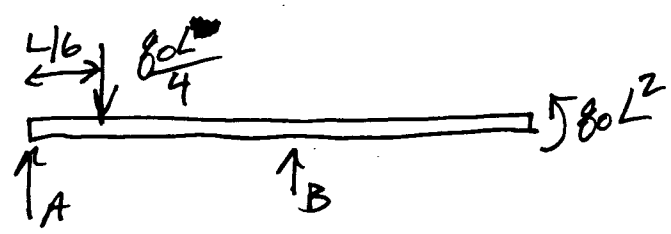
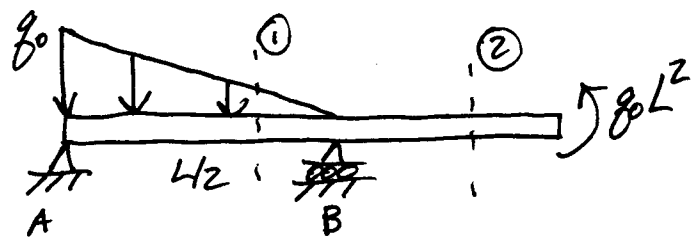


Supplement to Lecture on 11/6/2012

①



$$\sum M_z^A = q_0 L^2 + \frac{BL}{2} - \frac{q_0 L^2}{24} = 0 \rightarrow B = -\frac{23}{12} q_0 L$$

$$\sum F_y = A + B - \frac{q_0 L}{4} = 0 \rightarrow A = \frac{13}{6} q_0 L$$

① $0 < x < \frac{L}{2}$: $q(x) = q_0 - q_0 \frac{2x}{L}$

$$\frac{dV}{dx} = -q \rightarrow V(x) = q_0 \frac{x^2}{L} - q_0 x + C_1$$

$$V(x=0) = A = \frac{13}{6} q_0 L = C_1 \rightarrow V(x) = q_0 \frac{x^2}{L} - q_0 x + \frac{13}{6} q_0 L$$

$$\frac{dM}{dx} = V \rightarrow M(x) = \frac{1}{3} q_0 \frac{x^3}{L} - \frac{1}{2} q_0 x^2 + \frac{13}{6} q_0 L x + C_2$$

$$M(x=0) = 0 \rightarrow C_2 = 0$$

$$EI \frac{d\theta}{dx} = M \rightarrow EI \theta(x) = \frac{1}{12} q_0 \frac{x^4}{L} - \frac{1}{6} q_0 x^3 + \frac{13}{12} q_0 L x^2 + C_3$$

We do not know $\theta(x)$ anywhere in $0 < x < L/2$.

$\theta = \frac{dV}{dx}$
 Integrate again $\rightarrow EI v(x) = \frac{1}{60} q_0 \frac{x^5}{L} - \frac{1}{24} q_0 x^4 + \frac{13}{36} q_0 L x^3 + C_3 x + C_4$

(2)

$$v(x=0) = 0 \rightarrow c_4 = 0$$

$$v(x=L/2) = 0 \rightarrow \frac{1}{60 \cdot 32} f_0 L^4 - \frac{1}{24 \cdot 6} f_0 L^4 + \frac{13}{36 \cdot 8} f_0 L^4 + \frac{c_3 L}{2} = 0$$

$$\rightarrow c_3 = -0.08611 f_0 L^3 = -\frac{31}{360} f_0 L^3$$

$$(2) \quad \frac{L}{2} < x < L \rightarrow f(x) = 0$$

$$\frac{dV}{dx} = -f = 0 \rightarrow V(x) = d_1$$

$$V(x=L) = 0 \rightarrow d_1 = 0$$

$$\frac{dM}{dx} = V = 0 \rightarrow M(x) = d_2$$

$$M(x=L) = f_0 L^2 = d_2$$

$$M(x) = f_0 L$$

$$EI \frac{d\theta}{dx} = M \rightarrow EI \theta(x) = f_0 L^2 x + d_3$$

$$\text{Matching: } EI \theta(x=L/2^-) = EI \theta(x=L/2^+)$$

$$\frac{1}{12 \cdot 16} f_0 L^3 - \frac{1}{48} f_0 L^3 + \frac{13}{48} f_0 L^3 - \frac{31}{360} f_0 L^3 = \frac{1}{2} f_0 L^3 + d_3$$

$$\rightarrow d_3 = -\frac{953}{2880} f_0 L^3 = -0.331 f_0 L^3$$

$$EI \frac{dv}{dx} = EI \theta \rightarrow EI v(x) = \frac{1}{2} f_0 L^2 x^2 - \frac{953}{2880} f_0 L^3 x + d_4$$

Either use $EI v(x=L/2^+) = 0 \rightarrow \frac{f_0 L^4}{8} - \frac{953}{5760} f_0 L^4 + d_4 = 0$
 or matching: $EI v(x=L/2^-) = EI v(x=L/2^+)$

$$d_4 = \frac{233}{5760} f_0 L^4 = 0.04 f_0 L^4$$

EIV =

$$\text{If} \left[x < L/2, \frac{1}{60} q_0 \frac{x^5}{L} - \frac{1}{24} q_0 x^4 + \frac{13}{36} q_0 L x^3 - \frac{31}{360} q_0 L^3 x, \frac{1}{2} q_0 L^2 x^2 - \frac{953}{2880} q_0 L^3 x + \frac{233}{5760} q_0 L^4 \right];$$

EIθ = ∂_x EIV;

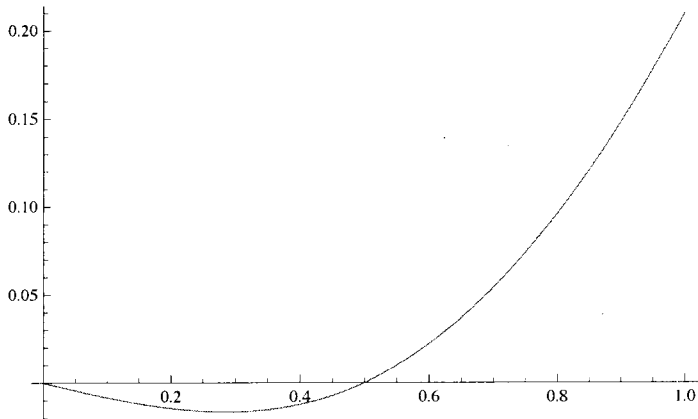
M = ∂_x EIθ;

V = ∂_x M;

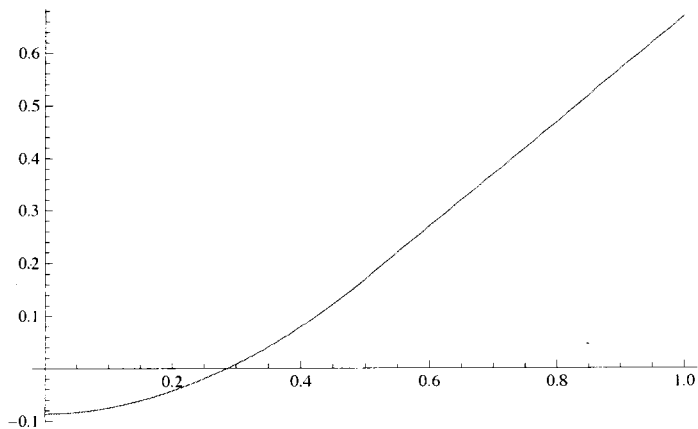
q0 = 1;

L = 1;

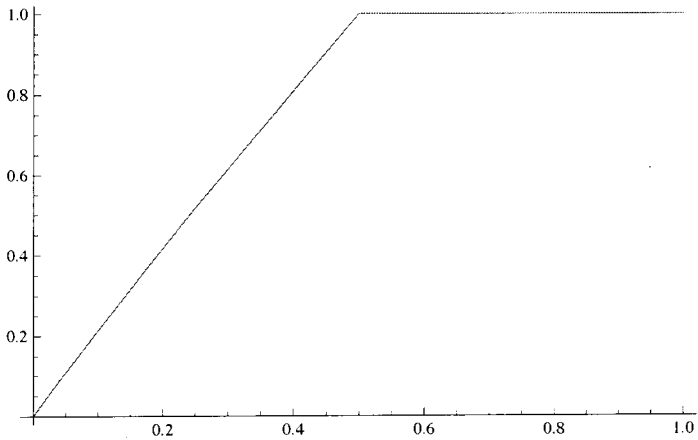
Plot[EIV, {x, 0, L}]



Plot[EIθ, {x, 0, L}]

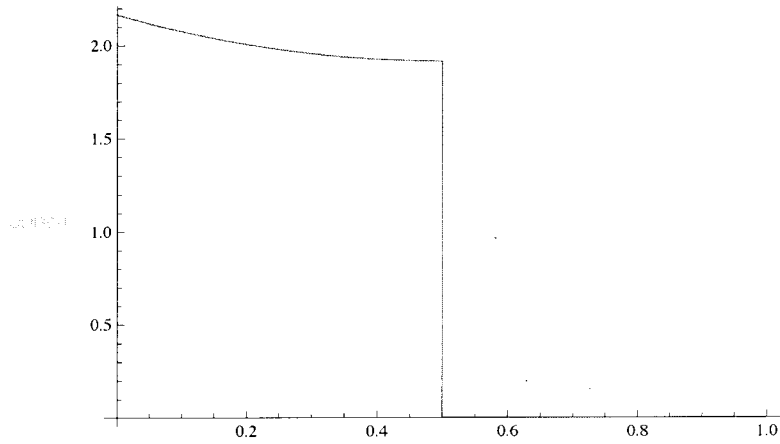


Plot[M, {x, 0, L}]



4

```
Plot[V, {x, 0, L}]
```



```
NSolve[EIθ == 0, x]
```

```
{{x → -0.275376}, {x → 0.287429}}
```

```
x = 0.28742893982308504;
```

```
EIv
```

```
-0.0164276
```