Mathematics at Charlemagne's Court and Its Transmission

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Introduction

By AD 400 copies of the great scientific works of antiquity were becoming very scarce in the Mediterranean world. A renewed interest in learning then led to the writing of digests of earlier work, based either on partial copies or more often on intermediate digests of the third century. These efforts of the late antique period were highly influential for the first generation of Islamic translations during the ninth century and for the revived interest in scholarship in the Christian West a century earlier. Although modest in their accomplishments, such episodes of intellectual revitalization are of interest today because they suggest that older works were sequestered during troubled times, then disseminated and studied under more stable conditions, especially with institutional or personal patronage. Further, we must surmise that information and ideas also were transmitted verbally, allowing a few gifted and reflective individuals to work effectively even in out of the way places.

There also was a great deal of practical information, part of it written, that was applied to pragmatic questions of everyday importance, such as monumental architecture. A large part of this history of ideas and their practical applications is within the purview of historical investigation. Another part calls for specialists in the sciences, who ideally can grasp the complexity and significance of such understanding from their own disciplinary perspectives. These viewpoints may be very different, but they are complementary.[1]

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[1] P. L. Butzer, 'Mathematics in West and East from the Fifth to the Tenth Centuries', in *Science in Western and Eastern Civilization in Carolingian Times*, ed. by P. L. Butzer and D. Lohrmann (Basel, 1993), pp. 443–81.

If one would consider the greatest works of antiquity as the sole norm for work in the mathematical sciences, then of course advances in scientific knowledge during Carolingian times might be regarded as insignificant. However, the complete works of such earlier authors only became available in western Europe during the eleventh century when the earliest Latin translations from the Arabic were carried out, many based on Greek manuscripts which have not survived. Thus an analysis of scientific studies in early medieval times must begin with different premises, in view of the limited sources available. This paper focuses on the mathematics and astronomy treated or applied at the *scola palatina*, the court school of Charlemagne, at Aachen, or as otherwise sponsored by Charlemagne and Louis the Pious at affiliated centres.[2] The period of major concern is between 794 and *c.* 840 when Aachen became the nexus of scientific currents that emerged in western Christendom during the eighth century. A number of themes can be identified that are treated below: 1) The science of computing the date for the movable feast of Easter, the *Computus*, which received specific attention at Aachen by Alcuin,[3] by a computistical conference of 809, in the *Aratea*, an astronomical poem, and by Dicuil; 2) the body of geometric transmission and understanding incorporated in the *Pseudo Boethian Geometry I*, probably assembled *c.* 780–810; 3) the collection of mathematical problems known as the *Propositiones ad acuendos iuvenes*, attributed to Alcuin, representing the oldest such collection in Latin and first mathematical 'primer' in western Europe; 4) the repertoire of practical geometry and surveying incorporated in the *Geometrica incerti auctoris*, probably compiled during the ninth century, perhaps by the anonymous Astronomer before 840; and 5) the geometry and the theory of proportions underlying construction of the royal chapel at Aachen, consecrated about 800.

The Computus

One of the most complex problems in mathematics and astronomy that engaged the Christian Church again and again was computation of the movable feast of Easter. Christians initially tied the celebration of Easter to the Jewish Passover, which was based on a lunar calendar. But time in the Roman Empire was reckoned by a solar calendar, so that early Christian scholars sought to coordinate the two calendars. No precise solution for this coordination has yet been found, and the available approximations, like those with the Islamic lunar calendar, require minor periodic adjustments. Further, liturgical planning in advance, to inform small or distant Christian communities, required 'prediction' of the date for Easter in future years.

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[2] See the various papers on mathematics and astronomy in *Science in Western and Eastern Civilization*, ed. by Butzer and Lohrmann, and in *Charlemagne and His Heritage: 1200 Years of Civilization and Science in Europe*, vol. II, *Mathematical Arts*, ed. by P. L. Butzer, H. Th. Jongen, and W. Oberschelp (Turnhout, 1998).

[3] For Alcuin see the outstanding Cambridge dissertation of 1997 by Mary D. Garrison, 'Alcuin's World through His Letters and Verse' (unpublished doctoral thesis, University of Cambridge, 1997), now forthcoming with Cambridge University Press.

That date required conception and calculation of the relations between sun, moon, and earth in a nineteen-year cycle, say. This was a challenge that even intrigued C. F. Gauss, who as late as 1800 offered a formula to reckon the date of Easter. Presumably the problem was approached graphically at the time, by plotting the intersection of lunar and solar cycles, which would not only require an observational database but also a clear conception of cycles and related graphic skills.[4]

The recurrent problem posed by predicting the date for Easter led to a series of efforts, first in the Greek East, especially Egypt, and later in the Latin West.[5] The *Laterculus*, based on Augustalis (*c.* 250) and a sixteen-year cycle of Hipplolytus in Rome, was in use in the British Isles until about AD 700. The continuous sequence of calendar years going back to the birth of Christ, rather than with reference to the reigns of successive emperors or the first Easter, is primarily due to Dionysius Exiguus (d. 550).

It was Alcuin of York who, through his work, teachings, and efforts in the transmission and dissemination of the basic writings on *computus* and natural history of the Venerable Bede, made the Carolingian reform of the calendar possible; it standardized calendrical reckoning and chronology for the following centuries. Alcuin, the head of the palace school at Aachen, was most probably the author of the *Calculatio Albini magistri* of 776, definitely of the *Ratio de luna* of 798, but not of the anonymous texts *De bissexto* and *De saltu lunae*, as has often been assumed. On top, nine of the fifty-seven letters of the correspondence between Alcuin and Charlemagne written between November 797 and March 799 deal with astronomical and computistical problems.[6]

The most important of these dealt with the date of the so-called 'moon-leap', *saltus lunae*, needed to bring the sun and moon in the nineteen-year cycle in agreement. Alcuin wanted to follow the Roman tradition, but Charlemagne's young advisors, the *aegyptiaci pueri*, preferred the Alexandrian.[7] Alcuin's own astronomical observations on the moon, Mars, and Sirius, in particular about the 'vanished Mars', hidden behind the sun about 18 July 798, inspired the Carolingian scholars to make further observations on the motions of the planets in the geocentric system.

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[4] For example, H. P. Lattin, 'The Eleventh-Century MS Munich 14436: Its Contribution to the History of Coordinates, of Logic, of German Studies in France', *Isis*, 38 (1947), 205–25.

[5] P. L. Butzer, 'Mathematics in Egypt and Its Connections with the Court School of Charlemagne', in *Mathematical Analysis, Wavelets, and Signal Processing*, Contemporary Mathematics, 190 (Providence, RI, 1995), 1–30.

[6] The minutes of this conference, probably due to Adalhard, still exist: *Epistolae Variorum Carlo Magno Regnante Scriptae*, no. 42, ed. by E. Dümmler, MGH, Epp., 4 (Berlin, 1895), pp. 565–67.

[7] For Alcuin and calendar reckoning during Charlemagne's reign see the outstanding doctoral thesis: K. Springsfeld (née Arendt), 'Alkuin Einfluss auf die Komputistik zur Zeit Karls' des Grossen' (unpublished doctoral thesis, Rheinisch-Westfälische Technische Hochschule, Aachen, 2000), published as *Beiheft zu Sudhoff's Archiv*, Heft 48 (Stuttgart, 2002); also her master's thesis: K. Arendt, 'Komputistik zur Zeit Karls' des Grossen' (unpublished master's thesis, Rheinisch-Westfälische Technische Hochschule, Aachen, 1993), p. 107.

There was an astronomical-computistical conference of sorts in Aachen in 809, almost certainly organized by Abbot Adalhard of Corbie (*c.* 750–826), a first cousin of Charlemagne, regarded as an expert on the computus after Alcuin's death.[8] The deliberations were elaborated as the Seven Book *Computus*, or the so-called 'Aachen Encyclopedia'. This work contains Alcuin's tracts *Calculatio* and *Ratio de luna*, and Bede's *De natura rerum*, together with a little of Martianus Capella. It even contains new astronomical diagrams visualizing Plinian planetary theory.[9] This synthetic effort brought the understanding of the five planets (Mercury, Venus, Mars, Jupiter, Saturn) into a coherent picture of the celestial realm more precisely and comprehensively than any Latin text since the fifth century. According to Arno Borst, this encyclopaedic work is 'so extensive and compact as is no book on *computus* before or after; [. . .] it dominated European time-studies for the next 300 years'.[10]

Dicuil of Iona, who came to the court at Aachen *c.* 806/12, wrote a *Liber de astronomia* 813/14, primarily a *computus* for Easter tables, possibly in connection with the basic conference of 809.[11] It includes a critical explanation of the lunar cycle, calculates the date of Easter on the basis of Victor of Aquitaine (fl. 457), and is independent of Bede's methods. Mathematical analysis of this work is still lacking, precluding more direct comment. There also is the *Aratea*, an epic poem in hexameter form dealing with astronomical phenomena. The most renowned copy of the *Aratea*, MS Cod. Voss. Lat. Q 79 of Leiden University Library, the Aratos-Cicero-Germanicus-

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[8] D. Ganz, *Corbie in the Carolingian Renaissance* (Sigmaringen, 1990); U. Winter, 'Die mittelalterlichen Bibliothekskataloge aus Corbie' (unpublished doctoral thesis, Humboldt Universität, Berlin, 1972); W. Hartmann, *Die Synoden der Karlingerzeit im Frankenreich und in Italien* (Paderborn, 1989); A. Borst, 'Computus: Zeit und Zahl im Mittelalter', *Deutsches Archiv*, 44 (1988), 1–82; A. Borst, *Die karolingische Kalenderreform*, Schriften der MGH, 46 (Hannover, 1998).

[9] Dated to 809–12, perhaps written at Metz or Prüm; Madrid Biblioteca Nacional 3307 (of 820–40); Vatican, Biblioteca Apostolica Vaticana Lat. 645 and Reg. lat. 309. The Three Book *Computus* of 812–20, possibly under Arno of Salzburg, which was supposed to amplify the Seven Book version, is found in Munich, Bayerische Staatsbibliothek, Clm 210, and Vienna, Österreichische National Bibliothek, Lat. 387. See W. Stevens, 'Astronomy in Carolingian Schools', in *Charlemagne and His Heritage*, vol. I, *Scholarship, Worldview and Understanding*, ed. by P. L. Butzer, M. Kerner, and W. Oberschelp (Turnhout, 1997), pp. 417–87.

[10] A. Borst, 'Alkuin und die Enzyklopädie von 809', in *Science in Western and Eastern Civilization*, ed. by Butzer and Lohrmann, pp. 53–78 (pp. 73 and 75): '[. . .] so ausgedehnt und geschlossen wie kein Lehrbuch der Zeitkunde davor und danach' and later '[. . .], dass die karolingische Enzyklopädie für die nächsten dreihundert Jahre das europäische Zeitdenken beherrschte'.

[11] W. Bergmann, 'Dicuils "De Mensura orbis terrae"', in *Science in Western and Eastern Civilization*, ed. by Butzer and Lohrmann, pp. 525–37.

Hyginus transmission, was perhaps compiled by the 'Astronomer' at the court of Louis the Pious, possibly *c.* 816.[12]

The Pseudo-Boethian Geometry I

The *Pseudo-Boethian Geometry I*, in five books, includes excerpts of Boethius's *De arithmetica* and important parts of Boethius's translation of at least the first four books of Euclid's *Elements*. As recently argued, the six earliest manuscripts contain all twenty-three definitions of Book I (for point, line, plane, angle, circle, rhomboid, etc.), including the five Postulates and the four Axioms, in complete form. Of the fifteen Definitions from Books II–III, fourteen are given, as well as seventy-six of the Enunciations of the 107 Propositions concerning triangles, diameters, tangents, and perpendiculars, accompanied by over a hundred constructions and illustrated with geometrical figures. But proofs are found only for Book I. Wesley Stevens has shown that it includes parts of Euclid's original arithmetic—including components that are not based on Nichomachos of Gerasa, the source of Boethius's *De arithmetica*.[13] It thus seems to include even excerpts from Books VII, VIII, and IX of the *Elements*.

This work also includes various gromatic tracts of the Roman *agrimensores* here rearranged to serve as geometrical material for the quadrivium.[14] These are based on the *corpus gromaticorum*, a collection of wide-ranging surveying and centuriation results,

[12] For the *Aratea* see *Aratea: Kommentar zum Aratea des Germanicus, MS Voss. Lat. Q 79* (Facsimile reproduction; Lucerne, 1989); A. von Euw, *Der Leidener Arateus: Antike Sternbilder in einer Karolingischen Handschrift* (Cologne, 1987); M. Erren, *Die Phainomena des Aratos von Soloi* (Wiesbaden, 1967). Concerning this date see R. Mostert and M. Mostert, 'Using Astronomy as an Aid to Dating Manuscripts: The Example of the Leiden Aratea', *Quaerendo*, 20.1 (1990), 248–61. They argue, 'From these (astronomical dates) we can deduce that the configuration, and probably the manuscript, can be dated 816, around 18 March.'

[13] W. Stevens, 'The Earliest Ninth-Century Text of Euclidean Geometry; ms Paris BN Lat. 13955' (in preparation); Stevens, 'Euclidean Geometry in the Early Middle Ages; a Preliminary Reassessment', in *Villard's Legacy: Studies in Medieval Technology, Science and Art*, ed. by M.-T. Zenner (Aldershot, in press); Stevens, 'Karolingische *renovatio* im Wissenschaften und Literatur', in *799 – Kunst und Kultur der Karolingerzeit: Karl der Grosse und Papst Leo III in Paderborn*, ed. by Christoph Stiegemann and Matthias Wemhoff, 3 vols (Mainz, 1999), III, 663–80.

[14] G. Chouquer and F. Favory, *Les arpenteurs romains: théorie et pratique* (Paris, 1992); L. Toneatto, *Codices artis mensoriae: i manoscritti degli antichi opuscoli latini d'agrimensura (V–XIX sec.)* (Spoleto, 1994); *Die römische Feldmesskunst; interdisziplinäre Beiträge zu ihrer Bedeutung für die Zivilisationsgeschichte Roms*, ed. by O. Behrends and L. Capogrossi Colognesi (Göttingen, 1992).

the archetype of which has been dated to about AD 450, with the earliest extant copies from the sixth or seventh century (in Wolfenbüttel).[15]

Dating of the *Pseudo-Boethian Geometry* is fluid. Ullman has argued that it was assembled at Corbie under Abbot Adalhard; the three oldest extant copies are dated by Stevens to about AD 825–40.[16] Indeed, Stevens tentatively ascribes the oldest of these copies to scribes at Fulda, perhaps to Walahfrid Strabo (who spent 827–29 there) and his friend Gottschalk (*c.* 803–69), who also worked at Corbie.

The Propositiones

In a letter of 800 to Charlemagne, Alcuin mentions sending him 'certain subtle figures of arithmetic, for pleasure'. These and others like them are assumed to be the source of the *Propositiones ad acuendos iuvenes.*[17] It is an eclectic collection of problems designed to sharpen logical and mathematical acuity. In part their interest stems from their diverse sources: a good number of them are familiar stock, originally derived from Greek or even Egyptian sources. Others are unusual variants of the same, but the nine types of *Hundred Fowls Problems* are first known from China *c.* AD 475, and a *Hound*

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[15] Wolfenbüttel, Herzog August-Bibliothek Aug. 2°. 36.23 (fols 2–83, 84–122, and 124–56). G. Thulin, *Die Handschriften des Corpus Agrimensorum Romanum* (Berlin, 1911).

[16] B. L. Ullman, 'Geometry in the Medieval Quadrivium', in *Studi di bibliografia e di storia in onoris di Tammaro de Marinis*, 4 vols (Vatican City, 1964), IV, 263–85; M. Folkerts, 'The Importance of the Pseudo-Boethian Geometria during the Middle Ages', in *Boethius and the Liberal Arts: A Collection of Essays*, ed. by M. Masi (Bern, 1981), pp. 187–209. Stevens, 'Karolingische *renovatio*'.

[17] Alcuin, *Epistolae*, 101, ed. by E. Dümmler, MGH, Epp., 4, pp 18–481. M. Folkerts has found thirteen manuscripts of the text, the earliest (Vatican Reg. lat. 309 from St Denis—not complete) being from the ninth century. The next oldest manuscript is from the late tenth century (Karlsruhe, Landesbibliothek, Cod. Augiensis 205) and is essentially the Alcuin text. M. Folkerts, *Die älteste mathematische Aufgabensammlung in lateinischer Sprache: Die Alkuin zugeschriebenen 'Propositiones ad acuendos iuvenes': Überlieferung, Inhalt, kritische Edition*, vol. VI, Österreichische Akademie der Wissenschaften, Mathematische-naturwissenschaftliche Klasse, Denkschriften, 116 (Vienna, 1978); M. Folkerts, 'Die Alkuin zugeschriebenen *Propositiones ad acuendos juvenes*', in *Science in Western and Eastern Civilization*, ed. by Butzer and Lohrmann, pp. 273–82. For translations into German and English see M. Folkerts and H. Gericke, 'Die Alkuin zugeschriebenen *Propositionen ad acuendos iuvenis*: Lateinischer Text und deutsche Übersetzung', in *Science in Western and Eastern Civilization*, ed. by Butzer and Lohrmann, pp. 283–362; D. Singmaster and J. Hadley, 'Problems to Sharpen the Young: An Annotated Translation of *Propositiones ad acuendos iuvenes*, the Oldest Mathematical Problem Collection in Latin, Attributed to Alcuin of York', *The Mathmatics Gazette*, 76 (1992), 102–26; D. Singmaster, 'The History of Some of Alcuin's *Propositiones*', in *Charlemagne and His Heritage*, vol. II, ed. by Butzer, Jongen, and Oberschelp, pp. 11–30.

and Hare Problem (an overtaking problem) is first recorded from China *c.* AD 150. A number—six of the fifty-three—also are completely new.[18] The genre is illustrated by one form of the *River Crossing Problem*:

> A man had to take a wolf, a goat and a bunch of cabbages across a river. The only boat he could find could only take two of them at a time. But he had been ordered to transfer all of these to the other side in good condition. How could this be done?
> Solution: I would take the goat and leave the wolf and the cabbage. Then I would return and take the wolf across. Having put the wolf on the other side I would take the goat back over. Having left that behind, I would take the cabbage across. I would then row across again, and having picked up the goat take it over once more. By this procedure there would be some healthy rowing, but no lacerating catastrophe.[19]

Another example is the *Crossing the Desert Problem*, the *Jeep Problem*, or the *Explorer's Problem*:

> A certain gentleman ordered that 90 measures of grain were to be moved from one of his houses to another, 30 leagues (= *leuwas*) away. One camel was to carry the grain in three journeys, carrying 30 measures on each journey. The camel eats one measure for each league. How many measures will remain?
> Solution: On the first journey the camel carries 30 measures over 20 leagues and eats one measure for each league, that is he eats 20 measures leaving 10. On the second journey he likewise carries 30 measures and eats 20 of these leaving 10. On the third journey he does likewise: he carries 30 measures, and eats 20 of these, leaving 10. Now there remain 30 measures and 10 leagues to go. He carries these 30 on his fourth [Problem requires 3] journey to the house, and eats 10 on the way, so there remains just 20 measures of the whole sum.[20]

It should be noted that there is also a still better solution, involving six journeys.

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[18] For a more detailed discussion of the different problem types see P. L. Butzer, 'Mathematics and Astronomy at the Court School of Charlemagne and Its Mediterranean Roots', *Cahiers de recherches médiévales (XIIIe–XVes)*, 5 (1998), 203–44.

[19] Folkerts, *Die älteste mathematische Aufgabensammlung*, no. 18. This problem is analysed in the setting of integer programming in every direction in R. Borndörfer, M. Grötschel, and A. Löbel, 'Alcuin's Transportation Problems and Integer Programming', in *Charlemagne and His Heritage*, vol. II, ed. by Butzer, Jongen, and Oberschelp, pp. 379–409.

[20] Folkerts, *Die älteste mathematische Aufgabensammlung*, no. 52. This optimization problem, the first appearance of the modern jeep problem in operations research, is treated in detail by W. Oberschelp, 'Alcuin's Camel and Jeep Problem', in *Charlemagne and His Heritage*, vol. II, ed. by Butzer, Jongen, and Oberschelp, pp. 411–22.

The last two problems played prominent roles in the centuries to follow; they shaped the subdisciplines of discrete mathematics (combinatorics and graph theory), optimization (linear and integer programming), and operations research, all fields of Applied Mathematics that have become especially popular only in the past fifty years. They encapsulate all the characteristics of most of today's large-scale real transportation problems. Other, more familiar problems deal with geometric and arithmetic progressions and with the areas of rectangles, quadrilaterals, and circles.

Several of Alcuin's problems have their models in a Byzantine collection of epigrams, the *Anthologia Palatina*. Its Book XIX contains forty-four arithmetical problems, ascribed to Metrodoros. These include the geometric 'Heap' problems (namely Alcuin's problems 2–4, 36, 40, 45, and 48), problem 7, the 'Cistern Problem' 8, as well as problems 16 and 44. Metrodoros was a brother of Anthemios of Tralles (d. 534), the master builder of the Hagia Sophia. Perhaps the epigrams became known to the court school via Ellisaios, the Greek teacher sent by the Byzantine emperor to tutor Rotrud, Charlemagne's oldest daughter, *c.* 781–806. There may possibly be an association between these epigrams and the earliest architectural plans of the Minster, since Metrodoros's brother was the builder of the Hagia Sophia and perhaps also of the Sergios and Bacchos Church, the 'little' Hagia Sophia. The latter has interesting similarities with San Vitale of Ravenna.[21] There may also be links between some of Alcuin's problems and the four included in *De arithmeticis propositionibus*, ascribed to Bede and largely transmitted in the same manuscripts as the *Propositiones*.

The Geometria incerti auctoris

Only Books III and IV of the *Geometria incerti auctoris* survive.[22] One book deals with the practical solution of surveying problems while the other uses sixty-one problems to deal with mensuration of triangles, circles, trapezoids, and other figures. Problems 30–39 are identical with ten in the *Propositiones* where they are presented in the same order; where there are differences, the treatment in the *Geometria* is much improved and the solution more precise, suggesting a slightly later compilation. The author was versed in the fundamentals of geometry and calculating with fractions; he probably drew from an *agrimensores* text not known to us. Although not yet re-edited according to modern standards, this practical geometry was once attributed to Gerbert of Aurillac (*c.* 945–1003), however Folkerts and Gericke suggest that it was probably written in the ninth century, perhaps at Aachen (and possibly *c.* 840).[23]

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[21] See below.

[22] G. Beaujouan, *'Part raison de nombres', L'art du calcul et les savoirs scientifiques médiévaux* (Aldershot, 1991); N. Bubnov, *Gerberti opera mathematica* (Berlin, 1899; repr. 1963)

[23] Folkerts, *Die älteste mathematische Aufgabensammlungin*, p. 30; Folkerts and Gericke 'Die Alkuin zugeschriebenen *Propositionen ad acuendos iuvenis*'; H. Gericke, *Mathematik im Abendland: Von den römischen Feldmessern bis zu Descartes* (Berlin, 1990).

The Geometry of the Aachen Minster

The facade and spires of the Cathedral of Cologne were completed 1863–80, following detailed drawings of *c.* 1300. Even in the 1880s its 157-metre-high towers were the tallest in the world, and they withstood the shocks of repeated heavy bombing of the adjacent railway station during the 1940s. This shows how medieval buildings were not only conceptualized as a whole and in detail, but that the immense load-bearing issues central to the stability of such a large slender building were well understood. No such plans exist for the octagonal court chapel of Aachen, built *c.* 792–800, an innovative structure which is attributed to Odo of Metz. At 32 m elevation, the domed vault represented, in its turn, the tallest building north of the Alps until the twelfth century. Most authors believe that this core element of the Aachen Minster was inspired by the Byzantine masterpiece, San Vitale, in Ravenna (522–47). Nonetheless the warped and domed forms of San Vitale, predicated on brick and light terracotta vault construction, are replaced in Aachen by more typically Roman barrel and groin vaults.[24]

Architectural styles and aesthetics are based on an explicit geometric conception, in turn implemented by particular construction materials and methods, applied with an empirical group of mechanical laws that relate to centre of gravity, mass, and load-bearing capacity. Even to lay out the base of a large, octagonal structure, such as that at the core of the Minster, required an elaborate exercise in practical geometry, initially on the designer's board and then on the ground. First, an exact square of the desired dimension had to be laid out, perhaps by initially tracing out a circle by means of a piece of string, followed by measuring the half-length of intersecting diagonals. Then, in both directions from each corner, that length had to be marked off on the square. The eight resulting points would mark the corners of the octagon. From that starting point, the architect had to carry out countless analogous operations to lay out the sixteen-sided perimeter walls, and to determine each detail of the network of supporting pillars and vaults, all according to a fairly explicit notion of proportionality, and in anticipation of the load distribution of the superstructure. Subsequently, each stage of the vertical construction would require similar planning, but in three dimensions.

The architect, therefore, had to be a good geometer and an experienced engineer, quite apart from his sense of aesthetics. Such qualifications may have become fairly commonplace during the twelfth century, but during early medieval times they were not. North of the Alps, domestic architecture was mainly in wood, while a rectangular stone church with attached, semicircular apses would pose few technical problems. Indeed, the drums and towers of the church of St Riquier (built in the 790s) were of wood, and basilican churches were generally covered with wooden roofs.[25]

The static problems of heavy stone construction are of an order of magnitude greater than for wood, and no stone-vaulted dome of any size had been built anywhere since the

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[24] K. J. Conant, *Carolingian and Romanesque Architecture 800 to 1200* (New York, 1978), p. 51.

[25] Conant, *Carolingian and Romanesque Architecture*, pp. 44–45 and 468, nn. 3, 5.

sixth century. The high, vaulted octagon of the Minster was then a daring structure for the period, and its architect will have required direct experience in the design and execution of complex stone structures. Elaborate masonry churches were being built in Lombardy or Southern Italy at the time, but of very different design, so that the architect commissioned to design the Minster would have first had to study San Vitale very closely.[26] But despite very similar dimensions, the different building materials available in Aachen required different constructional forms to carry the heavier loads, presumably contributing to certain adjustments of proportionality.

The Minster conformed to a system of proportions, in that the narthex diameter is identical to the elevation of the domical vault, and twice that of the octagon. For Michael Jansen, the octagon serves as a module not only to express a proportional harmony in the vertical dimensions but also in the horizontal.[27] As a design, the floor-plan of the octagon can be folded out from each of the eight sides to form a perfect arabesque that aligns surprisingly well with the intricate details of the vaulting in the narthex. That would imply a deliberate, symbolic subtext to the geometric figures. Jansen interprets this elaboration of an octagonal figure by a 'folding out' and reproducing indefinitely as a metaphor of evangelization in all directions.

Axel Hausmann, on the other hand, focuses on the proportional relations of various length dimensions that contribute to the structure of the Minster at various scales.[28] He argues that such dimensions increase in a progression determined by the module $\sigma = 1 + \sqrt{2}$, corresponding to the 'ideal' numbers 1, 2, 5, 12, 29, 70, 169, where each number is equal to twice its previous number plus its preprevious number (e.g.

[26] For basic information on San Vitale, see F. W. Deichmann, *Ravenna: Hauptstadt des spätantiken Abendlandes* (Wiesbaden, 1976). Concerning the architecture of the Aachen Minster see the recent substantial paper by Cord Meckseper, 'Wurde in der mittelalterlichen Architektur zitiert? Das Beispiel der Pfalz Karls des Grossen in Aachen', in *Jahrbuch 1998, Braunschweiger Wissenschaftliche Gesellschaft* (Braunschweig, 1999), pp. 65–85, and Uwe Lobbedey, 'Carolingian Royal Palaces: The State of Research from an Architectural Historian's Viewpoint', below in this volume. See also C. Meckseper, 'Über die Fünfeckkonstruktion bei Villard de Honnecourt und im späten Mittelalter', *Journal of the History of Architecture* (1985), 31–40, where he treats in a documentary fashion the geometrical knowledge of medieval master masons, emphasizing the later medieval period.

[27] M. Jansen, 'Concinnitas and venustas – weitere Überlegungen zu Mass und Proportion der Pfalzkapelle Karls des Grossen', in *Charlemagne and His Heritage*, vol. I, ed. by Butzer, Kerner, and Oberschelp, pp. 367–96.

[28] A. Hausmann, *Kreis, Quadrat und Oktogon: Struktur und Symbolik der Aachener Kaiserpfalz* (Aachen, 1994); idem, '. . . Inque pares numeros omnia conveniunt . . . Der Bauplan der Aachener Palastkapelle', in *Charlemagne and His Heritage*, vol. II, ed. by Butzer, Jongen, and Oberschelp, pp. 321–66; idem, *Cherubim und Kreuze, Karolingische Bronzen im Aachener Dom* (Aachen, 2000). In the latter volume the author argues that the bronze railing circling the top floor reveals the mathematical system used in the minster's construction.

12 = 2 x 5 + 2). The utility of this 'ideal cut' can be illustrated with a sheet of modern, European A4 paper, for which the proportion of the long to the short side is $\sqrt{2} \div 1$ or 0.4142, the 'ideal cut'. The paper can be folded in half along its longer axis as often as desired, the proportion remaining the same. In other words, the proportions can be maintained while expanding or contracting the dimensions, by doubling or halving respectively. That would allow for relatively simple geometric adjustments during the course of construction. This reproducibility has analogues with Jansen's octagonal module, and Hausmann also infers iconographic content for his method, much like that attached to the better known and older 'golden cut'.[29]

Whether or not one is prepared to accept these stimulating analyses and their inferred didactic implications, it is apparent that geometrical figures and reasoning represent a critical component in the construction of a complex monument such as the Minster. Further, given the time and context, geometry will have served as both a practical tool and a theological device. Progressions of numbers, angles, and geometric figures had come to signify scriptural events or Christian cosmology,[30] just as they also served to whet the mind or solve problems in the here and now. The same intimate interdigitation of science and religion is apparent in the related field of cartography, where the function of maps 'was primarily didactic and moralizing and lay not in the communication of geographical facts'.[31]

To embed such often subtle perspectives into an architectural design supposes repeated discussions within an elite scholarly group, from the initial, groundbreaking plans to subsequent conversations. Indeed different persons may well have had a primary say on the functional, the aesthetic, and the eschatological aspects of the design. Given the spirit of his biography, Charlemagne will have participated in such inferred discussions, and presumably he made the final decisions. It is tempting to think of Odo of Metz as the supervisor of construction, but he may just as well have been the primary inspiration for the religious metaphors incorporated into the design, a role that may have been seen by his contemporaries as more significant than the geometrical and engineering skills of the architect who made the palatine chapel a reality.

Who that architect was will remain a mystery. He would have had to be expert in the living tradition of practical geometry represented by the *agrimensores*. He understood the principles of harmonic proportions still current in the Mediterranean word.

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[29] For example, H. E. Huntley, *The Divine Proportion: A Study in Mathematical Beauty* (New York, 1970); A. Hausmann, *Der Goldene Schnitt, Göttliche Proportionen und Noble Zahlen* (Norderstedt, 2001).

[30] Hausmann, *Der Goldene Schnitt*; Jansen, 'Concinnitas and venustas -weitere Überlegungen'; also, with reservation, N. Hiscock, 'The Aachen Chapel: A Model of Salvation?', in *Science in Western and Eastern Civilization*, ed. by Butzer and Lohrmann, pp. 115–26.

[31] D. Woodward, 'Medieval *mappae mundi*', in *The History of Cartography*, vol. I, *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, ed. by J. B. Harley and D. Woodward (Chicago, 1987), pp. 286–370 (p. 342).

Given the stability of the octagon across 1200 years, he must have been a skilled engineer in regard to the distribution of massive masonry loading. That suggests an Italian origin, perhaps someone from the abbey of Montecassino, the monastic home of Paul the Deacon (*c.* 720–99). But no free-standing, round or octagonal structure had been built in the West for some 250 years, so that the choice of design is unlikely to have been his own. Charlemagne very probably visited and came to admire Ravenna during his Lombard campaigns or Italian visits, and it is more than likely that it was he who decided to recreate the splendour of Ravenna as the radial centre of a new, spiritual Jerusalem.

Transmission, Scholarship, and Patronage

The construction of the Minster suggests parallels to other forms of royal sponsorship that drew specialists from abroad to Aachen, where they practised their expertise while at the same time educating local persons of promise, in order to spawn a new generation of indigenous scholars and craftsmen. In this sense the Carolingian renewal was both organizational, bringing talented people together, and educational, sponsoring new talent.

As the classical Graeco-Roman and Islamic traditions of scholarship reveal, learning is a cumulative process that requires not only talent but a critical mass of like-minded scholars, active communication, and sustained interest by patrons or an elite audience.[32] In the scientific realm, fresh observations or experimentation lead to innovation and eventually synthesis, across timescales measured in generations and even centuries. Given politico-economic instability, or an intellectual environment uninterested in learning, or hostile to innovation or heterodoxy, such scholarly trajectories can be interrupted or snuffed out. Lacking the printing press, the paucity of manuscripts made a revival of learning very difficult, for a 'revival' required access to and digestion of earlier writing, understanding, and competing ideas. The recompilation and commentaries typical of revival efforts consequently deserve admiration rather than condescension.

With only erratic support from political figures, scholarly activity during the late period was primarily sustained by a few enlightened churchmen, who enjoyed the relative luxury and security to devote part of their time to intellectual pursuits. The survival of many of the works of antiquity, particularly those in Latin, owes much to their efforts. What is less fully appreciated is that the universalist goals of the Church led to an unprecedented network of communication, along the length of the Mediterranean and even to the British Isles. This networking is exemplified by periodic communications in regard to the liturgical date of Easter, by the early ecumenical councils and the later provincial synods, as well as by influential figures such as Cassiodorus, John of Biclar,

[32] K. W. Butzer, 'The Islamic Traditions of Agro-Ecology: Cross-Cultural Experience, Ideas, and Innovation', *Ecumene*, 1 (1994), 7–50.

Gregory the Great, and Maximus the Confessor, who worked and lived in both the East and West.[33]

Indeed the transnational experience of the churchmen of the late empire and early Byzantine period will have rivalled that of any prestigious occupation group until modern times. The interchange and presumed networking of these early Christians of learning will have, to some degree, compensated for their limited numbers or access to library sources. Both old and new manuscripts will probably have been exchanged—for example, the early dissemination of Isidorean manuscripts in the West—and potential scholars isolated by distance may have been sustained in the knowledge that they were not alone in their efforts. Above all, the introduction of monasticism in the West was commonly linked with intellectual aspirations, so that the best institutions of their kind became academic nuclei of sorts.

But all this should not detract from the fact that even in Charlemagne's day academic standards at some monastic schools were abysmal. In no era was there a substitute for individual excellence, such as exemplified by Bede and Alcuin. Charlemagne must have recognized this, judging by his efforts to bring top-flight scholars to the court school in Aachen, where they interacted with such positive results. As a by-product of the close relationship with Rome, new manuscripts continued to flow to the Frankish monasteries. The all-too-brief Carolingian *restauratio* was very much a consequence of the sustained patronage and inspired guidance of Charlemagne himself. Not since the interaction of Pliny with Vespasian and Titus had there been such enlightened sponsorship of learning.

[33] The problem of transmission of Greek and Latin learning and its Mediterranean roots has been treated in detail in Butzer 'Mathematics in Egypt', and Butzer 'Mathematics and Astronomy'. The former paper considers especially the transmission via England and Ireland, the latter via Visigothic and Suevic Spain, and Byzantium.