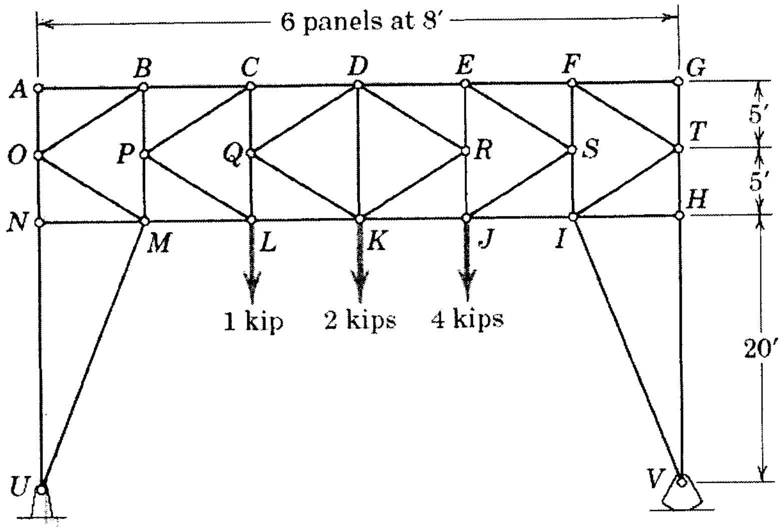
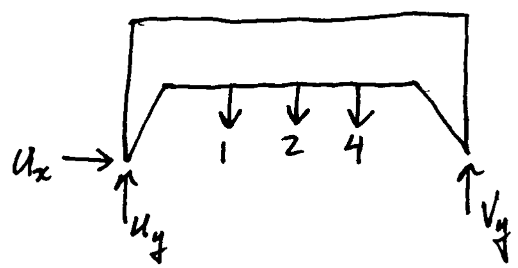


Determine the force in member DK.

The FBD & analysis of the entire structure.



$$\Sigma F_x = U_x = 0$$

$$\Sigma F_y = U_y + V_y - 7 = 0$$

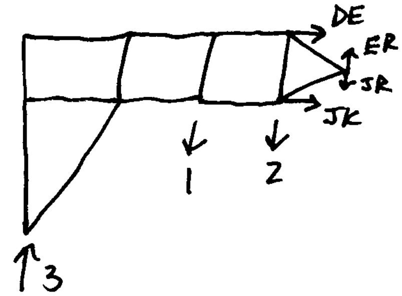
$$\Sigma M_z^U = -1 \cdot 16 - 2 \cdot 24 - 4 \cdot 32 + V_y \cdot 48 = 0$$

$$\rightarrow V_y = 4 \text{ kips}$$

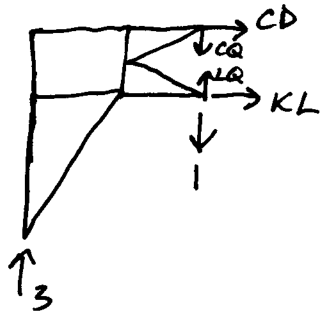
$$\rightarrow U_y = 3 \text{ kips}$$

Next we cannot make 1 cut that will expose DK and only 2 other member forces. So we will have to make more cuts. Let's cut through CD, DQ, DK, KR and JK (5 unknowns, 3 equations) and then through DE, DR, KR and JK (2 new unknowns, 3 new equations), and finally through CD, DQ, KQ and KL (2 new unknowns, 3 new equations) giving 9 unknowns & 9 equations. Let's draw our FBDs and try to determine if there is a quick way to get at DK.

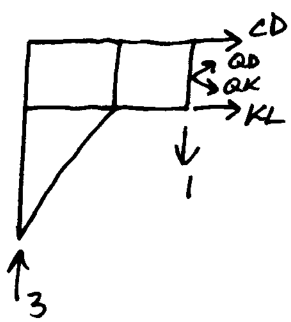
Consider the following sections:



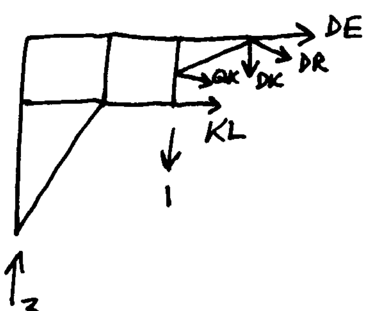
$$\Sigma F_x = DE + JK = 0$$
$$\Sigma M_z^R = JK \cdot 5 - DE \cdot 5 - 3 \cdot 32 + 1 \cdot 16 + 2 \cdot 8 = 0$$
$$\rightarrow JK = -DE = 6.4 \text{ kips}$$



$$\Sigma F_x = CD + KL = 0$$
$$\Sigma M_z^Q = KL \cdot 5 - CD \cdot 5 - 3 \cdot 16 = 0$$
$$\rightarrow KL = -CD = 4.8 \text{ kips}$$



$$\Sigma F_x = \overbrace{CD + KL}^{=0} + \frac{8}{\sqrt{89}} QD + \frac{8}{\sqrt{89}} QK = 0$$
$$\rightarrow QD = -QK$$
$$\Sigma F_y = 3 - 1 + \frac{5}{\sqrt{89}} QD - \frac{5}{\sqrt{89}} QK = 0$$
$$\rightarrow QD = -QK = \frac{-\sqrt{89}}{5} = -1.887 \text{ kips}$$

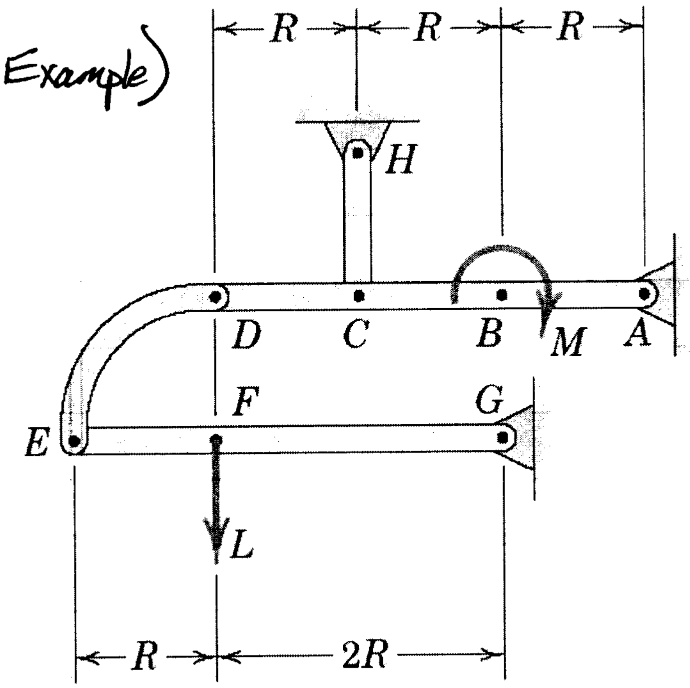


$$\Sigma F_x = KL + QK \frac{8}{\sqrt{89}} + DR \frac{8}{\sqrt{89}} + DE = 0$$
$$4.8 + \frac{8}{5} + DR \frac{8}{\sqrt{89}} - 6.4 = 0$$
$$\rightarrow DR = 0$$

$$\Sigma F_y = 3 - 1 - QK \frac{5}{\sqrt{89}} - DK - DR \frac{5}{\sqrt{89}} = 0$$
$$2 - 1 - DK = 0$$
$$\boxed{DK = +1 \text{ kips}}$$

Frames & Machines

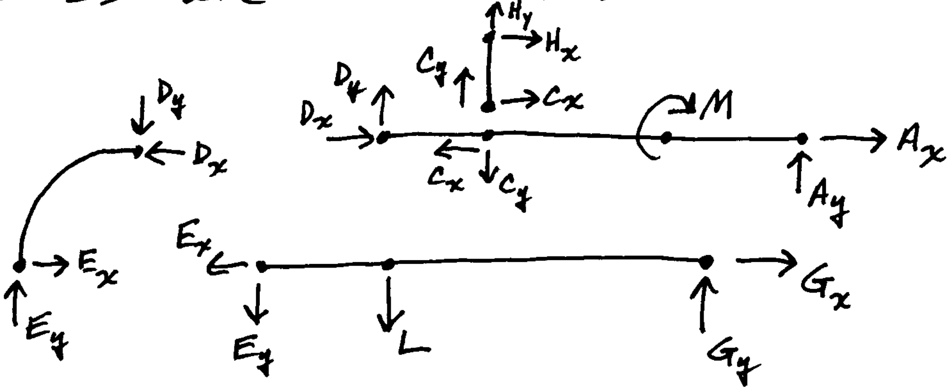
Example)



Given the values of the load L and dimension R, for what value of the couple M will the force in link CH be zero?

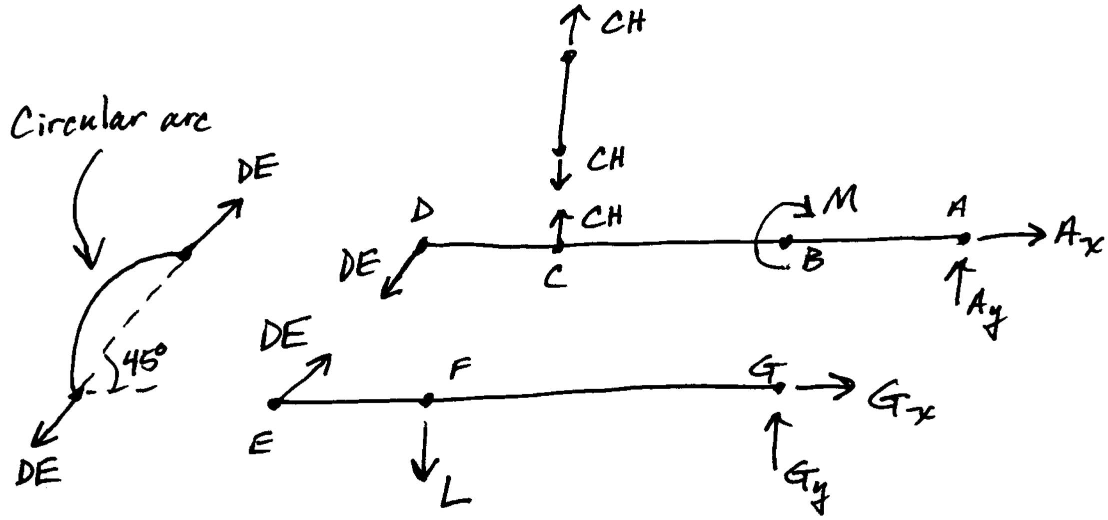
An inspection of the supports at H, A, and G reveals that there will be 6 (5 if you recognize that CH is a 2-force member) unknown support reactions. Hence, an analysis of the entire structure (3 equilibrium equations) will not get us too far. It will be more profitable to go directly to drawing FBDs of each component.

For the time being we will not use the fact that CH = 0, and we will not recognize that CH and ED are 2-force members.



4 FBDs → 12 equilibrium equations

Assuming L & M are known, we have A_x, A_y, C_x, C_y, D_x, D_y, E_x, E_y, G_x, G_y, H_x, H_y = 12 unknowns. So we can solve for all of these unknowns. Our analysis will be simplified if we recognize that CH and DE are 2-force members.



It is perfectly acceptable to start with these FBDs. However we must note that by identifying CH and DE as 2-force members, we have effectively already used the equilibrium equations for CH and DE. So now we are left with 2 FBDs → 6 equilibrium equations and A_x, A_y, G_x, G_y, CH and DE = 6 unknowns.

In our problem statement we are given CH=0, and asked to find M. So M is an unknown and CH is known.

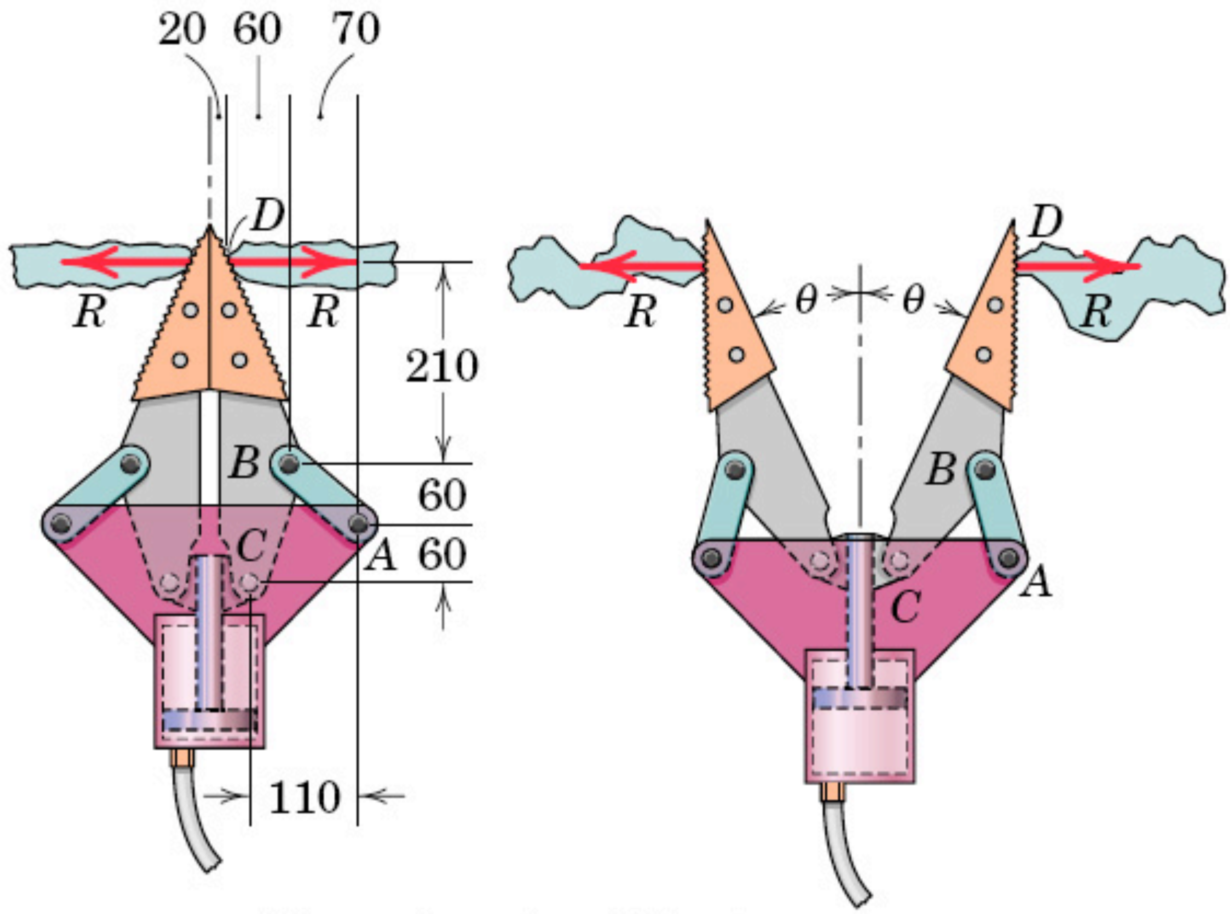
From our 2 FBDs we see that EFG has 3 unknown forces, and DCBA has 4 unknown forces/moments. Analyze EFG first.

EFG :
$$\Sigma F_x = DE\tfrac{\sqrt{2}}{2} + G_x = 0$$
$$\Sigma F_y = DE\tfrac{\sqrt{2}}{2} - L + G_y = 0$$
$$\Sigma M_z^G = -DE\tfrac{\sqrt{2}}{2}\cdot 3R + L\cdot 2R = 0$$
$$\to DE = \frac{2\sqrt{2}}{3}L$$

$$\to G_x = -\tfrac{2}{3}L \,,\quad G_y = \tfrac{1}{3}L$$

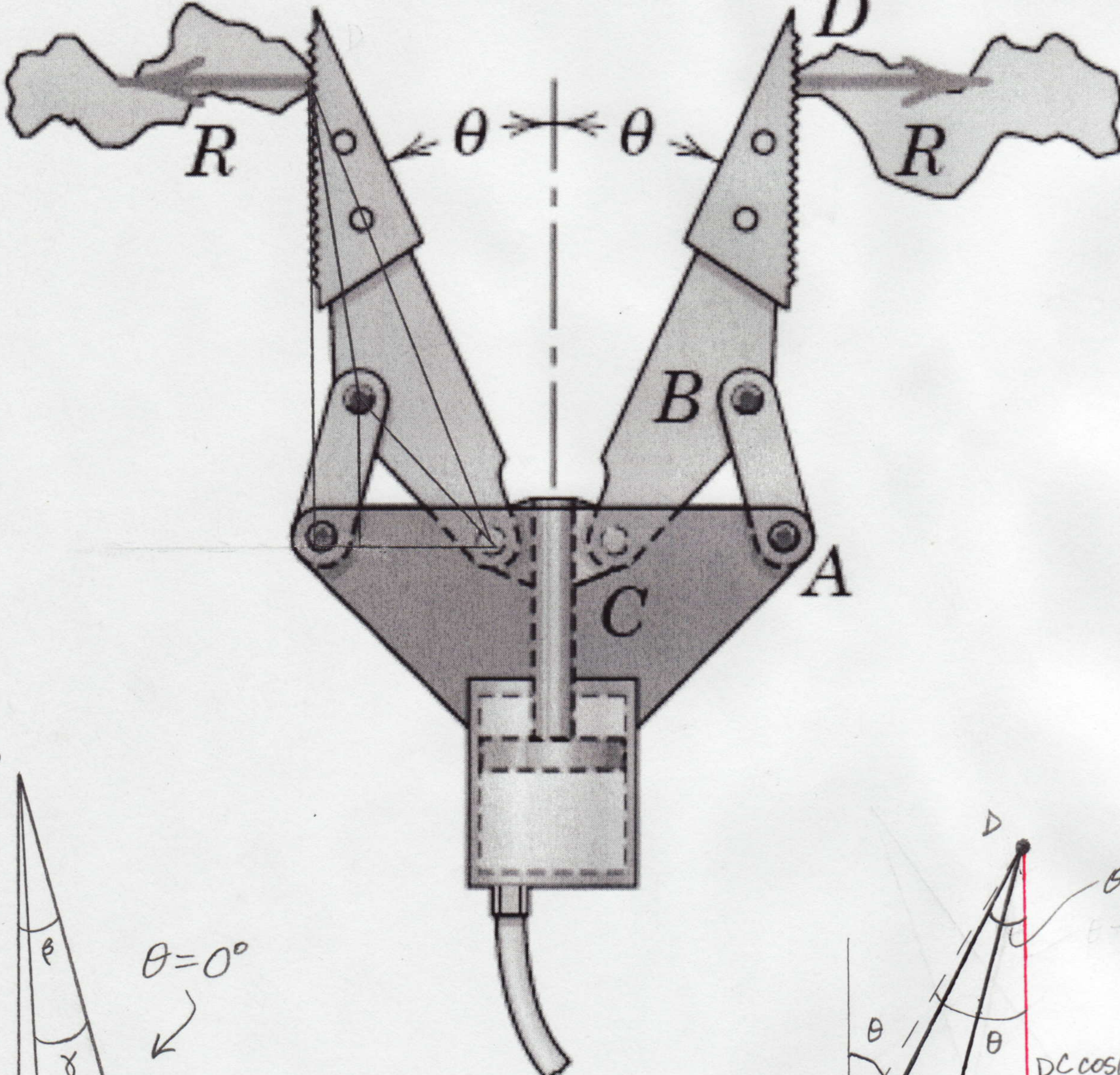
DCBA :
$$\Sigma F_x = -DE\tfrac{\sqrt{2}}{2} + A_x = 0 \to A_x = \tfrac{2}{3}L$$
$$\Sigma F_y = -DE\tfrac{\sqrt{2}}{2} + \overset{0\text{ (given)}}{\cancel{CA}} + A_y = 0 \to A_y = \tfrac{2}{3}L$$
$$\Sigma M_z^A = -M + \tfrac{\sqrt{2}}{2}DE\cdot 3R - \overset{0}{\cancel{CA}}\cdot 2R = 0$$
$$\to \boxed{M = 2LR}$$

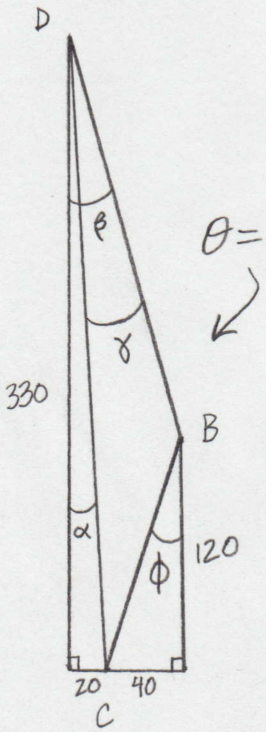
Note that we could have gotten to this answer quickly by doing ΣM_z^G for EFG to determine DE and then ΣM_z^A for DCBA to determine M. On a test I would be sure to ask for all support reactions, so you would have to go through all of these steps.



Dimensions in millimeters

The “jaws of life” device is used by rescuers to pry apart wreckage. A pressure of 35 MPa (35×10^6 N/m^2) is developed behind the piston with a 50-mm radius. Determining the prying force R, the force in link AB, and the horizontal force reaction at C for a general opening angle θ. Determine these forces when θ=0, and plot these expressions for $0\leq\theta\leq45^\circ$. Determine the minimum value for R and the angle θ where it occurs. Determine the work done by the hydraulics to move the piston, and the work done by the wreckage on the jaws.





$\theta = 0°$

$DC = \sqrt{20^2 + 330^2} = 330.606$

$BC = \sqrt{40^2 + 120^2} = 126.491$

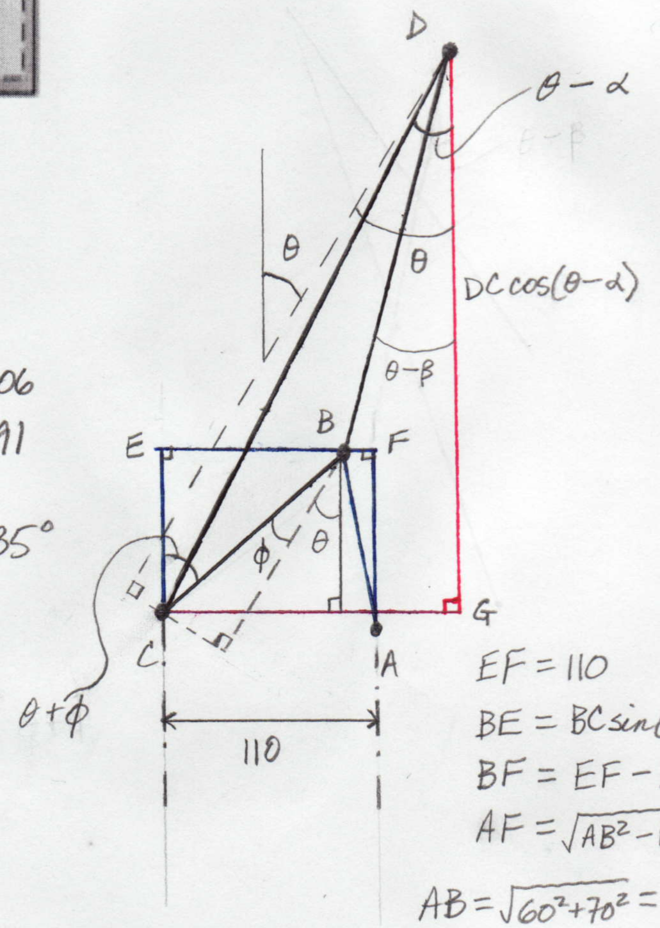
$\phi = \arctan \frac{40}{120} = 18.435°$

$BD = \sqrt{60^2 + 210^2} = 218.403$

$\alpha = \arctan \frac{20}{330} = 3.468°$

$\beta = \arctan \frac{60}{210} = 15.945°$

$\gamma = \beta - \alpha = 12.477°$



$EF = 110$

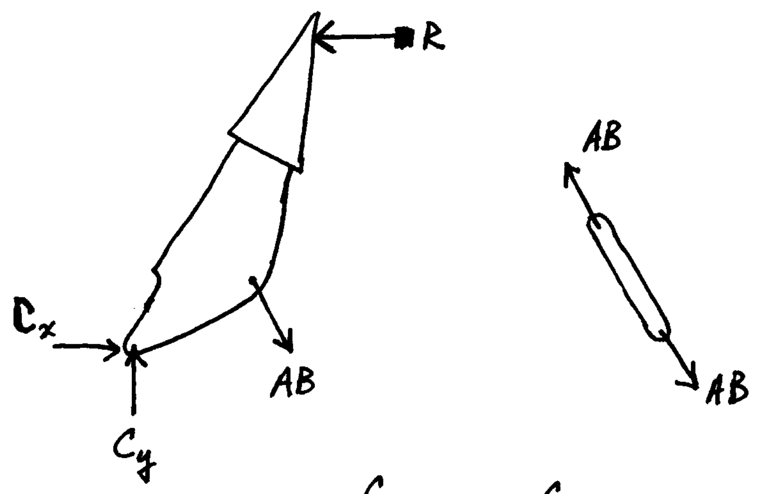
$BE = BC \sin(\theta + \phi)$

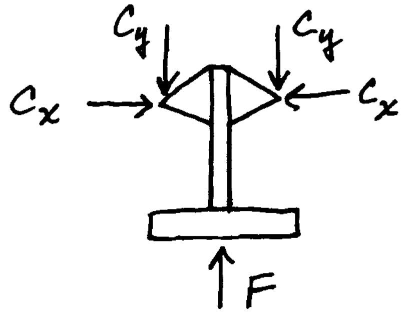
$BF = EF - BE$

$AF = \sqrt{AB^2 - BF^2}$

$AB = \sqrt{60^2 + 70^2} = 92.195$

FBDs:





(This FBD uses symmetry of the machine <u>and</u> loading.)

$$F = p \cdot \text{Area} = 35 \times 10^6 \frac{N}{m^2} \cdot \pi (0.050\,m)^2 = 274889\ N$$

Symmetry of piston (or ΣF_y and ΣM_z) implies that $C_y = F/2$.

Recognizing that AB is a 2-force member implies that we have already satisfied the equilibrium conditions for link AB.

<u>Equilibrium analysis of a jaw:</u>

$$\Sigma F_x = C_x + AB \frac{L_{BF}}{L_{AB}} - R = 0$$

$$\Sigma F_y = C_y - AB \frac{L_{AF}}{L_{AB}} = 0 \rightarrow AB = \frac{F}{2} \frac{L_{AB}}{L_{AF}}$$

$$\Sigma M_z^B = R\, L_{BD} \cos(\theta-\beta) + C_x L_{BC} \cos(\theta+\phi)$$

$$- C_y L_{BC} \sin(\theta+\phi) = 0$$

$$R = C_x + AB \frac{L_{BF}}{L_{AB}} = C_x + \frac{F}{2} \frac{L_{AB}}{L_{AF}} \frac{L_{BF}}{L_{AB}} = C_x + \frac{F}{2} \frac{L_{BF}}{L_{AF}}$$

$$\rightarrow \left(C_x + \frac{F}{2} \frac{L_{BF}}{L_{AF}}\right) L_{BD} \cos(\theta-\beta) + C_x L_{BC} \cos(\theta+\phi)$$

$$- \frac{F}{2} L_{BC} \sin(\theta+\phi) = 0$$

$$\rightarrow C_x = \frac{F}{2} \frac{L_{BC} \sin(\theta+\phi) - L_{BD} \frac{L_{BF}}{L_{AF}} \cos(\theta-\beta)}{L_{BD} \cos(\theta-\beta) + L_{BC} \cos(\theta+\phi)}$$

$$R = C_x + \frac{F}{2} \frac{L_{BF}}{L_{AF}}$$

$$AB = \frac{F}{2} \frac{L_{AB}}{L_{AF}}$$

$$C_y = \frac{F}{2}$$

We will plug these formulas into the computer to answer the questions in the problem statement.

In addition to these questions let's also look at the work done on the piston by the hydraulics and the work done on the wreckage by the jaws.

The work done by a force is $W = \int \vec{F} \cdot d\vec{r}$

$d\vec{r} \equiv$ increment of displacement of the point of application of the force

The dot product implies that forces only do work when they displace in the direction of their action.

For a constant force the work done is simply $F\Delta$.

→ Work done by hydraulics is $W_F = F[60 - (L_{CE} - L_{AF})]$

$$L_{CE} = L_{BC}\cos(\theta + \phi)$$

Since the jaw force R is not constant we have to do the integral.

$$W_R = 2\int_0^{\Delta x} R\,dx \quad \text{(factor of 2 is for 2 jaws)}$$

$$x = L_{CG} + 20$$
$$= L_{DC}\sin(\theta - \alpha) + 20$$

$$dx = L_{DC}\cos(\theta - \alpha)\,d\theta$$

$$W_R(\theta_{max}) = 2\int_0^{\theta_{max}} R(\theta)\,L_{DC}\cos(\theta - \alpha)\,d\theta$$

Our calculations show that $W_F = W_R$. Even though force inputs and outputs may differ, energy is conserved.

In[57]:= θ =.;

$F = 35.\ 10^6\ \pi\ (0.05)^2$;

$LBC = \sqrt{40.^2 + 120.^2}$;

ϕ = ArcTan[40. / 120.];

$LBD = \sqrt{60.^2 + 210.^2}$;

LBE = LBC Sin[$\theta + \phi$];

LBF = 110. - LBE;

$LAB = \sqrt{60.^2 + 70.^2}$;

$LAF = \sqrt{LAB^2 - LBF^2}$;

β = ArcTan[60. / 210.];

In[67]:= F

LBC

$\phi \frac{180}{\pi}$

LBD

LAB

$\beta \frac{180}{\pi}$

Out[67]= 274889.

Out[68]= 126.491

Out[69]= 18.4349

Out[70]= 218.403

Out[71]= 92.1954

Out[72]= 15.9454

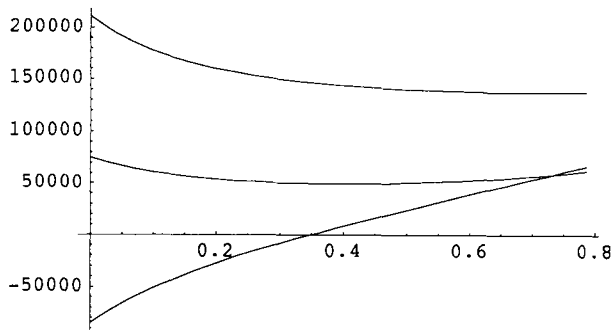
In[73]:= $Cx = \frac{F}{2} \frac{LBC\ Sin[\theta + \phi] - LBD \frac{LBF}{LAF} Cos[\theta - \beta]}{LBD\ Cos[\theta - \beta] + LBC\ Cos[\theta + \phi]}$;

$R = Cx + \frac{F}{2} \frac{LBF}{LAF}$;

$AB = \frac{F}{2} \frac{LAB}{LAF}$;

$Cy = \frac{F}{2}$;

```
In[77]:= θ = 0;
         Cx
         R
         AB
         Cy

Out[78]= -85382.3

Out[79]= 74969.8

Out[80]= 211196.

Out[81]= 137445.

In[82]:= θ =.;
         Plot[{Cx, R, AB}, {θ, 0, π/4}]
```




```
Out[83]= - Graphics -

In[86]:= dR = ∂θ R;
         FindRoot[dR == 0, {θ, 0.4}]

Out[87]= {θ → 0.404312}

In[88]:= θ = 0.40431234988201536;
         θ 180/π
         R

Out[89]= 23.1654

Out[90]= 49370.9
```

102

In[148]:=
```
θ =.;
θmax = 0.8 π / 4;
LDC = Sqrt[20^2 + 330^2];
LCE = LBC Cos[θ + ϕ];
α = ArcTan[20. / 330.];
WF = F (60 - (LCE - LAF));
θ = θmax;
WF
θ =.;
WR = 2 NIntegrate[R LDC Cos[θ - α], {θ, 0, θmax}]
```

Out[155]=

2.15378×10^7

Out[157]=

2.15378×10^7