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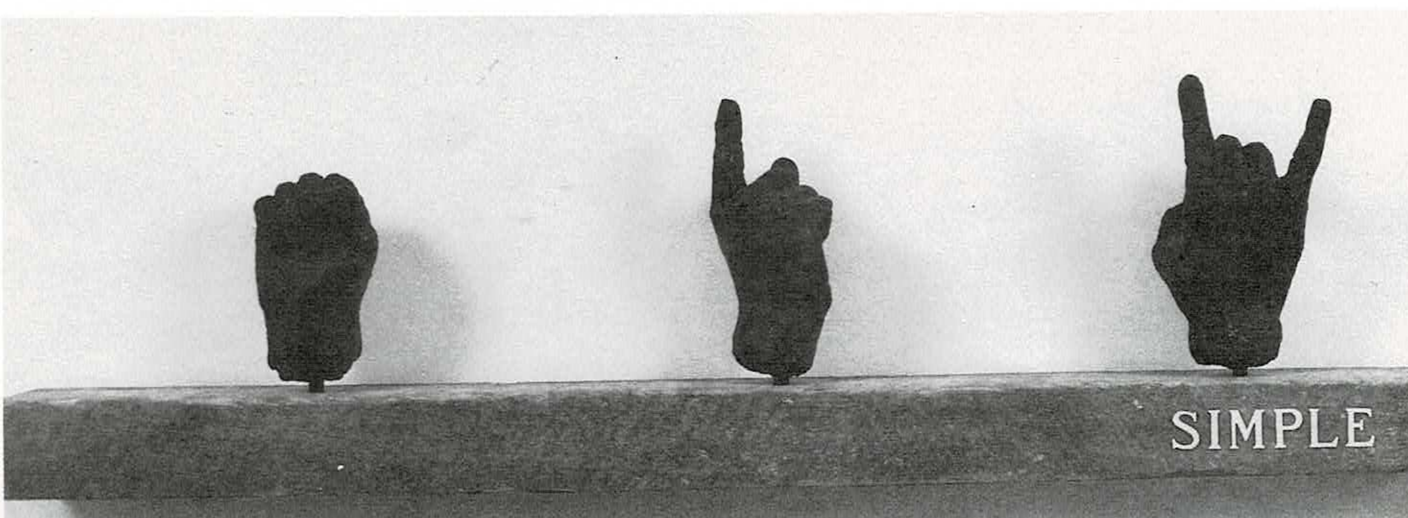
ONENESS, TWONESS, THREENESS

How Ancient Accountants Invented Numbers

by DENISE SCHMANDT-BESSERAT

PEOPLE APPEAR to be born to compute. The numerical skills of children develop so early and so inexorably that it is easy to imagine an internal clock of mathematical maturity guiding their growth. Not long after learning to walk and talk, they can set the table with impressive accuracy—one plate, one knife, one spoon, one fork, for each of the five chairs. Soon they are capable of *noting* that they have placed five knives, spoons, and forks on the table—and, a bit later, that this amounts to fifteen pieces of silverware. Having thus mastered addition, they move on to subtraction. It seems almost reasonable to expect that if a child were secluded on a desert island at birth and retrieved seven years later, he could enter a second-grade mathematics class without any serious problems of intellectual adjustment.

Of course, the truth is not so simple. This century, the



Andrew Menard, Simple Arithmetic, *1987* work of cognitive psychologists, notably Jean Piaget, has illuminated the subtle forms of daily learning on which intellectual progress depends. Piaget observed children as they slowly grasped—or, as the case might be, bumped into—concepts that adults take for granted, as they refused, for instance, to concede that quantity is unchanged as water pours from a short stout glass into a tall thin one. Psychologists have since demonstrated that young children, asked to count the pencils in a pile, readily report the number of blue or red pencils, but must be coaxed into finding the total. Such studies have suggested not only that the rudiments of mathematics are mastered gradually, and with effort, but that the very concept of abstract numbers—the idea of a oneness, a twoness, a threeness that applies to any class of objects and is a prerequisite for doing anything more mathematically demanding than setting a table—is itself far from innate.

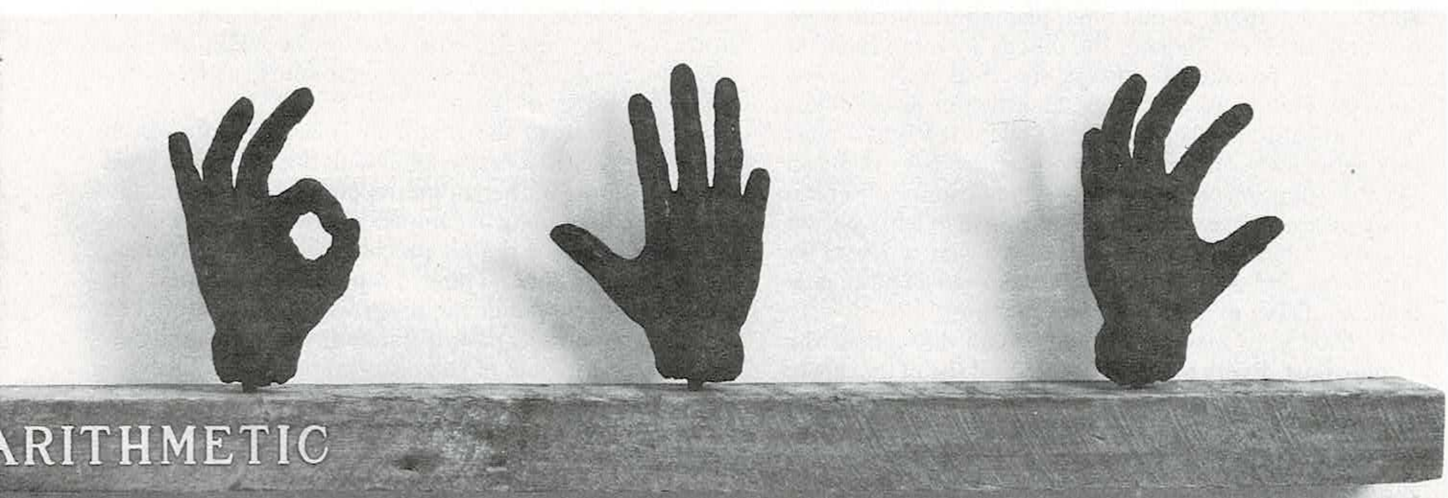
This observation draws support from linguistics and anthropology, in particular from the study of cultures that have evolved in isolation from modern society. Anthropologists have found that when a Vedda tribesman, of Sri Lanka, wanted to count coconuts, he would collect a heap of sticks and assign one to each coconut. Every time he added a new stick he said, "That is one." But if asked how many coconuts he possessed, he could only point to his pile of sticks and say, "That many," for the Vedda had no words devoted to expressing quantities. Thus, while capable of a kind of counting—counting in one-to-one correspondence, rather like the child setting the table—the Vedda apparently had no conception of numbers that exist independently of sticks and coconuts and can be applied to either without reference to the other.

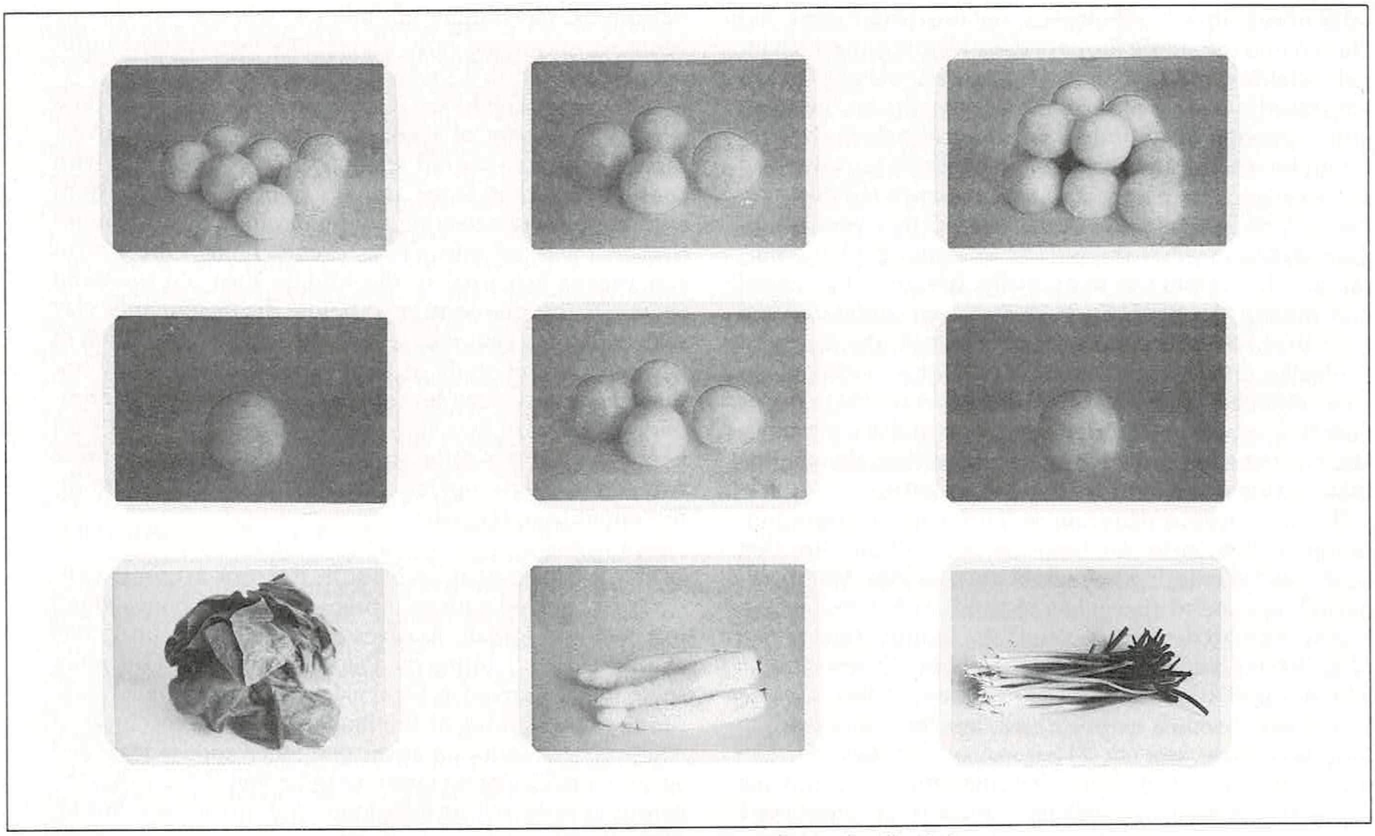
Indeed, even languages with numerous words for enumeration sometimes do not reflect a clear understanding of *abstract* numbers; they lack words, such as the English *two*, that are applicable to a wide range of objects. Franz Boas, studying the Tsimshians, of British Columbia, in the late nineteenth century, found that they denoted one, two, or three canoes, respectively, with the words *k'amaet, g'alpeeltk,* and *galtskantk,* whereas one, two, and three men were *k'al, t'epqadal,* and *gulal.* In short—to use a loose but instructive metaphor—societies appear to develop in the fashion of children; whereas one-to-one counting may come early, almost innately, abstract numbers do not.

In the case of children, it has been known for some time when the concept of abstract numbers typically arrives: the average six-year-old applies such words as *five* with considerable confidence and generality. But in Western culture, evidence bearing on the origin of abstract numbers has emerged only in recent years. It takes the form of clay tokens first used in the Middle East ten thousand years ago, five thousand years before the pictographic clay tablets that are generally associated with the invention of writing. Careful study of these tokens has radically improved our understanding not only of when abstract numbers arose but of how they arose. And it has shown with new clarity how writing itself originated and how these two seminal developments of Western culture were intertwined from the start.

THE EARLIEST Middle Eastern artifacts connected with counting are notched bones found in Israel and Jordan, in caves inhabited between 15,000 and 10,000 B.C., during the Paleolithic period. The bones seem to have served as lunar calendars, each notch representing one sighting of the moon. And such instances of counting by one-to-one correspondence appear again and again in preliterate societies: pebbles and shells aided the census in early African kingdoms, and, in the New World, cacao beans and kernels of maize, wheat, and rice were used as counters.

All such systems suffer from a lack of specificity. A collection of pebbles, shells, or notches indicates a quantity but not the items being quantified, and, thus, it cannot serve as a way of storing detailed information for long periods. The first means of enumeration known to have solved this problem was devised in the Fertile Crescent, the rich lowlands stretching from Syria to Iran. Clay tokens, usually an inch or less across, were used to represent, by virtue of their shapes, specific commodities: a cylinder stood for an animal; cones and spheres referred to two common measures of grain (approximately equivalent to a peck and a bushel), which the Sumerians would later





Marcel Broodthaers, Daguerre's Soup *(detail), 1976*

call the *ban* and the *bariga*. These tokens, like the notches of the Paleolithic period and the shells of African kings, did not reflect a clear conception of abstract numbers; two animals could be recorded only with two cylinders, and two bushels of grain with two spheres. Nonetheless, in their specificity, the tokens possessed a huge advantage over past technologies of information storage; they could be kept for years without any loss of meaning. This fusion of simple one-to-one counting with specific symbolic identification of the objects being counted might be called *concrete* counting.

It is no coincidence that this relatively sophisticated accounting practice appeared in the Fertile Crescent around 8000 B.C. For there, at that time, plants and animals were first domesticated. Indeed, the tokens are often found at sites where rectangular storage silos had been erected amid the round huts of villages, and the soil at these sites has an unnatural density of cereal pollen. It is only logical that agriculture, ushering in an economy dependent on the redistribution of foodstuffs and the planning of subsistence over many seasons, would require a reliable system of record keeping, and that the tokens were a device for organizing and storing economic data—an extrasomatic brain not liable to human memory failure.

By 6000 B.C., clay tokens had spread throughout the Middle East. Every major archaeological site of that era in Iraq, Iran, Syria, Turkey, and Israel yields somewhere between a handful of tokens and hundreds. Their nature and essential function seem to have changed little, if at all, over the millennia. As late as 3300 B.C., the same cones, spheres, and cylinders that had appeared five thousand years earlier were still in use. Then, near the end of the fourth millennium B.C., came a second, more complex type of token, one that bears varied, sometimes elaborate markings and assumes a wide assortment of shapes. There are ovoids, rhomboids, biconoids, bent coils, parabolas, quadrangles, and miniature representations of tools and animals.

Why the sudden profusion of complex tokens? The answer lies largely in the increasingly complex structure of the Sumerian society in which these tokens were used. By 3200 B.C., Sumer was seeing the birth of the first great cities, and complex tokens appear to represent the increasing number of finished goods that are the hallmark of an urban economy. Whereas plain tokens belonged to the pens and granaries, and stood for the several staple foodstuffs, complex tokens were used in the workplace and stood for finished merchandise—garments, metalworks, jars of oil, loaves of bread.

Another clue to the origin of this new information technology lies in its association with the rise of the great temples. In the Sumerian metropolis of Uruk, for example, the earliest group of complex tokens was found in the ruins of Eanna, the temple precinct dedicated to Inanna, the goddess of love. These temples—monumental, ornately decorated public buildings—signal the advent of a strong, economically powerful state government, and perhaps the first regime of coercive taxation and redistribution. Sumerian reliefs from this period depict processions of citizens delivering their dues to the temple in the form of foodstuffs—which, presumably, were then distributed among the public servants who maintained the temples and the laborers who built walls around the cities. There is even a hint that tribute was not always willingly paid;

Jasper Johns, 0–9, 1959–62

some of the carvings show scenes of punishment, perhaps being inflicted on the first tax delinquents. This possibility underscores the fact that taxation required tight administrative control, complete with a precise system of reckoning and record keeping, such as the tokens provided.

Even as complex tokens proliferated, the relatively plain tokens from an earlier age remained in use and kept their essential form and function, referring to agricultural commodities. And, though both plain and complex tokens are found in temple archives, they evolved separately, into two of the greatest inventions of the human species—abstract numbers and phonetic writing.

ONE CATALYST in both of these inventions was the habit of storing plain tokens in clay envelopes. About two hundred of these envelopes have been recovered, most still intact, at half a dozen sites. They typically bear the impression of one or more seals—the small, usually cylindrical stones engraved with a design that served as a sort of signature. (Just about all Sumerians—even slaves—appear to have possessed personal seals.) These imprints refer, presumably, to the person or government agency whose account was recorded within a clay envelope or to the parties who had agreed on a transaction that might have lain in the future or been recently completed. In other words, the sealed envelopes could have served as contracts or receipts.

A clay envelope has one obvious drawback as a means of storing information: it is not transparent; if you forget what is inside, the only way to find out is to break open the seal, which amounts to symbolically breaking the deal. That, presumably, is why Sumerian accountants began impressing the tokens on the soft exteriors of the envelopes before enclosing them, thus leaving a visible record of the number and shape of tokens held inside. At some point, accountants must have realized that the markings on the envelope—reflecting, as they did, everything significant about its contents—rendered the tokens superfluous. Thus were the first written tablets created, as two-dimensional representations of three-dimensional symbols; the circle replaced the sphere, and the wedge the cone.

Complex tokens, unlike plain ones, were not stored in envelopes. Rather, they were perforated and laced on a string that was then attached to a *bulla*, an oblong piece of clay. These strings of tokens were supplanted by two-dimensional symbols at roughly the same time that plain tokens suffered this fate, and this is almost surely no coincidence; once the economy of two-dimensional information storage had been demonstrated via plain tokens, accountants must have hastened to apply the lesson to complex ones. The means of rendering the new symbols differed, however, in part, perhaps, because it would have been difficult to impress the finely marked complex tokens on a clay tablet and get a sharp, legible image. Instead, these markings, along with the outline of the token, were replicated on tablets with a stylus—a technique that is scarcely surprising, since the stylus had been used to mark the tokens in the first place. Thus, an ovoid with an incision—the symbol for a jar of oil—became a neatly inscribed oval with a slash across it.

Two-dimensional renderings of plain and complex tokens were in a sense pictographs—pictures not of the items ultimately being represented but of the symbolic objects (cones, spheres, ovoids, quadrangles) that had in turn represented those items. Pictographic writing was thus a distinct step toward abstraction. The original symbols, the clay tokens, had been fundamentally concrete things; like the items they stood for, they could be grasped. The pictographs were much further removed—by an entire dimension, to be precise—from these items. This switch from three- to two-dimensional representation may seem an obvious step to the modern, who is steeped in the notion of abstraction, but the very fact that the step was not immediately taken—that for some time the unambiguously marked clay envelopes redundantly carried tokens—suggests that two-dimensional representation was a cognitive leap of some distance.

In the course of this transition, a second momentous step was taken. For a time, the images of tokens on clay tablets had maintained a kind of concreteness in their expression of plurality; a set of three jars of oil had been depicted with three identical slashed ovals in a row. But with the use of clay tablets, another kind of abstraction was attained; plurality was no longer expressed by repeating a pictograph in one-to-one correspondence. Instead, pictographs were preceded by numerals applicable to all kinds of objects.

In selecting symbols to stand for numbers, the Sumerians pressed into service shapes that stood for measures of grains. A wedge, which originally meant a small quantity of grain, now meant 1. A circle, which had represented a larger quantity, stood for 10. The two could be combined, somewhat as Roman numerals later would be: 23 was two circles and three wedges. (The same symbols continued to refer to grain as well, but differences of context were sufficient to avoid confusion.) A large wedge came to signify 60, and a large circle 360, giving the Sumerians a strangely hybrid system of numerical notation—elements of a base-six system mixed in with the decimal notation.

Why the symbols for grain were selected as abstract numerals can only be guessed, but there is no shortage of possible reasons. First, they were signs in common use, since grain was the staple crop in the ancient Middle East. Second, the circle and the wedge already referred to two different magnitudes, so their use to signify 1 and 10 was conceptually natural. Finally, since the circle and the wedge were derived from plain tokens, they were *impressed* signs, easily differentiated from the *incised* signs with which they were typically paired, so the numerals stood out clearly from a body of writing, much as they do today. This contrast underscores the remarkable feat the Sumerians had accomplished: with the invention of numerals, they developed a discrete category of signs, used exclusively to indicate quantity and capable of combination with any member of a second set of symbols representing tangible items. The Sumerians had invented abstract numbers—the concept of oneness, twoness, threeness.

NOT LONG AFTER the evolution of certain plain tokens into numerals, the pictographs derived from complex tokens crossed a second great threshold. By about 2900 B.C., these symbols could function phonetically, representing not objects but, in selected cases, sounds. Thus did the Sumerians invent the precursor of the modern alphabet—a syllabary, which represents each syllable with a single symbol.

The exact route of invention can only be guessed. As Sumerian society evolved, and the business of taxation and redistribution grew more complicated, the requisite systems of accounting became more elaborate. Accountants may have faced a problem: since a person's identity could be recorded only with his personal seal, his accounts could not be duplicated, nor substantially amended, in his absence. At some point—perhaps as a solution to a problem such as this—accountants began referring to individuals with separate sequences of symbols. This technique was facilitated by the fact that personal names were often built from concepts for which symbols existed. For instance, one name consisted of the word for *god* and the word for *life* and meant, literally, "The god that gives life." It was easy to represent this name with the symbols for *god* (a star) and *life* (an arrow). The crucial step came when names that resisted such easy depiction—names not built of concepts for which symbols already existed—were nonetheless recorded in comparable fashion. Thus, the slashed oval might be employed if a syllable in a name sounded like the Sumerian word for *oil*—even if the idea of oil had played no role in the name's origin. This was the beginning of phonetic writing—the use of symbols to stand not for objects or concepts but simply for the sounds they brought to mind.

The resultant Sumerian script, which led to the cuneiform writing that prevailed in the Middle East until the Christian era, was based on three major elements. Numerals were ideographs; each symbol stood for a distinct concept. Some words were pictographs, resembling the tokens they had replaced. And proper names usually were depicted phonetically, albeit with symbols that once had been strictly pictographic. In time, other words would also be phonetically encoded. But practicality eventually demanded changes in the nature of this coding. After all, it takes a cumbersomely large number of symbols to fill a syllabary, since any language has thousands of syllables. But if symbols are made to represent smaller units of sound—the sound of *o* or of *l* but not of *oil*—then a relatively small number of symbols can, in diverse combination, form all syllables and hence all words. The Phoenicians, by inventing the first truly phonetic alphabet, around 1500 B.C., fully exploited this fact (though their alphabet lacked vowels, which the Greeks added later).

Thus abstract numbers, sometimes thought of as a mere subset of the alphabet, actually ushered it in. Indeed, during the first three hundred years of writing, there were no literary, religious, or historical texts. Except for a small number of lexical texts—catalogues of signs, used to train scribes—the only written works produced between 3100 and 2900 B.C. were the ledgers of accountants: lists of the names and quantities of goods. There were no symbols for verbs, adverbs, or prepositions, which would have allowed the expression of entire sentences. The human species had to build its abstractions one block at a time. ●

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