Bond Graphs for Engineers
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An Epistemic Prehistory of Bond Graphs

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Abstract

This account traces the personal experience and knowledge-base which led to the development of bond graphs.

1. PROFESSIONAL INFLUENCES

Knowing this writer's early preconceptions and preoccupations may help readers to understand better the origin of bond graphs. In the critical decade 1949-1959 his professional focus spanned:

- -- hydroelectric plants & industrial processes;
- -- analog & digital computing;
- -- nonlinear dynamics & control.

These activities embraced civil, mechanical, electrical and electronic engineering and were respectively expressed through ASCE,ASME,AIEE, and IRE papers, discussions and meetings. It was thus impossible to avoid becoming a systems engineer and practicing generalist.

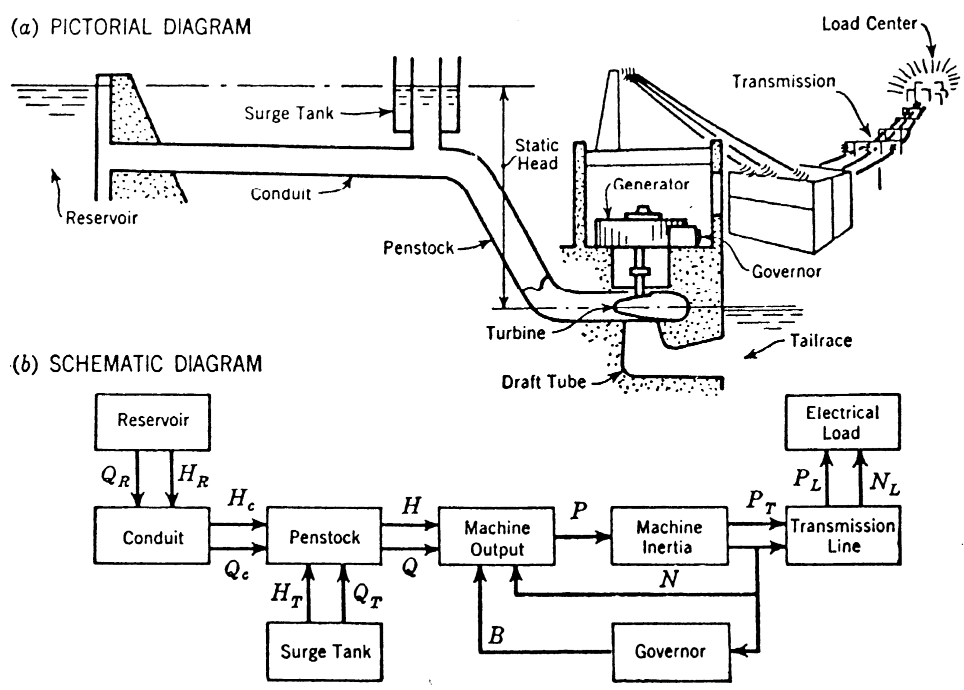
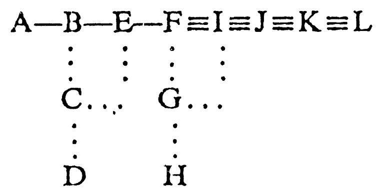
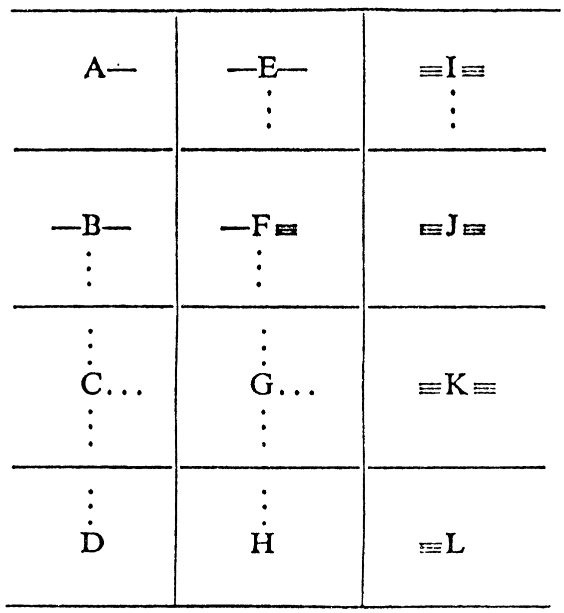


Figure 1. Hydroelectric Plant [1949]

An interconnected system



Primitive components or elements



Names of the elements

A: Energy supply;
B: Fluid turbine;
C: Speed governor;
D: Speed set;
E: Mechanical coupling;
F: Synchronous alternator;
G: Voltage regulator
H: Voltage set
I: Bus bars
J: Phase transformers
K: Transmission line
L: System bus

Figure 2. Hydroelectric Plant [1959]

To establish a timeline for the evolution of BGs an early benchmark is provided by Fig.1 which was originally drawn in 1949 and serves as a 'bad-but-beautiful' example of several misunderstandings regarding arrows immediately apparent to bondgraphers [1,2]. In contrast, Fig.2 represents this same system as schematized just a decade later in 1959 [3]. It is also significant that while Fig.2 can be readily communicated by last century's telephone or teletype, Fig.1 requires a fax machine only now becoming commonplace.

Clearly, this author's special training and experience in hydroelectric power actually forced certain insights upon him, most particularly an awareness of the strong analogies existing between:

TRANSMISSION: fluid pipes & electric lines;
TRANSDUCTION: turbines & generators;
CONTROL: speed governors & voltage regulators.

When these analogous devices were reduced to equations for computer simulation, the distinctions became completely blurred. Mathematical logic dictated that such compelling analogies implied an underlying common generalization from which other beneficial specializations might ensue. Bond graphs were the inevitable result.

After spending the eight years 1946 to 1954 in the MIT Civil Engineering Dept., the writer moved over to Mechanical Engineering to establish the first systems engineering subjects at MIT. It was this specific task which five years later produced bond graphs, drawing naturally upon all the attitudes and experience indicated above.

The point of this account is simply that any quite ordinary human being -- or perhaps even some system of artificial intelligence ! -- would have inexorably developed bond graphs if subjected to the same circumstances and influences as was this author.

2. INFLUENCE OF ENERGY LOCALIZATION

2.1. Energy Density and Energy Flux

The notion that field energy might be localized goes back at least to George GREEN early in the last century [4,5,6]. By the end of that century this concept had given rise to a new and fundamental energy continuity principle through the efforts of UMOV, POYNTING, HEAVISIDE and MIE. As a fundamental part of his dynamical theory of the electromagnetic field [7], MAXWELL had also localized energy in the form of energy densities existing even in empty space and had identified these same with the real electromagnetic stresses whose divergence produced local body forces upon ponderable matter. From MAXWELL's equations a general continuity theorem on the transfer of energy in the electromagnetic field was then derived in 1884 by John Henry POYNTING [8], and, independently, by Oliver HEAVISIDE [9], who further extended this principle to hold for continuum physics generally.

2.2. Continuity Equations

Rather than outlining here this rich, century-long elucidation of energy localization, including its relation to entropy, exergy and relativity, instead we excerpt only some key ideas, starting with a quote from Oliver HEAVISIDE [9]:

> " ...If we can localize energy definitely in space, then we are bound to ask how energy goes from place to place. [We may] enunciate [such a] principle thus:
>
> When energy goes from place to place
> it traverses the intermediate space."

Perhaps HEAVISIDE wrote this expression with tongue-in-cheek for he omitted any reference to the earlier scholastic aphorism of Thomas AQUINAS:

> Angels in travelling from place to place
> pass through the intervening space.

But HEAVISIDE went on to say [9]:

> " The idea that energy has position, therefore, naturally involves the idea of a flux of energy. Let $\mathbf{A}$ be the vector flux of energy ... and let T be the density of the energy. We may then write:
>
> $$\operatorname{div} \mathbf{A} + \dot{T} = 0."$$

This law is precisely analogous to EULER's continuity equation for fluid masses, a fact about to be explained below.

Then BERTRAND further extended this same idea to establish a continuity law for entropy [10], which now serves as a cornerstone for contemporary irreversible thermodynamics.

2.3. Conservation as Symmetry

Beginning with our own century, Hermann MINKOWSKI [11] expressed the local form of the relativistic conservation laws as the vanishing covariant 4-divergence of the matter-tensor, whose symmetrical components measure the mass, momentum and energy densities. Surprisingly, energy defined in this manner is not globally conserved, CLAUSIUS' dictum notwithstanding. Indeed, no fully-satisfactory global law has yet been discovered !

Light was early shed on this difficulty when, in launching a physics revolution still very much underway (e.g., string theory), the mathematician Emmy NOETHER demonstrated conclusively [12] that all local continuity-conservation laws are the direct consequence of purely local symmetries and correspond to the appropriate group invariants.

2.4. Steinmetz and EE

For engineers, Charles Proteus STEINMETZ then brought MAXWELL's electromagnetism with him when he emigrated to the U.S.. He used it actively at the turn of the century to help found modern electrical engineering and especially AC circuit theory. His influence upon multiport theory and bond graphs is best appreciated by quoting him [13]:

> "The component i, called the *current*, is defined as that factor of the electric power P which is proportional to the magnetic field, and the other component e, called the *voltage*, is defined as that factor of the electric power P which is proportional to the electrostatic field.
>
> Current i and voltage e, therefore, are mathematical fictions, factors of the power P, introduced to represent respectively the magnetic and the electrostatic phenomena."

2.5. Integral Theorems

Finally, then, applying the GREEN-GAUSS divergence theorem to the energy continuity equation relates the surface integral of energy flux in all forms to the volume integral of the time rate of change of energy density in all forms. But this surface integral of each energy-form in turn gives rise to the concept of an energy port, while the several component energy-forms in the volume integral establish the basis for circuit theory and for lumped element mechanics [14].

3. INFLUENCE OF ACOUSTICS AND MICROWAVES

Before mentioning this writer's own detailed investigations of transmission line transients, which drew equally from acoustics and from circuitry, we outline some important results which sprang from the forced and rapid development of microwave theory, provoked by WWII radar technology.

3.1. Ports and Multiports

In 1949 Harold WHEELER proposed the term 'port' [15] to fill a felt need in microwave theory and practice. We quote him [15]:

> "It has been customary to designate each entrance or exit of a network as a pair of terminals, based on the circuit concept of wires and conduction. The result was cumbersome terms such as 'four-terminal networks'. The ultimate confusion was caused by the term 'two-terminal-pair' with the unobvious meaning of a network with two pairs of terminals. Furthermore, the terminal-pair concept becomes artificial in the case of electromagnetic fields transmitting power within boundaries, through holes, and from one region to another in space.
>
> After considering many alternatives, the writer has adopted the term 'portal' or simply 'port' as the general designation of an entrance or exit of a network. A self-impedance becomes a 'one-port'. The usual transducer becomes a 'two-port' with one 'in-port' and one 'out-port'. The general network is designated a 'multi-port'. This plan has received a favorable reaction from the several engineers to whom it has been presented, and is first put to use in this monograph."

3.2. Scattering Matrix

Even before this proposal of WHEELER, specialists in the emergent microwave theory had seen fit to exploit many concepts from optics, acoustics, and other branches of physics, and most especially techniques employing the scattering matrix. The general history is indicated in a recent paper [16] by this writer and Ilene BUSCH-VISHNIAC, while the Radiation Laboratory microwave practice during WWII is well-documented in the book of MONTGOMERY, DICKE and PURCELL [17].

3.3. Transmission and Scattering

But for this writer, introduction to the use of wave-scattering methods first came via the transmission line problems mentioned previously. Particularly for the fluid line, ever since the BERNOULLIs, the dynamics had been customarily treated using forewaves and backwaves. Thus the relation to conservation laws and scattering techniques was quite natural while the connection with causality became readily apparent through analog computation.

Thus this author's extended series of treatments of fluid and other transmission lines [1,2,18,19,20,21,22] served as his own introduction to wave-scatter methods, while explicit treatment of scattering variables and matrices in the specific context of bond graphs occurs in later publications [16,23,24].

4. INFLUENCE OF GABRIEL KRON

4.1. Early Exposure

From the outset, this writer owes much to the work of Gabriel KRON, to which he was introduced even before returning to MIT in 1946 for graduate study. Early on, Ernest KELLER's book [25] supplied an excellent outline of KRON's powerful methods, especially as applied to electromechanical systems treated in full accord with Hamilton-Lagrange dynamics. Later, after KRON's death, Harvey HAPP assembled an excellent memorial volume [26] which contains an exhaustive bibliography.

4.2. Generalization Postulates

Most simply put, KRON's whole philosophy was to generalize yet further OHM's Law in several new directions, building upon the earlier extensions encompassing AC circuitry as previously cited [13,14]. KRON added mechanical translations and rotations to circuit theory, thus extending this technique to non-stationary networks, as well as to dynamics and electromechanics. In an early discussion [28] the writer indicated the promise of KRON's approach, while a later paper [29] written as a tribute after the death of KRON explicitly detailed some of the connections between bond graphs and diakoptics.

4.3. Diakoptics

Diakoptics as a piecewise method of analysis and solution was ultimately formalized by KRON in the early 1950's [27]. As in the precursor concepts of two decades earlier, the basic idea of diakoptics is to analyze a system by first tearing it into several parts or primitives, then in a second step going through an explicit interconnection process to put the system back together. KRON ultimately came to believe that the solutions, themselves, as well as the system equations could be so obtained.

After 1959 it was clear that KRON's connection matrix (Step 2) corresponded to the junction structure in bond graphs while his primitives were the multiport components. But in seeming to use matrix algebra for more than mere interconnection, KRON seemed to be assuming linear superposition beyond justification for nonlinear dynamics. On this point, multiport/bond graph theory, for now at least, parts company with diakoptics.

Nevertheless, without Gabriel KRON to break trail, this investigator would have wandered far astray !

5. INFLUENCE OF GRAPH THEORY

5.1. Circuits

Circuit theory and its graph basis originated early in the last century with Georg OHM [30,31,32] and Gustav KIRCHHOFF [33,34] and by 1950 had become well-developed, sophisticated and specialized. One notes that an early instance of a graph and its spanning trees occurs in the second cited KIRCHHOFF paper. But it was quite clear that for multiports conventional circuit graphing was inadequate and unsuitable. For example, it was not possible to represent 3-port gate valves or multiport heat exchangers as circuit elements.

5.2. Flowgraphs

So, until 1959, it was the custom in simulative computation to employ a form of duplex block diagram or signal flow graph as a system schematic. Signal flow graphs (SFGs) were invented by a late MIT colleague, Samuel MASON [35,36,37]. They achieved nearly instantaneous global success but are less commonly seen today; the reasons for this rapid rise and fall deserve study and reflection, especially by bond graphers. For physical system modeling, the enforcement of mandatory causality without conservation constraints is a limitation to which we return. But much of the decline in this age of Chaos Theory is due in part to the mistaken notion that the branches must be linear and the nodes must be summative. Neither assumption was embraced by MASON, himself !

Another important contributor to SFG theory and applications, Louis ROBICHAUD, become much concerned with the interconnection process and the associated junction constraints, especially in the context of direct analog simulation [38]. In particular, Fig.3 indicates his causal form of the KCL or 0-junction, although he seems to have made little use of the dual law. Here the black half-nodes represent sources or inputs, the white half-nodes, sinks or outputs. Whole-nodes must then be always half-black/half-white to assure causal compatibility during interconnection. Fig.5 from ROBICHAUD serves as an excellent example of the state of affairs just prior to bond graphs. It is clearly causally correct, in contrast to Fig.1 !

Also worthy of note is the use made of SFGs in terms of wave-scattering variables by another one-time MIT colleague, Charles LORENS [39,40].

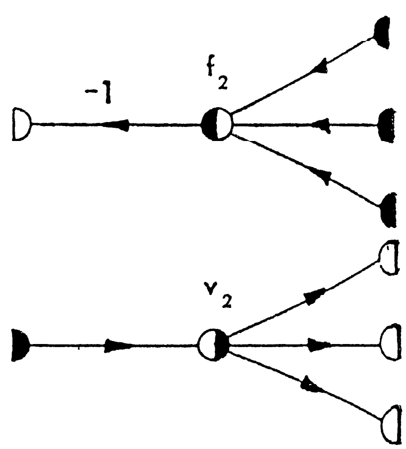


Figure 3. SFG Flow Junction
(ROBICHAUD [38])

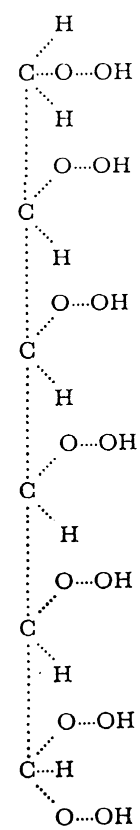


Figure 4. Glucose
(COUPER [41])

5.3. Clues from Chemistry

But for over a decade we remained troubled by three features of these directed graphs:

- -- they are unduly complex and confusing, especially for large systems;
- -- they force premature assignment of causality, unlike circuit diagrams;
- -- they do not enforce conservation constraints, again unlike circuit diagrams.

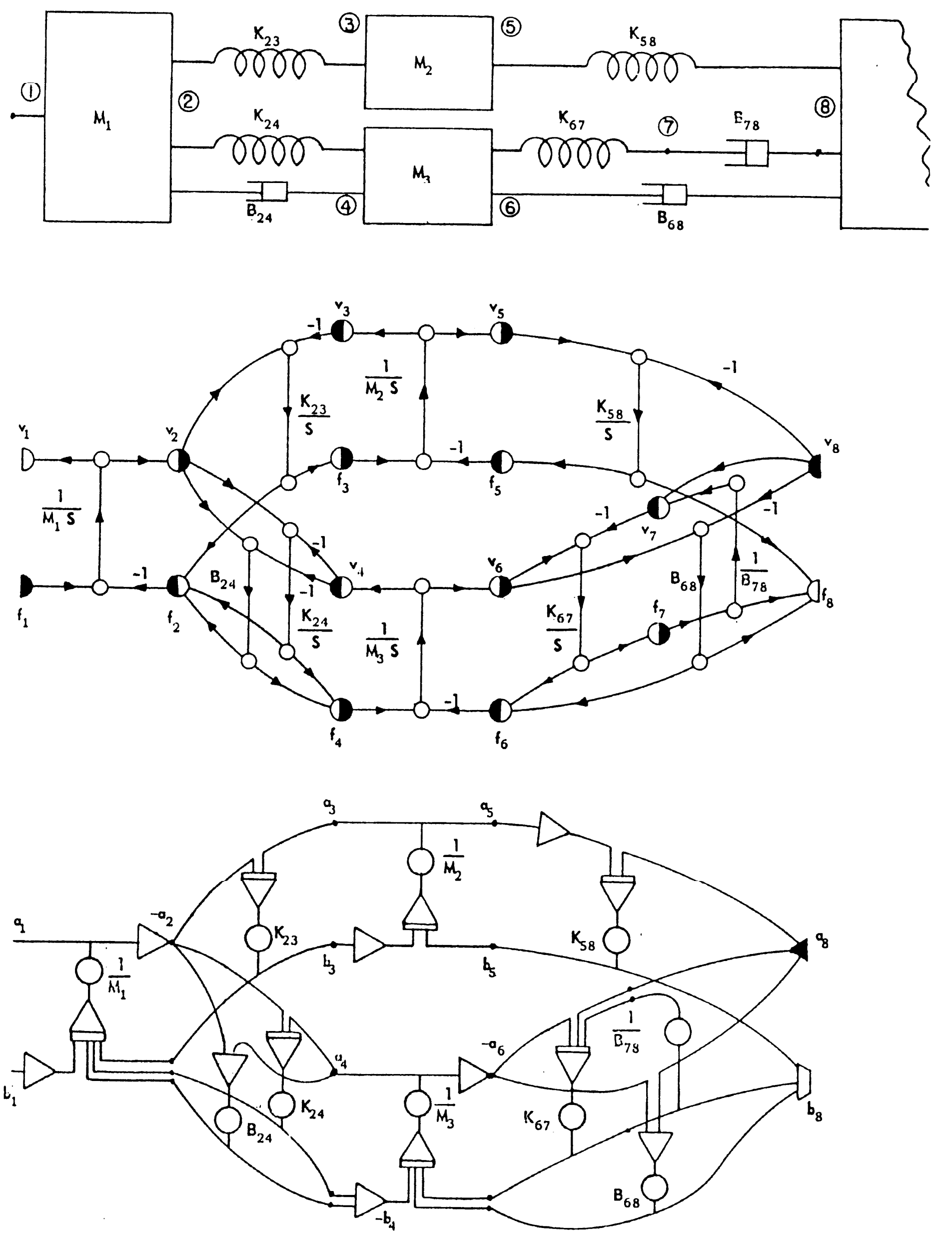
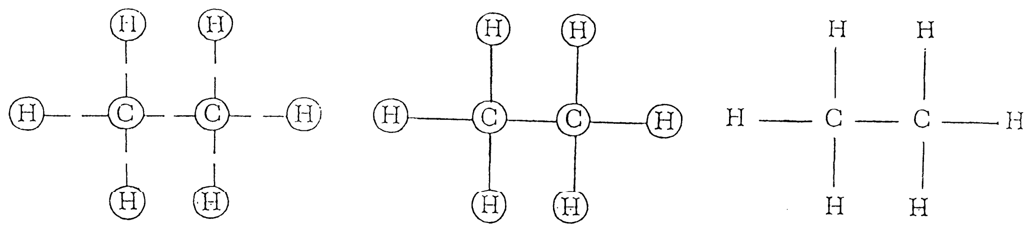


Figure 5. Direct Simulation *a la* SFG
(ROBICHAUD [38])

Clearly, we were searching for a new form of graph, like the circuit diagram, but without its limitations.



CRUM-BROWN [1864] FRANKLAND [1866] FRANKLAND [1870]

Figure 6. The Rapid Evolution of Structural Formulas

Yet the required form of graph had in fact been readily available for a full century since its introduction in 1858 by Archibald Scott COUPER in the form of structural formulas for chemistry [41]. Fig.4 shows one of COUPER's formulas, that for glucose. These graphs were further developed by Alexander CRUM BROWN [42], followed by Edward FRANKLAND [43,44], who had earlier put forward a doctrine of atomicity or chemical valence.

Fig.6 traces the rapid evolution of structural formulas to their present-day form. It is especially interesting for bond graphers to hear FRANKLAND on why the enclosing circles were abandoned [44]:

> " The graphic notation of Crum Brown which was adopted in the former edition...has been somewhat modified in the present, by the omission of the circles surrounding the symbols of the elements. These circles appeared likely to render the formulae more intelligible to beginners; in practice, however, I find that even young students prefer to draw the formulae without the circles, hence there is no reason for retaining them."

If this simplification served chemistry so well, surely bond graphs could benefit from the experience !

5.4. Clifford, Peirce and Graph Theory

In the 1870's, these same chemical structure graphs attracted the attention of the mathematician William Kingdom CLIFFORD in England and the scientist-logician-philosopher Charles Sanders PEIRCE in America. CLIFFORD's interest led shortly before his early death in 1879 to his abstraction of a 'linear graph' of 'nodes' and 'branches' as a topological mathematical system in its own right [45,46,47,48].

Throughout his longer life, PEIRCE had displayed a fascination with diagrammatic thinking, believing along with GAUSS that " algebra is a science of the eye", with the letters forming a diagram which can be observed and rearranged. So along with CAYLEY, SYLVESTER and CLIFFORD, PEIRCE soon put FRANKLAND's graphs to good use. A particularly fruitful application was concerned with the theory of systems of relations: PEIRCE's 'logic of relatives' [49]. In analogy to the chemical role of 4-valent carbon, PEIRCE now stressed the singular role of the triadic, or 3-term, relation in the following words [50]:

> "It is interesting to remark that while a graph with three tails cannot be made out of graphs each with two tails or one tail, yet combinations of graphs with three tails each will suffice to build graphs with every higher number of tails."

(Note the triadic character of the 0,1-junctions.)

PEIRCE also held that every conjunction of two or more branches could itself be considered a relation, commenting [51]:

> "...Thus it becomes plain that every node of bonds is equivalent to a relative..."

(More than any other single factor it was the long-delayed response to this one provocative idea which ultimately gave rise to the Kirchhoff 0,1-junctions for bond graphs.)

PEIRCE's use of graphs for relations was much stimulated upon the publication in 1886 of a remarkable paper by Alfred Bray KEMPE [52]. This immediately provoked a spirited correspondence between PEIRCE and KEMPE. For example, a decade later in his "Theory of Relatives" PEIRCE has this to say about KEMPE [53]:

> "Mr. A.B.Kempe has published...a profound and masterly 'Memoir on the Theory of Mathematical Form' which treats of the representation of relationships by 'Graphs', which is Clifford's name for a diagram consisting of spots and lines, in imitation of the chemical diagrams showing the constitution of compounds."

Further along PEIRCE states [54]:

> "In my method of graphs, the spots represent the relatives, their bonds the hecceities; while in Mr. Kempe's method, the spots represent the objects, whether individuals or abstract ideas, while their bonds represent the relations."

However, PEIRCE was actually quite mistaken in this judgment of KEMPE, who in fact employed multipartite graphs in general and bipartite graphs in particular, for which, in the latter case, one node-type stood for relations (e.g.,multiports) and a second node-type for terms (e.g., bonds).

5.5. Bigraphs to the rescue

These facts all became crystal-clear, shortly after "Analysis and Design" was published, when Volume 1 of "Mathematical Discovery" [55] appeared, where George POLYA, no stranger to graph theorists, embedded the theory of relations within a bigraph system. To quote POLYA [55]:

> "We represent an unknown by a small circle, a relation between unknowns by a small square, and we express the fact that a certain relation involves a certain unknown by joining the square representing the relation to the circle representing the unknown."

Fig.7 represents four relations between four unknowns. POLYA has simply rediscovered KEMPE !

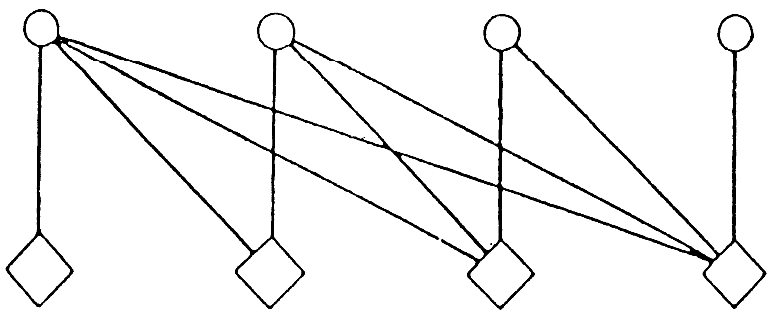


Figure 7. Relational Bigraph (POLYA [55])
SQUARES:Relations; CIRCLES:Terms

This new insight seemed so important to this writer that a short note was published [56] exploiting such bigraphs to derive all commonly used system diagrams (e.g.,block diagrams, signal flow graphs, circuit diagrams, bond graphs, etc.) from a single universal graph.

5.6. Advent of Bond Graphs

Following PEIRCE and the other trail-blazers, as early as 1958 we had adopted the convention of graphing each multiport as a nodal element, which represented a power constraint amongst the incident power bonds. But logic would then seem to dictate that we should also represent the two KIRCHHOFF Laws as manifest multiport nodes enforcing the appropriate power constraints.

So it was that on April 24, 1959, as the writer was about to give a seminar lecture at Case Institute (now Case-Western) on "Interconnected Engineering Systems", he awakened earlier that morning with the 0,1-junctions somehow finally planted in his head ! Thus on this date the BG system was complete and now constituted a formal discipline.

Incidentally, the numbers 0,1 were picked as symbols so as to reduce circuit duality to logical duality, a clear computational advantage.

The complete system was then first published during 1960 in an evolving series of copyrighted MIT Class Notes in the form of folio signatures which were made available to students and outsiders on a subscription basis. By 1961, the complete book had been published by MIT Press and more generally distributed [57]. The BG system was presented at the 1960 Moscow IFAC Congress to a large international audience [3] but it was also employed (without clarification) in the Handbook of Fluid Dynamics [22].

6. DEVELOPMENTS 1960-1992

Summarized here are certain significant contributions by the writer and his close colleagues as developed over the three decades following the advent of bond graphs.

6.1. Causal Stroke and Half Arrow

Both the causal-sense bit and the power-sign bit were from the very start considered augmentations of an initially unadorned bond graph, capable of being supplied either separately or else together, on a bond-by-bond basis, and requiring no more than a single mark (say | or \ ) for each bit.

The use of the causal stroke, |, pre-dates both the energy junctions and the Analysis and Design text. However, its full value in systematically providing a minimal set of state variables only became apparent later during the 1960's.

The half-arrow, \, for the bond power-sign was the joint creation of this writer and Ron ROSENBERG during his doctoral research [58].

Taken together, these simple supplementary marks have given good service over the past three decades.

6.2. TF and GY

The reason for using two letters for these ideal nonenergic elements is lost in pre-BG folklore. At one time, before energy junctions, this writer used one letter for 1-ports (e.g.:A,B,etc.),two letters for 2-ports (e.g.: KL,LM,etc.) and three letters for 3-ports (e.g.: UVW,XYZ,etc.). But with the advent of 0 and 1 this no longer made sense.

Sometime in the early 1960's, Forbes BROWN showed the good judgment to suggest -T- for transformer and -G- for gyrator (my contribution being to employ 'n' and 'g' for their respective moduli). Only BROWN bothered to accept this useful reform but now serious thought should be given to the use of single ASCII characters for all elemental multiports.

6.3. The ENPORT Language

The idea of a computer program which automatically assembles the system equations from component equations follows quite naturally from the work of KRON; WHEELER's terminology suggested the name ENPORT (both N and ENergy). A full description of such a program preceded the invention of BGs by several years but the actual first implementation occured as part of Project Entelechon, Project MAC, and as part of the doctoral thesis of ROSENBERG [58], who has continued to refine ENPORT ever since.

6.4. Irreducible Elements

Once the gyrator is introduced as a primitive element, the transformer can be eliminated and only one junction-type and one storage-type is needed. Moreover, using a unilateral active bond makes source elements unnecessary. The first publication of such an irreducible set occured in a paper with Dean KARNOPP [59] while a more recent commentary also exists [16].

6.5. Gyrostructure and Gyrographs

If a BG is converted to such irreducible form, the result was called by this writer a 'gyrograph' [60] and by ROSENBERG and others a 'gyrobond graph' [61]. Beside the thesis of ROSENBERG [58], such gyrostructures also occurred in the thesis of Joe FREE [62].

6.6. The Transactor

The strong analogy between solid-state physics and biochemistry gives rise to a common generalization in the form of the 'transactor' [63], which at the same time serves to model the reaction/diffusion process in a transistor and in a biomembrane. This new element directly suggests how sophisticated communication, computing and control can occur in living systems.

6.7. Resistors as Transducers

A paper considering dissipation as a lossless transduction process which completely converts available energy to unavailable forms was also published [60]. This concept suggests that the purely mathematical decomposition of scalar functions into odd and even parts is itself directly responsible for the Second Law of Thermodynamics.

6.8. From ERGS & BITS to BITS of JOULES

The still-unpublished MS of "Ergs and Bits" was initiated in 1952 and grew into fairly complete form within the next decade. This then required a massive re-write for bond-graph compatibility after 1959. Copies of sections and even whole chapters were subsequently distributed over the years to students and colleagues. During the Summer of 1965 Alistair MACFARLANE visited MIT and prepared an abstract of some of this material [64], featuring the scattering approaches used in E&B but otherwise employing his own 'transvariable/pervariable' terminology rather than our own efforts and flows.

Finally, ultimate publication was yet further delayed by the international adoption of SI units, which has even required a title change to "Bits of Joules". Perhaps this MS may yet see light in a final published form !

7. CONCLUDING PERSONAL REMARKS

I cannot close this commentary without relapsing to the first person singular in order to give thanks to all my teachers and mentors, most of whom are now deceased. None of this vital intellectual enterprise would have been possible without their inspiration and help.

Two of these benefactors deserve especial mention. It was George Arthur PHILBRICK who patiently taught me the causality catechism, by both direct and indirect means. Also a lifetime measure of logical wisdom I learned from the written remnants of Charles Sanders PEIRCE.

Readers of my Analysis and Design can see second-hand PEIRCE from cover-to-cover. But I urge you all to read him in the original, assisted by the now eleven volumes of his collected published and unpublished writings. Search and you will find therein such gemstones as this:

> "Try to verify any law of nature and you
> will find the more precise your observations,
> the more certain they will be to show
> irregular departures from the law."

The veritable Stone of Sisyphus !

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