Demand-System Asset Pricing: Theoretical Foundations

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Motivation

Koijen Yogo (2019) propose an influential new methodology for asset pricing.

1. Estimate demand systems for assets, as IO does for consumption goods.
2. Investors allowed to have **tastes over assets** (rather than just cash flows).

   For example, identity of issuer, dogmatic beliefs about payoffs, exogenous constraints.
3. Identify demand parameters from portfolio data; conduct counterfactuals.

We are interested in the applicability and interpretation of this approach.

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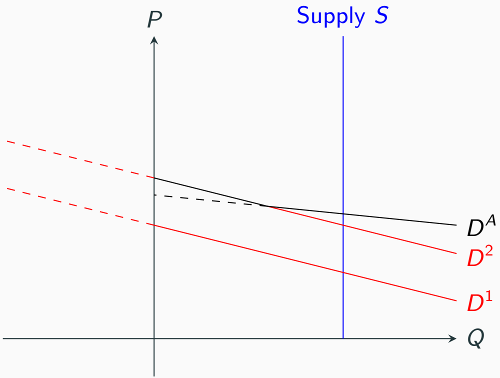
2. What are the theoretical implications of allowing for tastes?

   Tastes invalidate no arbitrage, with empirical and conceptual consequences.

3. Counterfactuals and estimation from general equilibrium play.

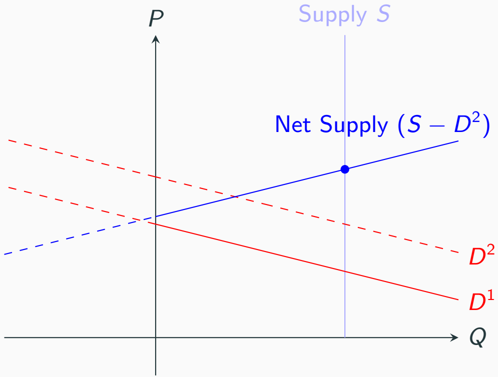
   Elasticities depend on security menu, tastes vs. constraints, outside goods.

The Identification Challenge



- Exogenous supply shocks can serve to identify demand. Not many of those...

A Potential Solution: Orthogonal Demand Shocks



- Identification requires **taste shocks that do not affect 1's demand**:
  (i) tastes over characteristics, (ii) dogmatic beliefs, (iii) constraints.

Thinking through this strategy in two steps

1. Tastes: non-pecuniary preferences over assets and/or dogmatic beliefs.

2. Adding in constraints.

Tastes: Modeling and Implications

Modeling Tastes: Two Approaches

1. **Payoff-augmenting:** Investor i evaluates asset j's payoff $y_j(z)$ as $\theta_i y_j(z)$.
   - Portfolio choice based on taste adjusted units of "effective consumption:"

$$\tilde{c}_1^i(z) \equiv \sum_j \theta_j^i y_j(z) a_j^i.$$

2. **Utility-augmenting:** Utility from portfolio $\mathbf{a}$ is $U(c(\mathbf{a})) + G(\mathbf{a})$.
   - Taste function G captures an asset's "warm glow" or "convenience yield."
   - Can impose regularity conditions. E.g: differentiable, $G(-\mathbf{a}) = -G(\mathbf{a})$.

1. Expected Utility $\Rightarrow$ Cardinal Tastes

Models of portfolio choice with expected utility require **cardinal** utility.

- Unlike most IO settings in which ordinal utility is sufficient.

Given a trade-off between risk-return and tastes, portfolio choices are sensitive to even simple **rank-preserving transformation of tastes.**

Implications for measurement:

1. Must identify intensity of tastes, not just ordinal rankings.
2. *Aggregation* (e.g., into ESG buckets) must be marginal utility-weighted.

2. No Arbitrage

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The *point* of tastes is that investors disagree about "something."

"Theorem." For sufficiently rich security menus and heterogeneous tastes, there do not exist price systems that preclude arbitrage for all investors.

⏩ Formal

Illustration with Payoff-Augmenting Tastes

There is a **green asset** and a **red asset** with prices p_g and p_r.

- Redundant cash flows: both deliver a unit payoff with certainty.
- Type 1 tastes are $\theta_g^1 > \theta_r^1$; Type 2 tastes are $\theta_g^2 < \theta_r^2$.

Now construct a long-short portfolio selling green and buying red.

- The cost of this portfolio is $p^* \equiv p_r - p_g$.
- Type i's taste-augmented payoff is $\theta_r^i - \theta_g^i$.

Absence of arbitrage requires $p^* < 0$ for Type 2, but $p^* > 0$ for Type 1.

NB: Do short positions inherit tastes?

Potential Remedies

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2. **Trading restrictions**, as commonly used in heterogeneous belief models.
   - Sensitivity to the precise form of (unobserved) security menu and constraints.
   - Speculative considerations complicate inference of demand parameters.

Trading Restrictions I: Dynamic Trading

Harrison Kreps (1978) is a close cousin of demand-based models:

- Dogmatic valuation differences and short-sales constraints.
- Only difference: dynamic rather than one-shot trading.

Result: demand has speculative component that depends on *others'* valuations.

Trading Restrictions II: Instruments

KY rely on portfolio restrictions to aid to construct *asset-level* price shocks.

Fix K constraints $F_k^i(\mathbf{a}^i, \mathbf{p}) \leq 0$ with Lagrange multipliers λ_k^i. The Lagrangian is

$$\mathcal{L} = \underbrace{\sum_{z \in \mathcal{Z}} \pi_z u_i(\tilde{c}_1^i(z))}_{\text{Preferences}} + \underbrace{\sum_k \lambda_k^i F_k^i(\mathbf{a}^i, \mathbf{p})}_{\text{Constraints.}}$$

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These restrictions do not cover important settings:

- Standard diversification concerns; indexing with tracking error; ...

Identification and Counterfactuals in GE

Our Approach

We explore a synthesis between neoclassical and demand-system asset pricing.

In particular, enrich Lucas '78 with payoff-augmenting tastes and mandates.

- Allows us to distinguish between *assets* and *state-contingent payoffs*.
- Delivers clear implications for portfolio determination in *general equilibrium.*
- Can directly mimic and evaluate current approaches.

Framework

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- Split Tree 1 into two smaller trees of size 1/2: **green** and **red**.
    - Green trees pays $y_g \in \{1+\epsilon, 1-\epsilon\}$; red tree pays $y_r \in \{1-\epsilon, 1+\epsilon\}$
    - The red tree pays more fruits with probability ρ; and vice versa w.p. $1-\rho$.

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- Type 1 prefers green. Type 2 prefers red. Effective consumption:

$$\tilde{c}^i(1, \iota) = \theta^i_g y_g(\iota) a^i_g + \theta^i_r y_r(\iota) a^i_r \qquad \text{and} \qquad \tilde{c}^i(2) = a^i_2.$$

Portfolio Choice Problem

Assume short-sales as in standard demand-system papers.

$$\max_{a_g^i, a_r^i \geq 0} \quad \pi_1 \left[\rho u\Big(\underbrace{\theta_g^i y_g(r) a_g^i + \theta_r^i y_r(r) a_r^i}_{=\tilde{c}^i(1,r)} \Big) + (1-\rho) u\Big(\underbrace{\theta_g^i y_g(g) a_g^i + \theta_r^i y_r(g) a_r^i}_{=\tilde{c}^i(1,g)} \Big) \right] + \pi_2 u\Big(\underbrace{e_2^i + p_g(e_g^i - a_g^i) + p_r(e_r^i - a_r^i)}_{=c_2^i} \Big)$$

$$\text{s.t.} \quad w_g^i = \frac{p_g a_g^i}{p_g a_g^i + p_r a_r^i} \in [\underline{w}_g^i, \overline{w}_g^i]. \qquad \text{(Mandate)}$$

Identification from Equilibrium Play

GE framework illustrates key identification concerns.

- Latent parameters: equilibrium allocations may obscure key parameters.
- Misspecification bias: hard to disentangle tastes and constraints.
- Unobserved substitutability: elasticities depend on (unobserved) security menu.

Case 1: Perfect Sorting Equilibrium

Guess that each type sorts into preferred color Under log utility, demand is:

$$\frac{\pi_1}{a_g^1} = p_g \frac{1-\pi_1}{a_2^1} \qquad \text{and} \qquad \frac{\pi_1}{a_r^2} = p_r \frac{1-\pi_1}{a_2^2}.$$

and prices are purely determined by green investor's wealth share ω:

$$\frac{p_r}{p_g} = \frac{1-\omega}{\omega}.$$

Identification concern: **taste parameters are latent** for all "sorted" types.

- One could identify ordinal preferences, but not cardinal intensities.
- Yet, intensities can be critical for counterfactuals.

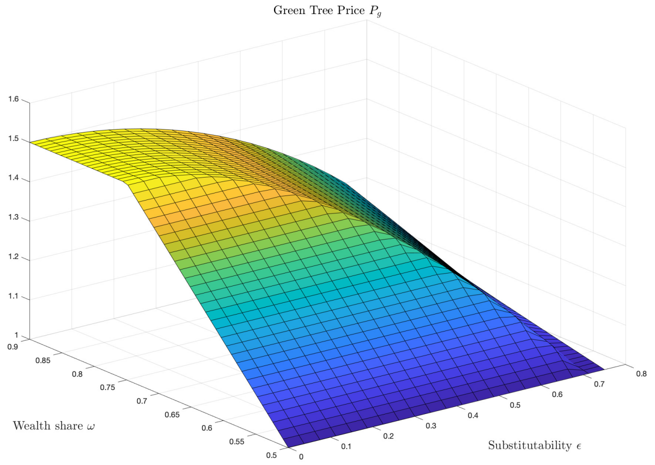
Case 2: Mixed Equilibrium

- Sorting is optimal only for some wealth distributions (and some ϵ).
- For $\omega \geq \bar{\omega}$, Type 1 buys red **and** green trees, Type 2 buys only red trees.
- If assets are perfect substitutes ($\epsilon = 0$), prices are:

$$p_g = \theta_g^1 \qquad \text{and} \qquad p_r = \theta_r^1.$$

These are the parameters that were previously unidentified.

The Pricing Function and Counterfactuals



What happens if money is allocated to ESG? Need to know taste intensity.

Micro-foundations: Mandates versus Tastes

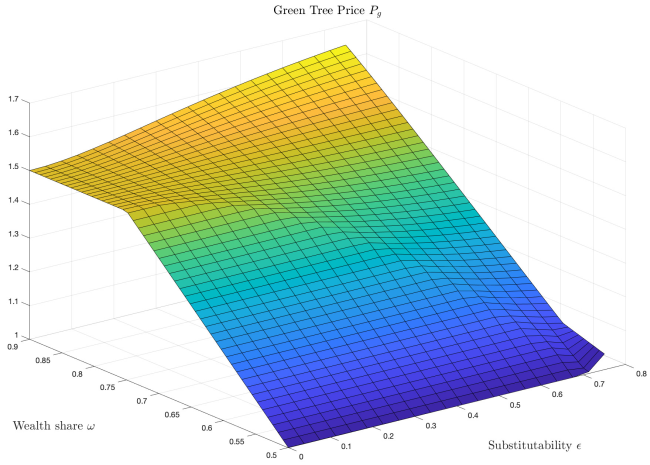
Simple price instruments generally do not distinguish tastes and constraints.

- E.g., share m of Type 1 agents have unobserved mandate to buy only green.

Observationally equivalent to tastes under sorting, but counterfactuals differ.

- Tastes are price-sensitive, mandates are not.

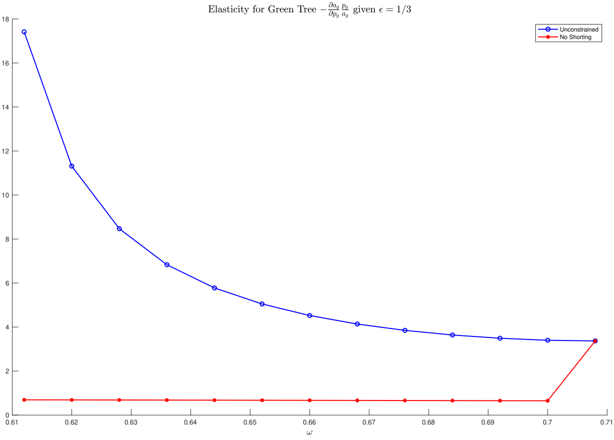
Green Price: High Mandate Share ($m = 0.85$)



Misspecification Bias

- Common data sources (e.g. 13F) do not cover short or bond positions.
  - Hence, we do not know whether a fund faces short sale constraints.
- This can introduce a bias in the demand for *complementary assets.*
- Example: green investor can short red trees, but you assume they can't.
  - Observed **green** elasticities much higher than without **constraints on red**.
  - Extent of bias depends on prices that are determined in GE.

Misspecification Bias: Implied Elasticities



Elasticities and the Security Menu

- Consider our red and green trees, and say cash flows are the same.

- When forming portfolios, individual investors only buy the cheaper color.
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$$\epsilon_g = -\frac{\partial a_g}{\partial p_g} \frac{p_g}{a_g} = \infty. \qquad \text{(Asset Elasticity)}$$

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Interpretation depends on **tastes, security menu, and outside goods.**

Conclusion

We provide a “neoclassical perspective” on demand-based asset pricing:

1. Demand elasticities are structural only under stringent assumptions.
2. Tastes may invalidate the organizing principle of no arbitrage.
3. These issues are amplified in general equilibrium.
4. Meaning of estimated elasticities depends on model structure.

These results imply challenges for identification and counterfactual analyses.

No Arbitrage

- Y: the $J \times Z$ matrix of cash flows.
- $\mathcal{A}$: the set of feasible portfolios.
- p_j: the price of asset j.
- Pricing function $P : \mathcal{A} \to \mathbf{R}$: $P(a) = \sum_{j \in \mathcal{J}} p_j a_j$.
- A taste function $v^i : \mathcal{A} \to \mathbf{R}^Z$ maps a portfolio a into a $1 \times Z$ vector of state-contingent taste-augmented payoffs.
  - In the absence of tastes, all investors care only about cash flows as in standard asset pricing, $v^i(a) = a \cdot Y$.

No Arbitrage

Definition (No Arbitrage with Tastes)

- Given a set of assets $\mathcal{J}$ and taste functions v^i for all investors i,
- the pricing function P leaves no arbitrage opportunities if,
- for any investor i and any portfolio such that the effective payoff is weakly positive, $v^i(a) \geq 0$, and strictly positive, $v^i(a) > 0$ with strictly positive probability,
- the associated price is positive, $P(a) > 0$.

No Arbitrage

Theorem (Formal)
Fix a set of assets and taste functions v^i for all investors.

There does not exist pricing function P that leaves no arbitrage opportunities for any investor if and only if:

$$\text{there exist } a, v^i, v^{i'} \text{ such that } v^i(a) > 0 \text{ and } v^{i'}(a) \leq 0. \tag{C}$$

A sufficient but not necessary condition for (C) is that there exist assets j and j' such that

(i) both assets have identical cash flows

$$y_j(z) = y_{j'}(z) \text{ for all } z \in \mathcal{Z};$$

(ii) there exist investors i and i' with sufficiently heterogeneous tastes with respect to these assets,

$$v_j^i \geq v_{j'}^i \text{ and } v_j^{i'} \leq v_{j'}^{i'} \text{ with at least one inequality strict.}$$

Back