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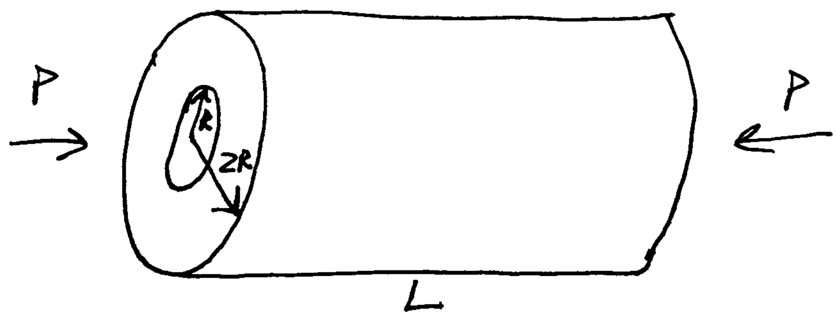
A tube of length L, inner radius R and outer radius $2R$ is compressed by an axial force P. The Young's modulus and Poisson's ratio of the material are E and ν.

a) Determine the change in volume of the tube material.

b) Determine the change in volume of the space inside the tube.



Assume that the load P is distributed uniformly over the area of the tube ends.

a) $$\varepsilon_a = \frac{\Delta L}{L} = \frac{\sigma}{E} = \frac{-P}{AE} = \frac{-P}{\pi(4R^2-R^2)E}$$

$$\rightarrow \Delta L = \frac{-PL}{3\pi R^2 E}$$

$$\varepsilon_t = \frac{R_i - R}{R} = \frac{R_o - 2R}{2R} = -\frac{\nu\sigma}{E} = \frac{+\nu P}{3\pi R^2 E}$$

$$\rightarrow R_i = R\left(1 + \frac{\nu P}{3\pi R^2 E}\right)$$

$$R_o = 2R\left(1 + \frac{\nu P}{3\pi R^2 E}\right)$$

$$\rightarrow \quad V_{material} = \pi (R_o^2 - R_i^2)(L + \Delta L)$$

$$V_{material} = 3\pi R^2 \Big(1 + \underbrace{\frac{\nu P}{3\pi R^2 E}}_{\varepsilon_t}\Big)^2 L \Big(1 - \underbrace{\frac{PL}{3\pi R^2 E}}_{\varepsilon_a}\Big)$$

Now, for structural materials prior to failure, strains are usually $\ll 1$, so we can expand the expression above as follows.

$$\begin{aligned} V_{material} &= 3\pi R^2 L\,(1+\varepsilon_t)^2(1+\varepsilon_a) \quad (^*\ \varepsilon_a < 0 \text{ for our problem}) \\ &= 3\pi R^2 L\,(1 + 2\varepsilon_t + \varepsilon_t^2)(1+\varepsilon_a) \\ &= 3\pi R^2 L\,(1 + 2\varepsilon_t + \varepsilon_a + \underbrace{\varepsilon_t^2 + 2\varepsilon_t\varepsilon_a + \varepsilon_t^2\varepsilon_a}_{\text{Higher order terms in } \varepsilon_t,\ \varepsilon_a}) \end{aligned}$$

$$V_{material} \approx \underbrace{3\pi R^2 L}_{V_o}\,(1 + 2\varepsilon_t + \varepsilon_a)$$

$$\rightarrow \boxed{\Delta V \approx V_o\,(2\varepsilon_t + \varepsilon_a) \text{ for the material}}$$

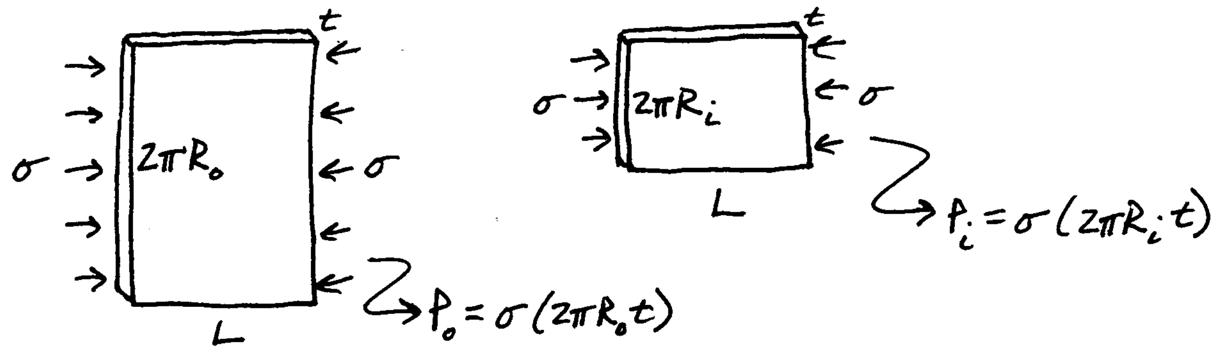
b) What about the void?

$$V_{void} = \pi R_i^2 (L + \Delta L) = \pi R^2 L \Big(1 + \underbrace{\frac{\nu P}{3\pi R^2 E}}_{\varepsilon_t}\Big)^2 \Big(1 - \underbrace{\frac{PL}{3\pi R^2 E}}_{\varepsilon_a}\Big)$$

So again $\quad \Delta V_{void} = \overbrace{V_{void}^o}^{\pi R^2 L} (2\varepsilon_t + \varepsilon_a)$

It looks as if the void has the same strains as the solid.

If you are still confused as to why the inner radius expands, think about the following. Take a thin layer of material off of the outer radius and one off of the inner radius and roll them out flat.



Note that if the stress is uniform in the tube then for the same thickness of these layers the net force will be different. So we are also cutting up the force when we cut these layers. Now let's compute the lateral strain in the circumference.

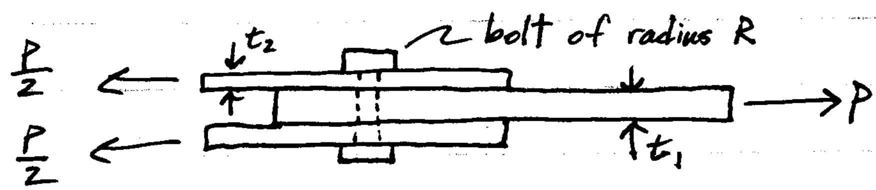
$$\varepsilon_t = \frac{-\nu\sigma}{E} = \frac{\Delta(2\pi R_o)}{2\pi R_o} = \frac{\Delta(2\pi R_i)}{2\pi R_i}$$

$$\frac{-\nu\sigma}{E} = \frac{\Delta R_o}{R_o} = \frac{\Delta R_i}{R_i}$$

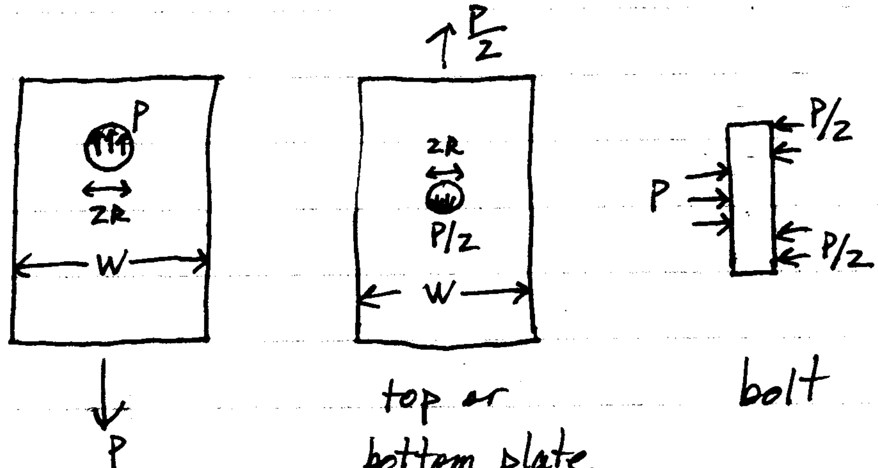
So if $\sigma < 0$ this says that both ΔR_o and ΔR_i are > 0.

Bearing and shear stresses in pins and bolts

We will study shear stress and shear strains in greater detail when we look at torsion. For now there are some relatively simple concepts that can be introduced with respect to fasteners. Consider the following lap joint configuration.



FBDs:



middle plate

top or bottom plate

bolt

Average bearing stress $= \dfrac{F_b}{A_b}$ where F_b is the bearing force being carried and A_b is the projected bearing area (i.e. the maximum area perpendicular to the force).

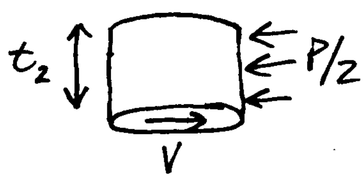
For the hole in the middle plate the bearing force is P and $A_b = 2Rt_1$, $\therefore\ \sigma_b = P/2Rt_1$.

For the top/bottom plates the bearing stress is $\sigma_b = \left(\frac{P}{2}\right)/2Rt_2$. Then for the bolt the bearing stresses will be equivalent to those in the plates but will change with position along the length of the bolt.

<u>Average shear stress</u> – First shear stress differs from normal stress in the sense that the force appearing in the numerator is tangential or parallel to the area in the denominator.

$$\tau_{average} = \frac{V}{A}, \quad V = \text{shear force}, \quad A = \text{area}$$

FBD of cut

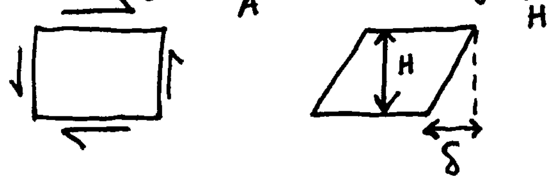


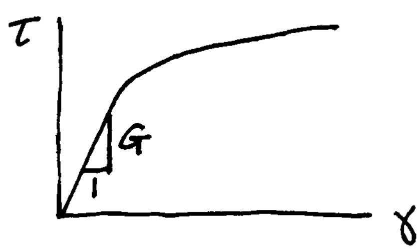
$$\Sigma F_x = -\frac{P}{2} + V = 0 \rightarrow V = P/2, \quad A = \pi R^2$$

$$\rightarrow \tau_{average} = \frac{P}{2\pi R^2}$$

<u>Shear stress vs. shear strain</u>

$\tau = \frac{V}{A}$ $\qquad$ $\gamma = \frac{\delta}{H}$





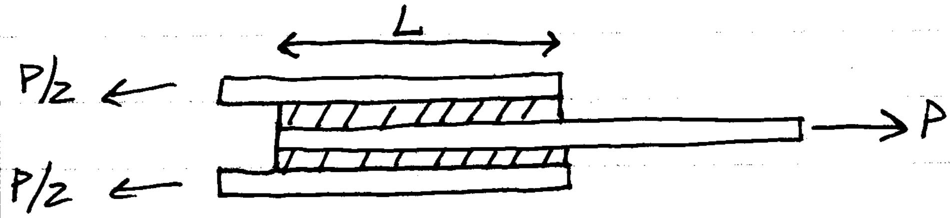
G = shear modulus

For isotropic materials $G = \frac{E}{2(1+\nu)}$

<u>Example</u> : Rubber pads are bonded to steel plates to make the flexible joint illustrated below. The shear strength of the joint bond is τ_{max}.

a) Using a factor of safety of 5, what is the maximum design load for this joint?

b) What is the relative displacement/deflection between the load points on the steel plates at the maximum design load? Assume the plates are rigid and the shear modulus of the rubber is G.

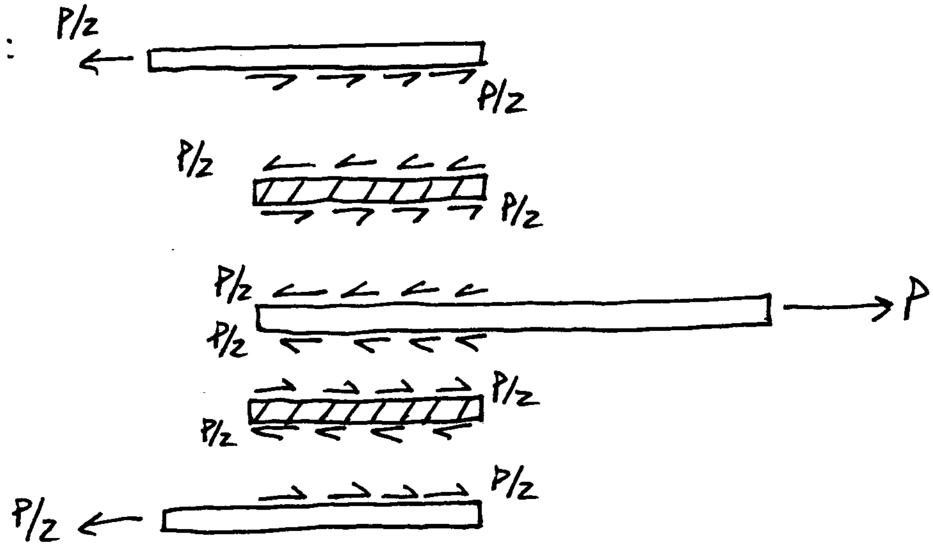


All thicknesses $= t$
All widths $= W$
Bond length $= L$

So what do we mean by "factor of safety"? Well, we want the average stress at our maximum design load to be the material strength divided by the factor of safety.

i.e. $\tau_{design} = \tau_{max} / SF$

FBDs:



$$\tau_{design} = \left(\frac{P}{2}\right)/WL = \tau_{max}/5$$

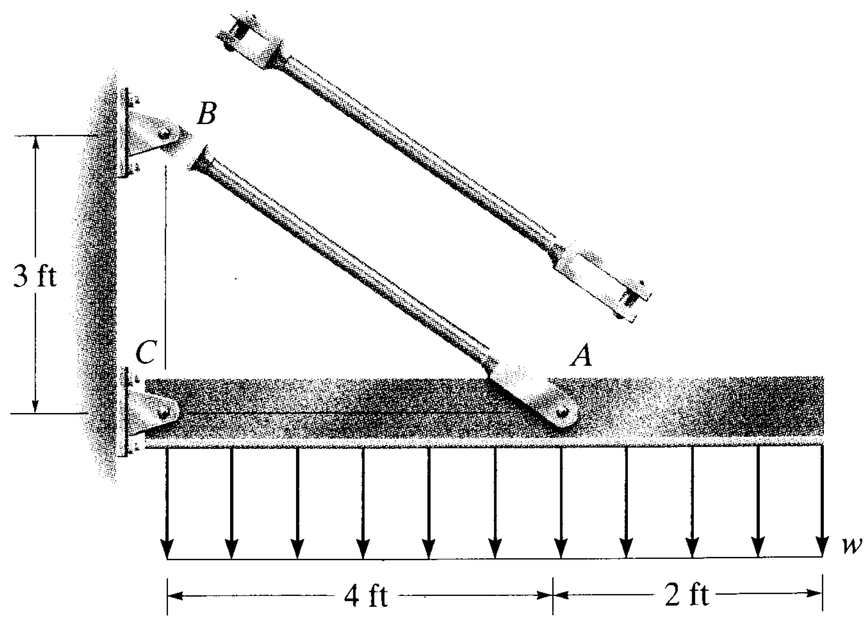
mechanics analysis $\tau = V/A$ — design criterion

a) $\rightarrow$ $\boxed{P_{max} = \frac{2}{5}\tau_{max} WL}$ ← Max design load

b) $\gamma_{average} = \tau_{average}/G = \tau_{max}/5G$

$$\gamma_{average} = \frac{\delta}{t} \rightarrow \boxed{\delta = \frac{\tau_{max}}{5G} t}$$

Each rubber pad/joint has this deflection.

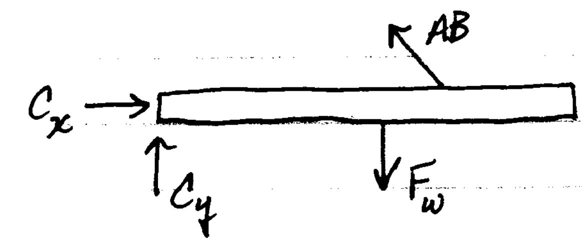


Determine the maximum magnitude of the distributed load w such that the allowable shear stress $\tau_{allow} = 13.5$ ksi is not exceeded in the bolts and the allowable tensile stress $\sigma_{allow} = 22$ ksi is not exceeded in the rod.

Bolt diameter = 0.40 in., Rod diameter = 0.50 in.

First, let's use statics to determine the force in the rod.

FBD:



$F_w = 6w$ (units of the 6 are ft)

$$\Sigma M_z^c = -\underbrace{6w}_{F_w}(3) + \underbrace{\tfrac{3}{5}AB}_{AB_y}(4) = 0 \rightarrow AB = 7.5w \text{ (ft)}$$

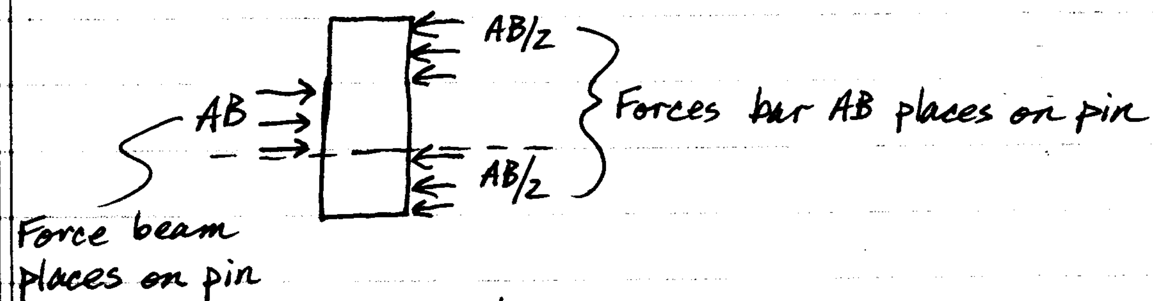
Now let's find the stress in the rod.

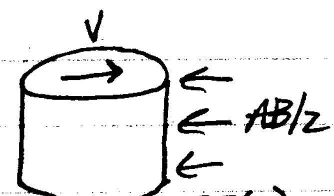
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{7.5\,w\,(ft)\,12\,\frac{in}{ft}}{\pi\,\frac{(0.5)^2\,in^2}{4}} = 458.4\,w\,\frac{1}{in}$$

For $\sigma_{AB} \leq \sigma_{Allow} \rightarrow 458.4\,w\,\frac{1}{in} \leq 22\,\frac{kips}{in^2}$

$$\rightarrow w \leq 0.048\,\frac{kips}{in} = 48\,\frac{lbs}{in}$$

How about the shear stresses in the pins?





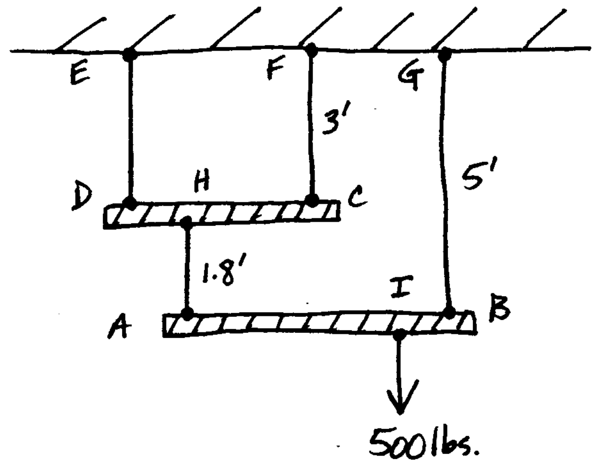
Equilibrium $\rightarrow V = \frac{AB}{2}$

$$\tau_{average} = \frac{V}{A} = \frac{\overset{7.5(12)}{90}\ [redacted]\ w\ in}{2\left[\pi\,\frac{(0.4)^2\,in^2}{4}\right]} = 358.1\,w\,\frac{1}{in}$$

$\tau_{average} \leq \tau_{allow} \rightarrow 358.1\,w\,\frac{1}{in} \leq 13.5\,\frac{kips}{in^2}$

$$\rightarrow w \leq 0.038\,\frac{kips}{in} = 38\,\frac{lbs}{in}$$

$$\rightarrow \boxed{w_{max} = 38\ lbs/in}$$

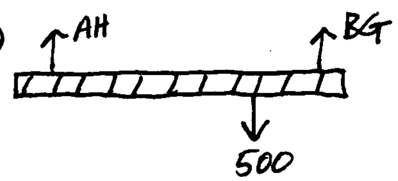


Steel wires $E = 30{,}000$ ksi connected to rigid beams. Determine the deflection of the 500 lb. load.

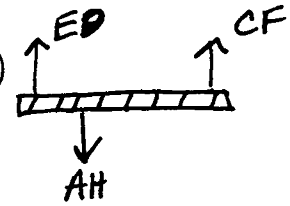
$A_{wires} = 0.025\ in^2$

$DH = 1'$, $HC = 2'$, $AI = 3'$, $IB = 1'$

FBDs: ①



②



① $\Sigma M_z^A = BG(4) - 500(3) = 0 \rightarrow BG = 375$ lbs.

$\Sigma F_y = AH + BG - 500 = 0 \rightarrow AH = 125$ lbs.

② $\Sigma M_z^D = CF(3) - AH(1) = 0 \rightarrow CF = \frac{125}{3}$ lbs.

$\Sigma F_y = CF + ED - AH = 0 \rightarrow ED = \frac{250}{3}$ lbs.

For each wire we have $\sigma = E\varepsilon \rightarrow \frac{P}{A} = E\frac{\delta}{L}$

$$\rightarrow P = \frac{EA}{L}\delta \text{ or } \delta = \frac{PL}{EA}$$

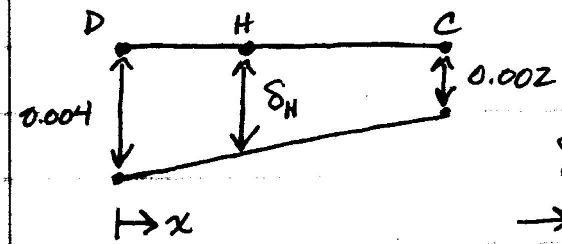
$$\rightarrow \delta_{BG} = \frac{375\,(\overbrace{60}^{5\cdot 12})}{30\times10^6\,(0.025)} = 0.03 \text{ in}$$

$$\delta_{AH} = \frac{125\,(1.8)12}{30\times10^6(0.025)} = 0.0036 \text{ in}$$

$$\delta_{CF} = \frac{\frac{125}{3}\,(3)12}{30\times10^6(0.025)} = 0.002 \text{ in}$$

$$\delta_{ED} = \frac{250/3\,(3)12}{30\times10^6\,(0.025)} = 0.004 \text{ in}$$

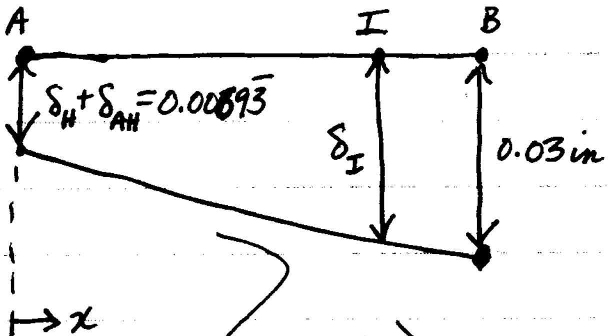
Next, let's determine how these deflections translate into the deflections of points on the beams.



note this is $\frac{\text{in}}{\text{ft}}$

$$\delta(x) = 0.004'' - \frac{0.002''}{3'}x$$

$$\rightarrow \delta_H = 0.004'' - \frac{0.002''}{3'}(1') = 0.00333 \text{ in}$$



$$\delta(x) = 0.00693'' + \frac{(0.03 - 0.00693)''}{4'}x$$

$$\rightarrow \delta_I = 0.00693'' + \frac{(0.03-0.00693)''}{4'}(3') = 0.0242''$$

Angle of tilt of beam DC $= \frac{0.002''}{36''} \cdot \frac{180°}{\pi} = 0.0032°$

Angle of tilt of beam AB $= \frac{(0.03-0.00693)''}{48''} \cdot \frac{180°}{\pi} = 0.0275°$

uses $\theta \approx \tan\theta$ for small angles