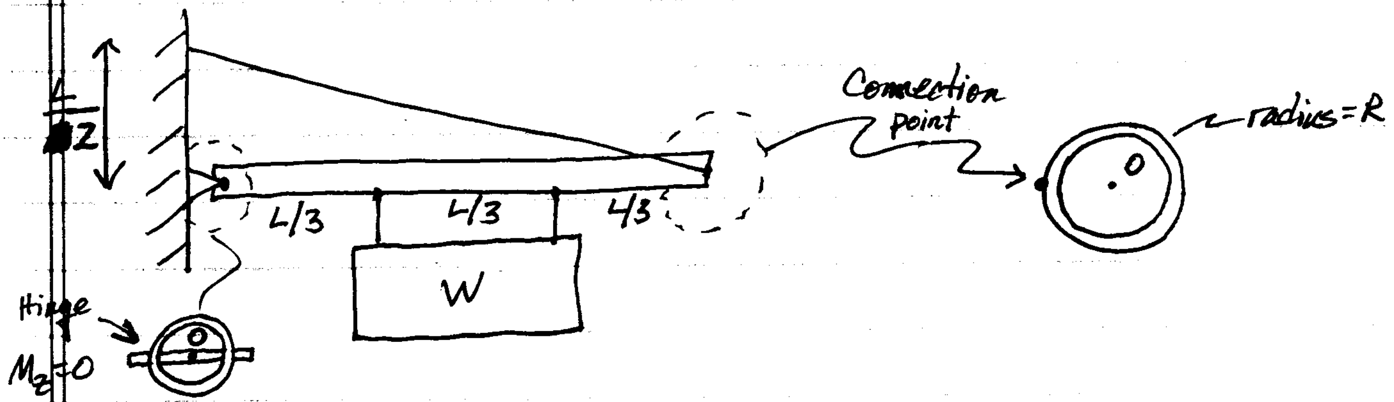
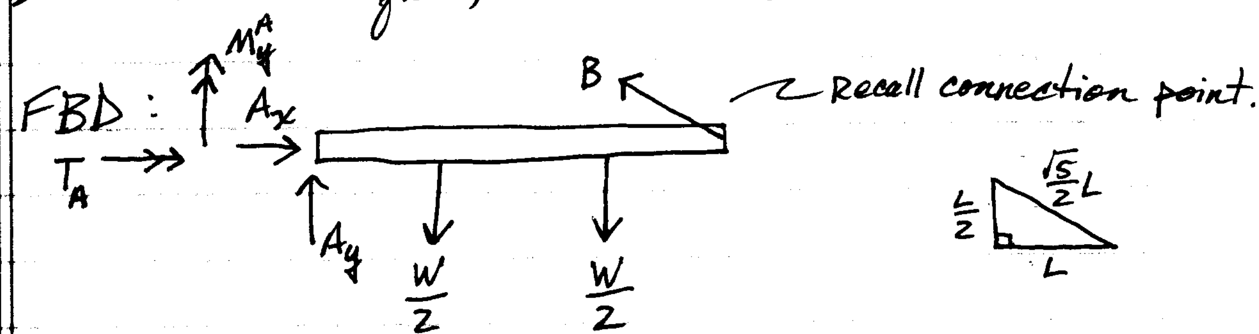
Combined Loadings



Determine torques, moments and forces in beam.



$$\Sigma F_x = A_x - \frac{2}{\sqrt{5}}B = 0 \rightarrow A_x = \frac{2}{\sqrt{5}}B$$

$$\Sigma F_y = A_y + \frac{1}{\sqrt{5}}B - W = 0$$

$$\Sigma M_z^A = -\frac{W}{2}\frac{L}{3} - \frac{W}{2}\frac{2L}{3} + \frac{1}{\sqrt{5}}BL = 0$$

$$B = \frac{\sqrt{5}}{2}W \rightarrow A_x = W \ , \ A_y = \frac{W}{2}$$

$$\Sigma M_x^o = T_A - \frac{1}{\sqrt{5}}BR = 0 \rightarrow T_A = \frac{WR}{2}$$

$$\Sigma M_y^o = M_y^A - \frac{2}{\sqrt{5}}BR = 0 \rightarrow M_y^A = WR$$

At any section of this beam the internal reactions can include an axial force, a shear force in y, an axial torque, a z bending moment, and a y bending moment. Each of these internal forces will contribute to the stresses in the beam.

At any cut the axial force is the same.

FBD for x-forces: $W \rightarrow \boxed{\quad\quad} \rightarrow F \quad \therefore F = -W$

$$\sigma_{Axial} = \frac{-W}{A}$$

FBD for x-torques: $\underset{\frac{WR}{2}}{\twoheadrightarrow} \boxed{\quad\quad} \twoheadrightarrow T \quad \therefore T = -\frac{WR}{2}$

$$\tau_{torque}^{max} = \frac{TR}{I_p} = -\frac{WR^2}{2I_p}$$

FBD for y-moments: $\overset{WR}{\uparrow} \boxed{\quad\quad} \downarrow M_y \quad \therefore M_y = WR$

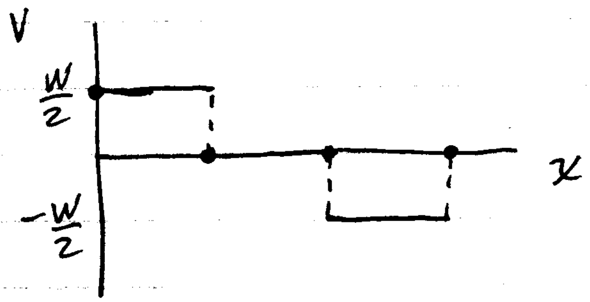
$$\sigma_{y\text{-bending}} = \frac{-M_y z}{I_y}$$

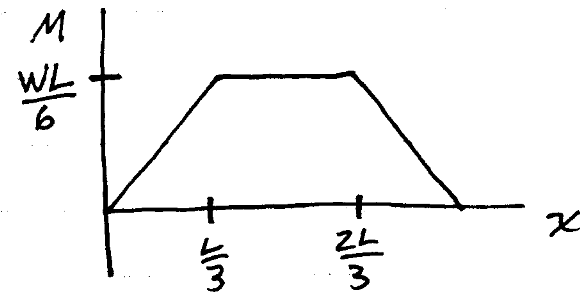
Finally, for z-moments we need several cuts or integration of $g(x)$.

$$q(x) = \frac{W}{2}\langle x - \tfrac{L}{3}\rangle^{-1} + \frac{W}{2}\langle x - \tfrac{2L}{3}\rangle^{-1}$$

$$V(x) = -\frac{W}{2}\langle x - \tfrac{L}{3}\rangle^{0} - \frac{W}{2}\langle x - \tfrac{2L}{3}\rangle^{0} + \underbrace{\frac{W}{2}}_{A_y}$$

$$M(x) = -\frac{W}{2}\langle x - \tfrac{L}{3}\rangle - \frac{W}{2}\langle x - \tfrac{2L}{3}\rangle + \frac{W}{2}x$$



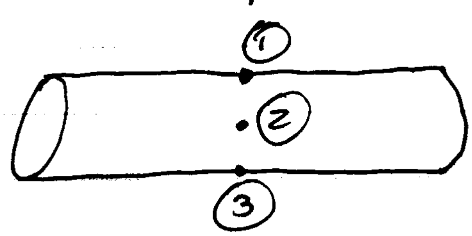


$$M_z^{max} = \frac{WL}{6} \qquad \sigma_{z\text{-bending}} = \frac{-M_z y}{I_z}$$

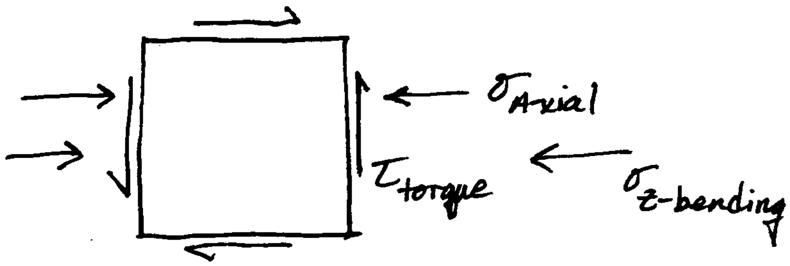
$$V^{max} = \frac{W}{2} \qquad \tau_{shear} = \frac{VQ}{I_z b}$$

Where do all of these stresses appear on a stress-element?

First, there are at least 3 interesting points to look at on any x-location along the bar.



1 : Looking down on top of beam



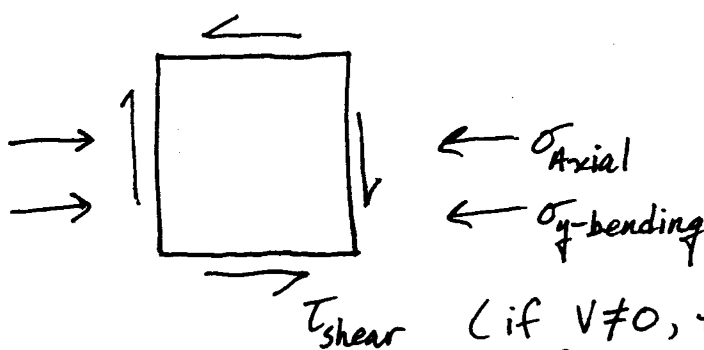
1 → $\sigma_{xx} = -|\sigma_{Axial}| - |\sigma_{z\text{-bending}}|$

$\sigma_{yy} = 0$, $\tau_{xy} = \tau_{torque}$

$\sigma_{y\text{-bending}} = 0$ b/c $z = 0$ at 1

$\tau_{shear} = 0$ b/c $\frac{Q}{b} = 0$ at 1

2



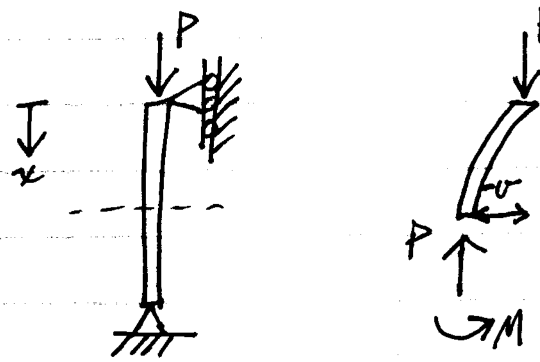
(if $V \neq 0$, this direction if $V > 0$)

3 → Same as 1 but change direction of $\sigma_{z\text{-bending}}$.

Buckling

Buckling is a competition between increasing the stored energy in a column due to bending stresses and decreasing the energy of the load by allowing it to move in its direction of application and do work.

The analysis requires us to include the beams deflection in the FBD.



$$\Sigma M_z = M + Pv = 0$$
$$M = -Pv$$

Note sign on v is due to convention.

But we also know $M = EI\chi = EI\dfrac{d^2v}{dx^2}$

$$\therefore \Sigma M_z = 0 \rightarrow EI\frac{d^2v}{dx^2} + Pv = 0$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

2nd order ODE with constant coefficients.

Try $v(x) = A\sin nx + B\cos nx$

$$\frac{d^2v}{dx^2} = -An^2 \sin nx + -Bn^2 \cos nx$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = A\left(\frac{P}{EI} - n^2\right)\sin nx + B\left(\frac{P}{EI} - n^2\right)\cos nx$$

$$= 0$$

$\sin nx$ and $\cos nx$ are orthogonal functions so either $A = B = 0$ (which is the non-buckled solution) or $n^2 = \frac{P}{EI}$.

$$v(x) = A\sin\left(\sqrt{\frac{P}{EI}}\,x\right) + B\cos\left(\sqrt{\frac{P}{EI}}\,x\right)$$

Boundary conditions: $v(x=0) = 0$
$v(x=L) = 0$

$$v(x=0) = B = 0$$
$$v(x=L) = A \sin\sqrt{\frac{PL^2}{EI}} = 0$$

$A = 0$ (non-buckled solution)

or $\sqrt{\frac{PL^2}{EI}} = \pi, 2\pi, 3\pi, \ldots, n\pi$

$$\boxed{P_{cr} = \frac{n^2\pi^2 EI}{L^2}}$$