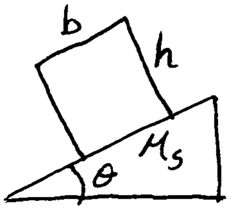
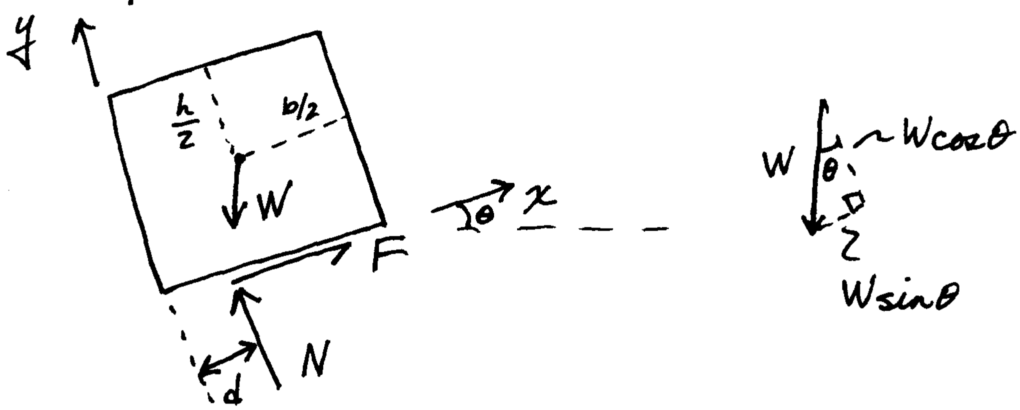
Tipping versus Sliding



Consider a block of weight W with base b and height h. The coefficient of friction between the block and the inclined plane is μ_s.

If we gradually increase the angle of the inclined plane from $\theta = 0$, what conditions are required such that the block will slide before tipping or visa versa?

FBD:



We know that the line of action of F is along the bottom surface of the block, so as far as equilibrium of the block is concerned we can move F anywhere along this line. However, the effective point of application of the unknown distribution of normal force does need to be located and a priori is unknown.

Analysis: $\sum F_x = F - W\sin\theta = 0$
$$F = W\sin\theta$$

$$\sum F_y = N - W\cos\theta = 0$$
$$N = W\cos\theta$$

The block will not slide if $F < \mu_s N$ and is just about to slide if $F = \mu_s N$.

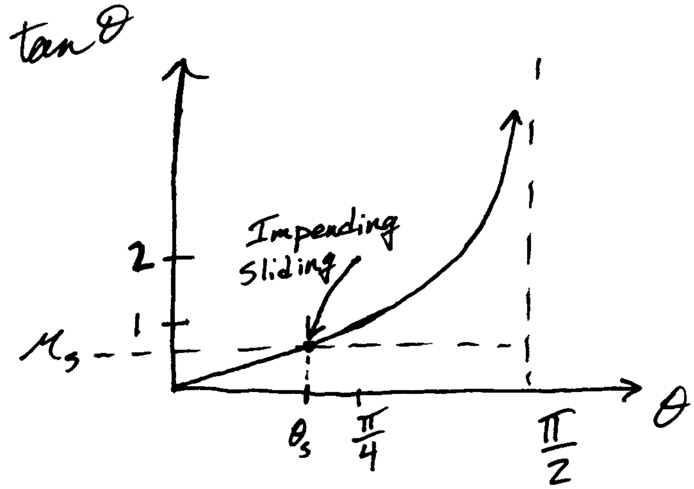
$F < \mu_s N \rightarrow \quad W\sin\theta < \mu_s W\cos\theta$

$\tan\theta < \mu_s \rightarrow$ no sliding

$\tan\theta = \mu_s \rightarrow$ impending sliding

Many materials $\mu_s \sim 0.5-0.6$
$\Rightarrow \theta \sim 26° - 31°$

Recall $\mu_s = 1$ is very high $\rightarrow$ almost anything slides at $\theta = 45°$



We have not considered if this condition can be reached before the block tips.

$$\sum M_z^o = -W\cos\theta \frac{b}{2} + W\sin\theta \frac{h}{2} + Nd = 0$$

We can solve this for d

$$d = \frac{1}{2} \frac{b\cos\theta - h\sin\theta}{\cos\theta}$$

$$d = \frac{h}{2}\left(\frac{b}{h} - \tan\theta\right)$$

When does tipping occur? There are several ways to think about this. If the block is just about to tip then the block is ~~[illegible]~~ losing contact with the surface at all points except the corner. This means N is at the corner and $d=0$.

or $d > 0 \rightarrow$ no tipping

$d = 0 \rightarrow$ impending tipping

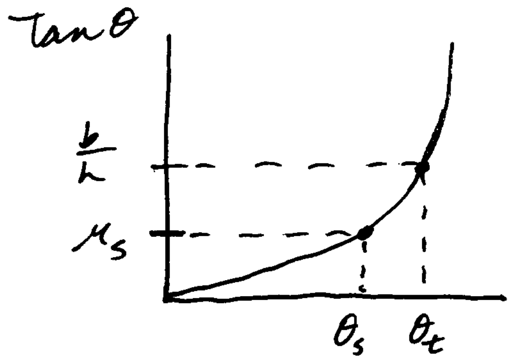
$d > 0 \rightarrow \frac{b}{h} > \tan\theta$ no tipping

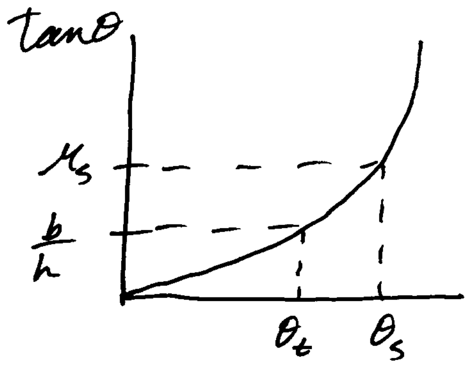
$\frac{b}{h} = \tan\theta$ impending tipping

What is the condition for sliding before tipping.

$\mu_s > \tan\theta$ no sliding

$\frac{b}{h} > \tan\theta$ no tipping





$\mu_s < \frac{b}{h} \rightarrow$ slides first

$\frac{b}{h} < \mu_s \rightarrow$ tips first

Does this make sense?

Slippery surface $\rightarrow \mu_s$ small $\rightarrow$ more likely to slide first

$\frac{b}{h}$ small $\rightarrow$ tall and/or narrow $\rightarrow$ more likely to tip

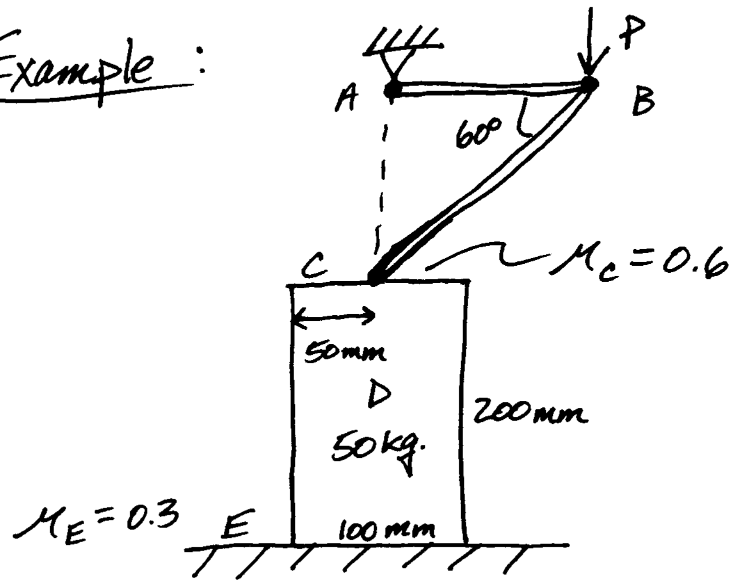
Example:

A & B have pins

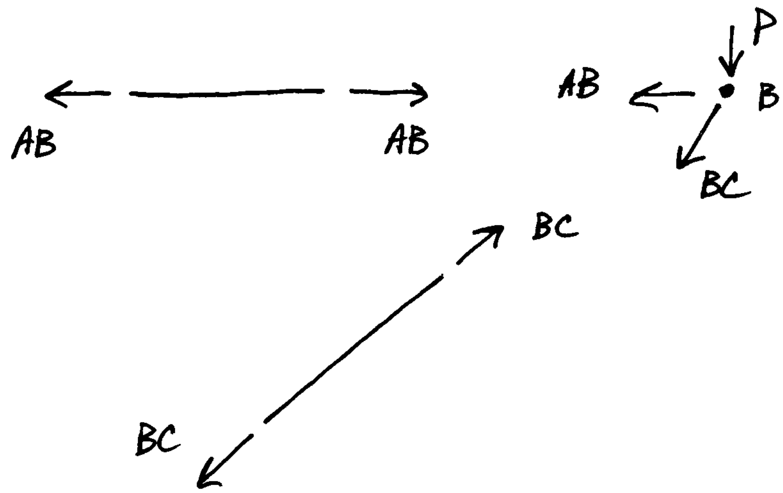
C is in contact (no pin)

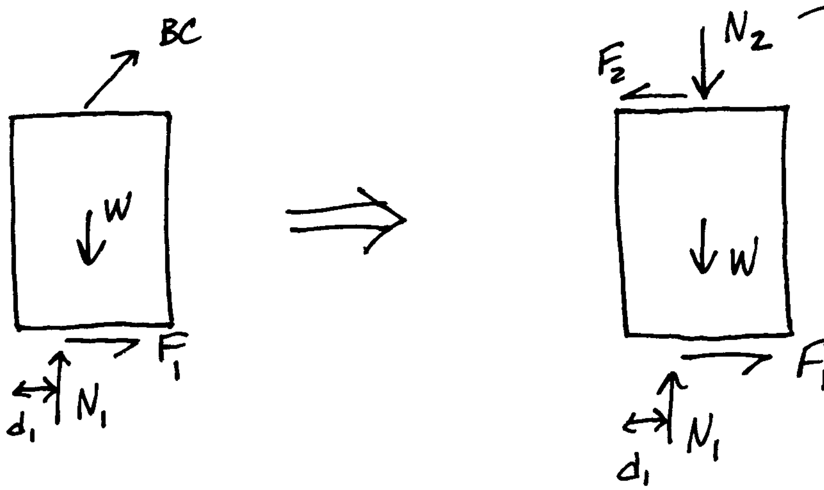
What is P_{max}?

What type of motion is impending at $P = P_{max}$?



Note that AB and BC are 2-force members.





Note F_2 and N_2 are directly related to BC.

Equilibrium of joint B: $\sum F_y = -P - \frac{\sqrt{3}}{2} BC = 0$

$$\rightarrow BC = -\frac{2}{\sqrt{3}} P$$

$$N_2 = -\frac{\sqrt{3}}{2} BC = P$$

$$F_2 = -\frac{1}{2} BC = \frac{1}{\sqrt{3}} P$$

We could find AB by using $\sum F_x$, but this is not necessary.

Check condition for sliding at C

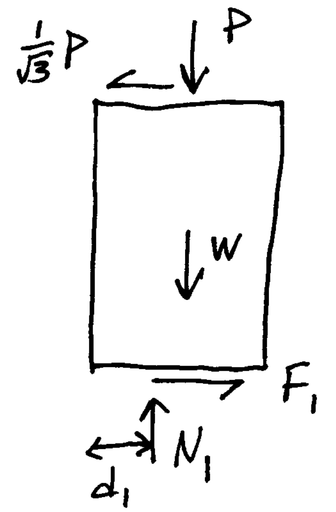
$$F_2 \le \mu_c N_2$$

$$\frac{1}{\sqrt{3}} P \le 0.6 P$$

$$0.577 \le 0.6 \rightarrow \text{Always true}$$

$\therefore$ no sliding at C

Block:



Equilibrium: $\Sigma F_x = F_1 - \frac{1}{\sqrt{3}} P = 0$

$$F_1 = \frac{1}{\sqrt{3}} P$$

$$\Sigma F_y = N_1 - P - 50(9.81) = 0$$

$$N_1 = P + 490.5$$

Check sliding: $F_1 \leq \mu_E N_1$

$$\frac{1}{\sqrt{3}} P \leq 0.3(P + 490.5)$$

$$0.577 P \leq 0.3P + 490.5(0.3)$$

$$0.277 P \leq 147.15$$

$$P \leq 531 \text{ N}$$ for no sliding at E

Tipping $\rightarrow d_1 = 0$ $\quad \Sigma M_z^o = -W(0.05) + \frac{1}{\sqrt{3}} P(0.2)$

$$- P(0.05) = 0$$

$$\rightarrow -0.05W + 0.115P - 0.05P = 0$$

$$P = 0.764 W$$

$$= 0.764(490.5)$$

$$P = 374.6 \text{ N}$$ for impending tipping

$$P \leq 374.6 \text{ N}$$ for no tipping

$\rightarrow$ $\boxed{P_{max} = 374.6 \text{ N}}$

at $P = P_{max}$ there is impending tipping at E

Review

1) Vectors
   - Dot products
   - Cross products
   - Unit vectors
   - Direction cosines
   - Determing a vector direction from two points or the normal to a plane

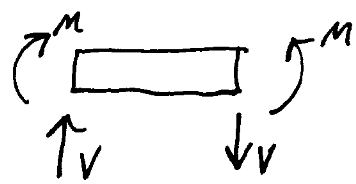
2) Forces and moments
   - Resolving a force in a given direction
   - Normal and tangential components of a force to a plane
   - Moment of a force about a point
   - Moment of a force about an axis

3) Equilibrium analysis
   - Free body diagrams
   - $\Sigma \vec{F} = 0$, $\Sigma \vec{M}_A = 0$

4) Trusses
   - Determination of reactions
   - Method of joints
   - Method of sections
   - Best practice is to assume tension and allow the sign to determine tension or compression.

5) Frames and machines
- FBDs of components
- Recognize 2-force members when possible
- Equal but opposite internal forces on adjoining components
- Equilibrium analysis
- Recognizing unknowns and the equations that can be solved to determine them is of primary importance

6) Shear force and bending moment diagrams
- Determining locations of cuts and number of cuts
- Convention

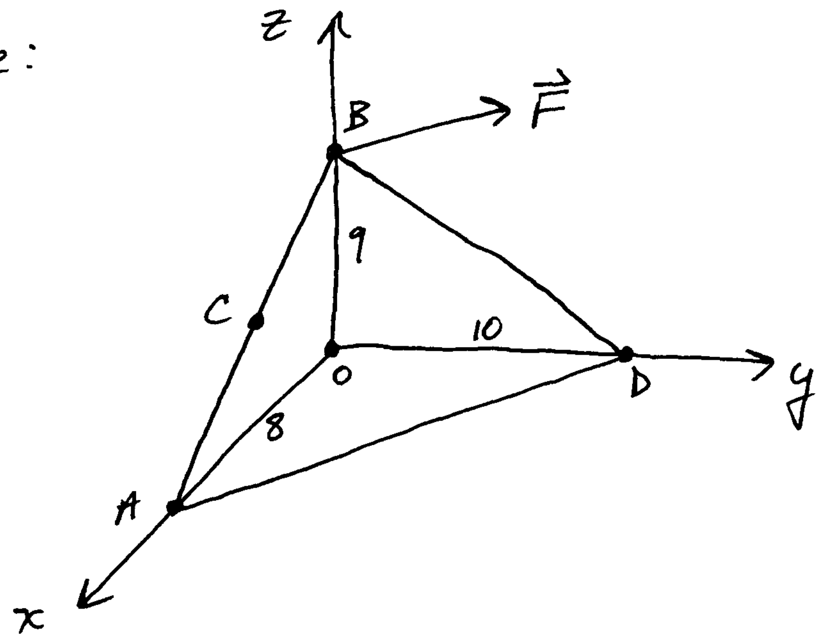


- Cut first, including the distributed load and then analyze
- FBDs of the part you analyze
- Equilibrium analysis

7) Centroids and moments of inertia
- Composite areas
- Parallel axis theorem

8) Friction and tipping
- FBDs and equilibrium analysis
- Conditions for sliding and tipping.

Example:



$\vec{F}$ is perpendicular to the plane with a positive x-component and magnitude π.

Determine the moment due to $\vec{F}$ about an axis passing through points O and C.

$$\vec{M}_{OC} = \left(\vec{M}_O \cdot \vec{e}_{OC}\right)\vec{e}_{OC}$$

$$\vec{e}_{OC} = \frac{\vec{OC}}{|\vec{OC}|} = \frac{4\vec{i} + 4.5\vec{k}}{\sqrt{4^2 + 4.5^2}} = 0.664\vec{i} + 0.747\vec{k}$$

$$\vec{M}_O = \vec{r}_{OB} \times \vec{F}$$

$$\vec{r}_{OB} = 9\vec{k}$$

$$\vec{F} = F\,\vec{e}_\perp$$

$$\vec{e}_\perp = \frac{\vec{AD} \times \vec{AB}}{|\vec{AD} \times \vec{AB}|} = \frac{(-8\vec{i} + 10\vec{j}) \times (-8\vec{i} + 9\vec{k})}{|(\text{ magnitude })|}$$

$$= \frac{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8 & 10 & 0 \\ -8 & 0 & 9 \end{vmatrix}}{|(\text{magnitude})|} = \frac{90\vec{i} + 72\vec{j} + 80\vec{k}}{\sqrt{90^2 + 72^2 + 80^2}}$$

$$= 0.641\vec{i} + 0.513\vec{j} + 0.570\vec{k}$$

$$\rightarrow \vec{F} = \pi \vec{e}_\perp = 2.014\vec{i} + 1.612\vec{j} + 1.791\vec{k}$$

$$\vec{M}_o = \vec{r}_{OB} \times \vec{F} = 9\vec{k} \times (2.014\vec{i} + 1.612\vec{j} + 1.791\vec{k})$$

$$= 9(2.014)\underbrace{\vec{k} \times \vec{i}}_{\vec{j}} + 9(1.612)\underbrace{\vec{k} \times \vec{j}}_{-\vec{i}}$$

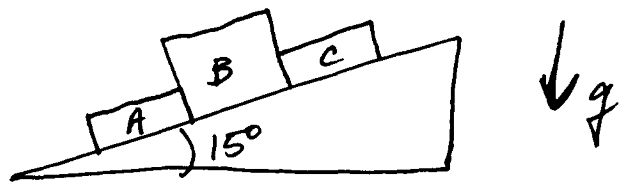
$$\vec{M}_o = -14.51\vec{i} + 18.13\vec{j}$$

$$\vec{M}_o \cdot \vec{e}_{oc} = -14.51(0.664) = -9.63$$

$$\vec{M}_{oc} = -9.63(0.664\vec{i} + 0.747\vec{k})$$

$$\boxed{\vec{M}_{oc} = -6.4\vec{i} - 7.2\vec{k}}$$

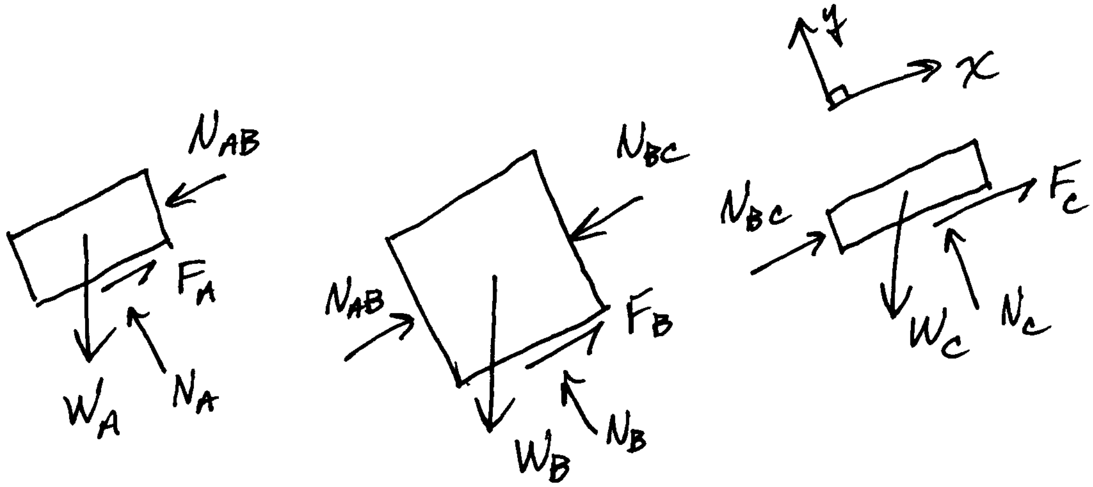
Example:



$W_A = 20$ lb $\mu_A = 0.3$
$W_B = 40$ lb $\mu_B = 0.2$
$W_C = 30$ lb $\mu_C = 0.35$

If the blocks are released from rest, what happens?

There are several ways to approach this problem, but they all begin with FBDs.



Assuming Equilibrium:

A: $\Sigma F_x = F_A - N_{AB} - W_A \sin 15° = 0$
$\Sigma F_y = N_A - W_A \cos 15° = 0$
$N_A = W_A \cos 15° = 19.32$ lbs.

B: $\Sigma F_x = N_{AB} - N_{BC} + F_B - W_B \sin 15° = 0$
$\Sigma F_y = N_B - W_B \cos 15° = 0 \rightarrow N_B = 38.64$ lbs.

C. $\Sigma F_x = N_{BC} + F_C - W_C \sin 15° = 0$
$\Sigma F_y = N_C - W_C \cos 15° = 0 \rightarrow N_C = 28.98$ lbs.

Here there are several approaches to determine what happens. Perhaps the most rational first step is to determine if each block can be in equilibrium on its own, i.e. are each of the friction conditions satisfied if $N_{AB}=0$ and $N_{BC}=0$?

$$N_{AB}=0 \Rightarrow F_A = W_A \sin 15^\circ = 5.176 \text{ lbs.}$$

$$F_A \overset{?}{\leq} \mu_A N_A \Rightarrow 5.176 \leq 5.796 \checkmark$$

A can be in equilibrium on its own

$$N_{AB}=0 \text{ and } N_{BC}=0 \Rightarrow F_B = W_B \sin 15^\circ = 10.353 \text{ lbs.}$$

$$F_B \overset{?}{\leq} \mu_B N_B \Rightarrow 10.353 \leq 7.728 \ \times$$

B cannot be in equilibrium on its own.

$$N_{BC}=0 \Rightarrow F_C = W_C \sin 15^\circ = 7.765 \text{ lbs.}$$

$$F_C \overset{?}{\leq} \mu_C N_C \Rightarrow 7.765 \leq 10.143 \checkmark$$

C can be in equilibrium on its own.

What happens? B wants to slide down the incline, so it will disconnect from C leaving $N_{BC}=0$ and bump into A causing $N_{AB} \neq 0$. Can A support B?

$N_{AB} \neq 0 \rightarrow \quad F_A - N_{AB} - W_A \sin 15° = 0$

$$F_B + N_{AB} - \cancel{N_{BC}}^{\,0} - W_B \sin 15° = 0$$

sum $\rightarrow \quad F_A + F_B - (W_A + W_B)\sin 15° = 0$

We know B wants to slide, so $F_B = \mu_B N_B$

$$F_B = 7.765 \text{ lbs.}$$

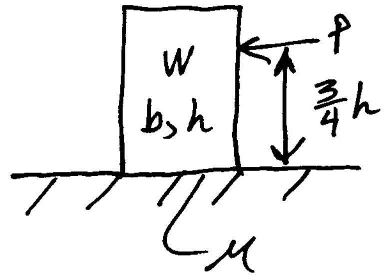
$$\rightarrow F_A = (W_A + W_B)\sin 15° - 7.765$$

$$F_A = 7.764 \text{ lbs.}$$

is $F_A \leq \mu_A N_A$? $\quad 7.764 \leq 5.796$ X

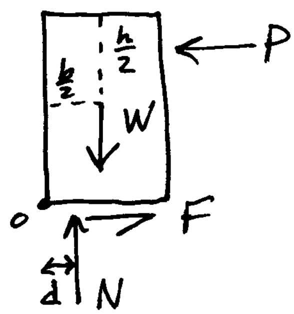
No. Therefore A cannot support B, and A & B will slide down the plane together.

Example:



Conditions for tipping versus sliding?

FBD:



$$\Sigma F_x = P - F = 0 \rightarrow F = P$$

$$\Sigma F_y = N - W = 0 \rightarrow N = W$$

$$\Sigma M_z^o = -W\frac{b}{2} + P\frac{3h}{4} + Nd = 0$$

Impending sliding $\rightarrow F = \mu N \rightarrow P = \mu W$

Impending tipping $\rightarrow d = 0 \rightarrow P = \frac{2b}{3h} W$

If P is increased from ϕ, sliding will occur first if $\mu W < \frac{2b}{3h} W$ and tipping will occur first if $\rightarrow \frac{2b}{3h} < \mu$ [redacted]