communication connection with DSN, which allows the ground to form a precise position estimate (using so-called two-way ranging and doppler measurements) as well as estimate the onboard clock bias and drift. While two-way communications is a common and precise way to relate the position and time of the spacecraft to the known position and time of the ground station(s), it also consumes ground resources and hence it is not an approach scalable to a very large number of spacecraft. Terrestrial and low-Earth orbit applications, on the other hand, typically rely on one-way ranging and doppler by processing signals from GNSS constellations. One-way ranging is possible because the large number of GNSS satellites generate many measurements with diverse geometry allowing for the estimation of both the receiver's position and its clock errors.

Much current work focuses on onboard deep space navigation without ground support.[1,2] Proposed techniques include optical navigation with respect to distant planets/moon ([3–5]), with respect to lunar craters ([6–9]), or using the apparent size and position of the Earth and Moon disks ([10–12]). Other recent approaches include interlink ranging between various satellites in a non-Keplerian orbit (typically via third body perturbations), the so-called LiASON approach[13,14] currently being tested in the Capstone mission.[15] Pulsar navigation is also a possible onboard deep space navigation approach, and encouraging preliminary results were obtained in a recent International Space Station experiment.[16,17] Another approach to onboard deep space navigation relies on one-way ranging from ground stations, this approach is made possible by the Deep Space Atomic Clock, which maintains an extremely accurate estimate of the onboard time and it is therefore able to form one-way ranging measurements.[18–21]

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The ability to precisely keep track of time onboard is crucial to perform accurate navigation, especially when a high level of autonomy is required. The Deep Space Atomic Clock is not an economically viable solution for many spacecraft, especially small ones, requiring the onboard time to be corrected to maintain the desired accuracy throughout the mission.

Different models for satellite clock offset exist, the most commonly accepted being a two– or three–state linear model.[22,23] The first state is the clock bias, or offset, also referred to as the time interval error (TIE),[24] and represents the difference between the onboard time and the true time. The clock bias evolves as the integral of the clock drift, also called fractional frequency offset (because it's physically caused by the clock oscillator frequency error). It's also possible to include a third state, to model the clock drift rate (also known as aging).

Once the clock offset model has been specified, its coefficients can be estimated through measurements relating the onboard time to a reference "true" time; for example using a least squares (LS) estimation process. Alternatively, a Kalman filter approach can be used. Reference 25 proposes a Kalman filter to estimate the clock states of an ensemble of clocks from observations of time differences between them. Reference 26 adds the capability to accept both time and frequency measurements, outliers rejection, and the ability to include any combination of white noises or random walk noises in all three clock states.

Clock estimation with a Kalman filter requires modeling of the process noise which must be deduced from the physical properties of the clock oscillator noise.[27,28] Initial state and covariance conditions must also be specified. References 29 and 30 propose to relax these requirements by employing a finite impulse response unbiased estimation algorithm with measurements of the time interval error. They also compare the performance with a classic two-state clock Kalman filter with data from a real clock.

An adaptive Kalman filter with classified adaptive factors for clock offset estimation has also been proposed.[31] A long short-term memory (LSTM) machine learning approach to accurately express the nonlinear characteristics of the navigation satellite clock bias has been developed as well.[32]

The aforementioned works estimate clock errors from time or frequency measurements from reference oscillators on the ground or on another satellite. They do not include a direct correlation between the clock offset and the position/velocity onboard propagation error. The aim of this work is to perform deep space onboard PNT without requiring an atomic clock onboard our vehicle nor onboard other satellites our vehicle communicates with. This aim is achieved by two novel contributions of this work. First, we derive how onboard clock errors correlate to onboard position and velocity propagation errors. Using this model, we build correlations between localization errors and timing errors. Optical measurements generated onboard and referenced to the spacecraft time are used as navigation aids but they are unable to estimate the onboard time bias. The second contribution of this work is relating one-way ranging signals to the spacecraft position and time, and using this information to update the onboard clock bias estimate.

The built-in correlation between timing and localization errors allows for the estimation of the onboard clock bias from range/position measurements time-tagged by an external source's accurate clock. This time update formulation is also a novel contribution of this work. As a use case, we examine the situation when two-way communication with DSN can be established (for example in conjunction with telemetry downloads) and DSN uploads a position estimate of the spacecraft accompanied with a precise time-tag.

We mechanize our approach with an extended Kalman filter; the filter state includes orbital position, velocity, clock bias and clock drift. The scenario in which simulations are carried out is the same as in Reference 9, in which a least squares solution is used to estimate a constant clock offset (that is, clock drift is assumed to be zero). We improve on the results of Reference 9 by accounting for clock drift and clock noise.

A similar problem and approach to this work was identified and investigated in Reference 33, although the authors do not include any direct correlation between the orbit states errors and the clock errors in the dynamics/error propagation.

CLOCK MODEL AND TIME PROPAGATION

Clock model

The clock model adopted here is a two states linear model, which includes the clock bias, $b(t) = t_{clock} - t$ (where t_{clock} is the uncompensated onboard time and t is the true time), and a constant clock drift $d(t)$. The derivative of the clock bias is modeled as the clock drift plus an additive Gaussian noise, the clock drift is modeled as a random walk. The clock dynamics equations in continuous time are then:

$$\begin{bmatrix} \dot{b}(t) \\ \dot{d}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} b(t) \\ d(t) \end{bmatrix} + \boldsymbol{\nu}_{clock} \tag{1}$$

where $\boldsymbol{\nu}_{clock} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}_{clock}(t))$, that is, it's a Gaussian noise with zero mean and covariance matrix $\boldsymbol{Q}_{clock}(t)$.

The onboard navigation system includes clock errors estimates. The filter equations are implemented in discrete time, therefore we define the true time at the k-th step as t_k, the estimated time as $\hat{t}_k = \hat{t}(t_k)$, and the estimated time bias and drift as $\hat{b}_k = \hat{b}(t_k)$ and $\hat{d}_k = \hat{d}(t_k)$, respectively.

The clock estimated state dynamics equations in discrete time become:

$$\begin{bmatrix} \hat{b}_{k+1} \\ \hat{d}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta\hat{t}_k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{b}_k \\ \hat{d}_k \end{bmatrix} \tag{2}$$

where $\Delta\hat{t}_k = \hat{t}_{k+1} - \hat{t}_k$ is the estimated elapsed time between the two subsequent time steps. This quantity can be related to the estimate of the time drift as follows:

$$\begin{aligned} \Delta\hat{t}_k = \hat{t}_{k+1} - \hat{t}_k &= t_{clock}(t_{k+1}) - \hat{b}_{k+1} - \big(t_{clock}(t_k) - \hat{b}_k\big) \\ &= t_{clock}(t_{k+1}) - t_{clock}(t_k) - \hat{d}_k \, \Delta\hat{t}_k \end{aligned} \tag{3}$$

After some re-arranging:

$$\Delta\hat{t}_k = \big(t_{clock}(t_{k+1}) - t_{clock}(t_k)\big)/(1 + \hat{d}_k) \tag{4}$$

This expression will be used in the next section to derive the state covariance propagation equations.

State and Covariance Propagation

Let $\boldsymbol{x}(t)$ be the state of a stochastic dynamic system (not including the clock bias and drift). The dynamics equations in continuous and discrete form can be written as:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), t) + \boldsymbol{\nu}(t) \tag{5}$$

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}(t_{k+1}) = \boldsymbol{x}(t_k) + \int_{t_k}^{t_{k+1}} \boldsymbol{f}(\boldsymbol{x}(t), t)\, dt + \boldsymbol{\nu}_k \tag{6}$$

In our case, $\boldsymbol{x}(t) = \left[\boldsymbol{r}(t)^T, \boldsymbol{v}(t)^T\right]^T \in \mathcal{R}^6$ and the derivative function is $\boldsymbol{f}(\boldsymbol{x}(t), t) = [\boldsymbol{v}(t), \boldsymbol{a}(t)]$, where $\boldsymbol{a}(t)$ contains all gravitational and non gravitational accelerations acting on the spacecraft. The process noise $\boldsymbol{\nu}_k$ is Gaussian with zero mean and covariance $\boldsymbol{Q}_{posvel_k}$, which is generated using the linear process noise model,[34] as:

$$\mathbf{Q}_{posvel_k} = \begin{bmatrix} \frac{1}{4}(\Delta\hat{t}_k)^4 \boldsymbol{q}\boldsymbol{I}_3 & \frac{1}{2}(\Delta\hat{t}_k)^3 \boldsymbol{q}\boldsymbol{I}_3 \\ \frac{1}{2}(\Delta\hat{t}_k)^3 \boldsymbol{q}\boldsymbol{I}_3 & (\Delta\hat{t}_k)^2 \boldsymbol{q}\boldsymbol{I}_3 \end{bmatrix}, \tag{7}$$

where $\boldsymbol{q} = [q_1 \; q_2 \; q_3]^T$ represents the process noise acceleration in inertial frame coordinates.

The estimated state at time t_k, $\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}(t_k)$, is propagated forward to t_{k+1} with our estimate of the time step $\Delta\hat{t}_k$:

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}(t_{k+1}) = \hat{\boldsymbol{x}}_k + \int_0^{\Delta\hat{t}_k} \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\, d\tau \tag{8}$$

where $\Delta\hat{t}_k$ has been derived in Eq. (4).

The state estimation error is defined as:

$$\delta\boldsymbol{x}_{k+1} = \boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1} = \boldsymbol{x}(t_{k+1}) - \hat{\boldsymbol{x}}(t_{k+1}) \tag{9}$$

The clock bias and drift estimation errors are similarly defined:

$$\begin{bmatrix} \delta b_k \\ \delta d_k \end{bmatrix} = \begin{bmatrix} b_k \\ d_k \end{bmatrix} - \begin{bmatrix} \hat{b}_k \\ \hat{d}_k \end{bmatrix} = \begin{bmatrix} b(t_k) \\ d(t_k) \end{bmatrix} - \begin{bmatrix} \hat{b}(t_k) \\ \hat{d}(t_k) \end{bmatrix} \tag{10}$$

Substituting Eq. (6) and Eq. (8) in Eq. (9), we have:

$$\begin{aligned} \delta\boldsymbol{x}_{k+1} &= \int_0^{\Delta t_k} \boldsymbol{f}(\boldsymbol{x}(t_k + \tau), t_k + \tau)\, d\tau + \boldsymbol{\nu}_k - \int_0^{\Delta\hat{t}_k} \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\, d\tau \\ &= \int_0^{\Delta t_k} \Big(\boldsymbol{f}(\boldsymbol{x}(t_k + \tau), t_k + \tau) - \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\Big)\, d\tau + \boldsymbol{\nu}_k - \int_{\Delta t_k}^{\Delta\hat{t}_k} \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\, d\tau \\ &= \int_0^{\Delta t_k} \Big(\boldsymbol{f}(\boldsymbol{x}(t_k + \tau), t_k + \tau) - \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\Big)\, d\tau + \boldsymbol{\nu}_k + \int_{\Delta\hat{t}_k}^{\Delta t_k} \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\, d\tau \\ &\approx \boldsymbol{F}(t_{k+1}, t_k)\, \delta\boldsymbol{x}_k - \left.\frac{\partial \boldsymbol{f}(\hat{\boldsymbol{x}}_k, t)}{\partial t}\right|_{\hat{t}_k} (t_k - \hat{t}_k)\, \Delta\hat{t}_k + \boldsymbol{\nu}_k + \boldsymbol{f}(\hat{\boldsymbol{x}}_{k+1}, \hat{t}_k + \Delta\hat{t}_k)\, \big(\Delta t_k - \Delta\hat{t}_k\big) \end{aligned} \tag{11}$$

where $\boldsymbol{F}(t_{k+1}, t_k)$ is the position-velocity state transition matrix from t_k to t_{k+1} and second and higher orders terms are neglected. The term $(t_k - \hat{t}_k)$ (i.e. the error on estimated time) in Eq. (11) is equal to the error on the clock offset estimate, that is, $(t_k - \hat{t}_k) = \delta b_k$.

The true time step Δt_k can be expressed as a function of the drift estimation error δd_k, by starting from Eq. (4) and using Eq. (2), as follows:

$$\begin{aligned}\Delta\hat{t}_k &= \big(t_{clock}(t_{k+1}) - t_{clock}(t_k)\big)/\big(1+\hat{d}_k\big) = \big(t_{k+1} + b_{k+1} - t_k - b_k\big)/\big(1+\hat{d}_k\big) \\ &= \big(\Delta t_k + d_k \Delta t_k\big)/\big(1+\hat{d}_k\big) = \Delta t_k\big(1+d_k\big)/\big(1+\hat{d}_k\big)\end{aligned} \tag{12}$$

hence

$$\Delta t_k = \Delta\hat{t}_k \frac{1+\hat{d}_k}{1+d_k} \approx \Delta\hat{t}_k - \frac{\Delta\hat{t}_k}{1+\hat{d}_k}\,\delta d_k \tag{13}$$

The term $\big(\Delta t_k - \Delta\hat{t}_k\big)$ in Eq. (11) can then be written as $\big(-\frac{\Delta\hat{t}_k}{1+\hat{d}_k}\,\delta d_k\big)$.

Therefore, it's possible to write the position-velocity estimation error propagation equation as:

$$\delta\boldsymbol{x}_{k+1} = \boldsymbol{F}(t_{k+1}, t_k)\,\delta\boldsymbol{x}_k + \left[\left.\frac{\partial\boldsymbol{f}(\hat{\boldsymbol{x}}_k, t)}{\partial t}\right|_{\hat{t}_k}\Delta\hat{t}_k\right]\delta b_k + \left[-\boldsymbol{f}(\hat{\boldsymbol{x}}_{k+1}, \hat{t}_{k+1})\,\frac{\Delta\hat{t}_k}{1+\hat{d}_k}\right]\delta d_k + \boldsymbol{\nu}_k \tag{14}$$

The clock bias error is:

$$\begin{aligned}\delta b_{k+1} &= b_{k+1} - \hat{b}_{k+1} = b_k + d_k\Delta t_k - \hat{b}_k - \hat{d}_k\Delta\hat{t}_k \\ &\approx \delta b_k + (\delta d_k + \hat{d}_k)(\Delta\hat{t}_k - \frac{\Delta\hat{t}_k}{1+\hat{d}_k}\delta d_k) - \hat{d}_k\Delta\hat{t}_k \\ &\approx \delta b_k + \Delta\hat{t}_k\delta d_k - \frac{\Delta\hat{t}_k}{1+\hat{d}_k}\,\hat{d}_k\,\delta d_k \\ &= \delta b_k + \frac{\Delta\hat{t}_k}{1+\hat{d}_k}\,\delta d_k\end{aligned} \tag{15}$$

where, in the step from the second to the third line, the term $\hat{d}_k\Delta\hat{t}_k$ cancels out, and the second order term $\frac{\Delta\hat{t}_k}{1+\hat{d}_k}\delta d_k^2$ is neglected.

Finally, since the clock drift is assumed with constant dynamics affected by process noise, its estimation error propagates as:

$$\delta d_{k+1} = \delta d_k \tag{16}$$

The estimation error propagation equations are then written in matrix form as below:

$$\begin{bmatrix}\delta\boldsymbol{x}_{k+1}\\ \delta b_{k+1}\\ \delta d_{k+1}\end{bmatrix} = \begin{bmatrix}\boldsymbol{F}(t_{k+1}, t_k) & \left.\frac{\partial\boldsymbol{f}(\hat{\boldsymbol{x}}_k, t)}{\partial t}\right|_{\hat{t}_k}\Delta\hat{t}_k & -\boldsymbol{f}(\hat{\boldsymbol{x}}_{k+1}, \hat{t}_{k+1})\frac{\Delta\hat{t}_k}{1+\hat{d}_k}\\ \mathbf{0} & 1 & \frac{\Delta\hat{t}_k}{1+\hat{d}_k}\\ \mathbf{0} & 0 & 1\end{bmatrix}\begin{bmatrix}\delta\boldsymbol{x}_k\\ \delta b_k\\ \delta d_k\end{bmatrix} + \begin{bmatrix}\boldsymbol{\nu}_k\\ \boldsymbol{\nu}_{clock,b,k}\\ \boldsymbol{\nu}_{clock,d,k}\end{bmatrix} \tag{17}$$

The above equation is in the form:

$$\boldsymbol{e}_{k+1} = \boldsymbol{\Phi}_k \boldsymbol{e}_k + \overline{\boldsymbol{\nu}}_k \tag{18}$$

Therefore, the predicted estimation error covariance at time $(k+1)$, $\bar{P}_{k+1}$, is obtained as:

$$\bar{P}_{k+1} = \boldsymbol{\Phi}_k \boldsymbol{P}_k \boldsymbol{\Phi}_k^{\mathrm{T}} + \boldsymbol{Q}_k \tag{19}$$

Where $\boldsymbol{Q}_k$ is the full (eight by eight) state process noise covariance, which comprises the six by six position/velocity block $\boldsymbol{Q}_{posvel_k}$, defined in Eq. (7), and the two by two clock process noise block $\boldsymbol{Q}_{clock_k}$, which is defined as:[27,28]

$$\boldsymbol{Q}_{clock} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \tag{20}$$

$$q_{11} = \frac{h_0}{2}\Delta\hat{t}_k + 2h_{-1}(\Delta\hat{t}_k)^2 + \frac{2}{3}\pi^2 h_{-2}(\Delta\hat{t}_k)^3 \tag{21}$$

$$q_{12} = q_{21} = h_{-1}\Delta\hat{t}_k + \pi^2 h_{-2}(\Delta\hat{t}_k)^2 \tag{22}$$

$$q_{22} = \frac{h_0}{2\Delta\hat{t}_k} + 4h_{-1} + \frac{8}{3}\pi^2 h_{-2}\Delta\hat{t}_k \tag{23}$$

In this work, the clock Allan parameters are set to $h_0 = 2e-19$, $h_{-1} = 7e-21$ and $h_{-2} = 2e-20$.[28]

SIMULATION SCENARIO

In our simulation scenario, the spacecraft orbits the Moon and performs optical navigation with respect to the Moon surface through an onboard camera. The measurements coming from the optical system are the bearing angles of the craters in the camera field of view, and the timestamps of these measurements are affected by the non-constant onboard clock error described in the previous Section.

Two types of external measurements are considered. The first one is a position measurement provided from ground, obtained by processing batches of DSN range and range rate measurements via two-way ranging; the second consists of DSN one-way ranging measurements.

Satellite-to-station visibility is modeled by taking into account the relative position of Earth, Moon, and the spacecraft. No atmospheric delays are modeled in the DSN measurements. The spacecraft's attitude is assumed to be such that the camera is always pointing downward to the Moon surface.

The dynamics model includes a spherical harmonic model for the Moon gravity, plus Earth and Sun as third body perturbations. The filter propagates the dynamics in the Moon Centered Inertial (MCI) frame, the onboard state is 8-dimensional (shown in Table 1).

Craters measurement update

The first type of measurement available to the spacecraft, time-tagged by the onboard clock, is given by bearing angles α and β that represent the location of crater centroids with respect to the camera.

Table 1: Filter estimated states

State	# of Elements	Description
$[\boldsymbol{r}]_{MCI}$	3	Spacecraft position in MCI frame
$[\boldsymbol{v}]_{MCI}$	3	Spacecraft velocity in MCI frame
b	1	Onboard clock offset
d	1	Onboard clock drift

Every time a new image is acquired, a certain number of craters are detected through a Mask R-CNN detector, and are then matched to craters of an onboard catalog. The reader is referred to Reference 9 for further details about the detection and matching algorithms.

In order to compute the expected measurement, which is needed to obtain the residual in the classic Kalman filter formulation, the bearing angles are calculated using the vector from the camera to the crater centroid at estimated time, $\boldsymbol{r}_{cr}^{CAM}(\hat{t}_k)$. This vector is first computed in the MCI frame, and then transformed in camera frame:

$$\left[\hat{\boldsymbol{r}}_{cr}^{CAM}(\hat{t}_k)\right]_{CAM} = [\hat{x}\ \hat{y}\ \hat{z}\,](\hat{t}_k) = \hat{\boldsymbol{T}}_{MCI}^{CAM}(\hat{t}_k)\left(\hat{\boldsymbol{T}}_{MCMF}^{MCI}(\hat{t}_k)\left[\boldsymbol{r}_{cr}(\hat{t}_k)\right]_{MCMF} - \left[\hat{\boldsymbol{r}}(\hat{t}_k)\right]_{MCI}\right) \tag{24}$$

$$\hat{\boldsymbol{T}}_{MCI}^{CAM}(\hat{t}_k) = \boldsymbol{T}_{B}^{CAM}\hat{\boldsymbol{T}}_{MCI}^{B}(\hat{t}_k) \tag{25}$$

where $\left[\boldsymbol{r}_{cr}(\hat{t}_k)\right]_{MCI}$ and $\left[\hat{\boldsymbol{r}}(\hat{t}_k)\right]_{MCI}$ are the crater centroid and spacecraft center of mass positions in the Moon Centered Inertial (MCI) frame at estimated time, respectively, $\hat{\boldsymbol{T}}_{MCI}^{CAM}(\hat{t}_k)$ is the rotation matrix from the MCI frame to the camera frame, $\hat{\boldsymbol{T}}_{MCMF}^{MCI}(\hat{t}_k)$ is the transformation matrix from the Moon Centered Moon Fixed (MCMF) frame to the MCI frame, and the subscript B represents the body frame. The spacecraft body frame is assumed aligned with the North-East-Down frame.

The expected measurement can then be computed as:

$$\hat{\boldsymbol{y}}_k = \hat{\boldsymbol{y}}(\hat{t}_k) = \begin{bmatrix}\hat{\alpha}_k \\ \hat{\beta}_k\end{bmatrix} + \boldsymbol{w}_k = \begin{bmatrix}\mathrm{atan}(\hat{x}/\hat{z}) \\ \mathrm{atan}(\hat{y}/\hat{z})\end{bmatrix} + \boldsymbol{w}_k \tag{26}$$

where the measurement noise $\boldsymbol{w}_k$ is assumed gaussian with zero mean and covariance matrix $\boldsymbol{R}_k$.

The partials of measurement equations with respect to the estimated position and clock bias are reported below.

$$\frac{\partial \boldsymbol{y}_k}{\partial \hat{\boldsymbol{r}}} = \frac{\partial \boldsymbol{y}_k}{\partial \hat{\boldsymbol{r}}_{cr}^{CAM}}\frac{\partial \hat{\boldsymbol{r}}_{cr}^{CAM}}{\partial \hat{\boldsymbol{r}}} \tag{27}$$

$$\frac{\partial \boldsymbol{y}_k}{\partial \hat{b}_k} = \begin{bmatrix}\frac{\partial \hat{\alpha}_k}{\partial \hat{b}_k} \\ \frac{\partial \hat{\beta}_k}{\partial \hat{b}_k}\end{bmatrix} = \frac{\partial \boldsymbol{y}_k}{\partial \hat{\boldsymbol{r}}_{cr}^{CAM}}\frac{\partial\ \hat{\boldsymbol{r}}_{cr}^{CAM}}{\partial \hat{b}_k} \tag{28}$$

with:

$$\frac{\partial \boldsymbol{y}_k}{\partial \hat{\boldsymbol{r}}_{cr}^{CAM}} = \begin{bmatrix} \hat{z}/(\hat{x}^2+\hat{z}^2) & 0 & -\hat{x}/(\hat{x}^2+\hat{z}^2) \\ 0 & \hat{z}/(\hat{y}^2+\hat{z}^2) & -\hat{y}/(\hat{y}^2+\hat{z}^2) \end{bmatrix} \tag{29}$$

$$\frac{\partial \hat{\boldsymbol{r}}_{cr}^{CAM}}{\partial \hat{\boldsymbol{r}}} = -\boldsymbol{T}_B^{CAM} \hat{\boldsymbol{T}}_{MCI}^{B}(\hat{t}_k) \tag{30}$$

$$\begin{aligned} \frac{\partial \boldsymbol{r}_{cr}^{CAM}}{\partial \hat{b}_k} &= \boldsymbol{T}_B^{CAM} \hat{\boldsymbol{T}}_{MCI}^{B}(\hat{t}_k) \frac{\partial \hat{\boldsymbol{T}}_{MCMF}^{MCI}(\hat{t}_k)}{\partial \hat{t}_k} \frac{\partial \delta t_k}{\partial \delta b_k} \left[\boldsymbol{r}_{cr}(\hat{t}_k)\right]_{MCMF} \\ &= \boldsymbol{T}_B^{CAM} \hat{\boldsymbol{T}}_{MCI}^{B}(\hat{t}_k) \frac{\partial \hat{\boldsymbol{T}}_{MCMF}^{MCI}(\hat{t}_k)}{\partial \hat{t}_k} \left[\boldsymbol{r}_{cr}(\hat{t}_k)\right]_{MCMF} \end{aligned} \tag{31}$$

The term $\frac{\partial \hat{\boldsymbol{T}}_{MCMF}^{MCI}(\hat{t}_k)}{\partial \hat{t}_k}$ is the derivative, with respect to the estimated time, of the MCMF to MCI rotation matrix, and $\frac{\partial \delta t_k}{\partial \delta b_k} = 1$, since $\delta t_k = \delta b_k$ as described previously. Essentially, the clock error affects the computation of this transformation matrix because of its dependence on the estimated time.

The partials with respect to the spacecraft velocity and the clock drift are zero.

The measurement partial matrix $\boldsymbol{H}_k$ is then:

$$\boldsymbol{H}_k = \begin{bmatrix} \hat{z}/(\hat{x}^2+\hat{z}^2) & 0 & -\hat{x}/(\hat{x}^2+\hat{z}^2) & 0 & 0 & 0 & \frac{\partial \hat{\alpha}_k}{\partial \hat{b}_k} & 0 \\ 0 & \hat{z}/(\hat{y}^2+\hat{z}^2) & -\hat{y}/(\hat{y}^2+\hat{z}^2) & 0 & 0 & 0 & \frac{\partial \hat{\beta}_k}{\partial \hat{b}_k} & 0 \end{bmatrix} \tag{32}$$

The Kalman gain and the updated full state are then computed as:

$$\mathbf{K}_k = \bar{\boldsymbol{P}}_k \mathbf{H}_k^T \left(\mathbf{H}_k \bar{\boldsymbol{P}} \mathbf{H}_k^T + \mathbf{R}_k\right)^{-1} \tag{33}$$

$$\hat{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_k + \boldsymbol{K}_k(\boldsymbol{y}_k - \boldsymbol{H}_k \bar{\boldsymbol{x}}_k) \tag{34}$$

where $\bar{\boldsymbol{x}}_k$ and $\bar{\boldsymbol{P}}$ are the predicted state and covariance at time k, obtained by propagating the estimated state and covariance from $\hat{t}_{k-1}$ to $\hat{t}_k$ through Eq. (17) and Eq. (19), respectively.

One-way ranging

When performing DSN one-way ranging, the ground station broadcasts a signal according to its precise ground clock while the spacecraft is subject to onboard clock errors. Hence the spacecraft receives a pseudorange measurement affected by the clock bias b. The range measurement $\boldsymbol{\rho}_j(t_j)$ is therefore modeled as follows:

$$\boldsymbol{\rho}_j(t_j) = ||\,[\boldsymbol{r}]_{ECI} - [\boldsymbol{r_{station}}]_{ECI}\,|| + b = ||\,[\boldsymbol{r}]_{MCI} + [\boldsymbol{r_{EM}}]_{ECI} - [\boldsymbol{r_{station}}]_{ECI}\,|| + b \tag{35}$$

where $[\boldsymbol{r_{station}}]_{ECI}$ is the DSN station position in ECI, $[\boldsymbol{r_{EM}}]_{ECI}$ is the Earth-Moon position vector in ECI, and b is the onboard clock offset.

The Kalman gain and state update equations are:

$$\boldsymbol{K}_k = \bar{\boldsymbol{P}}_k \boldsymbol{H}_\rho^{\mathrm{T}} \Big(\boldsymbol{H}_\rho \bar{\boldsymbol{P}}_k \boldsymbol{H}_\rho^{\mathrm{T}} + \boldsymbol{R}_j \Big)^{-1} \tag{36}$$

$$\hat{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_k + \boldsymbol{K}_k(\boldsymbol{\rho}_j - \boldsymbol{H}_\rho \hat{\boldsymbol{x}}_j) \tag{37}$$

where $\boldsymbol{H}_\rho \in \mathcal{R}^{1\times 8}$ is the matrix of partials of spacecraft-to-station range with respect to the filter state.

$$\boldsymbol{H}_\rho = \frac{\partial \boldsymbol{\rho}_j}{\partial \boldsymbol{x}} = \frac{\partial \boldsymbol{\rho}_j}{\partial \big[\, [\boldsymbol{r}]_{MCI}^T \quad [\boldsymbol{v}]_{MCI}^T \quad b \quad d \big]^T} = \begin{bmatrix} \frac{\big[[\boldsymbol{r}]_{MCI} + [\boldsymbol{r_{EM}}]_{ECI} - [\boldsymbol{r_{station}}]_{ECI}\big]^T}{\|[\boldsymbol{r}]_{MCI} + [\boldsymbol{r_{EM}}]_{ECI} - [\boldsymbol{r_{station}}]_{ECI}\|} & \boldsymbol{O}_{1\times 3} & 1 & 0 \end{bmatrix} \tag{38}$$

Ground position update

A possibility arising from explicitly accounting for the correlations between timing and localization errors is to estimate the clock errors from position measurements time tagged with true time rather than onboard time. This situation can occur, for example, when DSN tracks a satellite, performs orbit determination on it, and uploads the result which is then used by the satellite as an external measurement. Let $\boldsymbol{p}_j$ be the position measurement provided from the ground, time-tagged with the precisely kept ground time t_j.

The proposed approach starts by propagating the state and state transition matrix from the current estimated time $\hat{t}_k$ to time t_j. The equations used will be slightly different from the ones derived before, because t_j does not contain the onboard clock offset. Let Δt_j and $\Delta \hat{t}_j$ be the true and estimated elapsed times from current time to the ground update time:

$$\Delta t_j = t_j - t_k \tag{39}$$

$$\Delta \hat{t}_j = t_j - \hat{t}_k = t_j - t_k - \delta b_k = \Delta t_j - \delta b_k \tag{40}$$

The difference $(\Delta t_j - \Delta \hat{t}_j)$ is then given by the error on estimated clock bias δb_k.

Position, velocity, and clock states are propagated as:

$$\hat{\boldsymbol{x}}_j = \hat{\boldsymbol{x}}_k + \int_0^{\Delta \hat{t}_j} \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\, d\tau \tag{41}$$

$$\begin{bmatrix} \hat{b}_j \\ \hat{d}_j \end{bmatrix} = \begin{bmatrix} 1 & \Delta \hat{t}_j \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{b}_k \\ \hat{d}_k \end{bmatrix} \tag{42}$$

To derive the state transition matrix from $\hat{t}_k$ to t_j, we start again from the state estimation error:

$$\begin{aligned}
\delta \boldsymbol{x}_j = \boldsymbol{x}_j - \hat{\boldsymbol{x}}_j &= \int_0^{\Delta t_j} \boldsymbol{f}(\boldsymbol{x}(t_k + \tau), t_k + \tau)\, d\tau + \boldsymbol{\nu}_k - \int_0^{\Delta \hat{t}_j} \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\, d\tau \\
&= \int_0^{\Delta t_j} \Big(\boldsymbol{f}(\boldsymbol{x}(t_k + \tau), t_k + \tau) - \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau) \Big)\, d\tau + \boldsymbol{\nu}_k - \int_{\Delta t_j}^{\Delta \hat{t}_j} \boldsymbol{f}(\hat{\boldsymbol{x}}(t_k + \tau), \hat{t}_k + \tau)\, d\tau \\
&\approx \boldsymbol{F}(t_j, t_k)\, \delta \boldsymbol{x}_k + \left. \frac{\partial \boldsymbol{f}(\hat{\boldsymbol{x}}_k, t)}{\partial t} \right|_{\hat{t}_k} \delta b_k\, \Delta \hat{t}_j + \boldsymbol{\nu}_k + \boldsymbol{f}(\hat{\boldsymbol{x}}_j, \hat{t}_j)\, \big(\Delta t_j - \Delta \hat{t}_j \big) \\
&= \boldsymbol{F}(t_j, t_k)\, \delta \boldsymbol{x}_k + \left[\left. \frac{\partial \boldsymbol{f}(\hat{\boldsymbol{x}}_k, t)}{\partial t} \right|_{\hat{t}_k} \Delta \hat{t}_j + \boldsymbol{f}(\hat{\boldsymbol{x}}_j, \hat{t}_j) \right] \delta b_k + \boldsymbol{\nu}_k
\end{aligned} \tag{43}$$

The clock bias estimation error evolves as:

$$\begin{aligned}\delta b_j &= b_j - \hat{b}_j = b_k + d_k \Delta t_j - \hat{b}_k - \hat{d}_k \Delta \hat{t}_j \\ &= \delta b_k + (\hat{d}_k + \delta d_k)(\Delta \hat{t}_j + \delta b_k) - \hat{d}_k \Delta \hat{t}_j \\ &= \delta b_k + \hat{d}_k \Delta \hat{t}_j + \Delta \hat{t}_j \delta d_k + \hat{d}_k \delta b_k + \delta d_k \delta b_k - \hat{d}_k \Delta \hat{t}_j \\ &\approx \delta b_k + \hat{d}_k\, \delta b_k + \Delta \hat{t}_j \delta d_k = (1 + \hat{d}_k) \delta b_k + \Delta \hat{t}_j \delta d_k \end{aligned} \tag{44}$$

where the second order term $\delta d_k \delta b_k$ is neglected.

The drift estimation error evolves as:

$$\delta d_j = \delta d_k \tag{45}$$

The estimation error propagation equations can be written in matrix form as:

$$\begin{bmatrix} \delta \boldsymbol{x}_j \\ \delta b_j \\ \delta d_j \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}(t_j, t_k) & \left.\frac{\partial \boldsymbol{f}(\hat{\boldsymbol{x}}_k, t)}{\partial t}\right|_{\hat{t}_k} \Delta \hat{t}_j + \boldsymbol{f}(\hat{\boldsymbol{x}}_j, \hat{t}_j) & \boldsymbol{0} \\ \boldsymbol{0} & 1 + \hat{d}_k & \Delta \hat{t}_j \\ \boldsymbol{0} & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta \boldsymbol{x}_k \\ \delta b_k \\ \delta d_k \end{bmatrix} + \begin{bmatrix} \boldsymbol{\nu}_k \\ 0 \\ 0 \end{bmatrix} \tag{46}$$

which is in the form:

$$\boldsymbol{e}_j = \boldsymbol{\Phi}(t_j, t_k) \boldsymbol{e}_k + \bar{\boldsymbol{\nu}}_k \tag{47}$$

Therefore, when a ground measurement is received, Kalman gain computations and state update are performed as follows:

$$\boldsymbol{K}_k = \bar{\boldsymbol{P}}_k \big(\boldsymbol{H}_g \boldsymbol{\Phi}(t_j, t_k)\big)^{\mathrm{T}} \Big(\big(\boldsymbol{H}_g \boldsymbol{\Phi}(t_j, t_k)\big) \bar{\boldsymbol{P}}_k \big(\boldsymbol{H}_g \boldsymbol{\Phi}(t_j, t_k)\big)^{\mathrm{T}} + \boldsymbol{R}_{pos_j} \Big)^{-1} \tag{48}$$

$$\hat{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_k + \boldsymbol{K}_k (\boldsymbol{p}_j - \boldsymbol{H}_g \bar{\boldsymbol{x}}_j) \tag{49}$$

where $\boldsymbol{H}_g \in \mathcal{R}^{3 \times 8}$ is the matrix of partials of ground measurement equation,

$$\boldsymbol{H}_g = \frac{\partial \boldsymbol{p}_j}{\partial \boldsymbol{x}} = \frac{\partial\, [\boldsymbol{r}]_{MCI}}{\partial \big[\, [\boldsymbol{r}]_{MCI}^T \quad [\boldsymbol{v}]_{MCI}^T \quad b \quad d \big]^T} = \big[\boldsymbol{I}_3 \quad \boldsymbol{O}_{3 \times 5}\big] \tag{50}$$

and $\boldsymbol{R}pos_j$ is the ground position measurement noise covariance matrix.

SIMULATION RESULTS

This section presents simulations of a spacecraft in a 100 km circular and equatorial orbit around the Moon. The onboard camera takes an image of the Moon surface every 5 seconds. The data sources to generate images in our simulations is the LROC Global Morphologic Maps[35] and the Robbins lunar crater database.[36] Each image is given as an input to the Mask R-CNN detector, and is assigned a confidence value to remove false detections (detected craters with a confidence value lower than the threshold are not passed to the filter). Each detected crater is fitted to an ellipse

Table 2: Simulation parameters

Symbol	Description	Value
$[\boldsymbol{r}]_{MCI}\,(t_0)$	True initial spacecraft position in MCI frame [km]	$[1837.4 \quad 0.0 \quad 0.0]^T$
$[\boldsymbol{v}]_{MCI}\,(t_0)$	True initial spacecraft velocity in MCI frame [km/s]	$[\ 0.0 \quad 1.6335 \quad 0.0\ ]^T$
$b(t_0)$	True initial onboard clock offset [s]	0.1
$d(t)$	True onboard clock drift (constant) [ND]	1e-4
N	Image pixel length [pixels]	909
f	Camera focal length [pixels]	909
σ_α	Noise standard deviation on bearing angle α [pixels]	4.3921
σ_β	Noise standard deviation on bearing angle β [pixels]	4.3921
$\sigma_{\alpha\beta}$	Noise correlation between bearing angles α and β	0.0
σ_ρ	DSN range noise standard deviation [m]	43.4
$\sigma_{\dot{\rho}}$	DSN range rate noise standard deviation [m/s]	1
q	Process noise standard deviation [m/s^2]	$[1e-6 \quad 1e-6 \quad 1e-6]^T$

whose center gives the crater centroid pixel coordinates expressed in the camera frame, x_c and y_c. These coordinates are then used to match the detected craters to craters in the surface feature catalog available to the spacecraft (the Robbins lunar crater catalog). From this database, a local catalog is created at each time step by projecting the spacecraft camera's field of view pointing downward towards the surface according to the spacecraft's body frame (coinciding with the North-East-Down frame). After a local catalog is created, the Munkres or Hungarian Matching algorithm[37,38] is used to identify craters based on entries in the crater catalog.

The bearing angles α and β of the matched craters' centroids are then computed as:

$$\alpha = \arctan\left(\frac{x_c - 0.5N}{f}\right) \tag{51}$$

$$\beta = \arctan\left(\frac{y_c - 0.5N}{f}\right), \tag{52}$$

where N is the image pixel length and f is the camera focal length. Further details about the craters detection and matching algorithms can be found in Reference 9.

To simulate clock errors, images' timestamps are perturbed with a clock offset which increases with a constant drift.

Some of the parameters used in the simulations are reported in Table 2.

Neglecting the clock offset

Figure 1.a-d shows the estimation errors (red lines) with the 3σ error bounds (blue lines) over a 12 hours simulation, and the innovation residuals are shown in Figure 1.e. This simulation has been performed without including any of the terms related to the clock offset in the filter equations and assuming the spacecraft is processing craters measurements only, that is, no ground-based measurements are provided.

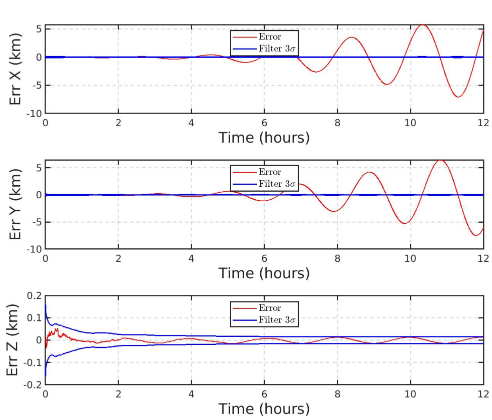
It is important noticing that the values for the clock bias and drift used in these simulations are those of the low-end oscillator from Ref.[28] This compensated crystal is not meant to keep un-aided spacecraft time, bur rather to be used in GPS receivers processing at least four pseudorange measurements consistently. While most spacecraft will probably select a more accurate onboard clock, the point to be taken is that in subsequent sections we are able to accurately keep an onboard time with this inexpensive oscillator and without GPS nor DSN tracking. Additionally, the chosen clock values allow to show the effectiveness of the proposed method within one-day simulations. More accurate clocks without ground steering would have the same trends, albeit after longer propagation times.

The position and velocity errors start diverging from the 3σ uncertainty bounds after about two hours because the filter is not taking into account that the measurements' timestamps are affected by the clock offset, whose estimation error also increases with time. The position and velocity errors are computed as the difference between the estimated state and the true state both evaluated at time t_k.

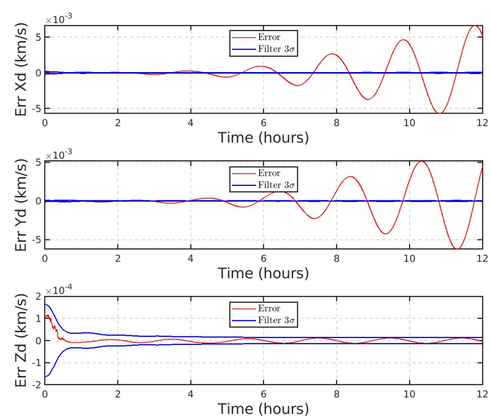
$$\delta \boldsymbol{x}_k = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k = \boldsymbol{x}(t_k) - \hat{\boldsymbol{x}}(t_k) \tag{53}$$

Let $\hat{t}_\ell$ be the estimated time whose numerical value coincides with the true time t_k, i.e. $\hat{t}_\ell = t_k$. Clearly $t_\ell = t_k$ only when the clock bias is zero. Filter divergence would slow down comparing the estimated state at t_ℓ with the true states at t_k. This asynchronous definition of localization error is not useful for an autonomous system that aims at making onboard decisions and it is therefore most concerned with the difference between the current true state and the current estimated state. If the onboard navigation state was downloaded to the ground to make decisions, on the other hand, the asynchronous error definition $\boldsymbol{x}(t_k) - \hat{\boldsymbol{x}}(t_\ell)$ would be appropriate because the ground would upload a command together with an execution time tag and a clock correction term. Hence remotely operated deep-space spacecraft typically do not necessitate the proposed time correlation terms.

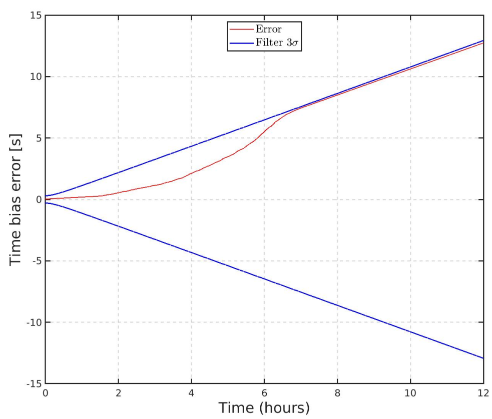
Figure 1.e. shows the measurement residuals. Clearly, the filter propagates the spacecraft dynamics between each measurement and the following one for an incorrect amount of time: the error on clock drift estimate causes the clock offset estimation error to grow in the order of seconds after two hours. This causes the measurement update to fail, hence the innovations of the α angle start diverging.



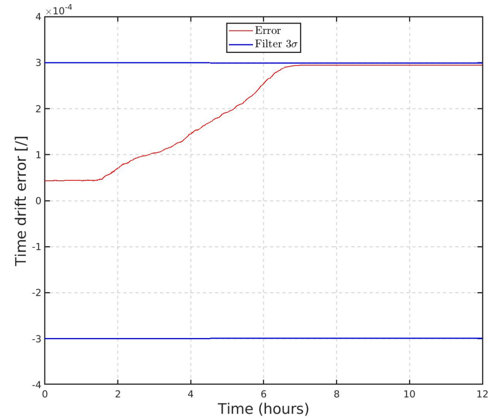
(a) Position estimation error.



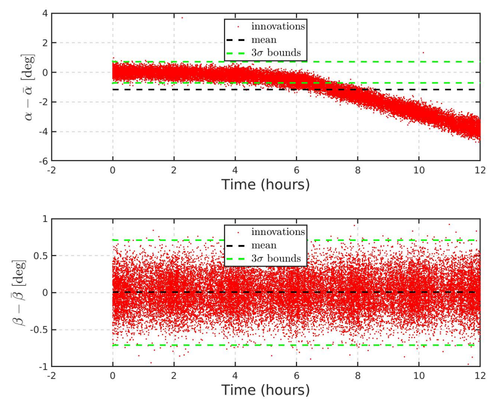
(b) Velocity estimation error.



(c) Clock bias estimation error.



(d) Clock drift estimation error.



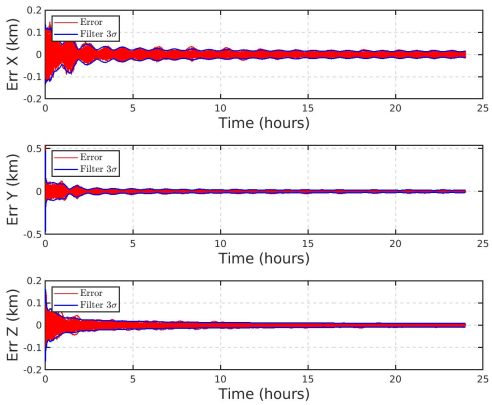
(e) Innovation residuals.

Figure 1: Simulation results without accounting for clock offset.

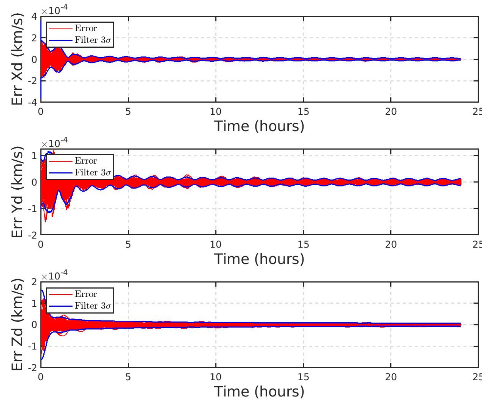
Craters only

Figure 2 shows 100 Monte Carlo runs when the new terms in the covariance propagation are included, and again processing craters measurements only. The duration of the simulation is of one day.

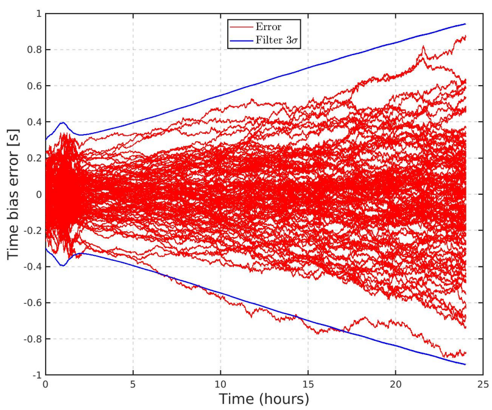
We note that even using only optical measurements time-tagged with the onboard clock, the filter is able to estimate the clock drift. This is due to the clock error terms included in the covariance propagation and in the crater measurement update equations. The filter is able to quickly reduce the error on clock drift (bottom right subfigure), preventing the error on clock offset to diverge too rapidly, as opposed to the previously shown scenario.



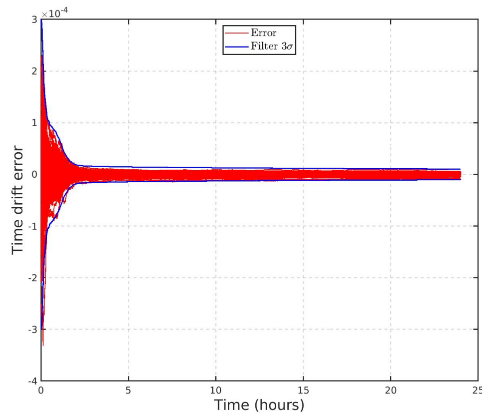
(a) Position estimation error.



(b) Velocity estimation error.



(c) Clock bias estimation error.

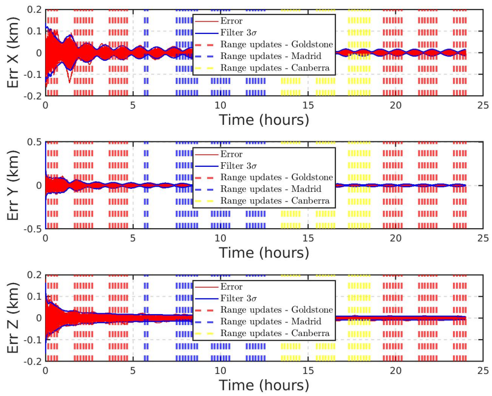


(d) Clock drift estimation error.

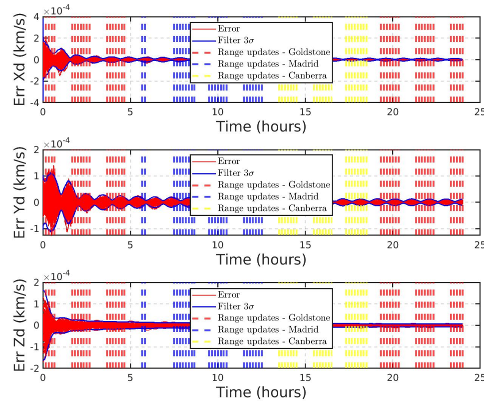
Figure 2: Estimation errors with craters measurements only.

Craters and one-way ranging

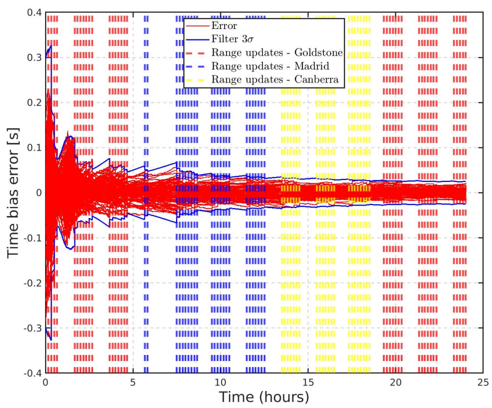
In this scenario, the filter processes craters measurements and one-way range measurements from DSN, using either all three stations (Figure 3), or only one station throughout the whole simulation (Figures 4, 5 and - 6 for Goldstone, Madrid and Canberra stations, respectively). Results show convergence of all state components with the uncertainty/error in the clock offset growing only with long periods without range measurements. Even in those phases the error on the onboard clock drift stays to low values so the error on clock bias does not grow too fast.



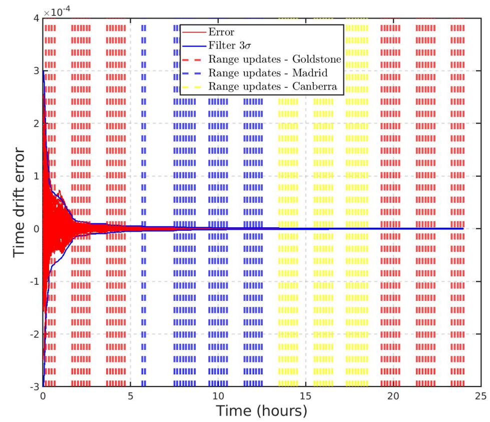
(a) Position estimation error.



(b) Velocity estimation error.

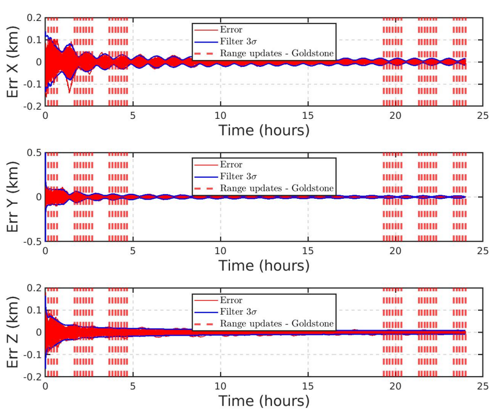


(c) Clock bias estimation error.

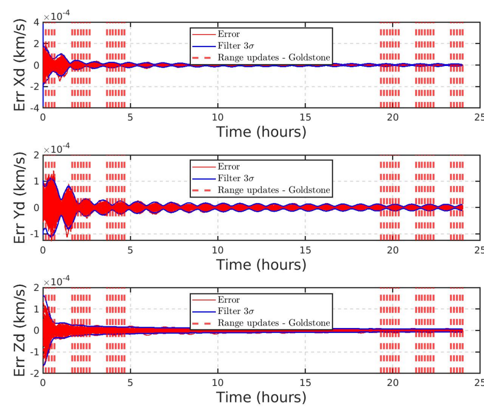


(d) Clock drift estimation error.

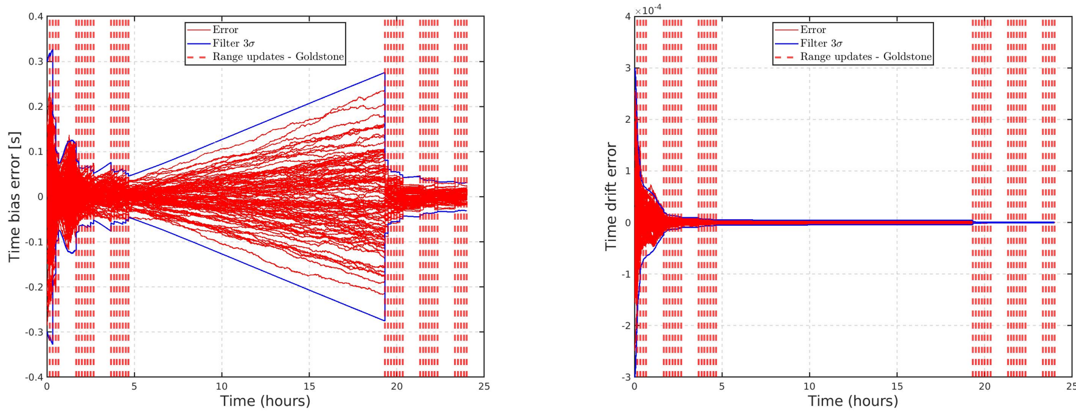
Figure 3: Estimation errors with one-way ranging measurements.



(a) Position estimation error.



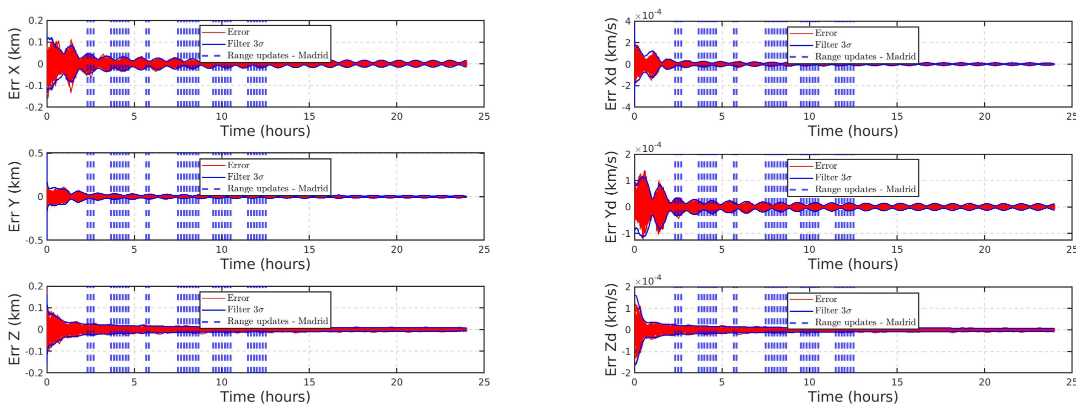
(b) Velocity estimation error.



(c) Clock bias estimation error.

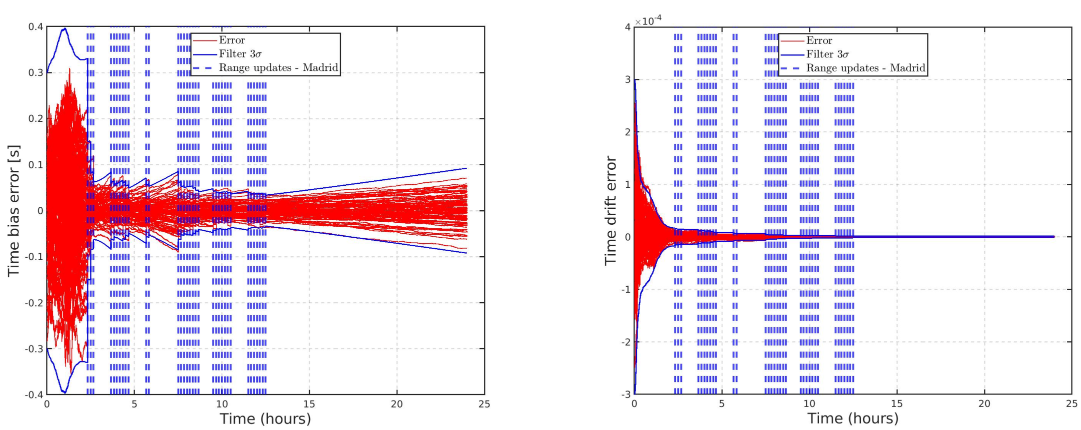
(d) Clock drift estimation error.

Figure 4: Estimation errors with one-way ranging measurements from Goldstone station only.



(a) Position estimation error.

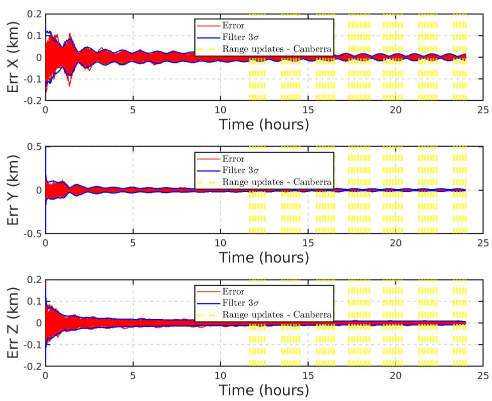
(b) Velocity estimation error.



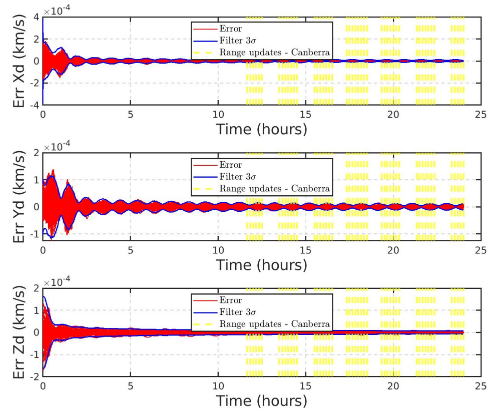
(c) Clock bias estimation error.

(d) Clock drift estimation error.

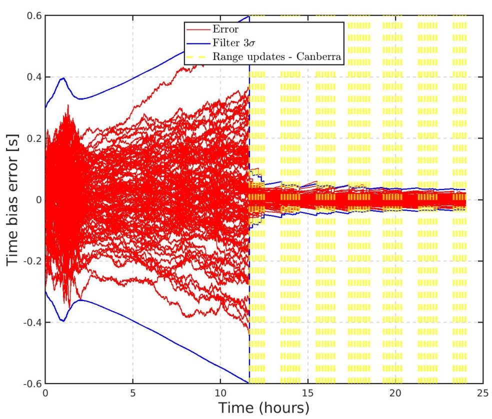
Figure 5: Estimation errors with one-way ranging measurements from Madrid station only.



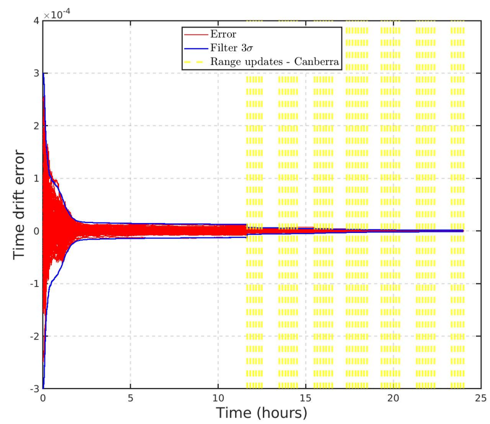
(a) Position estimation error.



(b) Velocity estimation error.



(c) Clock bias estimation error.



(d) Clock drift estimation error.

Figure 6: Estimation errors with one-way ranging measurements from Canberra station only.

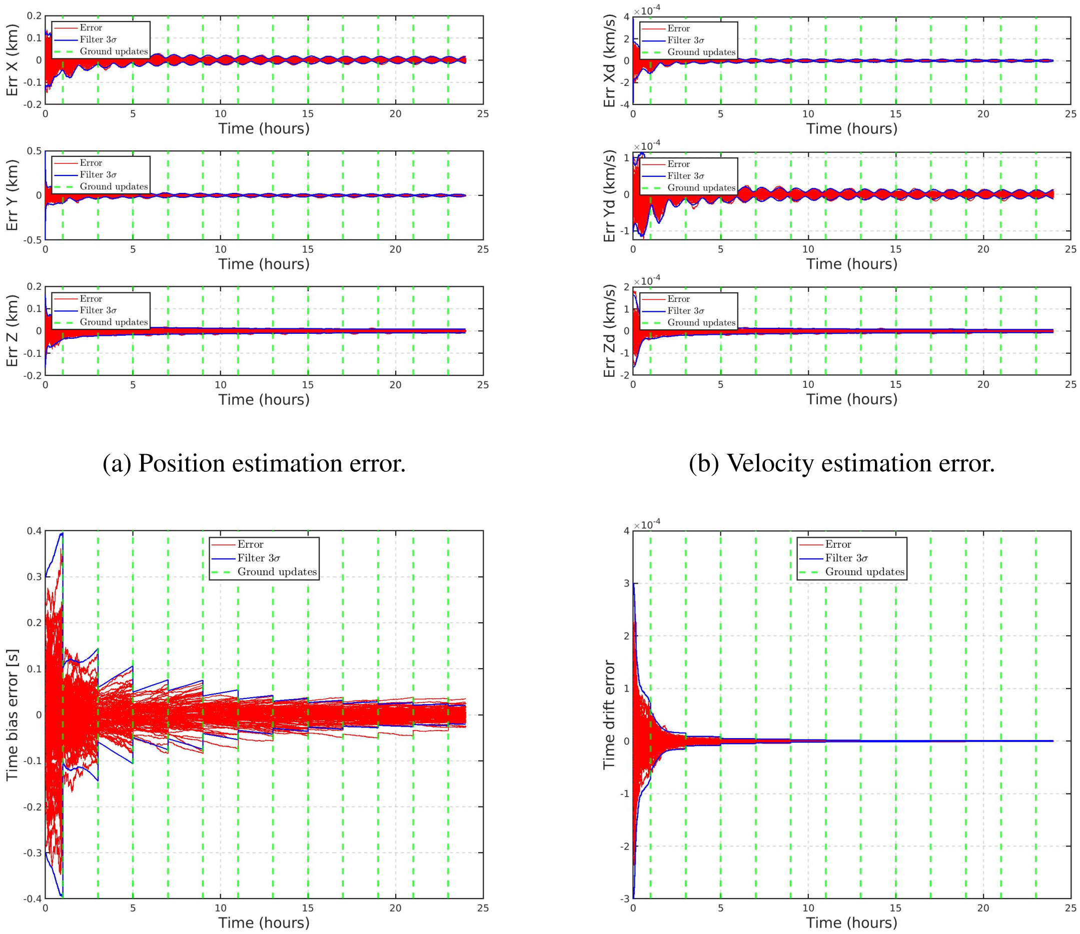
Craters and ground position updates

Our formulation allows for onboard clock estimation from an externally provided measurement accurately time-tagged with an external reference clock. As a motivating example, we employ two-way tracking from DSN, for example in conjunction with spacecraft data downloads.

Figure 7 shows a Monte Carlo simulation with a position ground update performed every two hours. Results show the filter performance is similar to the one-way ranging case shown in the previous section.

Table 3 shows the root mean square errors of position components and clock states for one Monte Carlo trial from each simulation case.

The performance of the one-way ranging and the two-way position update are very close to each other.



(a) Position estimation error.

(b) Velocity estimation error.

(c) Clock bias estimation error.

(d) Clock drift estimation error.

Figure 7: Estimation errors with craters measurements and position ground updates.

Table 3: RMS errors of one Monte Carlo trial for each simulation case

Case	RMS_x [km]	RMS_y [km]	RMS_z [km]	RMS_b [s]	RMS_d [/]
Neglecting clock offset	2.322	2.478	0.01	7.05	2.27e-4
Craters only	0.011	0.012	0.004	0.26	1.04e-5
Craters + DSN stations (all)	0.008	0.01	0.009	0.024	9.09e-6
Craters + DSN stations (Goldstone only)	0.008	0.01	0.009	0.024	9.07e-6
Craters + DSN stations (Madrid only)	0.008	0.01	0.009	0.048	1.00e-5
Craters + DSN stations (Canberra only)	0.008	0.01	0.009	0.078	9.95e-6
Craters + position from ground	0.01	0.015	0.005	0.039	1.28e-5

CONCLUSIONS

This work proposes a new approach to onboard position, navigation and timing in lunar orbit. Localization is performed via Moon crater relative navigation but no GPS measurements nor time correction are available to the spacecraft. Therefore, an alternative formulation to deal with discrepancies between the onboard time and true time is developed. The solution proposed consists of exploiting an Extended Kalman Filter algorithm with the addition of correlation terms between timing errors and state propagation errors. Results show that the inclusion of these terms ensure convergence in the estimation errors of all filter state's components in the scenario of Moon optical navigation without external time measurements and in two different configurations. The first one leverages one-way ranging from DSN to aid the onboard optical navigation, and the second one consists of a two-ways position update from DSN.

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