Feedback Guidance in Uncertain Spatiotemporal Wind Using a Vector Backstepping Algorithm

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This work deals with the problem of guiding a controlled object to a given target set in a three-dimensional configuration space in the presence of wind with an uncertain spatiotemporal velocity field. The proposed guidance law utilizes (imperfect) measurements of the velocity of the local wind along the ensuing path of the controlled object to a given target set. No information about the velocity gradients (temporal and spatial) of the wind is assumed to be available. The development of the proposed guidance feedback law is based on the utilization of a backstepping algorithm that forces the controlled object to follow closely the motion of a lower order kinematic model that is driven by a pure pursuit navigation law. In this way, the controlled object converges to its target set in finite time. Two kinematic models that describe the motion of the controlled object are considered. In the first model, the rate of change of the air velocity of the controlled object is unconstrained, whereas in the second one, it has to remain perpendicular to the air velocity at all times. Numerical simulations which are based on real wind data are presented.

Nomenclature

ξ = position vector of the controlled object, m
ν = air velocity vector of the controlled object, m/s
ν_g = ground velocity vector of the controlled object, m/s
α = control input of the fully-actuated controlled object, m/s^2
$|\cdot|$ = norm (magnitude) of a vector
$\bar{\alpha}$ = upper bound on the norm of the control input, m/s^2
ϖ = control input of the under-actuated controlled object, rad/s
ω = measurement of the wind's velocity, m/s
$\Delta\omega$ = measurement error of the wind's velocity, m/s
$\bar{\omega}$ = upper bound on the norm of ω, m/s
$\Delta\bar{\omega}$ = upper bound on the norm of $\Delta\omega$, m/s
CO = controlled object
LOS = line-of-sight (the ray from the controlled object to its destination)
t_{f} = arrival time, s
$\mathcal{T}_\epsilon$ = target set
ϵ = accuracy of convergence, m
$\mathbb{R}^n$ = set of n-dimensional real vectors
$\mathbf{C}$ = set of continuous functions
$\mathbf{C}^1$ = set of continuously differentiable functions

I. Introduction

This work deals with the problem of steering a controlled object (CO, for short) with second order kinematics to a given target set in finite time in the presence of uncertain wind, whose velocity

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varies both temporally and spatially. It is assumed that the CO obtains imperfect measurements of the velocity of the local wind along its ensuing path via onboard sensors. Alternatively, the CO can measure the local wind velocity by comparing the measurements of its air and ground velocities, where the ground velocity is measured by using, for example, a Global Positioning System (GPS) receiver. Yet, no information about the gradients, both temporal and spatial, of the wind's velocity field is available.

Traditionally, problems of steering a CO, such as an aerial vehicle or a missile, in the presence of wind have been addressed within the framework of optimal control [1–5]. The powerful techniques of optimal control are often based on a number of strong assumptions, which may not always be satisfied in practice. In particular, for the minimum-time steering problem of a CO with first order kinematics in the presence of wind, the so-called Zermelo navigation problem [1], one typically assumes that the spatiotemporal velocity field of the wind is globally and perfectly known a priori; an assumption that is very restrictive in practice. In Ref. [6], a number of different ways to overcome this problem for different information patterns about the local wind have been proposed for a CO with first order kinematics. In this work, the results presented in Ref. [6] are extended to the case when the motion of the CO is described by a second order kinematic model. In particular, the case that is considered first is when the CO can directly control the rate of change of its air velocity; the CO will be characterized as *fully actuated*. Note that the fully actuated CO can regulate both the direction and the magnitude of its air velocity along its ensuing path. It will be required, however, that the airspeed of the CO should not vary significantly, by means of a relevant soft constraint. Subsequently, the case when the CO can only control the rate of change of the direction of its air velocity will be considered; the CO will be characterized as *under-actuated*. Note that the under-actuated CO can only regulate the components of the time derivative of its air velocity that are perpendicular to the air velocity. Consequently, the under-actuated CO must travel at a constant airspeed at all times (hard constraint).

The problem of steering a CO with second order kinematics in the presence of wind has received a considerable amount of attention in the recent literature. The majority of the available results focus, however, on the two-dimensional problem of steering the so-called Dubins car or vehicle [7, 8] under the assumption that the wind field is either constant or time varying, yet spatially invariant, and perfectly known (see Refs. [5, 9, 10] and [11], respectively). The steering problem for simple extensions of the Dubins vehicle model in three dimensions in the presence of a priori known, spatially varying wind is addressed in [12] and [13] by means of numerical solution techniques. Tracking problems for similar kinematic models in the presence of wind have been studied, for example, in [14] and [15]. The problem of guiding the Dubins vehicle in the presence of stochastic wind in minimum expected time at a given target set can be found in [16]. The main advantage of the formulation of the steering problem as a stochastic optimal control problem has to do with the fact that in the latter case only the statistics of the stochastic velocity field of the wind are assumed to be available a priori. The latter assumption is less restrictive than the one that the CO has global knowledge of the wind's velocity field, as would be the case in the corresponding deterministic optimal control problem. Yet, the intensity and the mean of the noise that models the stochastic component of the wind in [16] are taken to be constant (spatially and temporally) and known a priori.

The problem of optimal interception of a moving target by a fully actuated CO with second order kinematics, which is equivalent to the steering problem in the presence of wind, is a well studied problem [17–19]. Typically, the performance index in the latter class of problems is taken to be the control effort required for the interception of the moving target by the CO at a given terminal time assuming that the target's velocity is an either constant or time-varying vector, which is a priori known to the CO, and the accuracy of convergence to the target is not explicitly prescribed. Recently, the minimum-time steering problem in the presence of wind with a time-varying velocity field and an explicit bound on the norm of the control input of the CO was addressed in [20].

The main contribution of this work is the presentation of the solution to the guidance problem of a CO to a given target set in the presence of uncertain wind under less restrictive assumptions than the ones typically made in the literature. In particular, it is explicitly required that the proposed feedback guidance law does not depend on a priori knowledge of the global velocity field of the wind. The target set of the CO is assumed to be a neighborhood of a fixed position vector, the target point, where the "size" of this neighborhood determines the accuracy of convergence of the CO to the target point. In addition, the requirement of finite time convergence has the following meaning: No matter how "small" the target set is taken to be, the CO will reach it after a finite amount of time, which is upper bounded by a finite bound that is independent of the "size" of the target set.

The approach adopted in this work is based on an integral backstepping algorithm [21, 22] in vector form, which furnishes a feedback control law that is continuous everywhere except from an isolated point, which is never attained in the time interval of interest, and drives the position vector of the CO to a given target set in finite time. The proposed feedback guidance law essentially forces the CO to track a reference velocity signal that depends explicitly on the position of the CO rather than the time. This position-dependent reference signal has the special property that it can steer the lower order kinematic model to its target set in finite time, when it acts as its control input. In this way, a feedback guidance law is developed that enforces finite time convergence of the CO's position vector to its target set in the presence of spatially and temporally varying disturbances. Because the proposed guidance law is (at least) continuous, it does not suffer from the chattering phenomenon that appears in the implementation of, say, discontinuous sliding mode controllers [23, 24], which can be employed alternatively to address similar classes of problems.

The rest of the paper is organized as follows. Section II highlights some of the key limitations of standard control techniques, namely the proportional-derivative and the proportional-integral-derivative control laws, which render them unsuitable for the problem treated herein. The steering problems for both the fully actuated and the under-actuated COs are formulated in Section III. Feedback guidance laws for the two kinematic models of the CO, which are based on an integral backstepping algorithm, are presented in Section IV. Numerical simulations, which utilize real wind data, are presented in Section V. Finally, Section VI concludes the paper with a summary of remarks.

II. Limitations of Standard Control Techniques for the Steering Problem in the Presence of Uncertain Spatiotemporal Wind

This section highlights some fundamental limitations of standard control techniques to address the problem of steering a controlled object (CO, for short) to a given target set in finite time, in the presence of uncertain wind. To this aim, the steering problem for a CO whose motion in the absence of wind is described by the double integrator kinematic model will be briefly discussed. In particular, let one assume that the equations of motion of the CO are given by

$$\dot{\xi} = \nu, \qquad \dot{\nu} = \alpha, \qquad\qquad \xi(0) = \bar{\xi}, \qquad \nu(0) = \bar{\nu}, \tag{1}$$

where $\xi \in \mathbb{R}^3$ ($\bar{\xi} \in \mathbb{R}^3$) and $\nu \in \mathbb{R}^3$ ($\bar{\nu} \in \mathbb{R}^3$) are the position and velocity vectors of the CO at time t (time $t = 0$), and α is the control input. The objective is to drive the position vector of the CO to the origin.

A popular feedback control law used in stabilization and tracking problems of mechanical systems is the so-called proportional-derivative control law [25], which is described, in its simplest form, by the following equation

$$\alpha_{\mathsf{PD}}(\xi, \nu) := -k_\xi \xi - k_\nu \nu, \tag{2}$$

where the gains k_ξ and k_ν are real constants (design parameters) or constant matrices, in general. One can show, by utilizing standard arguments from Lyapunov stability analysis, that in the absence of disturbances (induced by the wind, in this case), the CO's state vector (concatenation of position and velocity vectors) would converge to the origin asymptotically from any initial state, after selecting appropriately the gains k_ξ and k_ν. Because in the presence of disturbances, the inclusion of an integral term in the applied control law can improve the performance of the controller of the CO, it will now be assumed that the control input applied to the CO is not only a function of ξ and ν, but also ψ, where $\psi(t) := \int_0^t \xi(s)\mathrm{d}s$; one writes $\alpha(\xi, \nu, \psi)$. Next, let one consider a vector $\sigma(\xi, \nu, \psi) \in \mathbb{R}^3$, where $\sigma(\xi, \nu, \psi) := k_\nu \nu + k_\xi \xi + k_\psi \psi$, and where k_ξ, k_ν and k_ψ are positive constants [26]. It follows readily that

$$\dot{\sigma}(\xi, \nu, \psi) = k_\nu \alpha(\xi, \nu, \psi) + k_\xi \nu + k_\psi \xi. \tag{3}$$

Let $V(\xi,\nu,\psi) := \frac{1}{2}|\sigma(\xi,\nu,\psi)|^2$ be a candidate generalized Lyapunov function. Note that $V(\xi,\nu,\psi)$ is not a positive definite function (it does not vanish at the origin only); hence the "generalized" qualifier. It follows readily that

$$\dot{V}(\xi,\nu,\psi) = \langle \sigma(\xi,\nu,\psi), \dot{\sigma}(\xi,\nu,\psi)\rangle = \langle \sigma(\xi,\nu,\psi), k_\nu \alpha(\xi,\nu,\psi) + k_\xi \nu + k_\psi \xi\rangle. \tag{4}$$

Now, let one consider the following proportional-derivative-integral (PID, for short) control law

$$\alpha_{\mathrm{PID}}(\xi,\nu,\psi) := -\frac{k_\xi}{k_\nu}\nu - \frac{k_\psi}{k_\nu}\xi - \frac{\lambda}{k_\nu}\sigma(\xi,\nu,\psi), \tag{5}$$

where λ is a positive constant. By setting $\alpha(\xi,\nu,\psi) = \alpha_{\mathsf{PID}}(\xi,\nu,\psi)$, one takes

$$\dot{V}(\xi,\nu,\psi) = -\lambda\langle \sigma(\xi,\nu,\psi), \sigma(\xi,\nu,\psi)\rangle \leq 0. \tag{6}$$

It follows that the system trajectories will converge, as $t \to \infty$, to the sliding surface $\sigma(\xi,\nu,\psi) = k_\nu \nu + k_\xi \xi + k_\psi \psi = 0$, or equivalently,

$$\sigma(\xi,\nu,\psi) = k_\nu \ddot{\psi} + k_\xi \dot{\psi} + k_\psi \psi = 0. \tag{7}$$

Note that Eq. (7) describes an exponentially stable, and thus bounded-input bouded-output stable, linear time invariant system that is driven by an input σ, which is a bounded function of time; something that follows readily from Eq. (6) and the definition of V. Therefore, if $\sigma \to 0$ as $t \to \infty$, then $\dot{\psi} \to 0$ and $\psi \to 0$, and thus the position vector $\xi = \dot{\psi}$ will asymptotically converge to the origin. As shown in [27], Eqs. (3)-(6) imply that V is bounded, and σ, $\dot{V}$ are uniformily continuous functions of time. Application of the Barbalat's lemma yields $\dot{V}(\xi,\nu,\psi) \to 0$ as $t \to \infty$ and consequently, $\sigma \to 0$ as $t \to \infty$ as well. Moreover, in view of (7), it is also true that $\nu \to 0$ as $t \to \infty$, where $\nu = \ddot{\psi}$. Consequently, the motion of the CO becomes very slow as it approaches the origin. This is not necessarily a desirable situation, given that it is possible that as the CO approaches its target (the origin), its airspeed may not be sufficiently large to compensate any local wind; consequently, the arrival of the CO to its target may be delayed, if not completely prevented.

A possible modification of the PID control law given in (5) so that it explicitly accounts for the presence of wind is considered next. In particular, the motion of the CO is now described by the following set of equations

$$\dot{\xi} = \nu + \omega(t,\xi), \quad \dot{\nu} = \alpha(\xi,\nu,\psi), \qquad \xi(0) = \bar{\xi}, \quad \nu(0) = \bar{\nu}, \tag{8}$$

where $\nu \in \mathbb{R}^3$ ($\bar{\nu} \in \mathbb{R}^3$) is the air velocity of the vehicle at time t (time $t = 0$) and $\omega(t,\xi)$ denotes the velocity of the local wind. Let one assume, for the sake of the argument, that $\omega(t,\xi)$ is perfectly measured by the CO along its ensuing path to its target. In addition, let $\nu_g := \nu + \omega(t,\xi)$ and $\bar{\nu}_g := \bar{\nu} + \omega(0,\bar{\xi})$, where ν_g and $\bar{\nu}_g \in \mathbb{R}^3$ denote the inertial (or ground) velocities of the CO at time t and $t = 0$, respectively. Then, the equations of motion of the CO in terms of the state vector (ξ, ν_g) are given by

$$\dot{\xi} = \nu_g, \quad \dot{\nu}_g = \alpha_g(\xi,\nu_g,\psi), \qquad \xi(0) = \bar{\xi}, \quad \nu_g(0) = \bar{\nu}_g, \tag{9}$$

where $\alpha_g(\xi,\nu_g,\psi)$ is the (inertial) acceleration vector of the CO, which satisfies the following equation

$$\begin{aligned}\alpha_g(\xi,\nu_g,\psi) &= \frac{\mathrm{d}}{\mathrm{d}t}(\nu + \omega(t,\xi)) = \dot{\nu} + \frac{\mathrm{d}}{\mathrm{d}t}\omega(t,\xi)\\ &= \alpha(\xi,\nu_g - \omega(t,\xi),\psi) + \nabla_t \omega(t,\xi) + \nabla_\xi \omega(t,\xi)\dot{\xi}\\ &= \alpha(\xi,\nu_g - \omega(t,\xi),\psi) + \nabla_t \omega(t,\xi) + \nabla_\xi \omega(t,\xi)\nu_g.\end{aligned} \tag{10}$$

Therefore, the new equations describing the motion of the CO (Eq. (9)) have been brought to the double integrator form given in Eq. (1). This standard approach is often adopted in, for example, path following problems in the presence of wind (see, for example, [28]). Therefore, one could argue that the PID control law

$$\alpha_g(\xi,\nu_g,\psi) = \alpha_{\mathrm{PID}}(\xi,\nu_g,\psi) = -\frac{k_\xi}{k_\nu}\nu_g - \frac{k_\psi}{k_\nu}\xi - \frac{\lambda}{k_\nu}\sigma(\xi,\nu_g,\psi), \tag{11}$$

will solve the steering problem for the system described by Eq. (9) (in the presence of wind) similar to the way the PID control law (5) solves the steering problem for the system described by Eq. (1) (in the absence of wind). However, the realization of the control law (11) could only be achieved with the application of the actual control input $\alpha(\xi, \nu, \psi) = \alpha(\xi, \nu_g - \omega(t, \xi), \psi)$, which satisfies, in light of Eq. (10), the following equation

$$\alpha(\xi, \nu, \psi) = \alpha_{\mathsf{PID}}(\xi, \nu_g(\nu), \psi) - \nabla_t \omega(t, \xi) - \nabla_\xi \omega(t, \xi)\nu_g(\nu), \tag{12}$$

where $\nu_g(\nu) := \nu + \omega(t, \xi)$. Consequently, the realization of the control law (12) requires, in turn, not only knowledge of $\omega(t, \xi)$ (in order to compute ν_g) but its total time derivative $\frac{\mathrm{d}}{\mathrm{d}t}\omega(t, \xi) := \nabla_t \omega(t, \xi) + \nabla_\xi \omega(t, \xi)\nu_g(\nu)$ as well. In contrast with the assumption that the CO has access to (imperfect) measurements of the velocity of the local wind, $\omega(t, \xi)$, which is often more or less true in practice, the assumption that the total time derivative $\frac{\mathrm{d}}{\mathrm{d}t}\omega(t, \xi)$ can be accurately measured by the CO "on the fly" is hardly verifiable. Moreover, even if measurements of $\frac{\mathrm{d}}{\mathrm{d}t}\omega(t, \xi)$ were becoming available to the CO along its ensuing path, the objective that $\xi \to 0$ would only be achieved asymptotically, that is, as $t \to \infty$. It should also be mentioned that in many applications, it is desirable that the CO maintains a constant (either approximately or exactly) airspeed, $|\nu|$, rather than a constant ground speed, $|\nu_g|$. Consequently, once again the state vector (ξ, ν) constitutes a more natural choice for the state vector of the CO, in terms of enforcing the more intuitive constant speed constraint, than the state vector (ξ, ν_g).

Note that the control law given in Eq. (12) can better handle, in practice, the presence of uncertain wind than the PD control that results from (12) after setting $k_\psi = 0$ or the one defined in Eq. (2), because of the inclusion of the integral term. If the temporal and spatial gradients of the wind's velocity were also measurable "on the fly" and an upper bound on the measurement error of $\frac{\mathrm{d}}{\mathrm{d}t}\omega(t, \xi)$ was known, for example, $|\frac{\mathrm{d}}{\mathrm{d}t}\omega(t, \xi)| \leq \gamma$, for all $t \geq 0$ and $\xi \in \mathbb{R}^3$, where γ is a positive constant, then one could utilize instead the following guidance law [27]

$$\alpha(\xi, \nu, \psi) = \alpha_{\mathsf{PID}}(\xi, \nu_g(\nu), \psi) - \nabla_t \omega(t, \xi) - \nabla_\xi \omega(t, \xi)\nu_g(\nu) - \kappa(\gamma)\mathrm{sgn}(\sigma), \tag{13}$$

where $\mathrm{sgn}(\sigma)$ denotes the n-dimensional vector whose components are given by the signs of the corresponding components of the vector σ, and $\kappa(\gamma)$ is an appropriately chosen positive constant. The last term in (13) is a sliding control term that accounts for the uncertainty due to the measurement errors of the gradients (spatial and temporal) of the wind velocity. Alternatively, one can consider the term $-\hat{\kappa}(\gamma)\sigma/|\sigma|$, where again $\hat{\kappa}(\gamma)$ is a positive constant, in lieu of $-\kappa(\gamma)\mathrm{sgn}(\sigma)$. Both of these new control terms, which are purport to account for the effects of the unknown component of the time derivative of the wind's velocity, introduce discontinuities in the guidance law, that are often undesirable in practice (chattering phenomenon).

It should be noted here that in order to enforce the finite time convergence of the CO to its target set in the presence of an uncertain spatiotemporal field, one would typically utilize, for example, discontinuous sliding mode controllers (see [24] and references therein). The implementation of these discontinuous controllers, however, suffer from the chattering phenomenon [23]. It will be shown that the problem of guiding the position vector of the CO to a given target in finite time in the presence of uncertain wind can be solved by means of a controller that is continuous everywhere, except from an isolated point that is never attained during the time interval of interest.

III. The Guidance Problem in the Presence of Uncertain Wind

The main objective of this work is the design of (at least) continuous feedback guidance laws that, in contrast with the PD and the PID control laws presented in Section II, can drive the position vector of the CO to the origin in finite time in the presence of wind with an uncertain spatiotemporal velocity field, while the latter is traveling at an either approximately or exactly constant airspeed (soft and hard constraints, respectively). The characterization of the guidance law will be made under the assumption that the velocity field of the wind is not known a priori but is measured along the ensuing path of the CO to its target set by onboard sensors. In addition, the measurements of the local wind's velocity are imperfect and no information about its total time derivative (spatial and temporal gradients) is available to the CO at all times.

In particular, it is assumed that the wind velocity can be expressed as the vector sum of a known (measured) component, $\omega(t,\xi)$, and an uncertain one, $\Delta\omega(t,\xi)$ (measurement error). Alternatively, one can think of $\omega(t,\xi)$ and $\Delta\omega(t,\xi)$ as, respectively, the slowly varying (dominant wind) and the rapidly changing (gusting wind) components of the wind [29].

Assumption 1. *The CO located at the position vector ξ at time t obtains a measurement $\omega(t,\xi)$ of the local wind's velocity $\omega(t,\xi)+\Delta\omega(t,\xi)$, where $\Delta\omega(t,\xi)$ corresponds to the unknown measurement error, which is modeled as deterministic noise. It is assumed that the mapping $(t,\xi)\mapsto\omega(t,\xi)$ is continuous everywhere in $[0,\infty)\times\mathbb{R}^3$ and, in addition, the mapping $\xi\mapsto w(t,\xi)$ is locally Lipschitz continuous uniformly over t in any compact interval in $[0,\infty)$. In addition, there exist $\bar{\omega}$, $\Delta\bar{\omega}\geq 0$, where $\bar{\omega}+\Delta\bar{\omega}\in[0,1)$, and $\beta>0$, such that $|\omega(t,\xi)|\leq\bar{\omega}$ and $|\Delta\omega(t,\xi)|\leq\min\{\Delta\bar{\omega},\beta|\xi|\}$, for all $\xi\in\mathbb{R}^3$ and $t\geq 0$.*

Remark 1 Assumption 1 will facilitate the convergence analysis that will be presented in Section IV. In simple words, Assumption 1 states that the sum of the maximum magnitudes of the measured wind velocity and the measurement error never exceeds the "nominal" unit airspeed of the CO. This assumption is not necessary for the convergence analysis; milder assumptions can be made instead based on, for example, Propositions 8 and 10 from Ref. [6]. These assumptions, however, are either non-verifiable a priori or require that the wind velocity field has a particular structure.

Remark 2 Note that Assumption 1 requires that the measurement error, $\Delta\omega$, behaves as an *admissible uncertainty* (that is, it decreases as the CO approaches its target set); by contrast, the total wind velocity $\omega+\Delta\omega$ is not assumed to behave in the same way.

Two different kinematic models for the CO will be considered. The first model, that will be referred to as the *fully actuated* kinematic model, will be described by the following set of equations

$$\dot{\xi}=\nu+\omega(t,\xi)+\Delta\omega(t,\xi),\qquad \dot{\nu}=\alpha,\qquad\qquad \xi(0)=\bar{\xi},\quad \nu(0)=\bar{\nu}. \tag{14}$$

The situation is illustrated in Fig. 1. One observes that the input α can be expressed as the vector sum of three mutually perpendicular components, that is, $\alpha=\alpha_\nu+\alpha_\nu^\perp+\alpha_\nu^{\perp\perp}$, where $\alpha_\nu\in\mathbb{R}^3$ corresponds to the component of α that is parallel to ν, that is, $\alpha_\nu:=\langle\alpha,\nu\rangle\nu/|\nu|^2$, provided that $\nu\neq 0$, and $\alpha_\nu^\perp$ and $\alpha_\nu^{\perp\perp}$ correspond to two mutually orthogonal components of α that are both perpendicular to ν. In Fig. 1, the component $\alpha_\nu^{\perp\perp}$ is pointing into the page. An important observation here is that, on the one hand, the component α_ν is responsible for any changes on the airspeed of the CO, $|\nu|$, along its ensuing path. On the other hand, the components $\alpha_\nu^\perp$ and $\alpha_\nu^{\perp\perp}$, which are perpendicular to ν at all times, can only change the direction of motion of the CO, but they cannot affect its airspeed. Because in this particular model, the CO can control each of the three components of α independently, both its airspeed and the direction of its motion can be regulated simultaneously.

Remark 3 Although the kinematic model described by Eq. (14) neglects completely important system nonlinearites, including the dynamics of the vehicle in response to the wind, its utilization allows one to place the emphasis on the essentials of the geometry of the guidance problem. In particular, the guidance laws that will be presented next will correspond to "outer-loop" guidance laws, where the system nonlinearities are assumed to be handled by appropriate inner control loops [28]. In this way, one can gain useful insights into the geometry of the steering problem in the presence of wind.

The finite time guidance problem for the CO, whose motion is described by Eq. (14), to a given target set is formulated next.

Problem 1. *Suppose that Assumption 1 holds and let $\epsilon>0$ be given. Find a feedback control law $\alpha(\xi,\nu;\omega)\in\mathbf{C}(\mathbb{R}^3\backslash\{0\}\times\mathbb{R}^3)$, that will drive the system described by Eq. (14) to the target set $\mathcal{T}_\epsilon:=\{(\xi,\nu)\in\mathbb{R}^6:\ |\xi|\leq\epsilon\}$ in finite time $t_\mathsf{f}(\epsilon)<\bar{t}_\mathsf{f}$, where $0<\bar{t}_\mathsf{f}<\infty$, for any $\epsilon>0$.*

Remark 4 The requirement that the CO must reach its target set in finite time has the following meaning: For a given $\epsilon>0$, the CO driven by the feedback control law $\alpha(\xi,\nu;\omega)$ must converge to the target set $\mathcal{T}_\epsilon$ after $t_\mathsf{f}(\epsilon)$ units of time, where $t_\mathsf{f}(\epsilon)\leq\bar{t}_\mathsf{f}<\infty$, that is, $t_\mathsf{f}(\epsilon)$ is upper bounded by a finite bound, $\bar{t}_\mathsf{f}$, which is independent of ϵ.

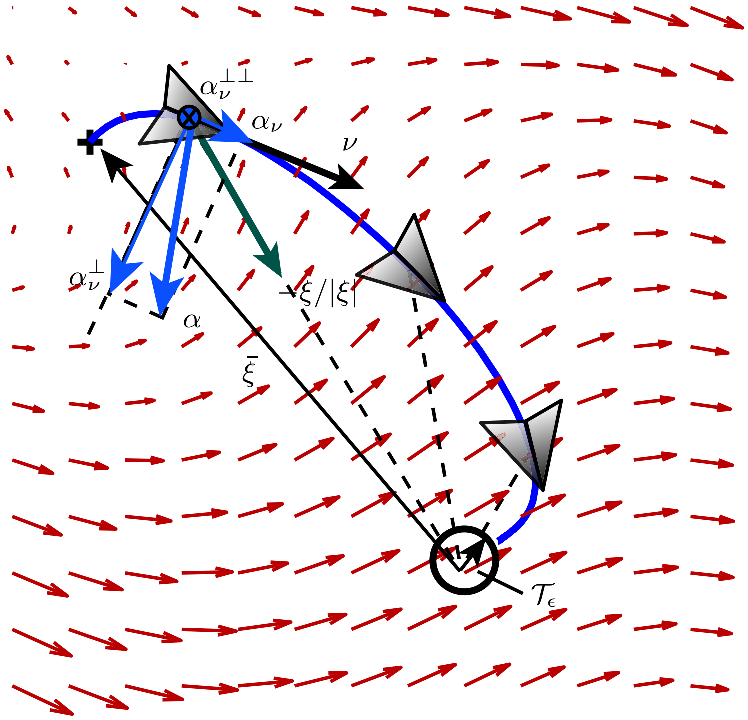


Fig. 1 The steering problem in the presence of wind for a controlled object whose motion is described by Eq. (14)**.**

Next, a different kinematic model that describes the motion of the underactuated CO is introduced. It will be referred to as the *under-actuated* kinematic model, which is described by the following equations [19, 30]

$$\dot{\xi} = \nu + \omega(t,\xi) + \Delta\omega(t,\xi), \quad \dot{\nu} = \varpi \times \nu, \qquad \xi(0) = \bar{\xi}, \quad \nu(0) = \bar{\nu}, \tag{15}$$

where ϖ is the new control input and $\times$ denotes the cross product operation. Note that $\dot{\nu} = \varpi \times \nu$ is, by definition, perpendicular to the air velocity ν at all times, and thus, it can be written as follows $\dot{\nu} = \varpi \times \nu = \dot{\nu}_\nu^{\perp} + \dot{\nu}_\nu^{\perp\perp}$, where $\dot{\nu}_\nu^{\perp}$ and $\dot{\nu}_\nu^{\perp\perp}$ are two mutually orthogonal components of $\dot{\nu}$, which span a plane that is perpendicular to the air velocity ν. The situation is illustrated in Figure 2 (note that $\dot{\nu}_\nu^{\perp\perp}$ is pointing into the page).

Note that only the components of ϖ that are perpendicular to ν can affect the motion of the under-actuated CO. It is also interesting to highlight that the state constraint: $|\nu(t)| = 1$, for all $t \in [0, t_{\mathsf{f}}]$ (hard constraint), is now encoded in the new equations of motion of the CO given in (15). In particular, $\dot{\nu}$ is now perpendicular to ν, and thus the CO is constrained to travel with constant airspeed (there is no component of $\dot{\nu}$ parallel to ν to change the CO's airspeed). Consequently, the control input can only affect the direction of motion of the under-actuated CO. The steering problem in this case is formulated as follows.

Problem 2. *Address Problem 1 for the system described by Eq.* (15)*. Equivalently, address Problem 1 under the additional (hard) state constraint:* $|\nu(t)| = 1$*, for all* $t \in [0, t_{\mathsf{f}}]$.

IV. Pure Pursuit Guidance via Vector Backstepping

In this section, Problems 1 and 2 are addressed. In particular, feedback laws aimed at making the system described by Eqs. (14) and (15) to evolve similarly to a lower order kinematic model, whose air velocity acts as the control input, will be presented next.

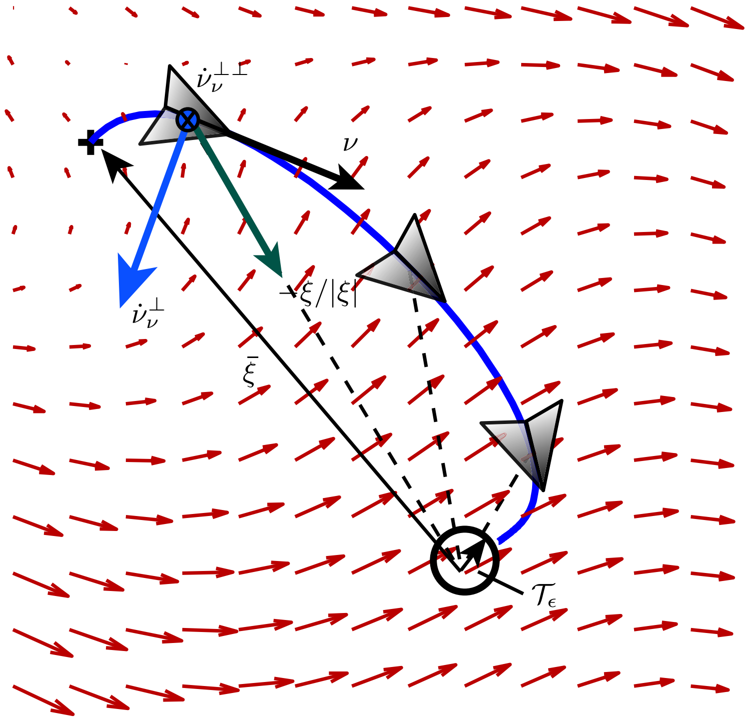


Fig. 2 The steering problem in the presence of wind with an uncertain spatiotemporal velocity field for a controlled object whose motion is described by Eq. (15)**.**

A. Pure Pursuit Navigation in Uncertain Spatiotemporal Wind

Let one consider the following lower order kinematic model

$$\dot{\xi} = \nu(\xi) + \omega(t,\xi) + \Delta\omega(t,\xi), \qquad \xi(0) = \bar{\xi}, \tag{16}$$

where $\nu(\xi)$ is the new control input. Let one assume that there exists $\nu_\star(\cdot) \in \mathbf{C}^1(\mathbb{R}^3\backslash\{0\})$, that will drive the system (16) to the target set $\mathcal{T}_\epsilon^\star := \{\xi \in \mathbb{R}^3 : \ |\xi| \leq \epsilon\}$ in finite time. The idea is to design a feedback law that will force the velocity of the actual CO, whose motion is described by either Eq. (14) or Eq. (15), to track the air velocity signal $\nu_\star(\xi)$, which is not an explicit function of time but depends instead on the position vector ξ of the CO. As will be shown next, one can exploit the special property of $\nu_\star(\xi)$ to steer the lower order kinematic model of CO described by Eq. (16) to the target set $\mathcal{T}_\epsilon^\star$ in finite time, in order to design guidance laws that solve Problems 1 and 2. This will be achieved with the utilization of an integral backstepping algorithm [21, 22] in vector form. In particular, let $\nu_\star(\xi) = \nu_{\mathsf{PP}}(\xi)$, where

$$\nu_{\mathsf{PP}}(\xi) := -\frac{\xi}{|\xi|}, \qquad |\xi| > 0. \tag{17}$$

The control law $\nu_{\mathsf{PP}}(\xi)$ will be referred to as the *pure pursuit* navigation law because of its intrinsic relation with the well known pure pursuit strategy [31, 32] from differential pursuit-evasion games (for more details, see Ref. [6]). In particular, with the application of the pure pursuit navigation law, the air velocity of the lower order CO points towards the target, that is, the direction $-\xi/|\xi|$ (known as the LOS direction), without explicitly accounting for the local wind. Because the local wind is not directly compensated, the pure pursuit navigation law $\nu_{\mathsf{PP}}(\xi)$ constantly tries to correct the error between the direction of the ground velocity $\dot{\xi}$ and the direction $-\xi/|\xi|$.

Proposition 1. *Let $\epsilon > 0$ and suppose that all conditions of Assumption 1 hold. Then, the pure pursuit navigation law $\nu_{\mathsf{PP}}(\xi)$ given by Eq.* (17)*, will drive the system described by Eq.* (16) *to the target set $\mathcal{T}_\epsilon^\star := \{\xi \in \mathbb{R}^3 : \ |\xi| \leq \epsilon\}$ in finite time from any initial state $\bar{\xi} \in \mathbb{R}^3\backslash\mathcal{T}_\epsilon^\star$. In addition,*

$$\langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \dot{\xi}\rangle = \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu_{\mathsf{PP}}(\xi) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle \leq -c_{\mathsf{PP}}, \quad \text{for all} \quad |\xi| > 0, \tag{18}$$

where $\mathcal{V}_{\mathsf{PP}}(\xi) := |\xi|$ *and* $c_{\mathsf{PP}} := 1 - \bar{\omega} - \Delta\bar{\omega}$.

Proof. The total time derivative of $\mathcal{V}_{\mathsf{PP}}(\xi) = |\xi|$ along the trajectories of the system (16), when $\nu = \nu_{\mathsf{PP}}(\xi)$, is given by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V}_{\mathsf{PP}}(\xi) &= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \dot{\xi}\rangle = \langle \xi/|\xi|, \nu_{\mathsf{PP}}(\xi) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle \\ &= -1 + \langle \xi/|\xi|, \omega(t,\xi) + \Delta\omega(t,\xi)\rangle,\end{aligned} \tag{19}$$

where the facts that $\nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi) = \xi/|\xi| = -\nu_{\mathsf{PP}}(\xi)$ and $|\nu_{\mathsf{PP}}(\xi)| \equiv 1$ have been used. By hypothesis, $|\omega(t,\xi)| \leq \bar{\omega}$ and $|\Delta\omega(t,\xi)| \leq \min\{\Delta\bar{\omega}, \beta|\xi|\} \leq \Delta\bar{\omega}$, for all $t \geq 0$ and $\xi \in \mathbb{R}^3\backslash\{0\}$, where $0 \leq \bar{\omega} + \Delta\bar{\omega} < 1$, which along with the Cauchy Schwarz inequality imply that

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V}_{\mathsf{PP}}(\xi) \leq -1 + \bar{\omega} + \Delta\bar{\omega} < 0, \tag{20}$$

and the result follows readily (note that the time derivative of $\mathcal{V}_{\mathsf{PP}}$ is upper bounded by a strictly negative number). □

Remark 5 Note that a control law $\tilde{\nu}_{\mathsf{PP}}(\xi) := -\vartheta(\bar{\omega}, \Delta\bar{\omega})\xi/|\xi|$, where $\bar{\omega} + \Delta\bar{\omega} < \vartheta(\bar{\omega}, \Delta\bar{\omega}) \leq 1$, will also work in this case. By taking $\vartheta(\bar{\omega}, \Delta\bar{\omega}) \equiv 1$, the CO will always travel with constant unit airspeed, which is taken to be the "nominal" airspeed in this work.

Lemma 1. *Let* $r \in \mathbb{R}^3$. *Then,*

$$\nabla_\xi \nu_{\mathsf{PP}}(\xi) r = \frac{1}{|\xi|^3}\langle \xi, r\rangle \xi - \frac{1}{|\xi|} r = \frac{1}{|\xi|}(\langle \nu_{\mathsf{PP}}(\xi), r\rangle \nu_{\mathsf{PP}}(\xi) - r). \tag{21}$$

In addition,

$$|\nabla_\xi \nu_{\mathsf{PP}}(\xi) r| \leq 2|r|/|\xi|. \tag{22}$$

Proof. Equation (21) follows from standard vector calculus. In addition,

$$\begin{aligned}|\nabla_\xi \nu_{\mathsf{PP}}(\xi) r| &\leq (|\langle \nu_{\mathsf{PP}}(\xi), r\rangle||\nu_{\mathsf{PP}}(\xi)| + |r|)/|\xi| \\ &\leq |r|(|\nu_{\mathsf{PP}}(\xi)|^2 + 1)/|\xi| \\ &\leq 2|r|/|\xi|,\end{aligned} \tag{23}$$

where the fact that $|\nu_{\mathsf{PP}}(\xi)| \equiv 1$ along with the Cauchy Schwarz and the triangle inequalities have been used. □

B. Pure Pursuit Feedback Guidance Law for the Fully Actuated Controlled Object

Next, a feedback guidance law that addresses Problem 1 is presented. As it has been mentioned already, the proposed guidance law will constantly attempt to steer the CO such that it follows closely the motion of a first order kinematic model driven by the pure pursuit navigation law introduced earlier.

Proposition 2. *Let* $\epsilon > 0$ *be given and suppose that all the conditions of Assumption 1 hold. Then, the guidance law*

$$\alpha_{\mathsf{PP}}(\xi, \nu; \omega) = -\frac{k}{\mu}(\nu - \nu_{\mathsf{PP}}(\xi)) + \frac{1}{\mu}\nu_{\mathsf{PP}}(\xi) + \frac{1}{|\xi|}\left(\langle \nu_{\mathsf{PP}}(\xi), \nu + \omega(t,\xi)\rangle \nu_{\mathsf{PP}}(\xi) - (\nu + \omega(t,\xi))\right), \tag{24}$$

where $0 < \mu < (1 - \bar{\omega} - \Delta\bar{\omega})/\beta$, $k > \mu\beta$ *and* $\nu_{\mathsf{PP}}(\xi) := -\xi/|\xi|$, *will drive the system described by Eq.* (14) *to the target set* $\mathcal{T}_\epsilon := \{(\xi, \nu) \in \mathbb{R}^6 : |\xi| \leq \epsilon\}$ *in finite time* t_{f} *from any initial state* $(\bar{\xi}, \bar{\nu}) \in \mathbb{R}^6\backslash\mathcal{T}_\epsilon$. *In addition, the airspeed* $|\nu(t)|$ *will remain bounded for all* $t \in [0, t_{\mathsf{f}}]$, *and in particular,*

$$|\nu(t)| \leq 1 + \sqrt{\tfrac{2}{\mu}(V_{\mathsf{PP}}(\bar{\xi}, \bar{\nu}) - \epsilon)}, \quad \text{for all } t \in [0, t_{\mathsf{f}}], \tag{25}$$

where $V_{\mathsf{PP}}(\xi,\nu) := |\xi| + \frac{\mu}{2}|\nu - \nu_{\mathsf{PP}}(\xi)|^2$, *for all* $(\xi,\nu) \in \mathbb{R}^6 \backslash \mathcal{T}_\epsilon$. *Finally, the travel time satisfies the following upper bound*

$$t_{\mathsf{f}} \leq \frac{V_{\mathsf{PP}}(\bar{\xi},\bar{\nu}) - \epsilon}{\lambda_{\mathsf{PP}}} < \frac{V_{\mathsf{PP}}(\bar{\xi},\bar{\nu})}{\lambda_{\mathsf{PP}}} < \infty, \tag{26}$$

where $\lambda_{\mathsf{PP}} := 1 - \bar{\omega} - \Delta\bar{\omega} - \mu\beta$.

Proof. In light of Lemma 1, $|\nabla_\xi \nu_{\mathsf{PP}}(\xi)\Delta\omega(t,\xi)| \leq 2|\Delta\omega(t,\xi)|/|\xi|$. By hypothesis $|\Delta\omega(t,\xi)| \leq \min\{\Delta\bar{\omega}, \beta|\xi|\} \leq \beta|\xi|$. Therefore,

$$|\nabla_\xi \nu_{\mathsf{PP}}(\xi)\Delta\omega(t,\xi)| \leq 2\beta, \quad \text{for all} \quad |\xi| > 0. \tag{27}$$

Moreover, in view of (21) and the fact that $\nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi) = -\nu_{\mathsf{PP}}(\xi)$, the guidance law $\alpha_{\mathsf{PP}}(\xi,\nu;\omega)$ given in Eq. (24) can be written as follows

$$\alpha_{\mathsf{PP}}(\xi,\nu;\omega) = -\frac{k}{\mu}(\nu - \nu_{\mathsf{PP}}(\xi)) - \frac{1}{\mu}\nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi) + \nabla_\xi \nu_{\mathsf{PP}}(\xi)(\nu + \omega(t,\xi)). \tag{28}$$

The time derivative of $V_{\mathsf{PP}}(\xi,\nu) := \mathcal{V}_{\mathsf{PP}}(\xi) + \frac{\mu}{2}|\nu - \nu_{\mathsf{PP}}(\xi)|^2$ evaluated along the trajectories of the system described by Eq. (14) is given by

$$\begin{aligned}
\dot{V}_{\mathsf{PP}}(\xi,\nu) &= \langle \nabla_\xi V_{\mathsf{PP}}(\xi,\nu), \dot{\xi}\rangle + \langle \nabla_\nu V_{\mathsf{PP}}(\xi,\nu), \dot{\nu}\rangle \\
&= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \dot{\xi}\rangle - \mu\langle \nabla_\xi \nu_{\mathsf{PP}}(\xi)\dot{\xi}, \nu - \nu_{\mathsf{PP}}(\xi)\rangle + \mu\langle \dot{\nu}, \nu - \nu_{\mathsf{PP}}(\xi)\rangle \\
&= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle \\
&\quad - \mu\langle \nabla_\xi \nu_{\mathsf{PP}}(\xi)(\nu + \omega(t,\xi) + \Delta\omega(t,\xi)), \nu - \nu_{\mathsf{PP}}(\xi)\rangle \\
&\quad + \mu\langle \alpha_{\mathsf{PP}}(\xi,\nu;\omega), \nu - \nu_{\mathsf{PP}}(\xi)\rangle \\
&= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu_{\mathsf{PP}}(\xi) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle + \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu - \nu_{\mathsf{PP}}(\xi)\rangle \\
&\quad - \mu\langle \nabla_\xi \nu_{\mathsf{PP}}(\xi)(\nu + \omega(t,\xi) + \Delta\omega(t,\xi)), \nu - \nu_{\mathsf{PP}}(\xi)\rangle \\
&\quad + \mu\langle \alpha_{\mathsf{PP}}(\xi,\nu;\omega), \nu - \nu_{\mathsf{PP}}(\xi)\rangle \\
&= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu_{\mathsf{PP}}(\xi) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle \\
&\quad - \mu\langle \nabla_\xi \nu_{\mathsf{PP}}(\xi)\Delta\omega(t,\xi), \nu - \nu_{\mathsf{PP}}(\xi)\rangle \\
&\quad - k|\nu - \nu_{\mathsf{PP}}(\xi)|^2 \\
&\leq -c_{\mathsf{PP}} - \mu\langle \nabla_\xi \nu_{\mathsf{PP}}(\xi)\Delta\omega(t,\xi), \nu - \nu_{\mathsf{PP}}(\xi)\rangle - k|\nu - \nu_{\mathsf{PP}}(\xi)|^2,
\end{aligned} \tag{29}$$

where $c_{\mathsf{PP}} := 1 - \bar{\omega} - \Delta\bar{\omega}$, and (18) and (28) have been used. In addition, by (27) and the Cauchy Schwarz inequality, it follows that

$$-\langle \nabla_\xi \nu_{\mathsf{PP}}(\xi)\Delta\omega(t,\xi), \nu - \nu_{\mathsf{PP}}(\xi)\rangle \leq |\nabla_\xi \nu_{\mathsf{PP}}(\xi)\Delta\omega(t,\xi)||\nu - \nu_{\mathsf{PP}}(\xi)| \leq \beta(1 + |\nu - \nu_{\mathsf{PP}}(\xi)|^2), \tag{30}$$

for all $t \in [0,\infty)$ and $\xi \in \mathbb{R}^3\backslash\{0\}$, where the fact that $2a \leq 1 + a^2$, $a \in \mathbb{R}$, has been used. In light of (30) along with the fact that $k > \mu\beta$, which is true by hypothesis, (29) implies that

$$\dot{V}_{\mathsf{PP}}(\xi,\nu) \leq -(c_{\mathsf{PP}} - \mu\beta) - (k - \mu\beta)\,|\nu - \nu_{\mathsf{PP}}(\xi)|^2 \leq -\lambda_{\mathsf{PP}},$$

where $\lambda_{\mathsf{PP}} := c_{\mathsf{PP}} - \mu\beta$, $\lambda_{\mathsf{PP}} > 0$ (by hypothesis). It follows that

$$V_{\mathsf{PP}}(\xi(t),\nu(t)) - V_{\mathsf{PP}}(\bar{\xi},\bar{\nu}) \leq -\lambda_{\mathsf{PP}} t. \tag{31}$$

In addition, by the triangle inequality,

$$\begin{aligned}
|\nu(t)| &\leq |\nu_{\mathsf{PP}}(\xi(t))| + |\nu(t) - \nu_{\mathsf{PP}}(\xi(t))| \\
&\leq 1 + \sqrt{\tfrac{2}{\mu}(V_{\mathsf{PP}}(\xi(t),\nu(t)) - |\xi(t)|)} \\
&\leq 1 + \sqrt{\tfrac{2}{\mu}(V_{\mathsf{PP}}(\xi(t),\nu(t)) - \epsilon)} \\
&\leq 1 + \sqrt{\tfrac{2}{\mu}(V_{\mathsf{PP}}(\bar{\xi},\bar{\nu}) - \epsilon)},
\end{aligned}$$

where $V_{\mathsf{PP}}(\xi,\nu) := |\xi| + \frac{\mu}{2}|\nu - \nu_{\mathsf{PP}}(\xi)|^2 > \epsilon$, for all $t \in [0, t_{\mathsf{f}}]$ and $(\xi,\nu) \in \mathbb{R}^6\backslash\mathcal{T}_\epsilon$. In addition, let t'_{f} be the first time instant at which $V_{\mathsf{PP}}(\xi(t'_{\mathsf{f}}),\nu(t'_{\mathsf{f}})) = \epsilon$. Then,

$$|\xi(t'_{\mathsf{f}})| \leq |\xi(t'_{\mathsf{f}})| + \tfrac{\mu}{2}|\nu(t'_{\mathsf{f}}) - \nu_{\mathsf{PP}}(\xi(t'_{\mathsf{f}}))|^2 = V_{\mathsf{PP}}(\xi(t'_{\mathsf{f}}),\nu(t'_{\mathsf{f}})) = \epsilon.$$

Therefore, $t_{\mathsf{f}} \leq t'_{\mathsf{f}}$. Furthermore, by setting $t = t'_{\mathsf{f}}$ in (31), it follows that

$$t_{\mathsf{f}} \leq t'_{\mathsf{f}} \leq \frac{V_{\mathsf{PP}}(\bar{\xi},\bar{\nu}) - \epsilon}{\lambda_{\mathsf{PP}}} \leq \frac{V_{\mathsf{PP}}(\bar{\xi},\bar{\nu})}{\lambda_{\mathsf{PP}}}.$$

□

Remark 6 Note that because the wind velocity is, in general, both temporally and spatially varying, the feedback guidance law $\alpha_{\mathsf{PP}}(\xi,\nu;\omega)$ is practically a time-varying feedback law.

Remark 7 One observes that by selecting a sufficiently large μ, and thus placing a large weight on the velocity error term $|\nu - \nu_{\mathsf{PP}}(\xi)|^2$ that appears in the definition of V_{PP}, the upper bound on the airspeed given by the right hand side of (25) becomes approximately equal to one.

Remark 8 In the proof of Proposition 2, the generalized Lyapunov function $V_{\mathsf{PP}}(\xi,\nu) = |\xi| + \frac{\mu}{2}|\nu - \nu_{\mathsf{PP}}(\xi)|^2$ has been used. Note that V_{PP} is a Lipschitz continuous function that does not belong to $\mathbf{C}^1$ in every neighborhood of the origin; hence, the "generalized" qualifier. It should be mentioned here that one can use a more general class of candidate (generalized) Lyapunov functions, which are given by $\tilde{V}_{\mathsf{PP}}(\xi,\nu) := \mathcal{V}_{\mathsf{PP}}(\xi) + \frac{1}{2}\mu(|\xi|)|\nu - \nu_{\mathsf{PP}}(\xi)|^2$, where $\psi \mapsto \mu(\psi)$ is a function in $\mathbf{C}^1([0,\infty))$ such that $\mu_1 \leq \mu(|\xi|) < \mu_2$, for all $|\xi| \geq 0$, where μ_1 and μ_2 are positive constants. One can show, by using similar arguments as in the proof of Proposition 2, that the corresponding feedback control law in this case will be given by

$$\begin{aligned}\tilde{\alpha}_{\mathsf{PP}}(\xi,\nu;\omega) := &-\frac{k}{\mu(|\xi|)}(\nu - \nu_{\mathsf{PP}}(\xi)) - \frac{1}{\mu(|\xi|)}\nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi) + \nabla_\xi \nu_{\mathsf{PP}}(\xi)(\nu + \omega(t,\xi)) \\ &- \frac{1}{2\mu(|\xi|)}\langle \nabla_\xi \mu(|\xi|), \nu + \omega(t,\xi)\rangle(\nu - \nu_{\mathsf{PP}}(\xi)). \end{aligned} \tag{32}$$

With the use of the control law (32), one has the flexibility of weighting the (soft) constraint $\nu = \nu_{\mathsf{PP}}(\xi)$ along the ensuing path of the CO, via the term $\frac{1}{2}\mu(|\xi|)|\nu - \nu_{\mathsf{PP}}(\xi)|^2$ that appears in the definition of $\tilde{V}_{\mathsf{PP}}$, based on the relative distance of the CO from its target instead of imposing a globally uniform weight.

It should be highlighted at this point that the pure pursuit guidance law $\alpha_{\mathsf{PP}}(\xi,\nu;\omega)$ given in Eq. (24) requires knowledge of the velocity of the local wind. This is in contrast with the pure pursuit navigation law $\nu_{\mathsf{PP}}(\xi)$ that drives (16), which is completely independent of the wind. One observes here that the use of the integral backstepping logic increased the required level of information about the local wind that should be available to the CO along its ensuing path. Now let $\nu^{\perp}_{\mathsf{PP}}(\xi)$ and $\nu^{\perp\perp}_{\mathsf{PP}}(\xi)$ be two mutually perpendicular unit vectors which along with $\nu_{\mathsf{PP}}(\xi)$ form an orthonormal triad of basis vectors that travels along the ensuing path of the CO. In particular, one has

$$\nu_g(\nu) = \langle \nu_{\mathsf{PP}}(\xi), \nu_g(\nu)\rangle\, \nu_{\mathsf{PP}}(\xi) + \langle \nu^{\perp}_{\mathsf{PP}}(\xi), \nu_g(\nu)\rangle\, \nu^{\perp}_{\mathsf{PP}}(\xi) + \langle \nu^{\perp\perp}_{\mathsf{PP}}(\xi), \nu_g(\nu)\rangle\, \nu^{\perp\perp}_{\mathsf{PP}}(\xi).$$

It is then interesting to note that (24) can be written as follows

$$\begin{aligned}\alpha_{\mathsf{PP}}(\xi,\nu;\omega) &= -\frac{k}{\mu}(\nu - \nu_{\mathsf{PP}}(\xi)) + \frac{1}{\mu}\nu_{\mathsf{PP}}(\xi) + \frac{1}{|\xi|}\left(\langle \nu_{\mathsf{PP}}(\xi), \nu_g(\nu)\rangle\, \nu_{\mathsf{PP}}(\xi) - \nu_g(\nu)\right), \\ &= -\frac{k}{\mu}(\nu - \nu_{\mathsf{PP}}(\xi)) + \frac{1}{\mu}\nu_{\mathsf{PP}}(\xi) - \frac{1}{|\xi|}\left(\langle \nu^{\perp}_{\mathsf{PP}}(\xi), \nu_g(\nu)\rangle\, \nu^{\perp}_{\mathsf{PP}}(\xi) + \langle \nu^{\perp\perp}_{\mathsf{PP}}(\xi), \nu_g(\nu)\rangle\, \nu^{\perp\perp}_{\mathsf{PP}}(\xi)\right),\end{aligned}$$

where $\nu_g(\nu) := \nu + \omega(t,\xi)$. One immediately concludes that the last term in the right hand side of Eq. (24) is always perpendicular to $\nu_{\mathsf{PP}}(\xi)$ and vanishes when $\nu_g(\nu)$ is parallel to $\nu_{\mathsf{PP}}(\xi)$ (the LOS direction or its opposite). Consequently, the role of the last term in the right hand side of Eq. (24) is to rotate the air velocity of the CO to point towards the correct direction that will allow the latter to converge to its target set.

Proposition 3. *The norm $|\alpha_{\mathsf{PP}}(\xi,\nu;\omega)|$ of the guidance law given by Eq.* (24) *satisfies the following upper bound*

$$|\alpha_{\mathsf{PP}}(\xi,\nu;\omega)| \leq (k/\mu + 2/\epsilon)\sqrt{\tfrac{2}{\mu}(V_{\mathsf{PP}}(\bar{\xi},\bar{\nu}) - \epsilon)} + 1/\mu + 2(\bar{\omega}+1)/\epsilon, \tag{33}$$

where $V_{\mathsf{PP}}(\xi,\nu) := |\xi| + \frac{\mu}{2}|\nu - \nu_{\mathsf{PP}}(\xi)|^2$, *for all* $(\xi,\nu) \in \mathbb{R}^6\backslash\mathcal{T}_\epsilon$ *and* $t \in [0, t_{\mathsf{f}}]$.

Proof. By Eq. (28), the triangle inequality and the fact that $|\nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi)| = |\nu_{\mathsf{PP}}(\xi)| = 1$, for all $|\xi| > 0$, it follows that

$$|\alpha_{\mathsf{PP}}(\xi,\nu;\omega)| \leq k/\mu|\nu - \nu_{\mathsf{PP}}(\xi)| + 1/\mu + |\nabla_\xi \nu_{\mathsf{PP}}(\xi)(\nu + \omega(t,\xi))|. \tag{34}$$

Similarly to the proof of Proposition 2, one can show that

$$|\nu(t) - \nu_{\mathsf{PP}}(\xi)| \leq \sqrt{\tfrac{2}{\mu}(V_{\mathsf{PP}}(\bar{\xi},\bar{\nu}) - \epsilon)}, \quad |\nu(t)| \leq 1 + \sqrt{\tfrac{2}{\mu}(V_{\mathsf{PP}}(\bar{\xi},\bar{\nu}) - \epsilon)}, \tag{35}$$

which along with (22) imply that

$$|\nabla_\xi \nu_{\mathsf{PP}}(\xi(t))(\nu(t) + \omega(t,\xi(t)))| \leq 2(|\nu(t)| + \bar{\omega})/\epsilon, \tag{36}$$

for all $t \in [0, t_{\mathsf{f}}]$. The result follows readily from (34)-(36). □

Remark 9 Note that, in practice, the norm $|\alpha_{\mathsf{PP}}(\xi,\nu;\omega)|$ is upper bounded by an a priori known bound $\bar{\alpha} > 0$, which reflects the limits of the operational envelope of the CO. The condition (33) allows one to tune the gains k and μ in order to achieve a given accuracy of convergence, ϵ, of the CO to its target set, while the condition $|\alpha_{\mathsf{PP}}(\xi,\nu;\omega)| \leq \bar{\alpha}$ is satisfied at all times. In particular, $|\alpha_{\mathsf{PP}}(\xi,\nu;\omega)|$ becomes smaller when one selects a sufficient large μ, or more precisely, a small ratio k/μ; something that places a large weight on the error $|\nu - \nu_{\mathsf{PP}}(\xi)|^2$ that appears in the definition of V_{PP}. Note, however, that in light of Proposition 2, the gains k and μ should be chosen such that $k/\mu > \beta$ and $\mu < (1 - \bar{\omega} - \Delta\bar{\omega})/\beta$; in addition, by selecting a large μ, the upper bound on the arrival time, $\bar{t}_{\mathsf{f}}$, will become large as well.

C. Pure Pursuit Feedback Guidance Law for the Under-Actuated Controlled Object

In the previous analysis, it has been assumed that the CO can control the time derivative of its air velocity under no explicit constraints. However, in many applications it is required that the CO should maintain a constant airspeed; consequently, the time derivative of its air velocity has to be perpendicular to the air velocity at all times.

In order to address Problem 2, the following approach is adopted. The control input vector ϖ_{PP} that steers the under-actuated CO is taken to be such that its cross product with the air velocity ν equals a third vector that, at each time t, is as "close" as possible to the vector α_{PP}, which is given by Eq. (24) (the control input of the fully actuated CO). In particular, the new control input ϖ_{PP} would ideally satisfy the following cross product (vector) equation $\varpi_{\mathsf{PP}}(\xi,\nu;\omega) \times \nu = \alpha_{\mathsf{PP}}(\xi,\nu;\omega)$, for all (ξ,ν) along the CO's ensuing path. However, unless $\alpha_{\mathsf{PP}}(\xi,\nu;\omega)$ is perpendicular to ν, the cross product (vector) equation does not admit a solution. One standard approach is to subtract the component $\langle \alpha_{\mathsf{PP}}(\xi,\nu;\omega), \nu\rangle \nu/|\nu|^2 = \langle \alpha_{\mathsf{PP}}(\xi,\nu;\omega), \nu\rangle \nu$ from $\alpha_{\mathsf{PP}}(\xi,\nu;\omega)$ and, subsequently, solve the resulting cross product (vector) equation

$$\varpi_{\mathsf{PP}}(\xi,\nu;\omega) \times \nu = \alpha_{\mathsf{PP}}(\xi,\nu;\omega) - \langle \nu, \alpha_{\mathsf{PP}}(\xi,\nu;\omega)\rangle \nu, \tag{37}$$

which now admits infinite solutions. The minimum norm solution to Eq. (37) is given by (see, for example, [33])

$$\varpi_{\mathsf{PP}}(\xi,\nu;\omega) = \frac{1}{|\nu|^2}\nu \times \alpha_{\mathsf{PP}}(\xi,\nu;\omega) = \nu \times \alpha_{\mathsf{PP}}(\xi,\nu), \tag{38}$$

where $\alpha_{\mathsf{PP}}(\xi, \nu; \omega)$ is given by Eq. (24). Note that Eq. (37) implies that there is an error $\Delta\dot{\nu}$ between the desired time derivative of the air velocity, $\alpha_{\mathsf{PP}}(\xi, \nu; \omega)$, and the actual one, $\varpi_{\mathsf{PP}}(\xi, \nu; \omega) \times \nu$. The error $\Delta\dot{\nu}$ corresponds to the projection of $\alpha_{\mathsf{PP}}(\xi, \nu; \omega)$ on the air velocity vector ν. In particular,

$$\Delta\dot{\nu}(\xi, \nu; \omega) := \alpha_{\mathsf{PP}}(\xi, \nu; \omega) - \varpi_{\mathsf{PP}}(\xi, \nu; \omega) \times \nu = \langle \nu, \alpha_{\mathsf{PP}}(\xi, \nu; \omega) \rangle \nu. \tag{39}$$

It can be shown, by using similar arguments as in the proof of Proposition 2, that the feedback guidance law (38) will (locally) solve Problem 2.

As will be illustrated in the numerical simulations that will be presented in Section V, the use of the proposed guidance law ϖ_{PP} gives good results in practice even when one considers challenging scenarios, where for example, the vectors ν and ν_{PP} are, at time $t = 0$, opposite. Next, a feedback control law, which guarantees (almost) global finite time convergence of the under-actuated CO to its target set for the two-dimensional case, is presented.

D. Pure Pursuit Feedback Guidance Law for the Under-Actuated Controlled Object in a Two-Dimensional Case

Next, it is shown that by using similar arguments to the ones employed for the development of the guidance law (24) for the fully actuated CO, but in a way that respects the fact that now the CO is under-actuated, one can develop a guidance law that solves the steering problem (almost) globally for the two-dimensional case.

In particular, let $\nu(\theta) := (\nu^{[1]}(\theta), \nu^{[2]}(\theta))$, where $\nu^{[1]}(\theta) = \cos\theta$, $\nu^{[2]}(\theta) = \sin\theta$ and θ denotes the heading angle of the CO, that is, $\theta = \mathrm{atan}_2(\nu^{[2]}, \nu^{[1]})$, $\theta \in [-\pi, \pi]$; in addition, $\bar{\theta} = \mathrm{atan}_2(\bar{\nu}^{[2]}, \bar{\nu}^{[1]})$, where $\bar{\nu} := (\bar{\nu}^{[1]}, \bar{\nu}^{[2]})$. Note that $|\nu(\theta)| = 1$ for all $\theta \in [-\pi, \pi]$. The input of the CO in this case is the rate of change of the heading angle, that is, $u = \dot{\theta}$. Then, the equations of motion of the CO in the two-dimensional plane can be written as follows

$$\dot{\xi} = \nu(\theta) + \omega(t, \xi) + \Delta\omega(t, \xi), \qquad \dot{\nu} = \alpha(\theta, u), \qquad\qquad \xi(0) = \bar{\xi}, \quad \nu(0) = \bar{\nu}, \tag{40}$$

where the acceleration $\alpha(\theta, u)$ is constrained to satisfy the following equation:

$$\alpha(\theta, u) = (-\dot{\theta}\sin\theta, \dot{\theta}\cos\theta) = u(-\sin\theta, \cos\theta).$$

Note that $\alpha(\theta, u)$ is always perpendicular to the air velocity vector $\nu(\theta)$.

Now, let $\xi = (\xi^{[1]}, \xi^{[2]})$ and $\omega(t, \xi) = (\omega^{[1]}(t, \xi), \omega^{[2]}(t, \xi))$. Then, the pure pursuit navigation law can be written as follows: $\nu_{\mathsf{PP}}(\xi) = (\cos\theta_{\mathsf{PP}}(\xi), \sin\theta_{\mathsf{PP}}(\xi))$, where $\theta_{\mathsf{PP}}(\xi) = \mathrm{atan}_2(-\xi^{[2]}, -\xi^{[1]})$, for all $\xi \in \mathbb{R}^2 \backslash \mathcal{T}_\epsilon^\star$. Note that, by definition, the function $\theta_{\mathsf{PP}}(\cdot)$ takes values on $[-\pi, \pi]$ and belongs to $\mathbf{C}^1(\mathbb{R}^2 \backslash \mathcal{T}_\epsilon^\star)$, for any $\epsilon > 0$. It is straightforward for one to show that

$$\nabla_\xi \theta_{\mathsf{PP}}(\xi) = 1/|\xi|^2(-\xi^{[2]}, \xi^{[1]}), \qquad \text{for all} \quad \xi \in \mathbb{R}^2 \backslash \mathcal{T}_\epsilon^\star. \tag{41}$$

In light of Assumption 1, it is also easy to show that

$$\langle \nabla_\xi \theta_{\mathsf{PP}}(\xi), \Delta\omega(t, \xi) \rangle \leq \beta. \tag{42}$$

Let one consider the following feedback control law

$$u_{\mathsf{PP}}(\xi, \theta; \omega) = u^\alpha_{\mathsf{PP}}(\xi, \theta; \omega) + u^\beta_{\mathsf{PP}}(\xi, \theta) + u^\gamma_{\mathsf{PP}}(\xi, \theta), \tag{43}$$

where

$$\begin{aligned}
u^\alpha_{\mathsf{PP}}(\xi, \theta; \omega) &:= \langle \nabla_\xi \theta_{\mathsf{PP}}(\xi), \nu(\theta) + \omega(t, \xi) \rangle, \\
u^\beta_{\mathsf{PP}}(\xi, \theta) &:= \begin{cases} -\dfrac{1 - \cos(\theta - \theta_{\mathsf{PP}}(\xi))}{\mu \sin(\theta - \theta_{\mathsf{PP}}(\xi))}, & \text{if} \ \ |\theta - \theta_{\mathsf{PP}}(\xi)| \in (0, \pi) \cup (\pi, 2\pi), \ \ \xi \in \mathbb{R}^2 \backslash \mathcal{T}_\epsilon^\star, \\ 0, & \text{if} \ \ \theta = \theta_{\mathsf{PP}}(\xi), \ \ \xi \in \mathbb{R}^2 \backslash \mathcal{T}_\epsilon^\star, \end{cases} \\
u^\gamma_{\mathsf{PP}}(\xi, \theta) &:= -\frac{k}{\mu} \sin(\theta - \theta_{\mathsf{PP}}(\xi)).
\end{aligned} \tag{44}$$

Note that, for every $\xi \in \mathbb{R}^2 \backslash \mathcal{T}_\epsilon^\star$, $u_{\mathsf{PP}}^\beta(\xi,\cdot)$ is continuous at $\theta = \theta_{\mathsf{PP}}(\xi)$, for any $\epsilon > 0$. To see this, it suffices to observe that $\lim_{\vartheta \to 0}(1-\cos\vartheta)/\sin\vartheta = 0$ by application of L'Hopital's rule. Note that $u_{\mathsf{PP}}^\beta(\xi,\theta)$ is not well defined when $|\theta - \theta_{\mathsf{PP}}(\xi)| = \pi$ (singular direction). The modification of the proposed control law such that this singular direction is always avoided is beyond the scopes of this paper. Let one consider the candidate (generalized) Lyapunov function

$$V_{\mathsf{PP}}(\xi,\theta) = \mathcal{V}_{\mathsf{PP}}(\xi) + \tfrac{\mu}{2}(\nu(\theta) - \nu_{\mathsf{PP}}(\xi))^2 = \mathcal{V}_{\mathsf{PP}}(\xi) + \mu(1 - \langle \nu(\theta), \nu_{\mathsf{PP}}(\xi)\rangle) \tag{45}$$

$$= \mathcal{V}_{\mathsf{PP}}(\xi) + \mu(1 - \cos(\theta - \theta_{\mathsf{PP}}(\xi)). \tag{46}$$

Moreover, $u_{\mathsf{PP}}^\beta(\xi,\theta)$ can be written alternatively as follows

$$u_{\mathsf{PP}}^\beta(\xi,\theta) = -\frac{1}{\mu\sin(\theta - \theta_{\mathsf{PP}}(\xi))}\langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu(\theta) - \nu_{\mathsf{PP}}(\xi)\rangle. \tag{47}$$

Then, provided that $|\theta - \theta_{\mathsf{PP}}(\xi)| \neq \pi$, the time derivative of $V_{\mathsf{PP}}(\xi,\theta)$ along the trajectories of the system (40) after closing the loop with the control law (43) is given by

$$\begin{aligned}
\dot{V}_{\mathsf{PP}}(\xi,\theta) &= \langle \nabla_\xi V_{\mathsf{PP}}(\xi,\theta), \dot{\xi}\rangle + \langle \nabla_\theta V_{\mathsf{PP}}(\xi,\theta), \dot{\theta}\rangle \\
&= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \dot{\xi}\rangle + \mu\sin(\theta - \theta_{\mathsf{PP}}(\xi))(u_{\mathsf{PP}}(\xi,\theta;\omega) - \langle \nabla_\xi \theta_{\mathsf{PP}}(\xi), \dot{\xi}\rangle) \\
&= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu(\theta) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle \\
&\quad + \mu\sin(\theta - \theta_{\mathsf{PP}}(\xi))(u_{\mathsf{PP}}(\xi,\theta;\omega) - \langle \nabla_\xi \theta_{\mathsf{PP}}(\xi), \nu(\theta) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle) \\
&= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu_{\mathsf{PP}}(\xi) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle + \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu(\theta) - \nu_{\mathsf{PP}}(\xi)\rangle \\
&\quad + \mu\sin(\theta - \theta_{\mathsf{PP}}(\xi))(u_{\mathsf{PP}}(\xi,\theta;\omega) - \langle \nabla_\xi \theta_{\mathsf{PP}}(\xi), \nu(\theta) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle) \\
&= \langle \nabla_\xi \mathcal{V}_{\mathsf{PP}}(\xi), \nu_{\mathsf{PP}}(\xi) + \omega(t,\xi) + \Delta\omega(t,\xi)\rangle \\
&\quad - \mu\sin(\theta - \theta_{\mathsf{PP}}(\xi))\langle \nabla_\xi \theta_{\mathsf{PP}}(\xi), \Delta\omega(t,\xi)\rangle - k\sin^2(\theta - \theta_{\mathsf{PP}}(\xi)) \\
&\leq -c_{\mathsf{PP}} + \mu\beta|\sin(\theta - \theta_{\mathsf{PP}}(\xi))| - k\sin^2(\theta - \theta_{\mathsf{PP}}(\xi)) \\
&\leq -c_{\mathsf{PP}} + \tfrac{\mu\beta}{2}(1 + \sin^2(\theta - \theta_{\mathsf{PP}}(\xi))) - k\sin^2(\theta - \theta_{\mathsf{PP}}(\xi)) \\
&\leq -c_{\mathsf{PP}} + \tfrac{\mu\beta}{2} - \left(k - \tfrac{\mu\beta}{2}\right)\sin^2(\theta - \theta_{\mathsf{PP}}(\xi)),
\end{aligned} \tag{48}$$

where (42), (43), (47), Proposition 1, and the fact that $2a \leq 1 + a^2$, $a \in \mathbb{R}$, were employed. By taking $0 < \mu < 2c_{\mathsf{PP}}/\beta$ and $k > \mu\beta/2$, it follows that

$$\dot{V}_{\mathsf{PP}}(\xi,\theta) \leq -c_{\mathsf{PP}} + \frac{\mu\beta}{2} < 0.$$

Therefore, for every $\epsilon > 0$, the CO whose motion is described by (40), when driven by the feedback guidance law (43), will reach the target set $\mathcal{T}_\epsilon$ from any initial condition $(\bar{\xi}, \bar{\nu}(\bar{\theta})) \in \mathbb{R}^4 \backslash \mathcal{T}_\epsilon$ in finite time t_{f}, provided that $|\theta(t) - \theta_{\mathsf{PP}}(\xi(t))| \neq \pi$, for all $t \in [0, t_{\mathsf{f}}(\epsilon)]$. In addition, $t_{\mathsf{f}}(\epsilon) \leq V_{\mathsf{PP}}(\bar{\xi},\bar{\theta})/(c_{\mathsf{PP}} - \mu\beta/2)$, for all $\epsilon > 0$.

V. Numerical Simulations

In this section, numerical simulations, which better illustrate the analysis developed so far, are presented. The simulations are based on a spatially varying velocity field from real wind data taken from the file `wind.mat`, which can be found in `MATLAB` [34]. The magnitude of the wind velocities from these data is properly scaled for simulation purposes. The quiver plot of the (normalized) wind velocity along with the contours of the (normalized) wind speed are illustrated in Fig. 3.

In addition, it is assumed that the measurement error $\Delta\omega(t,\xi)$, which is treated as deterministic noise, corresponds to 20% of the actual wind velocity, such that the actual wind is 20% stronger than the measured one; consequently, the guidance laws will account for a weaker wind than the actual one. Note that for these simulations, it will not be assumed that the measurement error behaves as an admissible uncertainty; consequently, $\Delta\omega(t,\xi)$ does not necessarily decrease as the CO approaches the origin, in contradistinction with Assumption 1. In this way, the proposed guidance laws will be

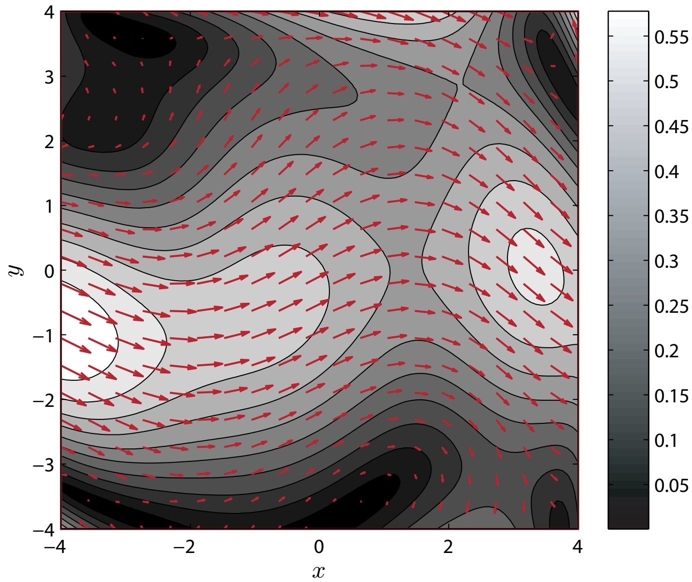
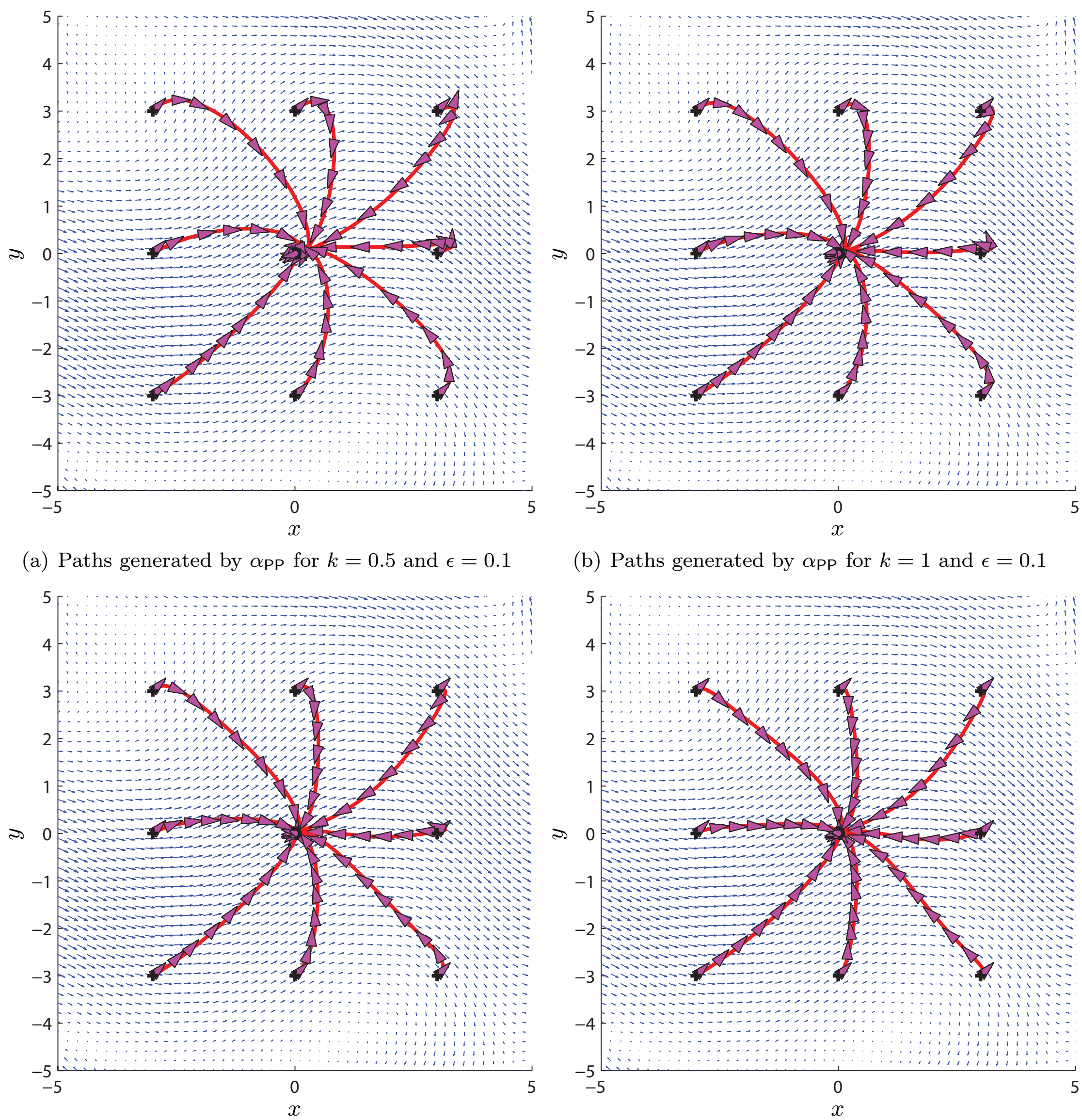


Fig. 3 The quiver plot of the (normalized) wind velocity along with the contours of the (normalized) wind speed based on real wind data.

tested in more challenging scenarios than those that are in accordance with Assumption 1. Figure 4 and Figures 5, 6 illustrate the trajectories of, respectively, the fully actuated CO (Eq. (14)) driven by the guidance law $\alpha_{\mathsf{PP}}(\xi, \nu; \omega)$ (Eq. (24)) and the under-actuated CO (Eqs. (15) and (40)) driven by the guidance laws $\varpi_{\mathsf{PP}}(\xi, \nu; \omega)$ (Eq. (38)) and $u_{\mathsf{PP}}(\xi, \theta; \omega)$ (Eq. (43)), for $\mu = 1$ and $\epsilon = 0.1$ and different values of the gain k. It is assumed that the CO starts from eight different locations along the perimeter of a square centered at the origin with initial airspeed $|\bar{\nu}| = 1\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}}$, whereas the direction of its air velocity measured from the x-axis is ${}^{\pi}\!/\!{}_{4}\,\mathrm{rad}$. One observes that, in all cases, the CO converges successfully to the target set in finite time while effectively accounting for the presence of the spatiotemporal wind as well as the significant measurement errors. Therefore, all the proposed guidance laws enjoy desirable robustness properties even when some of the technical conditions given in Assumption 1 do not hold.

As is illustrated in Figure 4, for larger values of the gain k, the fully actuated CO driven by the guidance law $\alpha_{\mathsf{PP}}(\xi, \nu; \omega)$ traverses paths that are "parallel" to the initial LOS direction, after a transient phase. During this transient phase, the CO appropriately corrects the error between the directions of its air velocity ν and ν_{PP} (LOS direction). On the other hand, when the gain k is small, $|\alpha_{\mathsf{PP}}(\xi, \nu; \omega)|$ also remains small along the ensuing path of the CO and consequently, the transient phase lasts longer and the turns that the CO makes are less sharp.

Similar observations can be made for the trajectories of the under-actuated CO driven by $\varpi_{\mathsf{PP}}(\xi, \nu; \omega)$ and $u_{\mathsf{PP}}(\xi, \theta; \omega)$, which are illustrated, respectively, in Figures 5 and 6. Due to the additional constraint that $|\nu(t)| \equiv 1$, the paths that the under-actuated CO traverses in this case consist of wider turns than the fully actuated CO driven by $\alpha_{\mathsf{PP}}(\xi, \nu; \omega)$ for the same values of the ratio k/μ. The most interesting case is when the CO starts from the point with coordinates $(3, 3)$ in the $x - y$ plane, in which case, the vectors $\bar{\nu}$ and $\nu_{\mathsf{PP}}(\bar{\xi})$ are initially opposite. One observes that both the guidance laws $\varpi_{\mathsf{PP}}(\xi, \nu; \omega)$ and $u_{\mathsf{PP}}(\xi, \theta; \omega)$ successfully drive the under-actuated CO to its target set, although the ensuing paths in the two cases differ significantly. Specifically, in the case when the CO is driven by $\varpi_{\mathsf{PP}}(\xi, \nu; \omega)$, one observes a noticeable transient phase at the beginning of the course of the CO, during which the CO moves away from the latter, while rotating its velocity vector to aim at the target set. By selecting larger values for the gain k, this transient phase becomes shorter. The transient phase is much shorter in the case when the CO is driven by the guidance law $u_{\mathsf{PP}}(\xi, \theta; \omega)$. In general, the paths of the CO that is driven by the guidance law $u_{\mathsf{PP}}(\xi, \theta; \omega)$ are more intuitive than those that result from the application of $\varpi_{\mathsf{PP}}(\xi, \nu; \omega)$, especially for small values of k/μ. On the other hand, the trajectories of the under-actuated CO driven by either $\varpi_{\mathsf{PP}}(\xi, \nu; \omega)$ or $u_{\mathsf{PP}}(\xi, \theta; \omega)$ look similar to those of the fully actuated CO (Eq. (14)) driven by $\alpha_{\mathsf{PP}}(\xi, \nu; \omega)$, for sufficiently large values of k.



(a) Paths generated by α_{PP} for $k = 0.5$ and $\epsilon = 0.1$

(b) Paths generated by α_{PP} for $k = 1$ and $\epsilon = 0.1$

(c) Paths generated by α_{PP} for $k = 2$ and $\epsilon = 0.1$

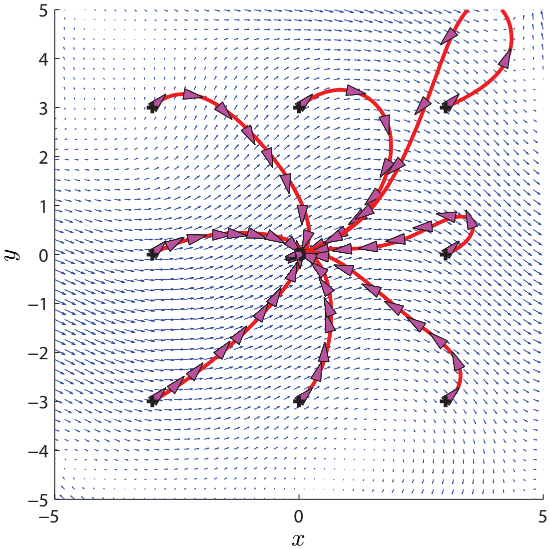
(d) Paths generated by α_{PP} for $k = 5$ and $\epsilon = 0.1$.

Fig. 4 Paths traversed by the fully actuated CO emanating from different initial states and driven by the guidance law $\alpha_{\mathsf{PP}}(\xi, \nu; \omega)$ in the presence of a spatially varying wind velocity field.

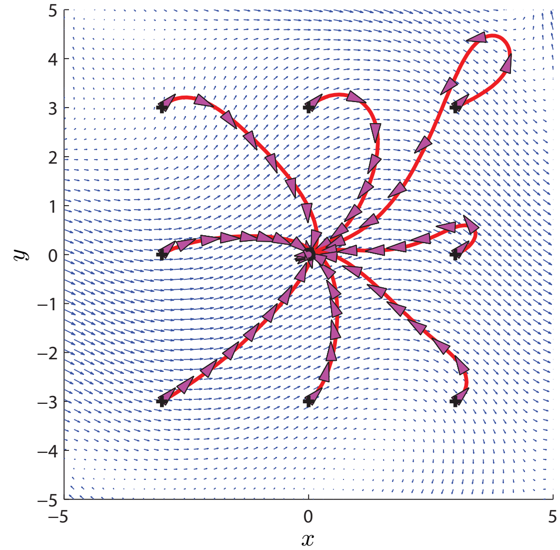
VI. Conclusions

In this work, feedback guidance laws that steer a controlled object to a given target set in the presence of uncertain wind, whose velocity varies both spatially and temporally, have been presented. The proposed approach was based on the use of a backstepping algorithm in vector form that has allowed the utilization of the solution techniques for the steering problem of a lower order kinematic model in the presence of uncertain wind, which were introduced in the author's prior work.

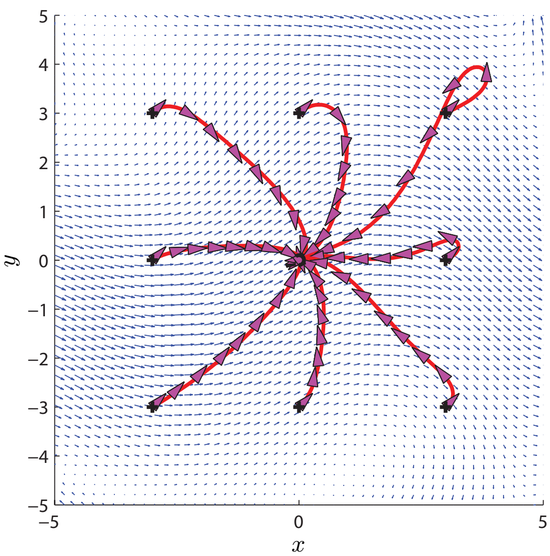
An interesting issue that has arisen in the analysis of the proposed guidance scheme had to do with the fact that the level of information about the wind required for the realization of the proposed control law had increased compared to what was needed for the characterization of the solution to the steering problem for the corresponding lower order kinematic model. In particular, the navigation law that successfully steers the lower order kinematic model to its target set does



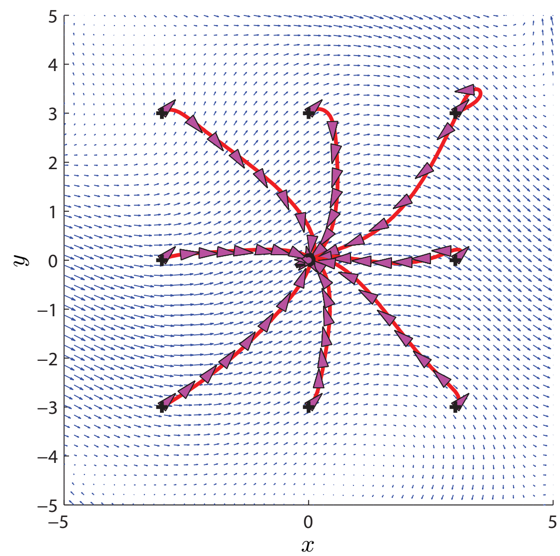
(a) Paths generated by ϖ_{PP} for $k = 0.5$ and $\epsilon = 0.1$



(b) Paths generated by ϖ_{PP} for $k = 1$ and $\epsilon = 0.1$



(c) Paths generated by ϖ_{PP} for $k = 2$ and $\epsilon = 0.1$

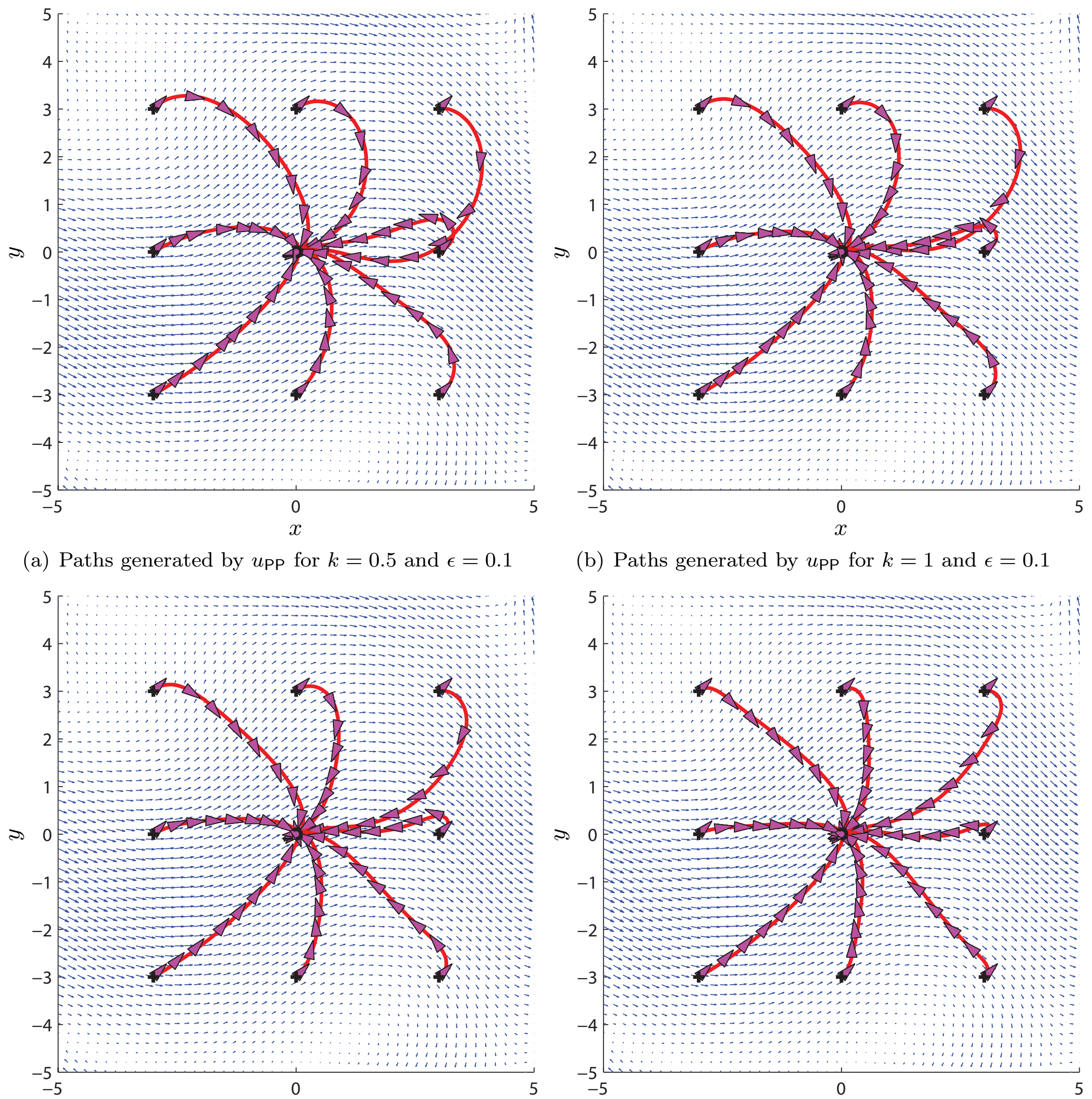


(d) Paths generated by ϖ_{PP} for $k = 5$ and $\epsilon = 0.1$.

Fig. 5 Paths traversed by the under-actuated CO emanating from different initial states and driven by the guidance law $\varpi_{\mathsf{PP}}(\xi, \nu; \omega)$ in the presence of a spatially varying wind velocity field.

not require any knowledge of the local wind. By contrast, the guidance law that results from this wind-independent navigation law via the application of the backstepping algorithm, requires explicit knowledge of the local wind. This observation rendered automatically some of the other navigation laws introduced in the author's previous work unsuitable for the problem treated in this work, given that their implementation requires knowledge of the wind's velocity. Consequently, the realization of their corresponding guidance laws, via backstepping algorithms similar to the ones presented in this work, would require knowledge of the gradient of the local wind velocity field, which is not always available in practice.

Another interesting issue was that the direct extension of the proposed approach for the steering of the fully actuated controlled object to the case when the latter is under-actuated cannot guarantee global convergence in finite time, in the three-dimensional case. It was shown, however, that by employing the backstepping control logic in a way that "respects" the kinematics of the under-actuated controlled object, one can design feedback control laws that solve (almost) globally the



(a) Paths generated by u_{PP} for $k = 0.5$ and $\epsilon = 0.1$

(b) Paths generated by u_{PP} for $k = 1$ and $\epsilon = 0.1$

(c) Paths generated by u_{PP} for $k = 2$ and $\epsilon = 0.1$

(d) Paths generated by u_{PP} for $k = 5$ and $\epsilon = 0.1$.

Fig. 6 Paths traversed by the under-actuated CO emanating from different initial states and driven by the guidance law $u_{\mathsf{PP}}(\xi, \theta; \omega)$ in the presence of a spatially varying wind velocity field.

finite-time guidance problem for the two-dimensional case.

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