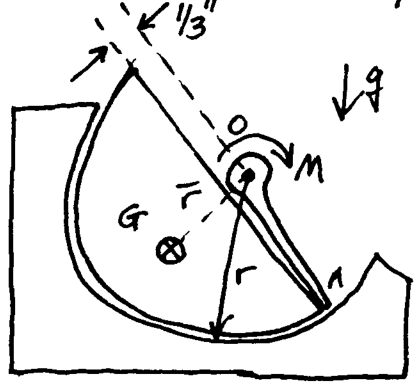
Example: Ice tray

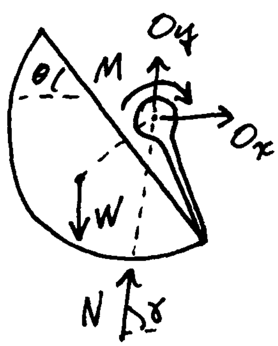


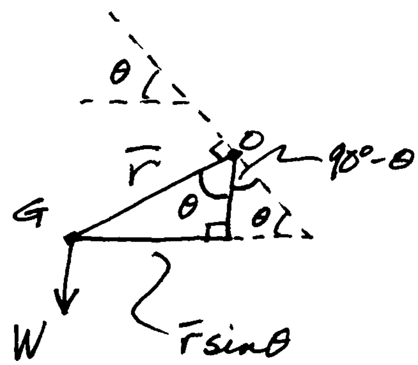
Arm OA ejects the ice cubes from the tray. There are 8 "cubes" with a total combined weight of 0.56 lb. Neglecting friction, and assuming that the resultant of the normal force of the tray on the cubes acts through O, determine the total moment M on all 8 cubes as a function of θ. (neglect weight of arm)

$r = 1.5''$

$\bar{r} = 0.55\,r$

FBD ("cube" – arm system)





This FBD has 5 unknowns, O_x, O_y, M, N and γ. In general we would not be able to solve for any of these quantities from an equilibrium analysis of this FBD alone. However, since O_x, O_y and N act through O, taking moments about O will yield a relationship between M and W.

$$\Sigma M_z^O = W\bar{r}\sin\theta - M = 0 \rightarrow M = W\bar{r}\sin\theta$$

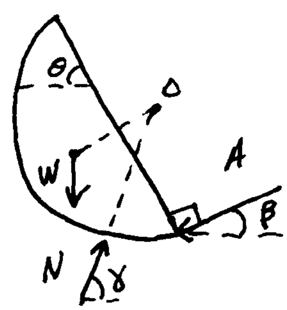
$$M = 0.56(1.5 \cdot 0.55)\sin\theta = 0.462\sin\theta$$

If we do want N, γ, O_x and O_y, $\Sigma F_x = 0$ and $\Sigma F_y = 0$ do not provide enough equations to solve for all of these quantities.

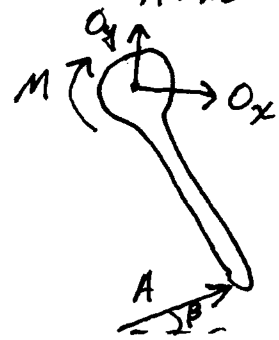
However, we can break the system into components to determine if we can find additional equations without introducing too many more unknowns.

Component FBDs

"Cube"



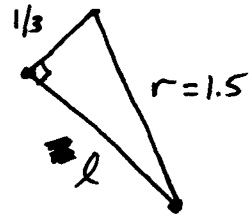
Arm



$$\beta = 90° - \theta$$

Now we could get A, N and γ from the equilibrium analysis of the "cube" and we could then determine O_x, O_y and M from the equilibrium analysis of the arm.

"Cube" equilibrium:



$$\ell = \sqrt{1.5^2 - (\tfrac{1}{3})^2} = 1.462''$$

$$\Sigma M_z^o = W\bar{r}\sin\theta - A\ell = 0$$

$$\rightarrow A = \frac{W\bar{r}\sin\theta}{\ell} = 0.316\sin\theta$$

$$\Sigma F_x = N\cos\gamma - A\underbrace{\cos\beta}_{=\sin\theta} = 0$$

$$\rightarrow N\cos\gamma = A\sin\theta = 0.316\sin^2\theta$$

$$\Sigma F_y = N\sin\gamma - A\underbrace{\sin\beta}_{=\cos\theta} - W = 0$$

$$\rightarrow N\sin\gamma = W + A\cos\theta = 0.56 + 0.316\sin\theta\cos\theta$$

$$\rightarrow N = \sqrt{(W + A\cos\theta)^2 + (A\sin\theta)^2}$$

$$\gamma = \arctan\left(\frac{W + A\cos\theta}{A\sin\theta}\right)$$

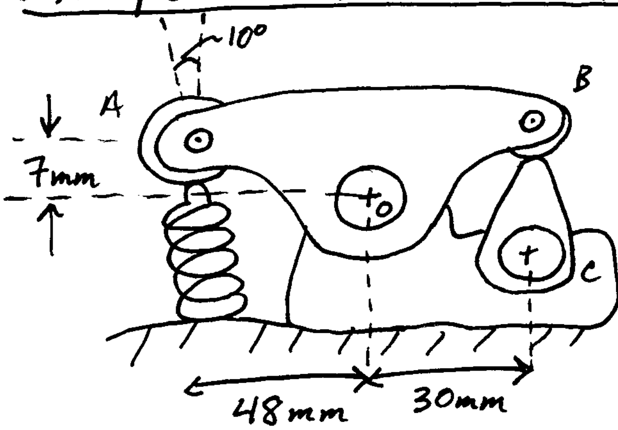
Arm Equilibrium: $\Sigma M_z^o = A\ell - M = 0$

$$\rightarrow M = W\bar{r}\sin\theta = 0.462\sin\theta \quad (\checkmark \text{ same as before})$$

$$\Sigma F_x = O_x + A\overbrace{\sin\theta}^{\cos\beta} = 0 \rightarrow O_x = -A\sin\theta$$

$$\Sigma F_y = O_y + A\overbrace{\cos\theta}^{\sin\beta} = 0 \rightarrow O_y = -A\cos\theta$$

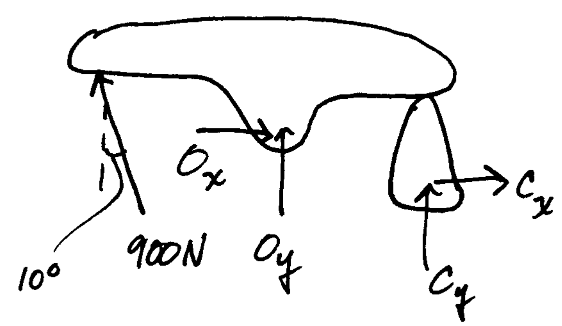
Example: Cam - Rocker - Valve



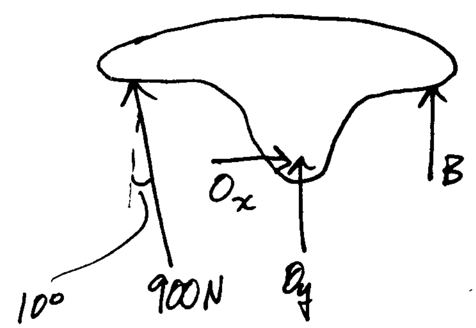
Spring force is 900 N. Determine the reaction forces at O and C.

FBDs

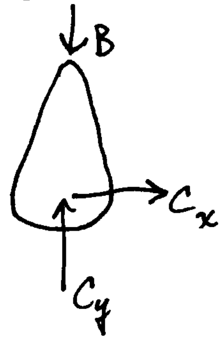
Rocker + Cam



Rocker



Cam



Equilibrium analysis: Rocker only

$$\Sigma M_z^O = B(30) - 900\cos 10^\circ (48) + 900 \sin 10^\circ (7) = 0$$

$$\rightarrow \quad B = 1382\ N$$

$$\sum F_x = -900 \sin 10^\circ + O_x = 0$$
$$\rightarrow O_x = 156\,N$$

$$\sum F_y = 900 \cos 10^\circ + B + O_y = 0$$
$$\rightarrow O_y = {}^{-}2268\,N$$

$$O = \sqrt{156^2 + 2268^2} = 2273\,N$$

<u>Cam</u>: $\sum F_x = 0 \rightarrow C_x = 0$

$$\sum F_y = C_y - B = 0 \rightarrow C_y = B = 1382\,N$$

<u>Cam-Rocker System</u>: $\sum F_x = -900 \sin 10^\circ + O_x + C_x = 0$

Same as $\sum F_x^{cam} + \sum F_x^{rocker} = 0$

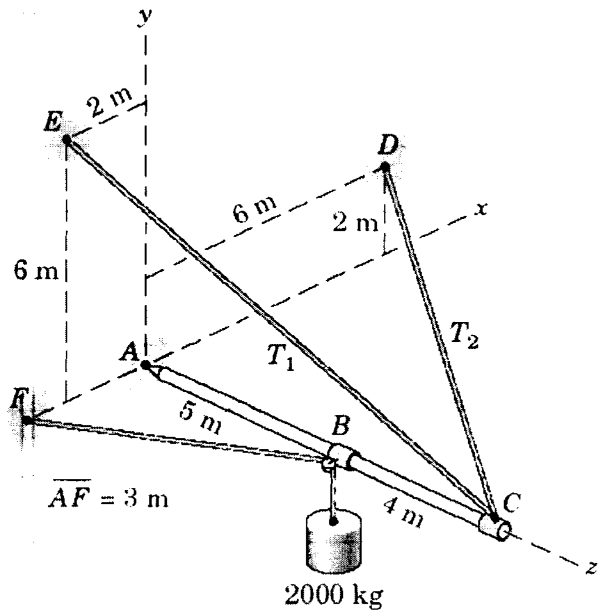
$$\sum F_y = 900 \cos 10^\circ + O_y + C_y = 0$$

Same as $\sum F_y^{cam} + \sum F_y^{rocker} = 0$

$$\sum M_z^o = -900 \cos 10^\circ (48) + 900 \sin 10^\circ (7) + C_y (30) + C_x d_\perp = 0$$

Same as $\sum M_z^{o,cam} + \sum M_z^{o,rocker} = 0$

Considering the equilibrium equations for the entire system <u>always</u> give 3 redundant equations (i.e. not new) if the equilibrium equations for each individual component have already been determined.



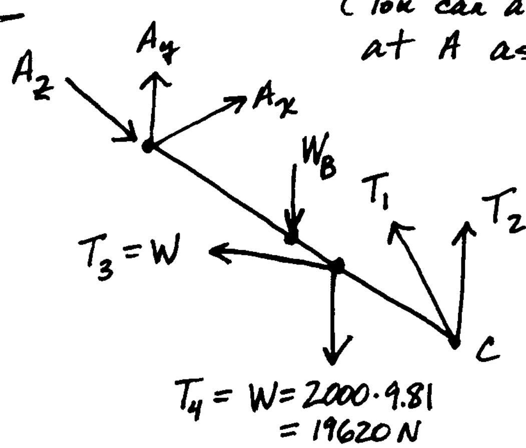
The 9-m steel boom has a mass of 600 kg with center of mass at midlength. It is supported by a ball and socket joint at A and the two cables under tensions T_1 and T_2.

a) calculate the tensions T_1 and T_2 and the reactions at A.

b) Determine a "quick" method of solution if only the tension T_1 is asked for.

a) FBD

(You can also represent the reaction at A as $\vec{A}$)



$$\vec{W}_B = -9.81(600)\vec{j} = -5886\vec{j}$$

Vector representations: $\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$

(All numerical values of forces are in Newtons)

$$\vec{T}_4 = -19620\vec{j}$$

$$\vec{T}_3 = 19620\,\frac{-3\vec{i} - 5\vec{k}}{\sqrt{3^2+5^2}} = -10094\vec{i} - 16824\vec{k}$$

$$\vec{T}_1 = T_1 \frac{-2\vec{i}+6\vec{j}-9\vec{k}}{\sqrt{2^2+6^2+9^2}} = T_1\left(-\frac{2}{11}\vec{i}+\frac{6}{11}\vec{j}-\frac{9}{11}\vec{k}\right)$$

$$\vec{T}_2 = T_2 \frac{6\vec{i}+2\vec{j}-9\vec{k}}{\sqrt{2^2+6^2+9^2}} = T_2\left(\frac{6}{11}\vec{i}+\frac{2}{11}\vec{j}-\frac{9}{11}\vec{k}\right)$$

We will be taking moments about A, so we need the moments of each of these forces about A.

$$\vec{M}_1^A = \vec{r}_{AC} \times \vec{T}_1 = 9\vec{k} \times T_1\left(-\frac{2}{11}\vec{i}+\frac{6}{11}\vec{j}-\frac{9}{11}\vec{k}\right)$$
$$= -\frac{18}{11}T_1 \underbrace{\vec{k}\times\vec{i}}_{\vec{j}} + \frac{54}{11}T_1 \underbrace{\vec{k}\times\vec{j}}_{-\vec{i}}$$
$$= -\frac{54}{11}T_1\vec{i} - \frac{18}{11}T_1\vec{j}$$

$$\vec{M}_2^A = \vec{r}_{AC}\times\vec{T}_2 = 9\vec{k}\times T_2\left(\frac{6}{11}\vec{i}+\frac{2}{11}\vec{j}-\frac{9}{11}\vec{k}\right)$$
$$= \frac{54}{11}T_2\,\vec{k}\times\vec{i} + \frac{18}{11}T_2\,\vec{k}\times\vec{j}$$
$$= -\frac{18}{11}T_2\vec{i} + \frac{54}{11}T_2\vec{j}$$

$$\vec{M}_{34}^A = \vec{r}_{AB}\times(\vec{T}_3+\vec{T}_4) = 5\vec{k}\times(-10094\vec{i}-19620\vec{j}-16824\vec{k})$$
$$= -5(10094)\vec{k}\times\vec{i} - 5(19620)\vec{k}\times\vec{j}$$
$$= 98100\vec{i} - 50470\vec{j}$$

$$\vec{M}_{W_B}^A = \vec{r}_{cm}\times\vec{W}_B = 4.5\vec{k}\times(-5886\vec{j}) = 26487\vec{i}$$

<u>Equilibrium</u>

$$\Sigma \vec{M}_z^A = -\frac{54}{11}T_1\vec{i} - \frac{18}{11}T_1\vec{j}$$
$$-\frac{18}{11}T_2\vec{i} + \frac{54}{11}T_2\vec{j}$$
$$+98100\vec{i} - 50470\vec{j}$$
$$+26487\vec{i} \qquad = 0$$

$\vec{i}$ components $\rightarrow \frac{54}{11}T_1 + \frac{18}{11}T_2 = 98100 + 26487$ (A)

$\vec{j}$ components $\rightarrow \frac{-18}{11}T_1 + \frac{54}{11}T_2 = 50470$ (B)

54 (A) $- 18$ (B) $\rightarrow \left(\frac{54\cdot 54}{11} + \frac{18\cdot 18}{11}\right)T_1 = 124587(54) - 50470(18)$

$$\rightarrow \boxed{T_1 = 19757\ N}$$

$$\rightarrow \boxed{T_2 = 16866\ N}$$

Then: $\Sigma\vec{F} = 19757\left(\frac{-2}{11}\vec{i} + \frac{6}{11}\vec{j} - \frac{9}{11}\vec{k}\right)$
$+ 16866\left(\frac{6}{11}\vec{i} + \frac{2}{11}\vec{j} - \frac{9}{11}\vec{k}\right)$
$- 10094\vec{i} \quad - 16824\vec{k}$
$- 19620\vec{j}$
$- 5886\vec{j}$
$+ A_x\vec{i} + A_y\vec{j} + A_z\vec{k} = 0$

$\vec{i} \rightarrow A_x = 10094 - 16866\left(\frac{6}{11}\right) + 19757\left(\frac{2}{11}\right) = 4487\ N$

$\vec{j} \rightarrow A_y = 5886 + 19620 - 16866\left(\frac{2}{11}\right) - 19757\left(\frac{6}{11}\right) = 11663\ N$

$\vec{k} \rightarrow A_z = 16824 + 16866\left(\frac{9}{11}\right) + 19757\left(\frac{9}{11}\right) = 46788\ N$

b) If we are only interested in T_1 then we want to find a point <u>or an axis</u> about which the moments due to all of the other <u>unknown</u> forces are zero.

Our unknown forces are those at A, T_1 and T_2. The forces at A have zero moment about point A and any axis passing through A, and T_2 has zero moment about any axis passing through D. (Note that all of the forces in this problem have zero moment about the axis AC.) If we take moments about the axis AD, the only unknown force that appears is due to T_1.

$$\vec{e}_{AD} = \frac{6\vec{i} + 2\vec{j}}{\sqrt{6^2+2^2}} = \frac{3}{\sqrt{10}}\vec{i} + \frac{1}{\sqrt{10}}\vec{j}$$

$$M_1^{AD} = \vec{M}_1^{A} \cdot \vec{e}_{AD} = \left(-\frac{54}{11}\frac{3}{\sqrt{10}} - \frac{18}{11}\frac{1}{\sqrt{10}}\right)T_1 = \frac{-180}{11\sqrt{10}}T_1$$

$$M_{34}^{AD} = \vec{M}_{34}^{A} \cdot \vec{e}_{AD} = 98100\frac{3}{\sqrt{10}} - 50470\frac{1}{\sqrt{10}} = \frac{243830}{\sqrt{10}}$$

$$M_{W_B}^{AD} = \vec{M}_{WB}^{A} \cdot \vec{e}_{AD} = 26487\frac{3}{\sqrt{10}} = \frac{79461}{\sqrt{10}}$$

$$\Sigma M^{AD} = \frac{-180}{11\sqrt{10}}T_1 + \frac{243830+79461}{\sqrt{10}} = 0$$

$$\rightarrow \boxed{T_1 = 19757\ N}$$

You do not need this in vector form because <u>all</u> of these moments are in the same $\vec{e}_{AD}$ direction.