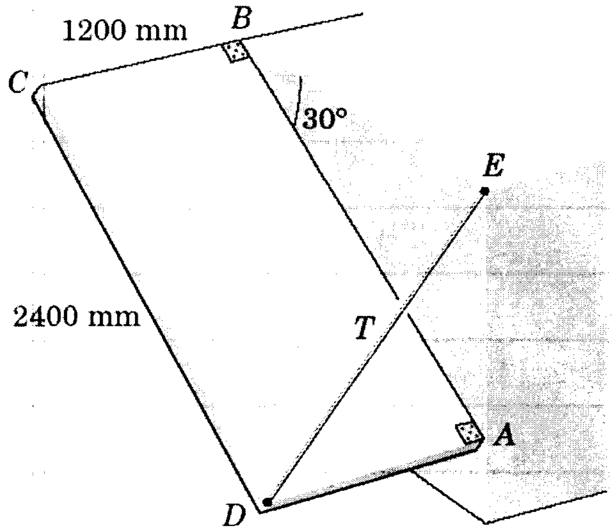
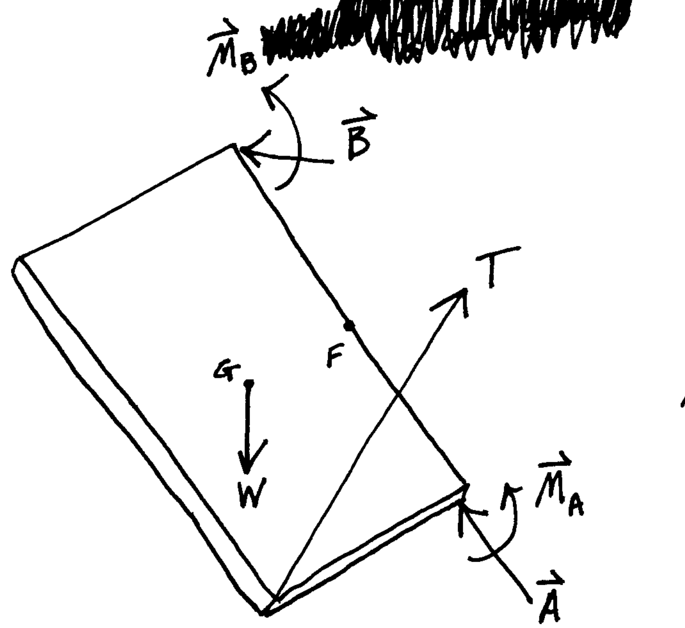
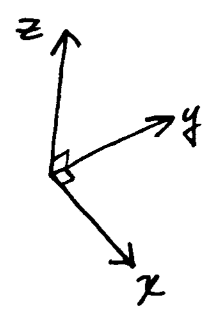


The uniform rectangular panel ABCD has a mass of 40 kg and is hinged at its corners A and B to the fixed vertical surface. A wire from E to D keeps edges BC and AD horizontal. Hinge A can support thrust along the hinge axis AB, whereas hinge B supports force normal to the hinge axis only. Find the tension T in the wire and the magnitude of the force supported by hinge B. (BEA is a right triangle.)

FBD





$\vec{M}_A$, $\vec{M}_B$ and $\vec{B}$ do not have components along AB.

All forces will be in N, lengths in mm, and moments in N·mm.

Vector representations: $\vec{W} = -9.81(40)\vec{k}$
$= -392.4\,\vec{k}$

$$\vec{T} = T\,\vec{e}_{DE} = T\,\frac{1200\vec{j} + 1200\vec{k}}{\sqrt{1200^2+1200^2}} = \frac{\sqrt{2}}{2}T\vec{j} + \frac{\sqrt{2}}{2}T\vec{k}$$

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

The important fact about $\vec{M}_A$ is $\vec{M}_A \cdot \vec{e}_{AB} = 0$, i.e. the moment due to $\vec{M}_A$ about the AB axis is zero. The same holds true for $\vec{M}_B \rightarrow \vec{M}_B \cdot \vec{e}_{AB} = 0$. Finally, the fact that $\vec{B}$ cannot resist motion along the hinge axis $\rightarrow \vec{B} \cdot \vec{e}_{AB} = 0$.

Let's determine T first by balancing the moments about the axis AB. The reactions at A and B cannot supply a moment about this axis.

$$M_{AB}^{T} = \vec{r}_{AD} \times \vec{T} \cdot \vec{e}_{AB}$$
$$= -1200\vec{j} \times T\left(\frac{\sqrt{2}}{2}\vec{j} + \frac{\sqrt{2}}{2}\vec{k}\right) \cdot \left(-\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{k}\right)$$
$$= -1200\frac{\sqrt{2}}{2}T\,\underbrace{\vec{j}\times\vec{k}}_{\vec{i}} \cdot \left(-\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{k}\right)$$
$$= 1200\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}T = 735\,T$$

$$M_{AB}^{W} = \vec{r}_{FG} \times \vec{W} \cdot \vec{e}_{AB}$$
$$= -600\vec{j} \times -392.4\vec{k} \cdot \left(-\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{k}\right)$$
$$= -600(392.4)\left(\frac{\sqrt{3}}{2}\right) = -203897$$

$$\Sigma M_{AB} = 735\,T - 203897 = 0$$

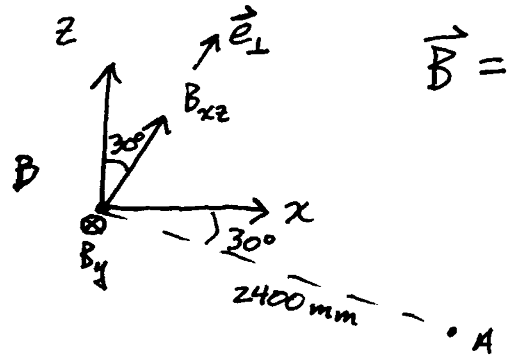
$$\boxed{T = 277\ N}$$

As I have assumed the reactions at the hinges to be, we cannot actually solve for $\vec{B}$. We have 2 components of $\vec{B}$, 2 of $\vec{M}_B$, 2 of $\vec{M}_A$ and 3 of $\vec{A}$ but only 5 equilibrium conditions remaining.

If $\vec{M}_A$ and $\vec{M}_B$ are zero, then we can solve for $\vec{A}$ or $\vec{B}$. (As drawn the problem statement is ambiguous.)

If $\vec{M}_A = \vec{M}_B = 0$ then we can take moments about A to determine $\vec{B}$.

Before taking moments about A we need to put $\vec{B}$ in vector form.



$$\vec{B} = \underbrace{B_{xz}\sin 30°\,\vec{i} + B_{xz}\cos 30°\,\vec{k}}^{B_{xz}\vec{e}_\perp} + B_y\,\vec{j}$$

From the previous sketch we could actually find the moments due to $\vec{B}$ about A with scalar methods.

$$\vec{M}_A^B = 2400\, B_{xz}\, \vec{j} - 2400\, B_y\, \underbrace{\vec{e}_\perp}_{\sin 30^\circ \vec{i} + \cos 30^\circ \vec{k} = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{k}}$$

By vector methods:

$$\vec{M}_A^B = \vec{r}_{AB} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1200\sqrt{3} & 0 & 1200 \\ \frac{1}{2}B_{xz} & B_y & \frac{\sqrt{3}}{2}B_{xz} \end{vmatrix}$$

Same as above

$$= -1200\, B_y\, \vec{i} + \underbrace{\left(600 + 1200\,\tfrac{3}{2}\right)}_{2400} B_{xz}\, \vec{j} - 1200\sqrt{3}\, B_y\, \vec{k}$$

$$\vec{M}_A^T = \vec{r}_{AD} \times \vec{T} = -1200\,\vec{j} \times T\left(\tfrac{\sqrt{2}}{2}\vec{j} + \tfrac{\sqrt{2}}{2}\vec{k}\right) = -600\sqrt{2}\, T\, \vec{i}$$

previous analysis $\rightarrow$ $T = \dfrac{600(392.4)\cancel{\frac{\sqrt{2}}{2}}}{1200\,\frac{\sqrt{2}}{2}\,\cancel{\frac{\sqrt{3}}{2}}} \rightarrow \vec{M}_A^T = -235440\,\vec{i}$

$$\vec{M}_A^W = \vec{r}_{AG} \times \vec{W} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -600\sqrt{3} & -600 & 600 \\ 0 & 0 & -392.4 \end{vmatrix} = 235440\,\vec{i} - 407794\,\vec{j}$$

$$\begin{aligned} \Sigma \vec{M}_A = \; & 1200\, B_y\, \vec{i} + 2400\, B_{xz}\, \vec{j} - 1200\sqrt{3}\, B_y\, \vec{k} \\ & - 235440\,\vec{i} \\ & + 235440\,\vec{i} - 407794\,\vec{j} \qquad = 0 \end{aligned}$$

$$\vec{i} \rightarrow B_y = 0\,, \quad \vec{k} \rightarrow B_y = 0\,, \quad \vec{j} \rightarrow B_{xz} = \frac{407794}{2400} = 170\,N$$

$$B = \sqrt{B_{xz}^2 + B_y^2} = 170\,N$$

Review

Vectors: $\vec{A} = A\,\vec{e}_A = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$

A = magnitude , $\vec{e}_A$ = unit vector in the same direction as $\vec{A}$

A_x, A_y, A_z = x, y and z components of $\vec{A}$

$$\boxed{\begin{aligned} A &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\ \vec{e}_A &= \frac{\vec{A}}{A} = \frac{A_x\vec{i} + A_y\vec{j} + A_z\vec{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \end{aligned}}$$

<u>Direction cosines</u>: $\cos\theta_x = \dfrac{A_x}{A}$, $\cos\theta_y = \dfrac{A_y}{A}$

$\cos\theta_z = \dfrac{A_z}{A} \rightarrow \vec{e}_A = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$

<u>Dot Product</u> : $\vec{A}\cdot\vec{B} = AB\cos\theta_{AB}$

$= A_xB_x + A_yB_y + A_zB_z$

$\vec{A}\cdot\vec{e}$ = "amount" of $\vec{A}$ in the $\vec{e}$ direction if $\vec{e}$ is a unit vector.

$\vec{A}\cdot\vec{i} = A_x$, $\vec{A}\cdot\vec{j} = A_y$, $\vec{A}\cdot\vec{k} = A_z$

$\vec{i}\cdot\vec{i} = 1$, $\vec{j}\cdot\vec{j} = 1$, $\vec{k}\cdot\vec{k} = 1$

$\vec{i}\cdot\vec{j} = \vec{i}\cdot\vec{k} = \vec{j}\cdot\vec{k} = 0$

Cross Product: $\vec{A} \times \vec{B} = AB \sin\theta_{AB}\, \vec{e}_\perp$

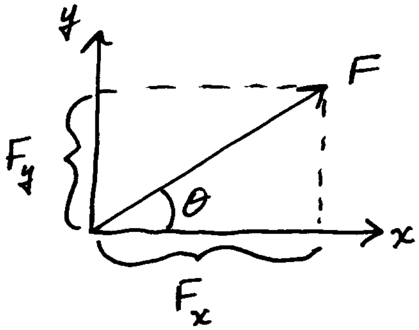
$\vec{e}_\perp$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ with direction chosen by the right-hand-rule curling $\vec{A}$ into $\vec{B}$.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{aligned} &(A_y B_z - A_z B_y)\vec{i} \\ &+ (A_z B_x - A_x B_z)\vec{j} \\ &+ (A_x B_y - A_y B_x)\vec{k} \end{aligned}$$

Note: $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

Forces: Force is a vector.

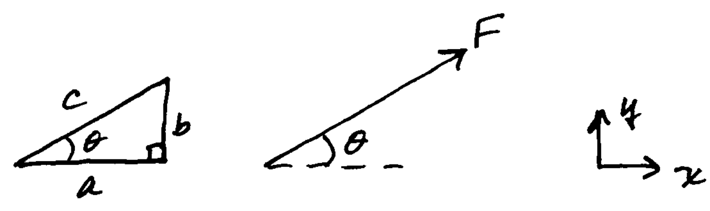
2-D:



$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

Similar triangles:



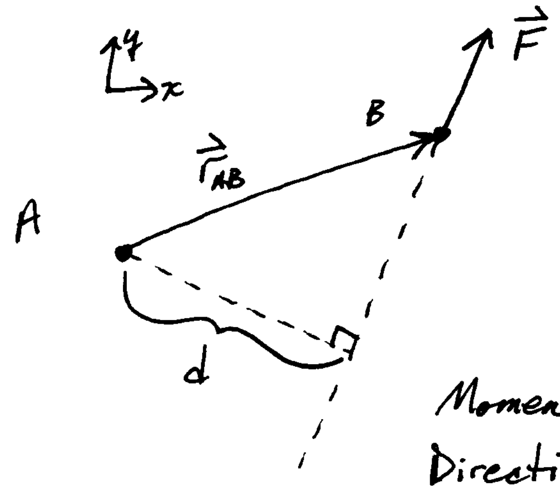
$$F_x = F\frac{a}{c}, \quad F_y = F\frac{b}{c}$$

3-D: $\vec{F} = F\,\vec{e}_F$

To determine $\vec{e}_F$, find any vector in the direction of $\vec{F}$ and normalize it.

Moments: $\vec{M}_A = \vec{r}_{AB} \times \vec{F}_B$

2-D:



d = "perpendicular distance" between $\vec{F}$ and A.

Moment magnitude = Fd
Direction is determine by right-hand-rule. In the case shown + z.

3-D: Can be done with scalar methods as shown above, but generally it is easier to do the cross product.

Moment about an axis CD: $\vec{M}_{CD} = (\vec{r}_{AB} \times \vec{F}_B \cdot \vec{e}_{CD})\vec{e}_{CD}$

$\vec{e}_{CD}$ = unit vector along CD axis
A = any point on the axis
B = point of application of the force

$\vec{r}_{AB} \times \vec{F}_B \cdot \vec{e}_{CD}$ gives the scalar amount of the moment that lies along the $\vec{e}_{CD}$ direction.

Equilibrium Analysis

1) Draw a free body diagram. This means that the system is isolated from its surroundings and all forces supplied by the surroundings are drawn and labeled.

2) Determine components of all forces.

3) Determine components of the moments due to all forces and all couples about a point of your choosing.

4) Set up equilibrium equations.

2-D: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z^A = 0$

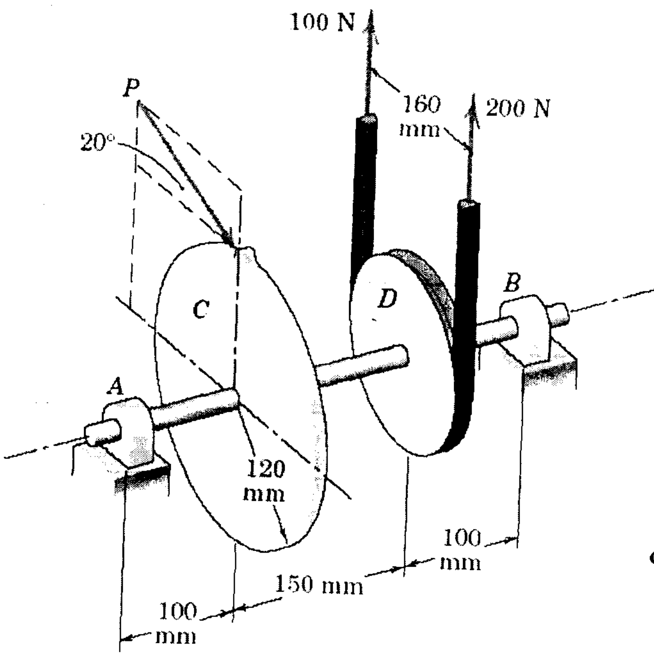
3-D: $\sum \vec{F} = 0$, $\sum \vec{M}_A = 0$

Then $\vec{i}$, $\vec{j}$ & $\vec{k}$ components yield 3 scalar equations for each vector equation.

5) Solve for unknowns.

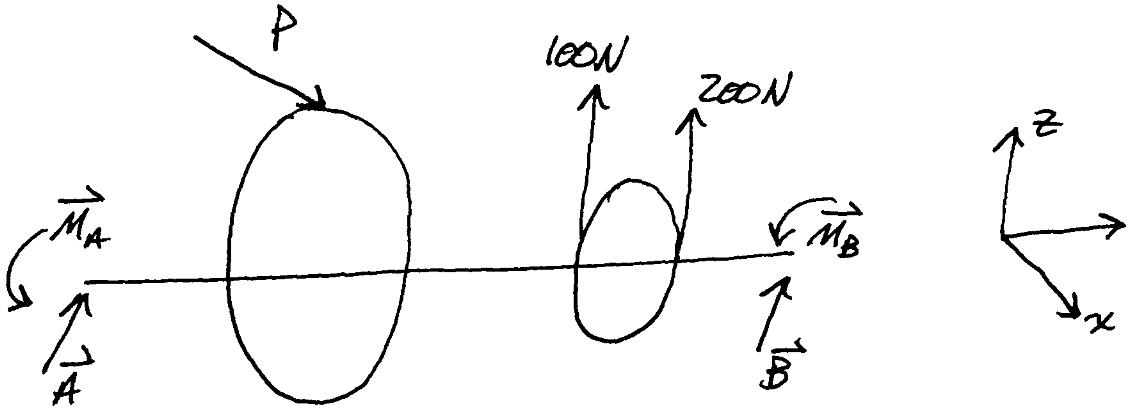
* 1 is mandatory !!!

2 - 5 is a suggested procedure which if done properly will always get the answer. However, there may be quicker methods for the solutions of certain problems.



Gear C drives the V-belt pulley D at a constant velocity. For the belt tensions shown calculate the gear-tooth force P and the magnitudes of the total forces supported by the bearings at A and B.

FBD



Note: $A_y = 0$, $M_y^A = 0$, $B_y = 0$, $M_y^B = 0$

To determine P we can balance the moments in the y-direction (i.e. about axis AB).

$$\Sigma M_y = \underbrace{P\cos 20^\circ}_{P_x} \cdot \underbrace{120}_{\text{z-moment arm}} + 100\cdot 80 - 200\cdot 80 = 0$$

$\rightarrow P = 70.9\ N$

To determine either $\vec{A}$ or $\vec{B}$ we must assume $\vec{M}_A = \vec{M}_B = 0$. This is unclear from the problem statement. In the following we will assume $\vec{M}_A = \vec{M}_B = 0$.

To determine $\vec{B}$ we can take moments about A.

$$\Sigma \vec{M}_A = \vec{M}_A^{P} + \vec{M}_A^{100} + \vec{M}_A^{200} + \vec{M}_A^{B} = 0$$

$$\vec{M}_A^{P} = \underbrace{P\cos 20^\circ}_{x\text{-force}} \cdot \underbrace{120}_{z\text{-arm}} \vec{j} - \underbrace{P\cos 20^\circ}_{x\text{-force}} \cdot \underbrace{100}_{y\text{-arm}} \vec{k} - \underbrace{P\sin 20^\circ}_{z\text{-force}} \cdot \underbrace{100}_{y\text{-arm}} \vec{i}$$

(note x-arm $= 0$)

or vector method: $\vec{M}_A^{P} = \vec{r} \times \vec{P} = (100\vec{j} + 120\vec{k}) \times (P\cos 20^\circ \vec{i} - P\sin 20^\circ \vec{k})$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 100 & 120 \\ P\cos 20^\circ & 0 & -P\sin 20^\circ \end{vmatrix} = \begin{array}{l} -100 P\sin 20^\circ \vec{i} \\ +120 P\cos 20^\circ \vec{j} \\ -100 P\cos 20^\circ \vec{k} \end{array}$$

$$\vec{M}_A^{100} = \underbrace{100}_{z\text{-force}} \cdot \underbrace{80}_{x\text{-arm}} \vec{j} + \underbrace{100}_{z\text{-force}} \cdot \underbrace{250}_{y\text{-arm}} \vec{i}$$

$$\vec{M}_A^{200} = -\underbrace{200}_{z\text{-force}} \cdot \underbrace{80}_{x\text{-arm}} \vec{j} + \underbrace{200}_{z\text{-force}} \cdot \underbrace{250}_{y\text{-arm}} \vec{i}$$

$$\vec{M}_A^{B} = \vec{r}_{AB} \times \vec{B} = 350\vec{j} \times (B_x \vec{i} + B_z \vec{k}) = -350 B_x \vec{k} + 350 B_z \vec{i}$$

$$\Sigma \vec{M}_A = -100P\sin 20^\circ \vec{i} + 120P\cos 20^\circ \vec{j} - 100P\cos 20^\circ \vec{k}$$
$$+25000\vec{i} \quad + 8000\vec{j}$$
$$+50000\vec{i} \quad - 16000\vec{j}$$
$$+350B_z\vec{i} \quad\quad - 350B_x\vec{k}$$
$$= 0\vec{i} + 0\vec{j} + 0\vec{k}$$

$\vec{j} \rightarrow \quad 120P\cos 20^\circ = 8000 \rightarrow P = 70.9$ (same as before)

$\vec{k} \rightarrow \quad B_x = \frac{-100}{350} P\cos 20^\circ = -19.05\,N$

$\vec{i} \rightarrow \quad B_z = -\left(\frac{-100P\sin 20^\circ + 75000}{350}\right) = -207.4\,N$

$$\rightarrow B = \sqrt{B_x^2 + B_z^2} = 208\,N$$

$$\Sigma \vec{F} = A_x\vec{i} + A_z\vec{k}$$
$$+B_x\vec{i} + B_z\vec{k}$$
$$+P\cos 20^\circ \vec{i} \quad - P\sin 20^\circ \vec{k}$$
$$+100\vec{k}$$
$$+200\vec{k}$$
$$= 0\vec{i} + 0\vec{j} + 0\vec{k}$$

$\vec{i} \rightarrow \quad A_x = -B_x - P\cos 20^\circ = -47.6\,N$

$\vec{k} \rightarrow \quad A_z = -B_z + P\sin 20^\circ - 300 = -68.3\,N$

$$\rightarrow A = \sqrt{A_x^2 + A_z^2} = 83.3\,N$$

* Note we did not have to do the first ΣM_x step.