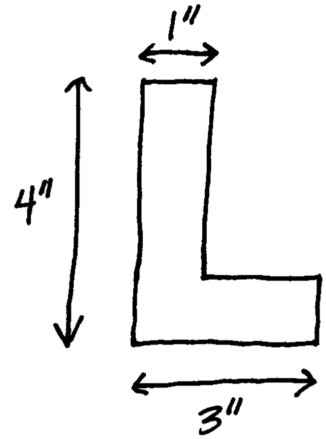
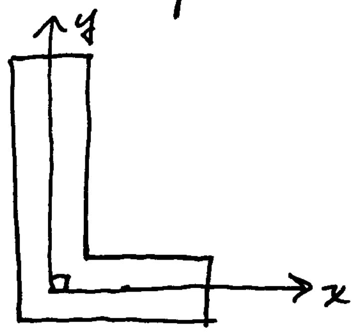
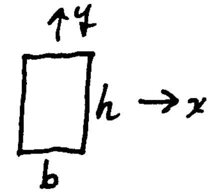
Example: Determine $\bar{x}$, $\bar{y}$, $\bar{I}_x$ and $\bar{I}_y$ for the following shape. The x-direction is horizontal and the y-direction is vertical.



(A) Set up x & y axes

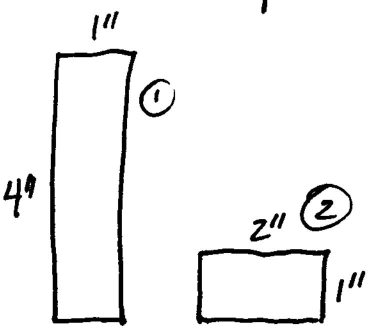




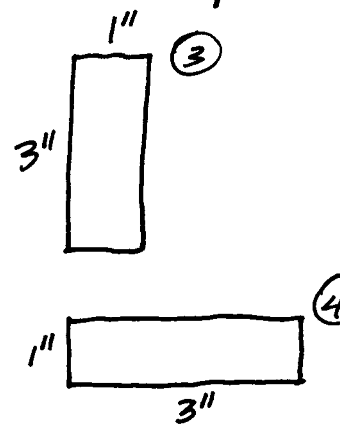
$\bar{I}_x = \frac{1}{12} b h^3$

$\bar{I}_y = \frac{1}{12} h b^3$

(B) Break composite shape into components.



or



(C) Determine $\bar{x}$, $\bar{y}$, I_x, I_y for each component

Component	$\bar{x}_{(i)}$	$\bar{y}_{(i)}$	$\bar{I}_x^{(i)}$	$\bar{I}_y^{(i)}$
1	0	1.5	$\frac{1}{12}\cdot 1\cdot 4^3 = 5\frac{1}{3}$	$\frac{1}{12}\cdot 4\cdot 1^3 = \frac{1}{3}$
2	1.5	0	$\frac{1}{12}\cdot 2\cdot 1^3 = \frac{1}{6}$	$\frac{1}{12}\cdot 1\cdot 2^3 = \frac{2}{3}$
3	0	2	$\frac{1}{12}\cdot 1\cdot 3^3 = \frac{27}{12}$	$\frac{1}{12}\cdot 3\cdot 1^3 = \frac{1}{4}$
4	1	0	$\frac{1}{12}\cdot 3\cdot 1^3 = \frac{1}{4}$	$\frac{1}{12}\cdot 1\cdot 3^3 = \frac{9}{4}$

D Determine $\bar{x}, \bar{y}$ with weighted average

1 & 2 $\rightarrow$
$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A_1 + A_2} = \frac{0 + 1.5 \cdot 2}{4+2} = \frac{1}{2}$$

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2} = \frac{1.5 \cdot 4 + 0}{4+2} = 1$$

3 & 4 $\rightarrow$
$$\bar{x} = \frac{\bar{x}_3 A_3 + \bar{x}_4 A_4}{A_3 + A_4} = \frac{0 + 1 \cdot 3}{3+3} = \frac{1}{2} \checkmark$$

$$\bar{y} = \frac{\bar{y}_3 A_3 + \bar{y}_4 A_4}{A_3 + A_4} = \frac{2 \cdot 3 + 0}{3+3} = 1 \checkmark$$

E Determine $\bar{I}_x$ & $\bar{I}_y$ using parallel axis theorem

1 & 2 :
$$\bar{I}_x = \bar{I}_x^{①} + A_1 d_{x_1}^2 + \bar{I}_x^{②} + A_2 d_{x_2}^2$$

$$= 5\tfrac{1}{3} + 4\underbrace{(\bar{y} - \bar{y}_1)^2}_{0.25} + \frac{1}{6} + 2\underbrace{(\bar{y} - \bar{y}_2)^2}_{1}$$

$$= 5\tfrac{1}{3} + 1 + \frac{1}{6} + 2 = 8.5 \text{ in}^4$$

$$\bar{I}_y = \bar{I}_y^{①} + A_1 d_{y_1}^2 + \bar{I}_y^{②} + A_2 d_{y_2}^2$$

$$= \frac{1}{3} + 4\underbrace{(\bar{x} - \bar{x}_1)^2}_{0.25} + \frac{2}{3} + 2\underbrace{(\bar{x} - \bar{x}_2)^2}_{1}$$

$$= \frac{1}{3} + 1 + \frac{2}{3} + 2 = 4 \text{ in}^4$$

3 & 4: $\bar{I}_x = \bar{I}_x^{(3)} + A_3 \underbrace{d_{x_3}^2}_{(\bar{y}-\bar{y}_3)^2} + \bar{I}_x^{(4)} + A_4 \underbrace{d_{x_4}^2}_{(\bar{y}-\bar{y}_4)^2}$

$= \frac{9}{4} + 3\cdot 1 + \frac{1}{4} + 3\cdot 1 = 8.5 \text{ in}^4$ ✓

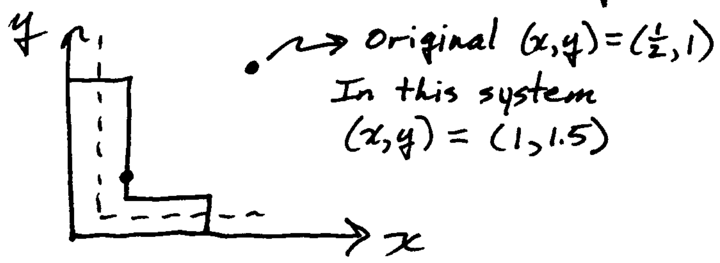
$\bar{I}_y = \bar{I}_y^{(3)} + A_3 d_{y_3}^2 + \bar{I}_y^{(4)} + A_4 d_{y_4}^2$

$= \frac{1}{4} + 3\cdot\frac{1}{4} + \frac{9}{4} + 3\cdot\frac{1}{4} = 4 \text{ in}^4$ ✓

* Steps A → E get to the solution for any choice of axes and any choice of component shapes.

Tricky part is d_x is a distance in the y-direction and d_y is a distance in the x-direction

What about I_x & I_y for these axes?



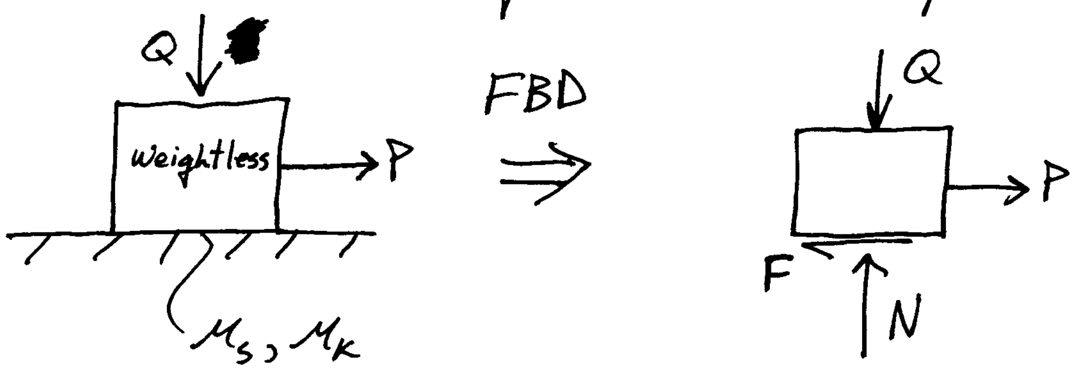
$I_x = \bar{I}_x + A d_x^2 = 8.5 + 6(1.5)^2 = 22 \text{ in}^4$

$I_y = \bar{I}_y + A d_y^2 = 4 + 6(1)^2 = 10 \text{ in}^4$

Friction

We will consider a simple model for dry friction that is commonly known as Coulomb friction.

Consider the following simple configuration.



N = normal force that the surface (ground) places on the block.

F = friction force that the surface places on the block. If the block is moving F acts in the direction opposite to that of the motion. If the block is stationary then F acts to equilibrate the other tangential forces in the system, and <u>impending</u> motion is in the opposite direction of F.

N is always normal to the plane of contact and F is always in (tangential to) the plane of contact.

The Coulomb friction law states that

$F \leq \mu_s N$ static case (equality implies impending sliding)

$F = \mu_k N$ moving case

μ_s = coefficient of static friction

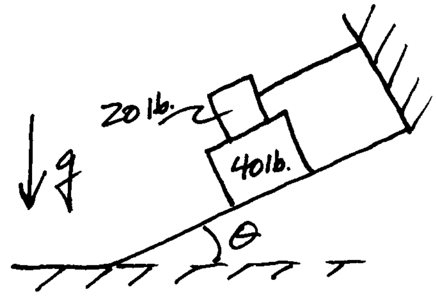
μ_k = coefficient of kinematic friction

Typical values for μ_s and μ_k

	μ_s	μ_k
Steel / Steel	0.78	0.42
Al / mild steel	0.61	0.47
Teflon / steel	0.04	very very close to ϕ
Nickle / Nickle	1.10	0.53
Cu / cast iron	1.05	0.29
Metal / wood	0.6	0.4
Wood / wood	0.6	0.5
Metal / Stone	0.7	0.4
Rubber tires / Dry Pavement	0.9	0.8

The analysis of problems with friction are no different than any other statics problems. We still draw FBDs and do equilibrium analysis.

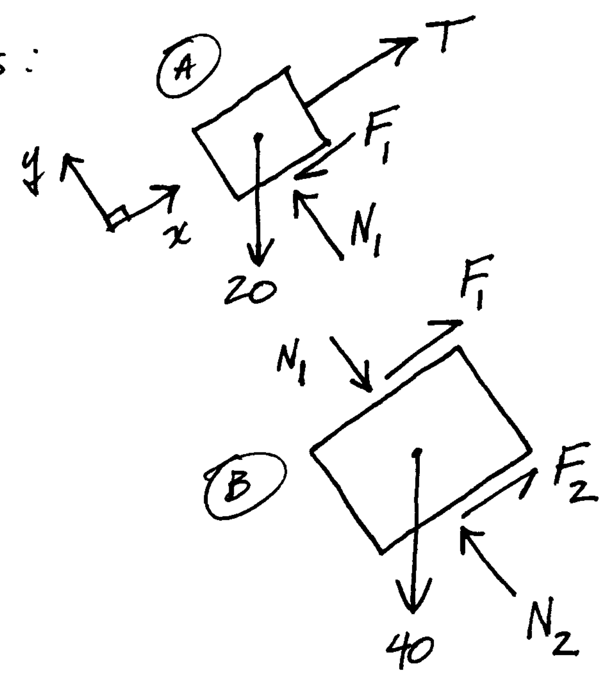
Example:



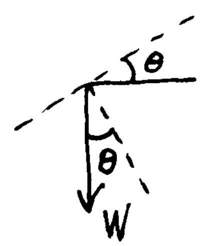
$\mu_s = \frac{1}{2}$ for all surfaces

What is θ if the 40 lb. block is just about to slide down the incline?

FBDs:



* For impending sliding problems it is <u>necessary</u> to get the direction of the friction force correct.



Equilibrium: A $\Sigma F_x = T - F_1 - 20\sin\theta = 0$

$$\Sigma F_y = N_1 - 20\cos\theta = 0$$

ΣM_z only helps us determine the <u>location</u> of N_1 and F_1 but not the magnitude.

B $\Sigma F_x = F_1 + F_2 - 40\sin\theta = 0$

$$\Sigma F_y = N_2 - N_1 - 40\cos\theta = 0$$

Right now we have $T, F_1, F_2, N_1, N_2, \theta$ as unknowns but only 4 equations. Again, the moment balances do not help us because we would need to introduce the unknown locations x_1 and x_2 of (N_1, F_1) and (N_2, F_2).

Since sliding is <u>impending</u> we also know

$$F_1 = \mu_s N_1 \quad \text{and} \quad F_2 = \mu_s N_2$$

$$F_1 = \tfrac{1}{2} N_1 \qquad\qquad F_2 = \tfrac{1}{2} N_2$$

$$\rightarrow \quad T - \tfrac{1}{2} N_1 - 20 \sin\theta = 0$$

$$N_1 = 20 \cos\theta$$

$$\rightarrow \quad T = 10 \cos\theta + 20 \sin\theta$$

$$\tfrac{1}{2} N_1 + \tfrac{1}{2} N_2 - 40 \sin\theta = 0$$

$$N_2 - N_1 - 40 \cos\theta = 0 \rightarrow N_2 = N_1 + 40\cos\theta$$

$$\rightarrow 10\cos\theta + \underbrace{(10\cos\theta + 20\cos\theta)}_{\frac{1}{2}N_2} - 40\sin\theta = 0$$

$$40\cos\theta - 40\sin\theta = 0$$

$$\cos\theta - \sin\theta = 0$$

$$1 - \frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = 1 \longrightarrow \theta = 45^\circ$$