Disagreement and Liquidity

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Abstract

Heavy, liquid trade of informationally sensitive securities is a puzzle for traditional models, which predict that asymmetric information should decrease trade and destroy liquidity. Empirically, trading of stocks, bonds, and options increase with asymmetric information. Additionally, trading and liquidity increase following high past returns. To resolve this puzzle, I propose a model in which trading is entirely generated by disagreement stemming from overconfident interpretation of private signals. The model predicts that overconfidence increases trading and enhances liquidity and that asymmetric information increases trading and can enhance liquidity. I also propose a more general model incorporating both disagreement and exogenous liquidity trading. The general model relates traditional intuition about asymmetric information destroying liquidity and trade to disagreement trading. Asymmetric information and overconfidence can at first destroy liquidity and then enhance it, potentially explaining why asymmetric information seems to destroy liquidity in money markets but not in informationally sensitive markets.
1 Introduction

Extensive trading of equities and other informationally sensitive securities is a puzzle. Standard asset pricing models have no role for trading, and models that consider trading typically predict that asymmetric information decreases trading and destroys liquidity, defined as the ability to trade an asset without significantly changing its price. Given the large potential for asymmetric information in stocks, corporate bonds, and stock options, it is counterintuitive that these securities are heavily traded in liquid markets. Holmstrom (2008) highlights this puzzle by comparing money (liquidity) markets to stock markets: "Markets for liquidity are very different than stock markets. In the stock market, uncertainty and adverse selection fears are present all the time, but this does not prevent the markets from functioning.... Differences in beliefs often alleviate adverse selection. Stock markets thrive on differences in beliefs. Markets for liquidity are killed by them."

Since Akerlof (1970), economists have recognized that asymmetric information has the potential to destroy trade. To overcome asymmetric information, at least some trade must be motivated by something other than rationally processed information, otherwise liquidity will dry up, markets will freeze, and the no-trade prediction of Milgrom and Stokey (1982) will prevail. Starting with the noisy rational expectations models of Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981) and including virtually all research on liquidity and market microstructure (most notably Kyle, 1985), this extra trading has been modeled as exogenous noise.

Traditional asymmetric information models make two basic predictions: (1) Significant noise trading is necessary to generate trading and liquidity; and (2) asymmetric information decreases trading and liquidity. Hong and Stein (2007) address the first prediction, noting that trading volume in traditional models is approximately pinned down by noise (non-informational) trading volume. For example, in Kyle’s (1985) model exogenous noise trading represents half of total order flow variance. In my moderate variance calibration of Diamond and Verrecchia’s (1981) model, liquidity trading is 95% of total trading volume. The New York Stock Exchange has daily volumes in excess of $30B. Liquidity trading of that magnitude seems implausible. One way out of this problem is to interpret noise trading more generally and claim that the vast majority of trading is uninformed noise from irrational traders. While theoretically possible, this is not particularly satisfying. If most trading is exogenously assumed, we aren’t really left with a model of trading. Moreover, traditional
models assume that noise trading is not just exogenous but also orthogonal to information. This is a critical assumption, and it is likely invalid if noise trading is driven by disagreement among market participants. For example, if disagreement comes from overconfidence in private information, the same trade is at once informative and noise, making noise trading perfectly correlated with information.

Prediction (2) presents even more fundamental problems. In contrast to traditional intuition, asymmetric information actually increases trading. I study asymmetric information, turnover, and liquidity of stocks, corporate bonds, and stock options.¹ My analysis supports three stylized facts: (1) Trade and liquidity are positively correlated; (2) asymmetric information increases trade and decreases liquidity; and (3) high past returns increase trade and liquidity.² Fact (1) supports the notion that trade and liquidity reinforce one another. Fact (2) contradicts the prediction of traditional models that asymmetric information destroys trading. Fact (3) shows that traditional models leave out something related to past returns.

To resolve the shortcomings of traditional models, I propose that trading is primarily driven by disagreement. I.e., people trade because they have different beliefs about an asset’s value. Counterparties essentially make zero-sum bets about asset values, and they do so fully aware that other parties disagree with them. Disagreement trading has become an increasingly popular explanation of trading volumes (Hong and Stein, 2007, summarize this view), but there is little existing research on the relationship between disagreement and liquidity and virtually none on how disagreement changes asymmetric information’s impact on trading and liquidity.

In my model of disagreement trading, belief differences stem from overconfident interpretation of private information. The overconfidence bias creates trade and liquidity even when prices fully reveal the beliefs of other agents. Moreover, because disagreement stems from private signals, trading and liquidity can increase with asymmetric information. Formally, I model disagreement trading among ex ante homogenous agents who simultaneously serve as informed traders, noise

¹I proxy for asymmetric information with analyst earnings forecast dispersion and also study periods around earnings announcements, which likely have elevated asymmetric information. For illiquidity, I use several measures of bid-ask spreads as well as Amihud’s (2002) \( \text{illiq} = \frac{|\text{Return}|}{\text{Volume}} \) measure.

²In equity markets, these evidence for these facts has been shown before. In particular, Sadka and Scherbina (2007) show that analyst forecast dispersion (one proxy for asymmetric information) decreases liquidity; Frazzini and Lamont (2007) show that turnover is elevated around earnings announcements (another proxy for asymmetric information); and Statman, Thorley, and Vorkink (2006) and Griffin, Nardari, and Stulz (2007) show that trading increases following high returns. I extend these findings to corporate bonds and stock options and show that analyst dispersion increases trading, earnings announcements decrease liquidity, and past returns increase liquidity.
traders, and market makers. Agents are risk averse, receive endowments of a risky asset, observe private signals about the asset’s value, and trade the asset with one another in an anonymous public market in which market price is visible to all agents. Agents are fully rational except for an overconfidence bias, which causes them to overestimate the precision of their own signals. I consider two versions of the model, a baseline model in which asset endowments are constant and a general model in which asset endowments are stochastic. The baseline model is an adaptation of Grossman (1976) with overconfidence. The general model is an adaptation of Diamond and Verrecchia (1981) with overconfidence.

The baseline model describes an environment in which trading is entirely driven by disagreement. Because supply is certain and there is no non-informational motive for trade, prices fully reveal aggregate information (as in Grossman, 1976). Nonetheless, overconfidence induces agents to disagree and trade, thereby generating liquidity (in contrast to Grossman, 1976). The model’s main predictions are: (1) Overconfidence increases trading and liquidity; (2) Private information increases trading and liquidity; and (3) Public uncertainty decreases liquidity without affecting trading volume.

The prediction that private information increases trade and liquidity conflicts with traditional intuition. To understand this contrast, I consider a general model that adds non-informational liquidity trading, modeled through stochastic endowments, to the baseline model. When some trading is uninformed, increasing private information can destroy liquidity by increasing the share of informed trade relative to uninformed trade. Private information increases a trade’s price impact by increasing the probability that the trade is informed. In the baseline model, all trade is informed so this channel is inoperative. More generally, the probability that a trade is informed is insensitive to the level of private information whenever uninformed trade is very large or very small relative to informed trade. In these settings, traditional intuition fails, and private information increases liquidity as in the baseline model.

My model explains trade and liquidity in the face of asymmetric information and conforms to the stylized facts described above. It also generates the additional testable prediction that stocks with the highest turnover response to asymmetric information should have the smallest illiquidity response to asymmetric information. I test this prediction and find that it is true in the data.

In the next section, I review related literature. Section 3 presents empirical analysis support-
ing the stylized facts introduced above, with details in an online appendix. Section 4 introduces, solves, and derives comparative statics for the baseline model. Section 5 presents the general model, including comparative statics and numerical examples of how overconfidence and asymmetric information affect trading and liquidity. Derivations and proofs for the general model are in an appendix. Section 6 discusses and tests the model’s empirical predictions. Section 7 concludes.

2 Literature Review

Disagreement

Disagreement models posit that a combination of private information (or private interpretation of public information) and behavioral biases causes otherwise rational investors to disagree about asset values. Disagreement models are motivated by two shortcomings of standard asset pricing models with homogenous beliefs. First, standard models (at least in their simplest forms) are at odds with well-established asset pricing anomalies like momentum (Jegadeesh and Titman, 1993), post-earnings drift (Bernard and Thomas, 1989), and long-term return reversion (Fama and French, 1992; Lakonishok, Shleifer, and Vishny, 1994). Second, standard models have no role for trading and thus cannot explain the high turnover observed in many financial markets. Disagreement is an intuitively appealing rationale for trade and, depending on what drives the disagreement, also has the potential to explain pricing anomalies.

Hong and Stein (2007) provide a nice summary and taxonomy of disagreement models. The first ingredient for these models is some manner of private information. One possibility is that private information comes from gradual information flow as in the gradual dissemination of information to newswatchers over time in Hong and Stein (1999). Another possibility is that investors have limited attention and thus process only a subset of available information, possibly for entirely rational reasons related to the cost of attention (e.g., Peng and Xiong, 2006). A final possibility is that investors see the same information but interpret it differently, possibly due to heterogeneous priors (e.g., Harris and Raviv, 1993; Kandel and Pearson, 1995).

The second ingredient for disagreement models is a behavioral bias in information processing. Private information alone does not generate disagreement in standard models. Instead, market prices aggregate and fully reveal information (Grossman, 1976), which causes investors to agree
about asset values and eliminates motives for trade (Milgrom and Stokey, 1982). Introducing random asset supply or exogenous liquidity trading (e.g., Hellwig, 1980; Grossman and Stiglitz, 1980; Diamond and Verrecchia, 1981; Kyle, 1985) mitigates information revelation and creates some disagreement, but trading is still largely pinned down by assumptions about exogenous trading. For example, in Kyle’s (1985) model exogenous noise trading represents half of total order flow variance. In my moderate variance calibration of Diamond and Verrecchia’s (1981) model liquidity trading is 95% of total trading volume. To yield more significant disagreement trading, investors must value their own information more highly than information extracted from market prices. Overconfidence is a convenient modeling device for achieving this result and is supported by substantial psychological evidence (see Odean, 1998; Daniel, Hirshleifer, and Subrahmanyam, 1998; and DeBondt and Thaler, 1995, for good discussions of overconfidence). Overconfidence can be modeled in different ways. A common approach is to assume that investors overestimate the precision of their own signals relative to the signals of other investors.

Dynamic models of overconfidence posit that investors learn to be overconfident based on past experience. In particular, investors are subject to a self-attribution bias that causes them to overestimate how much their own skill was responsible for past successes. As a result, overconfidence is highest following positive returns. Daniel, Hirshleifer, and Subrahmanyam (1998) and Gervais and Odean (2001) model this phenomenon. Because investors hold the market in aggregate, self-attribution bias predicts that overconfidence should be high following high market returns. Stock-level overconfidence should be high following high individual stock returns because owners of the stock just experienced high returns. Statman, Thorley, and Vorkink (2006) show that turnover is higher than normal following high market returns and high individual stock returns, consistent with high returns increasing overconfidence, which in turn increases trading intensity. Griffin, Nardari, and Stulz (2007) find similar evidence across countries.

Casual intuition suggests that disagreement trading should enhance liquidity. However, liquidity has not been a major focus of the disagreement literature and is not explicitly discussed in most disagreement papers. Exceptions include Odean (1998), Kyle, Obizhaeva, and Wang (2013), and Baker and Stein (2004). Odean (1998) adds overconfidence to the Kyle (1985) model of liquidity. When the informed insider receives a noisy signal and overestimates the precision of that signal, the insider’s overconfidence increases liquidity. Specifically, the market maker’s price function becomes
less sensitive to order flow as the insider becomes more overconfident. Though they don’t explicitly focus on liquidity, the duopoly model of Kyle and Wang (1997) produces a similar result with two overconfident insiders. Kyle, Obizhaeva, and Wang (2013) show that overconfidence mitigates price impact and generates disagreement trading. Baker and Stein (2004) also model overconfidence as increasing liquidity. However, this relationship is an ad hoc assumption based on the logic that the same behavioral biases that cause investors to overestimate the precision of their own signals will also cause them to underestimate the informational content of market prices. None of these papers address how overconfident disagreement changes the impact of asymmetric information on trading and liquidity.

**Liquidity**

The market microstructure literature aims to understand what causes illiquidity in financial markets. The backdrop is that in Walrasian equilibrium beliefs are independent from prices and market prices perfectly reflect the demands of all agents. In real financial markets investors learn from market prices, and prices can deviate from fundamental values creating costs to trading. Trading costs (illiquidity) come from two main sources. First, not all agents are active in markets at the same time. Thus, transactions must be facilitated by market makers. These market makers must cover whatever costs they incur by being constantly active in financial markets, and they must be compensated for the risk they take by holding assets while searching for a counterparty. Market makers might also extract profits from strategic behavior. The second source of illiquidity is asymmetric information. Uninformed buyers may worry that they are being exploited by better-informed counterparties and thereby demand lower prices. Biais, Glosten, and Spatt (2005) survey the microstructure literature. Vayanos and Wang (2009) propose a unified model encompassing multiple sources of illiquidity.

My focus is on asymmetric information illiquidity because asymmetric information is intimately tied to disagreement. Other sources of illiquidity may also be important, but they are likely to

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3Kyle, Obizhaeva, and Wang’s (2013) model is the closest to my own. The main main difference between their single period model and my baseline model is that investors in the Kyle, Obizhaeva, and Wang model have market power whereas all agents in my model are price takers. Despite the modeling similarities, Kyle, Obizhaeva, and Wang address different questions than I do. Their focus is on how market power and overconfidence combine to give large investors an incentive to smooth trading whereas I analyze how overconfident disagreement changes the impact of asymmetric information on trading and liquidity.
be largely orthogonal to changes in disagreement. The classic models of asymmetric information liquidity are Kyle (1985) and Glosten and Milgrom (1985). In both models, market makers see order flow and are unsure whether the order flow comes from an informed insider or an uninformed liquidity trader. Market makers rationally infer some probability that order flow reflects information and adjust prices accordingly. Kyle (1985) describes this process in terms of the impact of order flow on price. Glosten and Milgrom (1985) highlight that asymmetric information naturally leads to bid-ask spreads. More recent microstructure research extends these frameworks to consider strategic behavior by market makers and market design issues. The microstructure liquidity literature generally does not model overconfidence. Odean (1998) and Kyle, Obizhaeva, and Wang (2013), discussed above, are notable exceptions.

The microstructure literature typically considers asymmetric information illiquidity in settings where market makers set prices and bid-ask spreads. However, market makers are not necessary for the concept of asymmetric information illiquidity. For example, one can think about illiquidity in the rational expectations framework of Grossman (1976). All agents receive private signals, observe market prices, and form trading demands. Grossman’s conclusion is that prices fully reveal the average signal. As a result, there is no disagreement and no trading. In effect, markets are infinitely illiquid. Hellwig (1980); Diamond and Verrecchia (1981); and Grossman and Stiglitz (1980) introduce uncertain asset supply so that prices are no longer fully revealing. As a result, agents disagree about asset prices and trade with one another. Prices finitely react to order flow. None of these models explicitly considers liquidity, but asymmetric information illiquidity is just as present in them as in the market maker microstructure models.

3 Stylized Facts

I propose three stylized facts about stock, bond, and option markets:

1. Trade and liquidity are positively correlated;

2. Asymmetric information increases trade and decreases liquidity; and

3. High past returns increase trade and liquidity.
These facts are not new, especially with respect to the stock market. For example, Sadka and Scherbina (2007) show that analyst forecast dispersion (one proxy for asymmetric information) decreases liquidity. Frazzini and Lamont (2007) show that turnover is elevated around earnings announcements (another proxy for asymmetric information). Statman, Thorley, and Vorkink (2006) and Griffin, Nardari, and Stulz (2007) show that trading increases following high returns. Hong and Stein (2007) also observe that returns and trading volume are correlated.

I extend these findings and show that analyst dispersion increases trading, earnings announcements decrease liquidity, and past returns increase liquidity. I also show that the stylized facts hold across stock, corporate bond, and option asset classes. Finally, in the online appendix, I show that results are robust across different types of stocks by sorting stocks on size, book-to-market ratios, past returns, and other characteristics.

My sources for stock data are CRSP for return and volume data, Compustat for industry and earnings announcement data, I/B/E/S for analyst earnings forecast data, and TAQ for intraday trade and quote data. I limit my stock sample to New York Stock Exchange (NYSE) stocks with prices above $5 at the end of the previous month. The sample starts in 1926, but most of my analysis is limited by analyst earnings forecast data, which starts in 1976, and bid-ask spread data, which starts in 1993 for my favored measure based on intraday TAQ data. I measure stock turnover as monthly volume divided by shares outstanding. My primarily liquidity measure is the effective bid-ask spread (\(ebidask\)) of Chordia, Roll, and Subrahmanyam (2000), which I calculate at the transaction level as twice the difference between a trade’s price and the midpoint of the prevailing quote before the trade. Because this measure is only available starting in 1993, I also use Amihud’s (2002) \(illiq_{it} = \frac{|Return_{it}|}{Volume_{it}}\) measure for some analyses. In the online appendix, I consider two alternative bid-ask spread measures.

Bond data comes from TRACE, supplemented by Mergent FISD bond characteristics. The TRACE data starts in 2002. I limit my sample to investment grade U.S. corporate bonds and medium term notes without asset backing or security enhancement features that are at least one year old and have at least one year of maturity left. I also require that a bond be actively traded (defined as having at least two buy trades and two sell trades) on at least 15 days during the previous month. I consider only transactions between dealers and their customers. Turnover is the total par value of all trades in a bond scaled by the bond’s outstanding par value. Effective bid-
ask spread (*ebidask*) is the difference between the weighted average prices of a day’s buy and sell trades.\(^4\)

Option data comes from Ivy DB OptionMetrics, available starting in 1996. My analysis is at the stock (as opposed to option) level. I define turnover as total option dollar volume divided by stock market capitalization. Quoted end-of-day proportional bid-ask spreads are weighted by dollar volume when averaged to the stock level.

Figure 1 plots equally-weighted average monthly turnover and bid-ask spreads for stocks, bonds, and options. Stock turnover (panel A) averages 14% during the plotted 1993 to 2011 time period and reaches as high as 40% late in the sample. Stock effective bid-ask spreads (panel B) average 0.4% and decrease over the sample period. Corporate bonds also have significant trading activity and moderate bid-ask spreads. Bond turnover (panel C) is typically near the 5-10% range, and average bond effective bid-ask spreads (panel D) range from 0.5% to 2.5%. In contrast to stocks and bonds, dollar transaction volumes are relatively small for stock options. Average dollar option volume (panel E) is typically under 1% of stock market capitalization. Average option bid-ask spreads hover around 20% of contract value. However, these figures are for option contract value as opposed to stock price exposure. In practice, options often deliver large stock price exposure with minimal up-front contract value.

In the remainder of this section, I summarize the findings most relevant for each of the three stylized facts. Details and additional stock analysis are in an online appendix.

**Fact 1: Trade and liquidity are positively correlated**

Traditional reasoning predicts a strong positive relationship between trading and liquidity. The two quantities are mutually reinforcing. More noise trading improves liquidity, and enhanced liquidity attracts additional trading activity. Similar logic follows from my general disagreement model.

Positive correlation between trade and liquidity is clear in the stock, bond, and option data. As turnover increases, bid-ask spreads tend to decrease. Table 1 reports results for panel regressions of log bid-ask spreads on log turnover. The regressions include stock, bond, and time fixed effects.

\(^4\)The TRACE data I use is the enhanced dataset available on WRDS, which identifies whether a trade is with a customer or another dealer and which side of the transaction the reporting dealer was on. The enhanced dataset also includes all volume data instead of truncating large trades.
Because turnover and bid-ask spreads are both expressed as logs, the results can be interpreted as elasticities. The elasticity of stock bid-ask spreads with respect to turnover is -13%. The equivalent coefficients for bonds and options are -9% and -17%, respectively. All three estimates are highly significant. The online appendix includes cross-sectional tests and robustness checks with alternative measures of illiquidity.

**Fact 2: Asymmetric information increases trade and decreases liquidity**

While trading and liquidity are positively correlated, they do not always move in the same direction. In particular, asymmetric information tends to increase trading while reducing liquidity.

I identify changes in asymmetric information in two ways. First, periods around earnings announcements are likely to have elevated asymmetric information. Prior to announcements, private information can be in the form of leaks and insider trading. After announcements, investors process different pieces of information at different paces using different models, keeping private information high until the announcement is fully digested and reflected in prices. Public uncertainty is also high around earnings announcements because asset values are highly sensitive to the announcements. Second, I follow Sadka and Scherbina (2007) and use dispersion of analyst forecasts as a proxy for asymmetric information. Analyst dispersion may represent or cause public uncertainty. Dispersion could also stem from private information. My specific measure of dispersion is the standard deviation across analysts of current year earnings forecasts scaled by the mean forecast. Firms are included if they are covered by at least two analysts, have a non-zero mean earnings forecast, and have a December fiscal year end. The December fiscal year requirement ensures that all stocks have the same amount of time remaining in the current fiscal year.

For my earnings announcement analysis, I scale turnover and bid-ask spreads by average values over the three calendar months prior to an earnings announcement and analyze scaled turnover and bid-ask spreads over a 21-day trading window around earnings announcements. Figure 2 plots equally weighted average scaled turnover and bid-ask spreads in event time around earnings announcements. Day 0 is the announcement day or first trading day after the announcement. Other days represent trading days relative to the announcement. 95% confidence intervals are plotted in dashed lines.

Panels A and B plot stock data. Consistent with Frazzini and Lamont (2007), turnover starts to
increase the day before an announcement, spikes to 80% above normal levels on the announcement
day, stays at that level for another day, and then decays. I extend the analysis of Frazzini and
Lamont by also studying bid-ask spreads, which widen around earnings announcements. Bid-ask
spreads peak at 13% above normal levels on the day of the announcement, and are also elevated the
day before and after the announcement. The online appendix shows that the same pattern emerges
for alternative bid-ask spread measures. Moreover, the results are robust across different types of
stocks. In sorts on size, book-to-market ratios, past returns, and institutional ownership, the same
basic patterns emerge for all groups.

Though not always as pronounced, turnover and bid-ask spreads also tend to increase around
earnings announcements for bonds and options. Panel C shows that bond turnover peaks at 40%
above normal levels the day after an earnings announcement. Bond bid-ask spreads (panel D) are
slightly elevated around earnings announcements, particularly the day before an announcement.
Option volumes (panel E) surge to over 250% normal levels around earnings announcements, and
option bid-ask spreads (panel F) widen to 6% above normal levels, with a peak the day before the
earnings announcement.

Analyst earnings forecast dispersion is a second proxy for asymmetric information. Sadka and
Scherbina (2007) show that in sorts on analyst dispersion, high dispersion stocks tend to be less
liquid. I employ panel regressions to control for firm and time fixed effects and add a test of the
effect of analyst dispersion on trading volumes. I also extend the analysis to bonds and options.
Table 2 presents my results. The analyzed variables are logs so coefficients can be interpreted as
elasticities. For stocks, the turnover coefficient on lagged analyst dispersion (column 1) is 1.9% and
the bid-ask spread coefficient on lagged analyst dispersion (column 2) is 10.9%. Bonds and options
respond similarly to analyst dispersion. For bonds, the turnover coefficient (column 3) is 7.7%, and
the bid-ask spread coefficient (column 4) is 7.9%. For options, the turnover coefficient (column 5) is
14.5%, and the bid-ask spread coefficient (column 6) is 3.6%. All coefficients are highly significant.
The online appendix repeats this analysis for additional measures of stock illiquidity, performs
cross-sectional analysis on stocks, and confirms that my baseline results are robust across most
sorts on stock characteristics.
Fact 3: High past returns increase trade and liquidity

Statman, Thorley, and Vorkink (2006) use a market VAR and individual stock-level VARs to show that market turnover increases following high market returns and individual stock turnover increases following both high market returns and high individual stock returns. I add illiquidity to Statman, Thorley, and Vorkink’s market VAR methodology and apply it to stocks, bonds, and options. The market VAR model is:

\[ Y_t = \alpha + \sum_{k=1}^{2} A_k Y_{t-k} + \epsilon_t \]  

where \( Y_t \) is a 3 × 1 vector of detrended log turnover, detrended log illiquidity, and excess market returns. Using two lags is optimal according to the Bayesian information criteria. In all cases, the market return variable is excess stock market returns. Thus, I am assessing the impact of stock market returns on future trading and liquidity in stocks, bonds, and options. For my baseline stock analysis, I use Amihud’s illiq instead of bid-ask spread because it is available for the full sample instead of just after 1993. For bonds and options I use the same bid-ask spread measures as before.

Figure 3 plots impulse response functions for stock, bond, and option market VARs. The plots show how a one standard deviation unexpected stock market return shocks affects future realizations of turnover and illiquidity measures. Consistent with Statman, Thorley, and Vorkink (2006), panel A shows that stock turnover responds positively to market returns. A one standard deviation return shock increases turnover in the next month by about 5%. Turnover increases slightly in the following month and then decays toward normal levels. Panel B shows that positive market returns also enhance liquidity. A one standard deviation return shock decreases illiq by 8% in the next month. Positive stock return shocks also decrease bond and option bid-ask spreads (panels D and F). Bond and option volumes (panels C and E) do not significantly respond to past stock returns. The online appendix reports coefficients for the stock VAR and results for stock

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5 Turnover and illiquidity measures are detrended using a Hodrick and Prescott (1997) filter. Following Statman, Thorley, and Vorkink (2006) and common practice, I use a penalty value of 14,400 for the filter. Also following Statman, Thorley, and Vorkink (2006), I employ a 2-sided filter. The 2-sided filter, which makes use of future data, would problematic if I was using it for forecasting purposes, but I am not. To verify that my results are unaffected by the use of future data, I replicated my market VAR with a 1-sided HP filter proposed by Stock and Watson (1999). Results (which are untabulated but are available on request) were unchanged.

6 Other impulse-response combinations are omitted for brevity. Plots of all stock impulse response functions are included in the online appendix.
VARs using bid-ask spread measures instead of illiq.

In addition to being impacted by market returns, individual stock turnover and liquidity respond positively to past stock and industry level returns. Using separately estimated stock-level VARs, Statman, Thorley, and Vorkink (2006) show that stock turnover is positively influenced by both past market returns and past individual stock returns. I employ a different econometric strategy and estimate a single panel VAR that includes stock-level turnover, illiquidity, and returns as well as industry returns. The panel VAR specification allows me to employ stock and time fixed effects, eliminating the need for detrending the data. Specifically, I estimate:

\[
Y_{i,t} = \alpha_t + f_{i} + \sum_{k=1}^{2} A_k Y_{i,t-k} + \epsilon_t
\]

where \(Y_{i,t}\) is a 4 \times 1 vector of log stock turnover, log stock illiq, stock returns, and industry returns for stock \(i\) in month \(t\). \(\alpha_t\) and \(f_{i}\) are 4 \times 1 vectors of time and stock fixed effects for each variable. I employ two lags for consistency with the market model. The time fixed effects control for the effect of market returns as well as any other market-level time variation. Prior to estimation, I eliminate the time fixed effects by time de-meaning all variables. The stock fixed effects are trickier because stock demeaned lag variables are not orthogonal to the regression residual. Similarly, directly estimating stock fixed effects would produce biased and non-consistent estimates for all coefficients. Following Holtz-Eakin, Newey, and Rosen (1988), I take the first differences of all variables, resulting in:

\[
Y_{i,t} - Y_{i,t-1} = (\alpha_t - \alpha_{t-1}) + \sum_{k=1}^{2} A_k (Y_{i,t,k} - Y_{i,t-1,k}) + \epsilon_t
\]

which can be estimated using using \(Y_{i,t-2}\) and \(Y_{i,t-3}\) as instruments. I include all observations with at least three lagged observations. When there are breaks in the data, I treat observations before and after the break as if they were separate stocks. The only remaining complication is estimating standard errors. The lagged variables directly control for autocorrelation in the data, but there is likely cross-sectional correlation within time periods. To account for this I employ bootstrapped standard errors with a bootstrap that randomly samples (with replacement) time periods. When a time period is drawn, all observations in that time period are included. This preserves the data’s cross-sectional correlation structure.
Figure 4 plots the most relevant impulse response functions of the panel VAR. As before, the impulse shocks are all one standard deviation. In panel A, turnover is unaffected by individual stock returns. In the other three panels, past returns forecast increased turnover and decreased illiquidity. A one standard deviation shock to an individual stock’s return forecasts a future decline in *illiq* of 2.7% (panel B). A one standard deviation shock to an industry’s return forecasts a 0.7% increase in turnover (panel C) and a 0.4% decrease in *illiq* (panel D). Coefficient estimates are reported in the online appendix, which also includes cross-sectional momentum analysis.

**Past returns and overconfidence**

My interpretation of the past returns evidence is that high past returns increase overconfidence, which in turn increases trading and liquidity. The connection between past returns and overconfidence is based on the learning models of Daniel, Hirshleifer, and Subrahmanyam (1998) and Gervais and Odean (2001). Self-attribution bias causes investors to be particularly overconfident following high returns. Because investors hold the market on average, aggregate overconfidence should track market returns. Similarly, individual stock returns could affect stock-level overconfidence. To the extent that investors specialize in certain industries or have industry-specific confidence levels, past industry returns also affect stock-level overconfidence.

To test the overconfidence interpretation, I analyze investor-level returns and trading activity. Self-attribution bias predicts that an investor’s overconfidence will increase following positive returns to his own portfolio. I test this hypothesis using account-level trading records from a discount brokerage firm (this is the Barber and Odean, 2000, data). Specifically, I look at how trading intensity responds to market returns, individual stock returns, and an investor’s own portfolio returns. Using six months of trailing trade data, I estimate an investor’s portfolio to be the net positions his trades over that period would result in. I set all short positions to zero because shorting is uncommon for retail investors. The outcome variable of interest is whether buying intensity, measured as the total dollar value of all buy trades increases between month t and month t+2. The returns considered are excess returns over the risk-free rate in the interim month.

Table 3 reports the results. Column (1) regresses increases in overall buying intensity at the investor-month level on market and portfolio returns. Unconditionally, buying intensity increases 15% of the time. 1% shocks to portfolio and market returns increase this probability by 27 and
23 basis points, respectively. Column (2) regresses increases in stock specific buying intensity at
the investor-month-stock level on market, portfolio, and individual stock returns. Individual stock
buying intensity increases 4% of the time, and 1% shocks to all three return variables increase this
probability by 5 to 7 basis points. All coefficient estimates are statistically significant. The results
are consistent with past returns increasing investor confidence, causing investors to trade more
aggressively.

4 Baseline Model

Setup

I consider a model in which agents have fixed endowments of a risky asset and receive private
signals about the asset’s value. The agents trade the asset in a public market in which all agents
see the market-clearing price. Informed by their private signals and the observed market price,
agents form beliefs about the asset’s payoff and decide how much of it to buy or sell. The only
departure from full rationality is that agents are overconfident about the precision of their own
signals. All agents are identical other than their realizations of the private signal. Thus, the same
agents simultaneously act as informed traders, behavioral traders, and market makers. The model is
an adaptation of Grossman’s (1976) rational expectations model in which the one significant change
is that agents are overconfident about the precision of their signals. As in Grossman’s model, the
lack of supply variance or any other noise results in prices that are fully revealing. Nonetheless,
overconfidence induces disagreement and trade.

In the next section, I introduce stochastic endowments. This adds liquidity trading to the model
and makes prices only partially revealing. The baseline model described here is a limiting case of
the more general model. I develop the baseline model separately both for expositional simplicity
and because the baseline model corresponds to a setting in which trade is entirely generated by
informed disagreement. Since my goal is to understand how liquidity interacts with disagreement
trading, this is a natural place to start.
Assumptions

There are two assets, a risk-free asset in elastic supply that yields 1 unit of consumption and a risky asset in fixed supply that yields \( \theta \) units of consumption and has a price of \( P \) (determined in equilibrium), where the risk-free asset is the numeraire. All agents share a common prior that \( \theta \) is normally distributed with a mean of \( \mu \) and precision of \( \tau_p \) (i.e., \( \theta \sim N(\mu, \frac{1}{\tau_p}) \)). In addition to the common prior, there are private signals with precision \( \tau_s \) about the asset’s value. The signals are \( y_m = \theta + \varepsilon_m \), where \( \varepsilon_m \sim iid N(0, \frac{1}{\tau_s}) \). \( \tau_p \) and \( \tau_s \) are both positive and finite. \( \mu \) is finite, and it is most natural to think of it as positive.

The economy has \( N \) agents divided into \( M \) equal groups, each of size \( \frac{N}{M} \). Agent \( i \) in group \( m(i) \) sees signal \( y_{m(i)} \). All agents have a known and certain endowment of \( \frac{\mu_X}{M} \) units of the risky asset. Thus, each group has an aggregate endowment of \( \frac{\mu_X}{M} \) and total asset supply is \( \mu_X \).\(^7\) \( M \) is greater than one and finite. More than one group is required to create disagreement and trade. A finite number of groups is required to prevent \( \theta \) from being perfectly revealed by the aggregate of the signals. I consider the limiting case in which \( N \to \infty \) to ensure that individual agents have negligible impact on the price of the risky asset.\(^8\) This limiting case is equivalent to a continuum of agents divided into \( M \) groups of equal mass.

All agents have constant absolute risk aversion preferences and risk tolerance of \( \frac{\eta}{N} \) (i.e., \( U_i(c_i) = -\exp\left\{\frac{-N}{\eta}c_i\right\} \)). Thus, aggregate risk tolerance is \( \eta \). Agents are overconfident about the precision of their own signals. They believe \( \varepsilon_m \) are independent and \( \varepsilon_m \sim i.id N(0, \frac{1}{\psi_{i,m} \tau_s}) \), where \( \psi_{i,m} = \psi \) if agent \( i \) is a member of group \( m \) and \( \psi_{i,m} = 1 \) otherwise. Unless otherwise noted, I assume \( \psi \) is greater than one and finite (i.e., agents are finitely overconfident). Agents know all model parameters including the overconfidence of other agents. Agents observe price but do not observe the signals of other agents.

\(^7\)The \( \mu_X \) notation is a little unnatural here, but it allows for directly comparable notation in the general model where total asset supply will be normally distributed with mean \( \mu_X \) and variance \( V \).

\(^8\)This is in contrast to the traditional market microstructure literature (e.g., Kyle, 1985), which models insiders as risk-neutral monopolists. Risk aversion and monopolistic behavior both have the effect of limiting asset demands. Incorporating both would unnecessarily complicate the model. Adding market power (as in Kyle, Obizhaeva, and Wang (2013)) increases the amount of overconfidence necessary to induce investors to trade, decreases trading volumes, and increases liquidity but does not alter the three propositions derived from the baseline model.
**Equilibrium**

I consider an equilibrium in which the asset’s price is a linear function of the average private signal. I.e., I assume:

$$P = \alpha + \beta \bar{y}$$  \hspace{1cm} (4)

where $\bar{y}$ is the average of the $M$ private signals.

Because price is a 1:1 function of the average private signal, all agents effectively see the average signal, from which they can extract the average signal of agents in groups other than their own, $\bar{y}_{-m(i)}$. Specifically, $\bar{y}_{-m(i)} = \frac{M}{\beta(M-1)} (P - \alpha) - \frac{1}{M-1} y_{m(i)}$. Using Bayesian updating, agent $i$’s posterior beliefs as a function of $y_{m(i)}$ and $P$ are:

$$E_i[\theta|y_{m(i)}, P] = \frac{\tau_p \mu + (\psi - 1) \tau_s y_{m(i)} + \frac{M \tau_s}{\beta} (P - \alpha)}{\tau_p + (M + \psi - 1) \tau_s}$$  \hspace{1cm} (5a)

$$Var_i[\theta|y_{m(i)}, P] = \frac{(\tau_p + (M + \psi - 1) \tau_s)^{-1}}{\eta}$$  \hspace{1cm} (5b)

Given his CARA utility, agent $i$’s asset demand is:

$$D_i = \frac{E_i[\theta|y_{m(i)}, P] - P}{\frac{\eta}{N} Var_i[\theta|y_{m(i)}, P]} = \frac{\eta}{N} \left( \frac{\tau_p \mu + (\psi - 1) \tau_s y_{m(i)} - \left( \tau_p + \left( M + \psi - 1 - \frac{M}{\beta} \right) \tau_s \right) P}{\tau_p + (M + \psi - 1) \tau_s} \right)$$  \hspace{1cm} (6)

The market clearing price must solve $\mu_X = \sum_i D_i$, which results in a price that is a linear function of $\bar{y}$. Equating its coefficients with the coefficients of (4) yields:

$$P = \frac{\tau_p \mu + (M + \psi - 1) \tau_s \bar{y}}{\tau_p + (M + \psi - 1) \tau_s} - \frac{\mu_X}{\eta (\tau_p + (M + \psi - 1) \tau_s)}$$  \hspace{1cm} (7)

This result should not be surprising. $\frac{\mu_X}{\eta (\tau_p + (M + \psi - 1) \tau_s)}$ is the average posterior expectation of the agents and $\frac{\mu_X}{\eta (\tau_p + (M + \psi - 1) \tau_s)}$ is the risk premium required to hold asset supply $\mu_X$ with posterior variance $(\tau_p + (M + \psi - 1) \tau_s)^{-1}$ and aggregate risk tolerance $\eta$. Compared to the price that would prevail without overconfidence ($\psi = 1$), equation (7) shows that overconfidence biases price toward the private signals and decreases the required risk premium. For $\psi$ that is small relative to $M$, both of these effects are modest.
Trading and Liquidity

Each agent’s directed trading volume is his net asset demand, \( D_i - \frac{\mu X}{N} \). Aggregate trading is
\[
Vol = \frac{1}{2} \sum_m |Trade_m|,
\]
where \( Trade_m \) is the net asset demand of group \( m \):
\[
Trade_m = \sum_{i:\text{m(i)=m}} \left[ D_i - \frac{\mu X}{N} \right] = \frac{\eta}{M} (\psi - 1) \tau_s (y_m - \overline{y})
\]
(8)

\(|Trade_m|\) is a folded mean-zero random variable so its expectation is
\[
E[|Trade_m|] = \sqrt{\frac{2 \text{Var}(Trade_m)}{\pi}}.
\]

Expected aggregate trading is:
\[
E[Vol] = \frac{1}{2} \sum_m E[|Trade_m|] = \frac{\eta (\psi - 1)}{2} \sqrt{\frac{2 (1 - \frac{\psi}{\pi}) \tau_s}{\pi}}
\]
(9)

From equation (9) it is clear that trading increases with \( \psi \) and \( \tau_s \) and is unaffected by \( \tau_p \).

In this model there is no liquidity trading, but the concept of liquidity is still operative. Analogous to Kyle’s (1985) lambda, I define illiquidity as the price impact of trade resulting from an exogenous shock. Intuitively, I am interested in how price would respond to an exogenous buy or sell trade. However, there is no exogenous trading in this model (nor is there in the real world). Rather, trade is an endogenous response to underlying shocks received by agents. Illiquidity is the ratio of a shock’s price impact to its impact on the shocked agents’ trades (i.e., their net asset demand). Because agents only interact through their trades, this ratio exactly represents the trades’ price impact. A shock affects an entire group of agents so the relevant net asset demand is the group’s, \( Trade_m \).\footnote{In the baseline model, the only shock is to a group’s signal so this is the shock I use to define illiquidity. More generally, one could also consider the impact of group endowment shocks or even shocks to individual agents. In all cases the resulting illiquidity is the same because in an anonymous market any trade must have the same price impact regardless of its source.}

Formally, illiquidity is:
\[
\lambda \equiv \frac{dP}{dy_m} \frac{dy_m}{dTrade_m}
\]
(10)
Taking derivatives of Trade\(_m\) (eq. 8) and \(P\) (eq. 7) with respect to \(y_m\) yields:

\[
\lambda = \frac{M(M + \psi - 1)}{\eta(M - 1)(\psi - 1)(\tau_p + (M + \psi - 1)\tau_s)}
\]
\[
= \left\{ \frac{M}{(M - 1)\eta(\tau_p + (M + \psi - 1)\tau_s)} \right\}
\]
\[
+ \left\{ \frac{\tau_s}{\tau_p + (M + \psi - 1)\tau_s} \right\} \left[ \frac{\eta}{M} \left( \psi - \frac{(M + \psi - 1)}{M} \right) \tau_s \right]^{-1}
\]
\[
= \{S\} + \{B\}
\]

Equation (11) shows that a buy trade affects price in two ways. First, it decreases the net asset supply that must be held by the non-shocked agents. The supply effect \((S)\) is the decline in the risk premium required by the non-shocked agents.\(^{10}\) Second, the trade increases non-shocked agents’ posterior expectations. \(B\) captures this belief channel. The trade’s impact on posterior expectations is the informational value of a \(y_m\) shock, \(\left[ \frac{\tau_p}{\tau_p + (M + \psi - 1)\tau_s} \right]\), divided by how aggressively the shocked agents trade on the shock, \(\left[ \frac{\eta}{M} \left( \psi - \frac{(M + \psi - 1)}{M} \right) \tau_s \right]^{-1}\).\(^{11}\)

Note that trading aggression is proportional to private signal precision and private signal value is the ratio of private signal precision to total precision. As a result, illiquidity depends only on aggregate information \((\tau_p + (M + \psi - 1)\tau_s)\), not its component parts. Public and private information both enhance liquidity. By contrast, overconfidence more than proportionally increases trading aggression without having much impact on signal value or aggregate information. Thus, overconfidence enhances liquidity, primarily through the belief channel.

The above equations and discussion support three propositions about how overconfidence and asymmetric information affect trading and liquidity.

**Proposition 1.** As overconfidence increases (\(\psi\) increases), trading and liquidity both increase.

**Proposition 2.** As private information becomes more precise (\(\tau_s\) increases), trading and liquidity both increase.

**Proposition 3.** As public uncertainty increases (\(\tau_p\) decreases) trading is unaffected and liquidity

\(^{10}\)The non-shocked agents have posterior variance \((\tau_p + (M + \psi - 1)\tau_s)^{-1}\) and risk tolerance \(\frac{(M - 1)\eta}{M}\), resulting in a risk premium (price reduction) of \(Supply \ast \frac{M}{(M - 1)\eta(\tau_p + (M + \psi - 1)\tau_s)}\).

\(^{11}\)Taking the derivative of equation (8) with respect to \(y_m\) establishes that \(\frac{d\text{Trade}_m}{dy_m} = \frac{\eta}{M} \left( \psi - \frac{(M + \psi - 1)}{M} \right) \tau_s\).
decreases.

Of the three propositions, proposition 2 is probably most surprising because it directly contradicts the traditional intuition that private information destroys liquidity and causes markets to break down. The general model develops this contrast in more detail and shows that private information enhances liquidity whenever the mix of informed and uninformed trade is insensitive to increases in private information. This is clearly the case in the baseline model because it only includes informed trade.

Propositions 2 and 3 concern different aspects of asymmetric information. Asymmetric information can be high because private signals are precise or because there is little public information. In either case, private information is more valuable and beliefs rely more heavily on private signals. In combination, propositions 2 and 3 imply that asymmetric information (weakly) increases trading and has an ambiguous impact on liquidity.

5 General Model

Setup and Assumptions

The general model is identical to the baseline model except that endowments are uncertain and unknown to agents in other groups. Specifically, agent $i$ in group $m$ ($i$) is endowed with $\frac{M}{N} x_{m(i)}$ units of the risky asset, where $x_{m} \overset{iid}{\sim} \mathcal{N}(\frac{\mu_X}{M}, \frac{V}{M})$ is the total endowment of group $m$. The resulting total asset supply is $X = \sum_{m} x_{m} \sim \mathcal{N}(\mu_X, V)$. Agents not in group $m$ know the distribution of $x_{m}$, but do not observe $x_{m}$.

The model is an adaptation of Diamond and Verrecchia’s (1981) noisy rational expectations model in which the one significant change is that agents are overconfident about the precision of their signals. The main distinguishing characteristic of Diamond and Verrecchia (1981) compared to other noisy rational expectations models (e.g., Hellwig, 1980) is that Diamond and Verrecchia consider endowment shocks to modeled agents whereas Hellwig and others consider direct, unobserved shocks to aggregate asset supply. The Diamond and Verrecchia approach is more natural because it makes liquidity trading an endogenous response to underlying endowment shocks instead of modeling liquidity trades as direct exogenous shocks to external asset supply. Diamond and Verrecchia’s
approach is also analytically convenient because it yields closed form solutions.

Results from the model are presented and discussed below. Derivations are in the appendix.

**Equilibrium**

I assume that price is a linear function of the average private signal and aggregate asset supply:

\[ P = \alpha + \beta \bar{y} - \gamma (X - \mu_X) \]  \hspace{1cm} (12)

Given the price function described by equation (12), agent \( i \) extracts a noisy signal \( A_{m(i)} \) for the average private signal of other groups \( (\bar{y}_{-m(i)}) \) from observing price, \( y_{m(i)} \), and \( x_{m(i)} \):

\[
A_{m(i)} = \frac{M}{\beta (M - 1)} (P - \alpha) - \frac{1}{M - 1} y_{m(i)} + \frac{\gamma M}{\beta (M - 1)} (x_{m(i)} - \frac{\mu_X}{M}) \\
= \bar{y}_{-m(i)} - \frac{\gamma M}{\beta} \left( \bar{x}_{-m(i)} - \frac{\mu_X}{M} \right) 
\]  \hspace{1cm} (13)

Note that \( A_{m(i)} \) is independent of \( y_{m(i)} \) and \( x_{m(i)} \) and \( A_{m(i)} \sim \mathcal{N} \left( \theta, \frac{1}{\tau_A} \right) \), where \( \tau_A \) is the precision agent \( i \) attributes to \( A_{m(i)} \):

\[
\tau_A = \left( \frac{1}{(M - 1) \tau_s} + \left( \frac{\gamma}{\beta} \right)^2 \frac{M}{M - 1} V \right)^{-1} 
\]  \hspace{1cm} (14)

Agent \( i \) forms posterior beliefs about the asset’s payoff (\( \theta \)) using Bayesian updating with signals \( y_{m(i)} \) and \( A_{m(i)} \). All agents use their posterior beliefs to determine their asset demands. Setting total asset demand equal to total asset supply results in a market-clearing price that is a linear function of the average private signal and aggregate asset supply just as I assumed in equation (12). The resulting fixed point problem has the unique solution:

\[
\alpha = \frac{(\eta^2 \psi^2 \tau_p \tau_s + MV \tau_p) \mu - \left( \eta \psi^2 \tau_s \frac{MV}{\eta} \right) \mu_X}{\eta^2 \psi^2 \tau_s \tau_p + (M + \psi - 1) \tau_s + (\tau_p + \psi \tau_s) MV} 
\]  \hspace{1cm} (15a)

\[
\beta = \frac{\psi \tau_s \left( \eta^2 \psi (M + \psi - 1) \tau_s + MV \right)}{\eta^2 \psi^2 \tau_s \tau_p + (M + \psi - 1) \tau_s + (\tau_p + \psi \tau_s) MV} 
\]  \hspace{1cm} (15b)

\[
\gamma = \frac{\eta^2 \psi (M + \psi - 1) \tau_s + MV}{\eta \left( \eta^2 \psi^2 \tau_s \tau_p + (M + \psi - 1) \tau_s + (\tau_p + \psi \tau_s) MV \right)} 
\]  \hspace{1cm} (15c)
For many applications, the ratio of $\gamma$ to $\beta$ is an important quantity. I define this ratio as:

$$\Gamma \equiv \frac{\gamma}{\beta} = \frac{1}{\eta \psi \tau_s} \quad (16)$$

To see that the baseline model is a limiting case of the general model, note that as $V \to 0$, $X \xrightarrow{P} \mu_X$ and $P \xrightarrow{P} \frac{\tau_p \mu + (M + \psi - 1) \tau_s \bar{y}}{\tau_p + (M + \psi - 1) \tau_s} - \frac{\mu_X}{\eta (\tau_p + (M + \psi - 1) \tau_s)}$, the baseline model price.

**Price Informativeness**

In the baseline model, price fully revealed all relevant information about the asset’s value. When endowments are stochastic, this is no longer the case. How much less informative is price?

One measure of price informativeness, used by Diamond and Verrecchia (1981), is the posterior precision an agent achieves relative to what his posterior precision would be if he observed all signals. Under full information, each agent has a posterior precision of $\tau_p + (M + \psi - 1) \tau_s$. This is the same posterior precision achieved in the baseline model. In the general model each agent knows the public prior and observes $y_{m(i)}$ and $A_{m(i)}$, resulting in posterior precision:

$$\tau_p + \psi \tau_s + \tau_A = \tau_p + \psi \tau_s + \left( \frac{1}{M - 1} \tau_s + \Gamma^2 \frac{M}{M - 1} V \right)^{-1} \quad (17)$$

More recently, Bai, Philippon, and Savov (2012) propose measuring price informativeness from the econometrician’s point of view. Specifically, they measure price informativeness as the $R^2$ of a regression of price on future asset value. In the general model, $P = \alpha + \beta \bar{y} - \gamma (X - \mu_X)$ has a variance of $\beta^2 \left( \frac{1}{\tau_p} + \frac{1}{M \tau_s} \right) + \gamma^2 V$; asset value ($\theta$) has a variance of $\frac{1}{\tau_p}$; and the covariance of price with asset value is $\frac{\beta}{\tau_p}$, resulting in:

$$R^2 = \frac{\tau_p^{-1}}{\tau_p^{-1} + (M \tau_s)^{-1} + \Gamma^2 V} \quad (18)$$

By contrast, if agents saw all private signals (as they effectively do in the baseline model), the $R^2$ of this regression would be $\frac{\tau_p^{-1}}{\tau_p^{-1} + (M \tau_s)^{-1}}$.

Under both measures of price informativeness, deviations from baseline full revelation price
informativeness are a function of $\Gamma^2V = \frac{V}{\eta^2\psi^2\tau_s^2}$. As noise ($V$) increases, price informativeness decreases. As risk tolerance ($\eta$), overconfidence ($\psi$), and private information ($\tau_s$) grow, prices become more informative. Roughly speaking, price informativeness is determined by the relative levels of informed and liquidity trading. Liquidity trading is increasing in $V$. Informed trading is increasing in $\eta$, $\psi$, and $\tau_s$.

**Trading**

As in the baseline model, group $m$’s directed trading volume is its net asset demand. The difference is that group $m$’s trading now depends on two random shocks instead of just one. Specifically, 

$\text{Trade}_m = \left\{ \frac{\eta}{M} \left( \psi \tau_s - \frac{\tau_A}{M-1} \right) (y_m - \bar{y}) \right\} - \left\{ \left( 1 - \frac{\eta \tau_A}{(M-1)\beta} \right) (x_m - \bar{x}) \right\}$, where the $(y_m - \bar{y})$ term represents informed trading and the $(x_m - \bar{x})$ term represents liquidity trading. Informed trading is greater than in the baseline model.\(^{13}\) Agents now have two motives for informed trade. First, overconfidence ($\psi > 1$) causes them to overweight their own signals as in the baseline model. Second, price no longer fully reveals the average signal, giving agents another reason to trade on their own signal. Liquidity trading is less than the endowment shocks themselves because endowment shocks are partially offset by demand changes. Agents realize that endowment shocks affect price and take this into account when determining their asset demands.\(^{14}\) As a result, liquidity, asset riskiness, and risk tolerance all influence liquidity trading.

\(^{13}\)To see that informed trade is greater than in the baseline model, note that $\tau_A < (M-1)\tau_s$. Thus, $\frac{\eta}{M} \left( \psi \tau_s - \frac{\tau_A}{M-1} \right) < \frac{\eta}{M} (\psi - 1) \tau_s$, the trading coefficient on $(y_m - \bar{y})$ in the baseline model.

\(^{14}\)Agents are atomistic and do not have any price impact by themselves (thus no monopoly pricing motive is present), but endowment shocks are shared by a positive mass of agents. The group’s endowment shock drives a wedge between price and group’s posterior value, which agents in the group exploit by changing asset demand in the opposite direction of the endowment shock.
Expected trading volume increases in the variance of group $m$’s trading:\(^{15}\)

\[
\text{Var}[\text{Trade}_m] = \left\{\text{Var}[\text{Informed}_m \text{Trade}_m]\right\} + \left\{\text{Var}[\text{Liquidity}_m \text{Trade}_m]\right\}
\]

\[
= \left\{\frac{\eta^2 (M - 1) \psi^2 \tau_s (\eta^2 \psi - 1) \psi \tau_s + MV}{M^3 (\eta^2 \psi^2 \tau_s + MV)^2}\right\}
\]

\[
+ \left\{\frac{(M - 1) (\eta^2 (\psi - 1) \psi \tau_s + MV)^2 V}{M^2 (\eta^2 \psi^2 \tau_s + MV)^2}\right\}
\]

\[
= \frac{(M - 1) (\eta^2 (\psi - 1) \psi \tau_s + MV)^2}{M^3 (\eta^2 \psi^2 \tau_s + MV)}
\]

(19)

From equation (19), it is clear that public uncertainty ($\tau_p^{-1}$) has no impact on trading volume, just as in the baseline model. Taking derivatives with respect to $\psi$, one can also see that overconfidence increases overall and informed trading (again consistent with the baseline model). Overconfidence can initially decrease liquidity trading if $V$ is large, but as $\psi$ increases it eventually decreases liquidity trading as well.\(^{16}\) These effects are small. In practice, overconfidence has very little impact on liquidity trading.

The relationship between private information ($\tau_s$) and trade is more complicated. Liquidity trading always decreases with $\tau_s$. When overconfidence is high ($\psi > \frac{9}{8}$ for informed trade and $\psi > 2$ for overall trade), informed and overall trading increase monotonically with $\tau_s$. When overconfidence is moderate ($1 < \psi < \frac{9}{8}$ for informed trade and $1 < \psi < 2$ for overall trade), informed and overall trading increase with $\tau_s$ if $\tau_s$ is large relative to $V$. Without overconfidence, all types of trading decrease with $\tau_s$. Essentially, private information always induces trade when there is high overconfidence. Under moderate overconfidence, private information decreases trade in liquidity trading (high $V$) environments but increases trade in disagreement trading (low $V$) environments. The Akerlof (1970) logic that private information destroys trade applies when agents are fully rational and in liquidity trading environments when agents are only moderately overconfident.

When trade primarily stems from overconfident disagreement, the opposite effect prevails, and

\(^{15}\)Specifically, expected volume is $E[Vol] = \frac{1}{2} \sum_m E[\text{Trade}_m]$ and $E[\text{Trade}_m] = \sqrt{\frac{\text{Var}[\text{Trade}_m]}{\pi}}$ just as in the baseline model.

\(^{16}\)Specifically, $\frac{d\text{Var}[\text{Liquidity}_m \text{Trade}_m]}{d\psi} < 0$ if $1 < \psi < \frac{1}{\sqrt{\pi \tau_s}}$ and is positive for larger $\psi$.

\(^{17}\)Assuming $\psi > 1$, $\frac{d\text{Var}[\text{Informed}_m \text{Trade}_m]}{d\tau_s} < 0$ if $\psi < \frac{9}{8}$ and $\tau_s < \frac{MV (3 - 2\psi - \sqrt{3 - 8\psi})}{2\eta^2 \psi (\psi - 1)}$ and $\frac{d\text{Var}[\text{Trade}_m]}{d\tau_s} < 0$ if $\psi < 2$ and $\tau_s < \frac{(2 - \psi)MV}{\eta^2 \psi^2 (\psi - 1)}$.\(^{24}\)
private information increases trade.

**Liquidity**

I employ the same definition and measure of liquidity that I introduced in the baseline model. Illiquidity is the price impact of a trade resulting from an exogenous shock, formally measured as:

\[ \lambda = \frac{dP}{dym} \frac{dTrade_m}{dym} \]

I could just as easily define \( \lambda \) using endowment \( (x_m) \) shocks instead of information \( (y_m) \) shocks. The resulting \( \lambda \) is the same. In either case, the shocked agents interact with the rest of the market only through their trading demand. Thus, a given shock to trading must have the same price impact regardless of what motivated the trade. This can be verified algebraically by taking derivatives with respect to \( x_m \) instead of \( y_m \). Even more generally, I could consider shocks to individual agents or exogenous trades external to the model. Regardless, price will always have the same response to a unit trade shock.

Taking derivatives of price and net asset demand with respect to underlying shocks yields:

\[
\lambda = \left\{ \frac{M}{(M-1) \eta (\tau_p + \psi \tau_s + \tau_A)} \right\} + \left\{ \frac{\eta^2 \psi^2 \tau_s}{\eta^2 \psi^2 \tau_s + MV} \right\} \left\{ \frac{\tau_s}{\tau_p + \psi \tau_s + \tau_A} \right\} \left\{ \frac{\eta}{M} \left( \psi \tau_s - \frac{\beta}{M} (\tau_p + \psi \tau_s + \tau_A) \right) \right\}^{-1} = \{S\} + \{B1\} \{B2\} \{B3\}^{-1}
\]

Equation (20) expresses lambda as the sum of a supply channel and a belief channel. The belief channel is further decomposed into the probability that a trade is informed \( (B1) \) times the impact a known shock to \( y_m \) would have on the posteriors of other agents \( (B2) \) divided by how aggressively shocked agents trade on \( y_m \) shocks \( (B3) \). \( S, B2, \) and \( B3 \) were present in the baseline model. As before, \( S \) and \( \frac{B2}{B3} \) tend to decrease as total information \( (\tau_p + \psi \tau_s + \tau_A) \) increases regardless of whether the information is public or private. \( S \) and \( \frac{B2}{B3} \) also decrease with overconfidence.

\( B1 \) is new and deserves special consideration. First note that \( B1 = \frac{\eta^2 \psi^2 \tau_s}{\eta^2 \psi^2 \tau_s + MV} \) is the probability that a trade is informed. Specifically, \( B1 \) is the ratio of informed trading variance to total trading variance (see equation (19) to verify this). \( B1 \) is increasing in \( \psi \) and \( \tau_s \) and represents the channel
through which they can destroy liquidity. Private information and overconfidence increase informed trading as a share of overall trading. When $V$ is close to $\frac{\psi^2 \tau_s}{M}$, this ratio is highly sensitive to $\psi$ and $\tau_s$. By contrast, for very small or large $V$, $B_1$ is close to 1 or 0 and fairly stable. The traditional logic that private information destroys liquidity applies only when increases in private information have a large impact on the ratio of informed to total trade. In particular, the traditional logic does not apply to disagreement trading environments because in those environments trade is primarily informed regardless of the exact level of private information. Similarly, overconfidence and private information enhance liquidity in extreme liquidity trading environments. The appendix formally considers derivatives of $\lambda$ with respect to $\psi$, $\tau_s$, and $\tau_p$ and derives parameter regions in which the derivatives are positive and negative. The appendix also shows that private information can enhance liquidity even without overconfidence. The basic result hinges on how much trading is informed, not on the presence of overconfidence.

**Numerical Examples**

To better understand how overconfidence and asymmetric information affect trading and liquidity, it is useful to consider numerical examples. I use the following baseline parameter values:

\[
\begin{align*}
\mu &= 1 \\
\mu_X &= 1 \\
M &= 10 \\
\tau_p &= 100 \\
\tau_s &= 10 \\
\psi &= 2 \\
\eta &= 0.1 \\
V &\in \{0.0000001, 0.1, 10\}
\end{align*}
\]

$\mu$ and $\mu_X$ are normalized to one. I consider 10 groups of agents. Prior precision of 100 yields a public prior standard deviation of 10%. Private precision of 10 makes the private signals in aggregate as valuable as the public prior. Overconfidence of 2 means that agents attribute twice as much value to their own signals as they do to the signals of agents in other groups. Aggregate risk tolerance of
produces a risk premium of 10% under public information without supply shocks.

I start by considering a low variance environment \((V = 0.0000001)\), which roughly corresponds to the baseline constant endowment model. At baseline values, expected turnover is 12% and \(\lambda = 58\%\) (meaning an exogenous trade of 1% of aggregate asset supply would change the asset’s price by 58 basis points). The first row of figure 5 plots these quantities as functions of overconfidence, varying \(\psi\) from 1 to 10 while holding all other parameters at baseline values. Panel A shows that trading is approximately zero when \(\psi = 1\) and trading increases close to proportionally with \(\psi\). When \(\psi = 2\), expected turnover is 12%. When \(\psi = 10\), expected turnover is 108%. The solid line in Panel B plots \(\lambda\) as a function of \(\psi\). As predicted by the baseline model, the market is highly illiquid when \(\psi = 1\) (approaching infinity as \(V \to 0\)) and becomes more liquid as \(\psi\) increases. When \(\psi = 2\), \(\lambda = 58\%\). When \(\psi = 10\), \(\lambda = 8\%\). Beyond \(\psi = 10\), \(\lambda\) continues to decrease with \(\psi\), approaching 0 as \(\psi \to \infty\). The dotted and dashed lines in panel B decompose \(\lambda\) into its supply and belief channels. The belief channel (dashed line) is the dominant source of illiquidity. The second row of figure 5 repeats the same exercise, varying private signal precision from 0 to 100. As private signals become more precise, trading increases (panel C) and illiquidity decreases (panel D). The final row of figure 5 considers public prior precision. As predicted, turnover is unaffected and illiquidity decrease as \(\tau_p\) increases.

Figure 6 plots expected turnover and illiquidity as functions of overconfidence, private information precision, and public prior precision in a moderate supply variance environment. Baseline parameter values are the same as before except that endowment variance is now 0.1, which corresponds to an aggregate asset supply standard deviation of 0.32, compared to its mean of 1. These examples capture a market in which trading comes from both informational and liquidity motives. Panels A, C, and E plot expected total turnover (solid line), expected informed turnover (dashed line), and expected liquidity turnover (dotted line). Informed turnover is the turnover that would prevail if agents differed only in their information shocks (i.e., if \(x_m = \bar{x} \forall m\)). Analogously, liquidity turnover is the turnover that would prevail if agents differed only in their supply shocks. Note that total turnover is less than the sum of informed and liquidity turnover because these two types of trade partially offset one another.

The \(\psi = 1\) starting point of panel A plots turnover in the absence of overconfidence, which recreates the Diamond and Verrecchia (1981) model. As alluded to earlier in the paper, liquidity
turnover represents 95% of total turnover. As overconfidence (panel A) and private information precision (panel C) increase, informed turnover increases, driving up total turnover. By contrast, private information decreases liquidity turnover as \( \tau_s \) increases. Liquidity turnover is fairly insensitive to overconfidence. At first it slightly decreases with \( \psi \), then it slightly increases with \( \psi \). Both forms of turnover are unaffected by public prior precision.

As expected, liquidity trading enhances liquidity. The baseline \( \lambda \) decreases from 58% to 20% when \( V \) increases from 0.0000001 to 0.1. The reduction in \( \lambda \) is entirely driven by the belief channel. Belief and supply illiquidity now have similar magnitudes. Most interestingly, liquidity trading changes the relationships between \( \psi \), \( \tau_s \), and illiquidity. Illiquidity is now a hump-shaped function of \( \psi \) (panel B) and \( \tau_s \) (panel D). Overconfidence and private information precision initially decrease liquidity before eventually enhancing it. Panels A and C show why. Initial increases in \( \psi \) and \( \tau_s \) dramatically increase the ratio of informed trading to total trading, which increases the probability that any given trade is informed. At higher levels of \( \psi \) and \( \tau_s \) this ratio and probability are fairly stable.

Figure 7 replicates the moderate variance example under a no-overconfidence (\( \psi = 1 \)) baseline. Panel A shows that without overconfidence private information decreases turnover. This is an illustration of the general result derived above. Overconfidence is a necessary ingredient for private information to increase trading. Nonetheless, panel B shows that private information enhances liquidity for \( \tau_s \) above 32. Overconfidence amplifies this liquidity enhancement but is not necessary for the basic result.

Figure 8 plots expected turnover and illiquidity in a high supply variance (\( V = 10 \)) environment in which trading is primarily liquidity-driven. In this environment, liquidity is enhanced (the baseline \( \lambda \) is 9.4%, and this is almost entirely from the supply channel). \( \lambda \) decreases in \( \psi \), \( \tau_s \), and \( \tau_p \) (see panels B, D, and F) because only the supply channel is really in play. Panels A, C, and E show that expected total turnover is high (its baseline value is 378%) and insensitive to \( \psi \), \( \tau_s \), and \( \tau_p \) even though \( \psi \) and \( \tau_s \) increase informed trading.
6 Model Assessment

In contrast to traditional models, disagreement trading generates significant trading and liquidity in the baseline model even without exogenous noise. Moreover, disagreement trading is consistent with the stylized facts developed in Section 2.

First, high turnover is generally associated with high liquidity. This is easiest to see by comparing the low, medium, and high variance numerical examples. As liquidity trading increases across the scenarios, trading and liquidity increase dramatically. Overconfidence also typically moves trading and liquidity in the same direction.

Second, the disagreement model is consistent with asymmetric information increasing trade while decreasing liquidity. Proposition (2) of the baseline model predicts that private information increases trade and liquidity. Proposition (3) predicts that public uncertainty decreases liquidity while having no impact on trading. Jointly, propositions (2) and (3) predict that asymmetric information (which consists of private information and public uncertainty) can only increase trading and has an ambiguous effect on liquidity. Unlike traditional models, these predictions are consistent with observed empirical evidence.

Third, overconfidence increases trade and liquidity. If overconfidence increases following high past returns, as self-attribution bias theory predicts, this delivers the prediction that turnover and liquidity will increase following high past returns.

In addition to conforming with the stylized facts, the model generates new predictions about how the impact of asymmetric information on trading and liquidity varies with the type of asymmetric information shock, the level of liquidity trading, and the existing level of private information. Testing these predications is challenging because they involve unobservable quantities, and I have not found a way to separately identify private information, public uncertainty, and liquidity trading. Fortunately, the model itself provides guidance for differentiating private information changes from public uncertainty changes. In the baseline model, private information increases trading, whereas public uncertainty has no impact on it. Thus, asymmetric information changes with a larger trading impact are more likely to be driven by changes in private information. The baseline model predicts that private information enhances liquidity whereas public uncertainty reduces it. Thus, we should expect asymmetric information to have the least negative impact on liquidity (and potentially even
enhance it) when it has the most impact on trading.

I test this prediction in the data by sorting stocks based on their past turnover responses to asymmetric information changes. Specifically, I estimate stock-level rolling 5-year regressions of turnover on lagged analyst forecast dispersion, controlling for aggregate dispersion. Stocks are annually sorted into low, medium, and high responsiveness groups based on 30th and 70th percentile breakpoints. I then replicate my analyst forecast dispersion panel regressions with interactions between analyst forecast dispersion and past turnover responsiveness groups.

Table 4 reports the results. Column (1) shows that turnover responsiveness to analyst forecast dispersion is persistent. Low past turnover responsiveness stocks have no turnover response to analyst forecast dispersion (the coefficient of log turnover on log lagged analyst dispersion is an insignificant -0.6% for the low responsiveness group). As past turnover responsiveness increases, this coefficient increases by 2.4 ppt for the medium responsiveness group and 5.8 ppt for the high responsiveness group. Both results are highly significant. The model predicts that as turnover responsiveness increases, illiquidity responsiveness should decrease. This is what I find in the data. Column (2) presents results for illiq. High turnover response stocks have a highly significant 4.2 ppt lower illiquidity response to analyst dispersion compared to low turnover response stocks. Medium turnover response stocks have about the same dispersion coefficient as low turnover response stocks. Column (3) to (5) present results for different bid-ask spread measures. The pattern is the same. As turnover responsiveness increases, bid-ask responsiveness decreases. In part due to the shorter sample, most of the bid-ask spread results are not significant. The exception is quoted intraday bid-ask spreads (column 4) for high response stocks, which have a significant (at the 10% level) 1.1 ppt lower dispersion coefficient compared to low response shocks.

7 Conclusion

Liquidity plays an increasingly important role in asset pricing and macro finance. Yet, we lack a clear understanding of some of the most basic drivers of liquidity in informationally sensitive markets. Existing models and intuition suggest that asymmetric information destroys trading and liquidity. Though less well understood, overconfidence is generally associated with enhanced liquidity. The theory and empirics supporting these contentions are not satisfying. In particular, existing
models rely heavily on exogenous noise trading, usually ignore overconfidence, and are unable to explain the empirical reality of large stock, corporate bond, and option trading volumes that are positively correlated with asymmetric information.

The disagreement literature proposes that overconfidence-driven disagreement provides a rationale for trade that does not require exogenous noise traders or uncertain asset supply. I show that overconfidence is also sufficient for generating and thinking about liquidity. In my baseline model, agents differentially weight their own signals even though prices perfectly reveal the average signal. This causes them to disagree about the asset’s value and trade. The market is liquid even without liquidity trading. All of this is within an intentionally simple market setup. All agents are homogeneous until receiving signals. No outside parties are needed for noise trading or market making. The same agents simultaneously serve as informed traders, noise traders, and market makers.

The baseline model rationalizes heavy trading and liquidity despite asymmetric information with no exogenous liquidity trading. The model also produces three predictions: (1) as overconfidence increases ($\psi$ increases), trading and liquidity both increase; (2) as private information becomes more precise ($\tau_a$ increases), trading and liquidity both increase; and (3) as public uncertainty increases ($\tau_p$ decreases) trading is unaffected and liquidity decreases. Consistent with stylized facts about stocks, corporate bonds, and stock options, these predictions jointly imply that asymmetric information increases trading and to the extent that past returns increase overconfidence they also increase trading and enhance liquidity. The predictions also imply that asymmetric information shocks with the largest trading impact should have the smallest (and potentially even negative) illiquidity impact. I test this prediction in the data and find that it is true.

The baseline model’s predictions are at odds with some of the recent literature on financial crises. Kacperczyk and Schnabl (2010) and Gorton and Metrick (2011) document the collapse of trade in asset backed commercial paper and repurchase agreements during the 2007-2009 financial crisis. Both of these markets previously facilitated liquidity trading in instruments that were perceived to be safe and information-insensitive. The authors reasonably argue that the market collapses were at least in part driven by increases in asymmetric information. Dang, Gorton, and Holmstrom (2009) propose a model of liquidity in which debt contracts optimally facilitate trade in part by minimizing asymmetric information. Consistent with traditional intuition, if the debt contracts become informationally sensitive trade and liquidity dry up. This narrative of the financial
crisis contradicts my baseline model and is also difficult to reconcile with the liquid trade observed in equity markets despite significant asymmetric information. My general model provides a way to bridge this gap. In the face of moderate liquidity trading, adding private information to a market at first destroys liquidity by increasing the likelihood any given trade is informed. Once most trades are already informed, further increases in private information enhance liquidity.

One aspect of disagreement trading I don’t address is welfare. Overconfidence-driven disagreement clearly has some negative implications. Unequal risk sharing causes optimistic agents to hold higher variance portfolios than they would without overconfidence, which diminishes welfare under the criterion of Brunnermeier, Simsek, and Xiong (2012). On the other hand, overconfidence facilitates liquidity, which is likely beneficial. Overconfidence also makes prices more informative by causing agents to trade on their information more aggressively. Additional work connecting the microfoundations of liquidity and trading to their welfare implications is necessary to fully understand these trade-offs.

\[\text{Additional work connecting the microfoundations of liquidity and trading to their welfare implications is necessary to fully understand these trade-offs.}\]

\[\text{Though not in my model, overconfidence likely also incentivizes gathering more information. Rubinstein (2001)}\]

\[\text{makes this point and argues that irrational investors may enhance market rationality by increasing price informative-ness.}\]

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A Appendix

Solution to General Model

I assume that the price function is linear:

\[ P = \alpha + \beta \bar{y} - \gamma (X - \mu_X) \]  

(21)

Claim 1. \( \beta \neq 0 \).

Proof. Assume \( \beta = 0 \). Thus, price is uninformative about private signals, and all agents have posterior beliefs of \( E_i[\theta | y_{m(i)}, x_{m(i)}, P] = \frac{\tau_p \mu + \psi \tau_s y_{m(i)}}{\tau_p + \psi \tau_s} \) and \( \text{Var}_i[\theta | y_{m(i)}, x_{m(i)}, P] = \frac{\tau_p \mu + \psi \tau_s y_{m(i)}}{\tau_p + \psi \tau_s} \) resulting in asset demand \( D_i[y_{m(i)}, x_{m(i)}, P] = E_i[\theta | y_{m(i)}, x_{m(i)}, P] - P \) for the market clearing price implies \( P = \frac{\tau_p \mu + \psi \tau_s y_{m(i)}}{\tau_p + \psi \tau_s} \). Thus, \( \beta = \frac{\psi \tau_s}{\tau_p + \psi \tau_s} \neq 0 \), a contradiction.

Given the price function described by equation (21), agent \( i \) extracts a noisy signal for \( \bar{y}_{-m(i)} \) from observing price, \( y_{m(i)} \), and \( x_{m(i)} \):

\[ A_{m(i)} = \frac{M}{\beta (M - 1)} (P - \alpha) - \frac{1}{M - 1} y_{m(i)} + \frac{\gamma M}{\beta (M - 1)} \left( x_{m(i)} - \frac{\mu_X}{M} \right) \]

\[ = \bar{y}_{-m(i)} - \frac{\gamma M}{\beta} \left( \bar{x}_{-m(i)} - \frac{\mu_X}{M} \right) \]

(22)

Note that \( A_{m(i)} \sim \mathcal{N} \left( \theta, \frac{1}{(M - 1)\tau_s} + \left( \frac{\gamma}{\beta} \right)^2 \frac{M}{M - 1} V \right) \) and \( A_{m(i)} \) is independent of \( y_{m(i)} \) and \( x_{m(i)} \). Using Bayesian updating with signals \( y_{m(i)} \) and \( A_{m(i)} \) and substituting \( P, y_{m(i)}, x_{m(i)} \) for \( A_{m(i)} \) using (22), agent \( i \)'s posterior beliefs as a function of \( P, y_{m(i)}, x_{m(i)} \) are:

\[ E_i[\theta | y_{m(i)}, x_{m(i)}, P] = \left( \frac{\tau_p \mu - \frac{M \tau_A}{\beta (M - 1)} \alpha + \left( \frac{\psi \tau_s - \frac{\tau_A}{M}}{\tau_p + \psi \tau_s + \tau_A} \right) y_{m(i)} }{\tau_p + \psi \tau_s + \tau_A} \right) \]

\[ \text{Var}_i[\theta | y_{m(i)}, x_{m(i)}, P] = \left( \frac{\tau_p + \psi \tau_s + \tau_A}{\tau_p + \psi \tau_s + \tau_A} \right)^{-1} \]

(23a)

(23b)

where \( \tau_A = \left( \frac{1}{(M - 1)\tau_s} + \left( \frac{\gamma}{\beta} \right)^2 \frac{M}{M - 1} V \right)^{-1} \) is the precision agent \( i \) attributes to \( A_{m(i)} \).

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Agent $i$’s asset demand is:

$$D_i = \frac{E_i[\theta | y_m(i), x_m(i), P] - P}{\frac{1}{N} \text{Var}_i[\theta | y_m(i), x_m(i), P]} \begin{cases} \eta \\ \frac{\alpha}{\beta} \\ \gamma \end{cases} \left( \tau_p \mu - \frac{M \tau_A}{\beta(M-1)} \alpha + \left( \psi \tau_s - \frac{\tau_A}{\beta(M-1)} \right) y_m(i) + \frac{\gamma M \tau_A}{\beta(M-1)} \left( x_m(i) - \frac{\mu}{M} \right) - \left( \tau_p + \psi \tau_s + \left( 1 - \frac{M}{\beta(M-1)} \right) \tau_A \right) P \right) \right) \tag{24}$$

The market clearing price must solve \( X = \sum_i D_i \). Thus,

$$P = \frac{\tau_p \mu - \frac{1}{\eta} \mu_X - \frac{M \tau_A}{\beta(M-1)} \alpha + \left( \psi \tau_s - \frac{\tau_A}{\beta(M-1)} \right) \bar{y} - \left( \frac{1}{\eta} - \frac{\gamma \tau_A}{\beta(M-1)} \right) (X - \mu_X)}{\tau_p + \psi \tau_s + \left( 1 - \frac{M}{\beta(M-1)} \right) \tau_A} \tag{25}$$

Equations (21) and (25) yield the following system of equations:

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \left( \tau_p + \psi \tau_s + \left( 1 - \frac{M}{\beta(M-1)} \right) \tau_A \right) = \begin{bmatrix} \tau_p \mu - \frac{1}{\eta} \mu_X - \frac{M \tau_A}{\beta(M-1)} \alpha \\ \psi \tau_s - \frac{\tau_A}{\beta(M-1)} \\ \frac{1}{\eta} - \frac{\gamma \tau_A}{\beta(M-1)} \end{bmatrix} \tag{26}$$

\textbf{Claim 2.} The unique solution to equations (26) is:

$$\alpha = \frac{\left( \eta^2 \psi^2 (\tau_p \tau_s + MV \tau_p) \mu - \left( \eta^2 \psi^2 \tau_s + \frac{M \psi \tau_p}{\eta} \right) \mu_X \right)}{\eta^2 \psi^2 \tau_s (\tau_p + (M + \psi - 1) \tau_s) + (\tau_p + \psi \tau_s) MV} \tag{27a}$$

$$\beta = \frac{\psi \tau_s \left( \eta^2 \psi (M + \psi - 1) \tau_s + MV \right)}{\eta^2 \psi^2 \tau_s (\tau_p + (M + \psi - 1) \tau_s) + (\tau_p + \psi \tau_s) MV} \tag{27b}$$

$$\gamma = \frac{\eta \psi^2 \tau_s (\tau_p + (M + \psi - 1) \tau_s) + (\tau_p + \psi \tau_s) MV}{\eta \left( \eta^2 \psi^2 \tau_s (\tau_p + (M + \psi - 1) \tau_s) + (\tau_p + \psi \tau_s) MV \right)} \tag{27c}$$

\textbf{Proof.} Consider \( \Gamma \equiv \frac{\alpha}{\beta} \). We already established that \( \beta \neq 0 \) so \( \Gamma \) is finite. Substituting \( \Gamma \) into (26) and dividing the \( \gamma \) equation by the \( \beta \) equation yields:

$$\Gamma = \left( \frac{1}{\eta \psi \tau_s} - \frac{\tau_A}{\beta(M-1)} \right) \frac{\tau_A}{M-1} = \frac{1}{\eta \psi \tau_s} \tag{28}$$
Plugging (28) into the β equation of (26) yields:

\[
\beta = \frac{\psi \tau_s \left( \eta^2 \psi (M + \psi - 1) \tau_s + MV \right)}{\eta^2 \psi^2 \tau_s (\tau_p + (M + \psi - 1) \tau_s) + (\tau_p + \psi \tau_s) MV} \tag{29}
\]

Plugging (28) and (29) into the γ equation of (26) yields:

\[
\gamma = \frac{\eta^2 \psi (M + \psi - 1) \tau_s + MV}{\eta \left( \eta^2 \psi^2 \tau_s (\tau_p + (M + \psi - 1) \tau_s) + (\tau_p + \psi \tau_s) MV \right)} \tag{30}
\]

Finally, plugging (28), (29), and (30) into the α equation of (26) yields:

\[
\alpha = \frac{\left( \eta^2 \psi^2 \tau_p \tau_s + MV \tau_p \right) \mu - \left( \eta \psi^2 \tau_s + \frac{MV}{\eta} \right) \mu_X}{\eta^2 \psi^2 \tau_s (\tau_p + (M + \psi - 1) \tau_s) + (\tau_p + \psi \tau_s) MV} \tag{31}
\]

Trading

Using equation (24),

\[
Trade_m \equiv \sum_{i:m(i)=m} \left[ D_i - \frac{M}{N} x_m \right] = \left\{ \eta \left( \frac{\psi \tau_s}{M} - \frac{\tau_A}{M - 1} \right) (y_m - \bar{y}) \right\} - \left\{ \left( 1 - \frac{\eta \gamma \tau A}{(M - 1) \beta} \right) (x_m - \bar{x}) \right\} \tag{32}
\]

All other trading derivations are in the main text of the paper.

Liquidity

Recall that illiquidity is defined as:

\[
\lambda \equiv \frac{dP}{dym} \left( \frac{dT_{\text{Trade}}}{dym} \right)
\]
Taking derivatives of $P$ (eq. 21) and $Trade_m$ (eq. 32) with respect to $y_m$ and plugging in $\gamma$ from (27c) and $\Gamma$ from (28) yields:

$$
\lambda = \frac{-\gamma}{(N_M)(\eta_M)} \left( \frac{\gamma M \tau_s}{\beta(M-1)} + \gamma \left( \tau_p + \psi \tau_s + \left( 1 - \frac{M}{M-1} \right) \tau_A \right) \right) - 1
$$

$$
= \frac{M (\eta^2 \psi^2 \tau_s + MV) (\eta^2 \psi (M + \psi - 1) \tau_s + MV)}{\eta (M - 1) (\eta^2 (\psi^2 - \psi) \tau_s + MV) (\eta^2 \psi^2 \tau_s (\tau_p + (M + \psi - 1) \tau_s) + (\tau_p + \psi \tau_s) MV)}
$$

$$
= \left\{ \left\{ \frac{\eta^2 \psi^2 \tau_s}{\eta^2 \psi^2 \tau_s + MV} \right\} \left\{ \frac{\tau_s}{\tau_p + \psi \tau_s + \tau_A} \right\} \left\{ \frac{\eta}{M} \left( \psi \tau_s - \frac{\beta}{M} (\tau_p + \psi \tau_s + \tau_A) \right) \right\}^{-1}
$$

$$
= \{ S \} + \{ B1 \} \{ B2 \} \{ B3 \}^{-1}
$$

(33)

Consistent with the baseline model, $\lim_{V \to 0} \frac{d\lambda}{d\psi} < 0$, $\lim_{V \to 0} \frac{d\lambda}{d\tau_s} < 0$, and $\lim_{V \to 0} \frac{d\lambda}{d\tau_p} < 0$. From (33) one can see that $\frac{d\lambda}{d\tau_p} < 0$ for all $V$. However, $\frac{d\lambda}{d\psi}$ and $\frac{d\lambda}{d\tau_s}$ are not always negative. Their signs are determined by complicated functions of the parameters. Considering limiting cases is instructive. We have already seen that $\frac{d\lambda}{d\psi}$ and $\frac{d\lambda}{d\tau_s}$ are negative in the limit as $V \to 0$. Both are also negative in the limit as $V \to \infty$. For interim values of $V$ (i.e., positive, finite $V$), $\frac{d\lambda}{d\psi}$ and $\frac{d\lambda}{d\tau_s}$ can be positive or negative. Both follow a similar pattern. As $\psi \to 0$ or $\tau_s \to 0$, $\lambda \to \frac{M}{(M-1)\eta \tau_p}$, which is solely a supply impact – it includes no belief price response.$^{19,20}$ $\frac{d\lambda}{d\psi}$ and $\frac{d\lambda}{d\tau_s}$ initially have the same sign as $\eta^2 \tau_p - V$ (i.e., $\text{sign} \left[ \lim_{\psi \to 0} \frac{d\lambda}{d\psi} \right] = \text{sign} \left[ \lim_{\tau_s \to 0} \frac{d\lambda}{d\tau_s} \right] = \text{sign} [\eta^2 \tau_p - V]$). As $\psi$ and $\tau_s$ increase, they eventually decrease $\lambda$, driving it to approach zero as $\psi \to \infty$ or $\tau_s \to \infty$.

**Liquidity without overconfidence**

Private information precision can enhance liquidity even without overconfidence. When $\psi = 1$ (which reproduces the model of Diamond and Verrecchia (1981)), illiquidity is:

$$
\lambda_{\psi=1} = \frac{M (\eta^2 \tau_s + MV) (\eta^2 \tau_s + V)}{\eta (M - 1) (\eta^2 \tau_s (\tau_p + M \tau_s) + (\tau_p + \tau_s) MV) V}
$$

$^{19}$Though I restrict my attention to overconfidence ($\psi > 1$) in other parts of the paper, it is useful to generalize and consider underconfidence ($\psi < 1$) here to get a full picture of the relationship between $\lambda$ and $\psi$.

$^{20}$The total risk tolerance of agents not receiving the shock is $\frac{M(M-1)\psi}{M}$ and their posterior variance is $\tau_p^{-1}$. 

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and its derivative with respect to private information precision is:

\[
\frac{d\lambda}{d\tau_s} = 1 = \frac{M \left( \eta^6 \tau_p \tau_s^2 + \eta^4 \tau_s (2\tau_p - M\tau_s) MV + \eta^2 (\tau_p - 2\tau_s) M^2 V^2 - M^2 V^3 \right)}{\eta (M - 1) \left( \eta^2 \tau_s (\tau_p + M\tau_s) + (\tau_p + \tau_s) MV \right)^2 V}
\]

(35)

The \( V \to 0 \) limit is uninteresting because without endowment shocks or overconfidence, illiquidity is infinite.

As \( V \to \infty \), \( \lambda_{\psi=1} = \frac{M}{(M-1)\eta(\tau_p + \tau_s)} \) and \( \frac{d\lambda}{d\tau_s}_{\psi=1} = \frac{-M}{(M-1)\eta(\tau_p + \tau_s)} < 0 \). Under infinite supply variance, liquidity trading swamps informed trading so trades carry no information. Thus, only the supply channel is operative, and the supply illiquidity channel always decreases as information (public or private) increases.

For interim values of \( V \), \( \lambda \) starts off as solely a supply effect: \( \lim_{\tau_s \to 0} \lambda_{\psi=1} = \frac{M}{(M-1)\eta\tau_p} \). As \( \tau_s \) increases, the supply illiquidity channel decreases, but the belief illiquidity channel increases at least initially. For large \( V \), the decreasing supply channel is more powerful. For small \( V \), the increasing belief channel is more powerful. Specifically, \( \lim_{\tau_s \to 0} \frac{d\lambda}{d\tau_s}_{\psi=1} = \frac{M \left( \eta^2 \tau_p - V \right)}{\eta^2 \tau_p (M-1)V} \). For large \( \tau_s \), only the belief channel is operative, and \( \lim_{\tau_s \to \infty} \lambda_{\psi=1} = \frac{\eta}{(M-1)V} \). Note that this is a positive constant whereas \( \lim_{\tau_s \to \infty} \lambda = 0 \) when \( \psi > 1 \). The belief channel consistently increases with \( \tau_s \) when \( V \) is small, but when \( V \) is large, \( \tau_s \) eventually decreases the belief channel, thereby decreasing overall illiquidity as well. Specifically, \( \text{sign} \left[ \lim_{\tau_s \to \infty} \frac{d\lambda}{d\tau_s}_{\psi=1} \right] = \text{sign} \left[ \eta^2 \tau_p - M^2 V \right] \). Another point of interest is to compare illiquidity at the two limits of \( \tau_s \): \( \lim_{\tau_s \to \infty} \lambda_{\psi=1} = \frac{\eta}{M V} \).

The overall relationship between private information and illiquidity without overconfidence is as follows: For high supply variance \( (V > \eta^2 \tau_p) \), private information decreases illiquidity; for low supply variance \( (V < \eta^2 \tau_p/M^2) \), private information increases illiquidity; and for moderate supply variance \( (\eta^2 \tau_p/M^2 < V < \eta^2 \tau_p) \), illiquidity is a hump-shaped function of private information. Within the moderate case, \( \tau_s \) decreases illiquidity overall when \( V > \frac{\eta^2 \tau_p}{M} \) and increases illiquidity overall when \( V < \frac{\eta^2 \tau_p}{M} \).
References


Table 1: Turnover Panel Regressions

Results are for stock- and bond-level regressions of log bid-ask spread measures on log turnover. Stock data is for NYSE stocks with lagged prices greater than $5. Bond data is for actively traded U.S. corporate bonds without credit enhancements. Option data is for all traded stock options. Bid-ask spreads are proportional to security value. Stock and bond bid-ask spreads are intraday effective spreads. Option bid-ask spreads are end of day quoted spreads. Robust clustered (by firm) standard errors are in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

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<th>Stocks</th>
<th>Bonds</th>
<th>Options</th>
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<td>(3)</td>
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<td>(0.006)</td>
<td>(0.011)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Log Turnover</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Stock/Bond FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Month FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Table 2: Analyst Dispersion Panel Regressions

Results are for stock- and bond-level regressions of log turnover and log bid-ask spread measures on lagged (by one month) log dispersion of analyst earnings forecasts. Data is limited to NYSE stocks with lagged prices greater than $5, at least 2 analyst forecasts, and fiscal years that end in December. Bid-ask spreads are proportional to security value. Stock and bond bid-ask spreads are intraday effective spreads. Option bid-ask spreads are end of day quoted spreads. Robust clustered (by firm) standard errors are in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

<table>
<thead>
<tr>
<th>Stocks</th>
<th>Bonds</th>
<th>Options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>Log Turnover</td>
<td>Log Bid-Ask</td>
</tr>
<tr>
<td>Lagged Log Analyst Dispersion</td>
<td>0.019***</td>
<td>0.109***</td>
</tr>
<tr>
<td>Stock/Bond FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Month FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Table 3: Impact of Returns on Buying Intensity

Dependent variables are indicators for an investor increasing his dollar buying activity over a two-month period. A value of 1 indicates that the dollar sum of all an investor’s buy trades in month t+2 is greater than the dollar sum of all his buy trades in month t. Column (1) analyzes total buys at the investor-month level. Column (2) analyzes stock-specific buys at the investor-month-stock level. To be included, the investor must have at least two buy trades in month t (overall for column (1) and of the specific stock for column (2)). The explanatory variables represent excess returns over the risk free rate in the interim month (t+1). The portfolio return is the return to a portfolio consisting of the investor’s cumulative net trades over the six months leading up to month t. Implied short positions are set to zero. Standard errors are in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance. The analyzed data is discount brokerage trades in 100,000 accounts between 1991 and 1996.

<table>
<thead>
<tr>
<th></th>
<th>(1) Overall Buy Increase</th>
<th>(2) Stock-Specific Buy Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio Return</td>
<td>0.268*** (0.009)</td>
<td>0.065*** (0.009)</td>
</tr>
<tr>
<td>Market Return</td>
<td>0.231*** (0.033)</td>
<td>0.069** (0.032)</td>
</tr>
<tr>
<td>Stock Return</td>
<td></td>
<td>0.046*** (0.007)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.151*** (0.001)</td>
<td>0.039*** (0.001)</td>
</tr>
</tbody>
</table>
Table 4: Analyst Dispersion Panel Regressions by Lagged Turnover Response

Results are for stock-level regressions of log turnover and log illiquidity measures on lagged (by one month) log dispersion of analyst earnings forecasts. Medium and high turnover responsiveness indicator variables are based on turnover responsiveness to dispersion changes over the past five years relative to 30th and 70th percentile breakpoints. Data is limited to NYSE stocks with lagged prices greater than $5, at least 2 analyst forecasts, and fiscal years that end in December. Robust clustered (by firm) standard errors are in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

<table>
<thead>
<tr>
<th></th>
<th>(1) Log Turnover</th>
<th>(2) Log illiq</th>
<th>(3) Log bidask</th>
<th>(4) Log qbidask</th>
<th>(5) Log ebidask</th>
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</thead>
<tbody>
<tr>
<td><strong>Lagged Log Analyst Dispersion</strong></td>
<td>-0.006</td>
<td>0.210***</td>
<td>0.100***</td>
<td>0.109***</td>
<td>0.111***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.010)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td><strong>Lagged Log Dispersion</strong></td>
<td>0.024***</td>
<td>-0.002</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.005</td>
</tr>
<tr>
<td>* Medium Turn Response</td>
<td>(0.006)</td>
<td>(0.010)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td><strong>Lagged Log Dispersion</strong></td>
<td>0.058***</td>
<td>-0.041***</td>
<td>-0.007</td>
<td>-0.011*</td>
<td>-0.008</td>
</tr>
<tr>
<td>* High Turn Response</td>
<td>(0.007)</td>
<td>(0.013)</td>
<td>(0.007)</td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td><strong>Medium Turn Response</strong></td>
<td>0.031</td>
<td>0.033</td>
<td>0.0002</td>
<td>0.009</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.036)</td>
<td>(0.021)</td>
<td>(0.020)</td>
<td>(0.021)</td>
</tr>
<tr>
<td><strong>High Turn Response</strong></td>
<td>0.190***</td>
<td>-0.126***</td>
<td>0.023</td>
<td>0.007</td>
<td>0.021</td>
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<tr>
<td></td>
<td>(0.026)</td>
<td>(0.048)</td>
<td>(0.026)</td>
<td>(0.025)</td>
<td>(0.025)</td>
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<tr>
<td><strong>Stock FE</strong></td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td><strong>Year FE</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Month FE</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Figure 1: Monthly Time Series. Stock turnover is relative to stock market capitalization. Bond turnover is relative to outstanding bond value. Option turnover is option contract value relative to stock market capitalization. Bid-ask spreads are all proportional to the value of the security being traded. Stock and bond bid-ask spreads are intraday effective spreads. Option bid-ask spreads are end of day quoted spreads.
Figure 2: Turnover and Liquidity Around Earnings Announcements. Turnover and bid-ask spreads are scaled by average daily values over the three calendar months before the earnings announcement. Solid lines are equally weighted averages. Dashed lines are 95% confidence intervals. Day 0 is the day of the earnings announcement.
Figure 3: Market VAR Responses to Market Return Impulse. Each VAR includes detrended log market turnover (\(\text{turn}\)), CRSP value-weighted market returns in excess of the risk free rate (\(\text{rmrf}\)), and a detrended log measure of market illiquidity (\(\text{illiq}\) for the stock VAR in the first row, effective bid-ask spread for the bond VAR in the second row, and quoted bid-ask spread for the option VAR in the third row). The solid lines are responses to one standard deviation shocks to \(\text{rmrf}\) after the number of lags indicated on the horizontal axis. The dashed lines are 95% confidence intervals.
Figure 4: Stock Panel VAR Impulse Response Functions. The panel VAR includes stock log turnover, stock log iliq (an illiquidity measure), stock returns, industry returns, and stock and time fixed effects. The first variable in each panel title is the impulse variable. The second variable is the response variable. The solid lines are responses to one standard deviation shocks to the impulse variables after the number of lags indicated on the horizontal axis. The dashed lines are 95% confidence intervals. For brevity only the most relevant impulse response functions are shown.
Figure 5: Trading and Illiquidity Under Low Variance. Expected turnover and illiquidity ($\lambda$) are calculated using values indicated on the horizontal axis for the parameter in parentheses and low variance baseline values ($\mu = 1$, $\mu_s = 1$, $M = 10$, $\tau_p = 100$, $\tau_s = 10$, $\psi = 2$, $\eta = 0.1$, $V = 0.0000001$) for all other parameters. Vertical lines represent the baseline. Expected total turnover is a solid line; expected informed turnover is a dashed line; and expected liquidity turnover is a dotted line. Overall illiquidity is a solid line; the illiquidity belief channel is a dashed line; and the illiquidity supply channel is a dotted line.
Figure 6: Trading and Illiquidity Under Moderate Variance. Expected turnover and illiquidity ($\lambda$) are calculated using values indicated on the horizontal axis for the parameter in parentheses and moderate variance baseline values ($\mu = 1, \mu_x = 1, M = 10, \tau_p = 100, \tau_s = 10, \psi = 2, \eta = 0.1, V = 0.1$) for all other parameters. Vertical lines represent the baseline. Expected total turnover is a solid line; expected informed turnover is a dashed line; and expected liquidity turnover is a dotted line. Overall illiquidity is a solid line; the illiquidity belief channel is a dashed line; and the illiquidity supply channel is a dotted line.
Figure 7: Trading and Illiquidity Without Overconfidence. Expected turnover and illiquidity (λ) are calculated using values indicated on the horizontal axis for the parameter in parentheses and moderate variance baseline values without overconfidence (μ = 1, μx = 1, M = 10, τp = 100, τs = 10, ψ = 1, η = 0.1, V = 0.1) for all other parameters. Vertical lines represent the baseline. Expected total turnover is a solid line; expected informed turnover is a dashed line; and expected liquidity turnover is a dotted line. Overall illiquidity is a solid line; the illiquidity belief channel is a dashed line; and the illiquidity supply channel is a dotted line.
Figure 8: Trading and Illiquidity Under High Variance. Expected turnover and illiquidity ($\lambda$) are calculated using values indicated on the horizontal axis for the parameter in parentheses and high variance baseline values ($\mu = 1$, $\mu_x = 1$, $M = 10$, $\tau_p = 100$, $\tau_s = 10$, $\psi = 2$, $\eta = 0.1$, $V = 10$) for all other parameters. Vertical lines represent the baseline. Expected total turnover is a solid line; expected informed turnover is a dashed line; and expected liquidity turnover is a dotted line. Overall illiquidity is a solid line; the illiquidity belief channel is a dashed line; and the illiquidity supply channel is a dotted line.