

## Extracting Rate Dependent Traction Separation Relations for Cracks/Interfaces in Viscoelastic Media

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### Introduction

Methods for characterizing and predicting crack growth in linearly or non-linearly elastic materials are well established both theoretically and experimentally. However, fundamental work relating to fracture in polymers and other time dependent materials is relatively limited. Complexity in characterizing crack development, growth and propagation in viscoelastic media, stems from two different yet unique challenges. Firstly, typical energy based methods, widely used in characterizing traction separation relationships in elastic media, are not applicable anymore due to inherent bulk energy dissipation, a characteristic of viscoelastic media. Furthermore, the load dependent response of viscoelastic materials makes it difficult to quantify any independent parameters which would be indicative of fracture process. For example, loss of stiffness at particular load level in an elastic body can be very well attributed to crack development/propagation, whereas the same cannot be hypothesized for a viscoelastic material.

### Objectives

The primary objective of this study is to establish a theoretical framework for developing a simple experimental procedure aimed at quantifying traction separation relations, a vital fracture parameter used in numerical modelling of cohesive or interfacial cracks in viscoelastic media. The procedure is designed to extract traction-separation relationships for both cohesive and adhesive cracks, without making any assumptions on their form. Numerical experiments were carried out on two strips of polyvinyl acetate bonded together and conclusive results produced demonstrate the efficiency of the framework developed in this work.

### Methodology

Similar to J integral proposed by Rice [1], which is applicable for Cauchy-elastic media and pseudo J-integral developed by Schapery [2], which is applicable for linear viscoelastic (VE) material with constant Poisson ratio, a modified J-integral is first developed. This development is preceded by the development of the displacements, strains and stresses in the tilde domain. Thus the displacements become

$$\tilde{u}_i = \int E(t-\tau) \frac{du_i}{d\tau} d\tau, \quad (1)$$

where  $E(t)$  is the relaxation modulus of the viscoelastic material (assuming constant Poisson's ratio). Then the tilde displacement gradients are

$$\tilde{u}_{i,j} = \int E(t-\tau) \frac{du_{i,j}}{d\tau} d\tau. \quad (2)$$

The strain-displacement relationship is given by

$$\tilde{\varepsilon}_{ij} = \frac{1}{2}(\tilde{u}_{i,j} + \tilde{u}_{j,i}) \quad (3)$$

The stresses are the actual stresses, so that equilibrium in the absence of body forces and tractions remain as

$$\sigma_{ij,j} = 0 \text{ and } T_i = \sigma_{ij}n_j. \quad (4)$$

The constitutive relationship between these stresses and the strains in the tilde domain are equivalent to the stress-strain relationship of an elastic medium with  $E = 1$  with a Poisson's ratio that is the same as that of the viscoelastic medium. To further elucidate, applying this transformation to the actual displacements, while maintaining the stresses as before, converts the problem of solving linear viscoelasticity problems to problems of linear elasticity. Given these transformed quantities in conjunction with the actual stresses, a J-integral in the tilde domain can be developed as

$$\tilde{J} = \oint T_i \frac{d\tilde{u}_i}{dx} ds - \oint \tilde{W} dy, \quad (5)$$

where

$$\tilde{W} = \frac{\sigma_{11}^2}{2} + \frac{\sigma_{22}^2}{2} + (1+\nu)\sigma_{12}^2 - \nu\sigma_1\sigma_2 \quad (6)$$

and  $\nu$  is the Poissons ratio. This development maintains the path independence property exhibited by Rice's J-integral. To articulate, the  $\tilde{J}$ -integral evaluated on any closed path in the viscoelastic media defined here will be zero. Although the  $\tilde{J}$ -integral equation provides us with means to determine any traction-tilde displacement relation that exists at a crack or an interface, the equation itself, in its base form (5), cannot be used to extract traction-separation relations, without making any assumptions on the form of  $T_i$ - $\tilde{u}_i$  relationship. This warrants the development of modified field projection method, an idea similar to the 'Field Projection Method' [3] developed for extracting the functional form of tractions in elastic media.

The core premise of field projection method lies in the applicability of Betti's reciprocity theorem for elastic medium. As the tilde transformation renders the viscoelastic problem to an elastic problem, the field projection method can be modified to enable the extraction of traction-separation relations in viscoelastic media.

To begin with, a new quantity  $\tilde{J}^{int}$  is defined as follows

$$\tilde{J}^{int} = \tilde{J}(S + \hat{S}) - \tilde{J}(S) - \tilde{J}(\hat{S}), \quad (7)$$

where  $S[\sigma_{\alpha\beta}, \tilde{u}_{\alpha,\beta}]$  and  $\hat{S}[\sigma_{\alpha\beta}, \tilde{u}_{\alpha,\beta}]$  are two admissible fields obtained when actual traction-separation relationship and a probe traction-separation field are respectively present in the cohesive zone. Using the definitions of  $\tilde{J}$  and  $\tilde{J}^{int}$  on a closed contour we obtain:

$$\int T_i \tilde{u}_i' dx + \int f_i \tilde{U}_i' dx = \left[ \oint W^{int} dy \right], \quad (8)$$

where  $\tilde{T}, \tilde{U}'$  corresponds to traction and tilde-displacement perpendicular to the interface in the physical problem and  $f_i, \tilde{u}_i'$  correspond to those in the test problem.

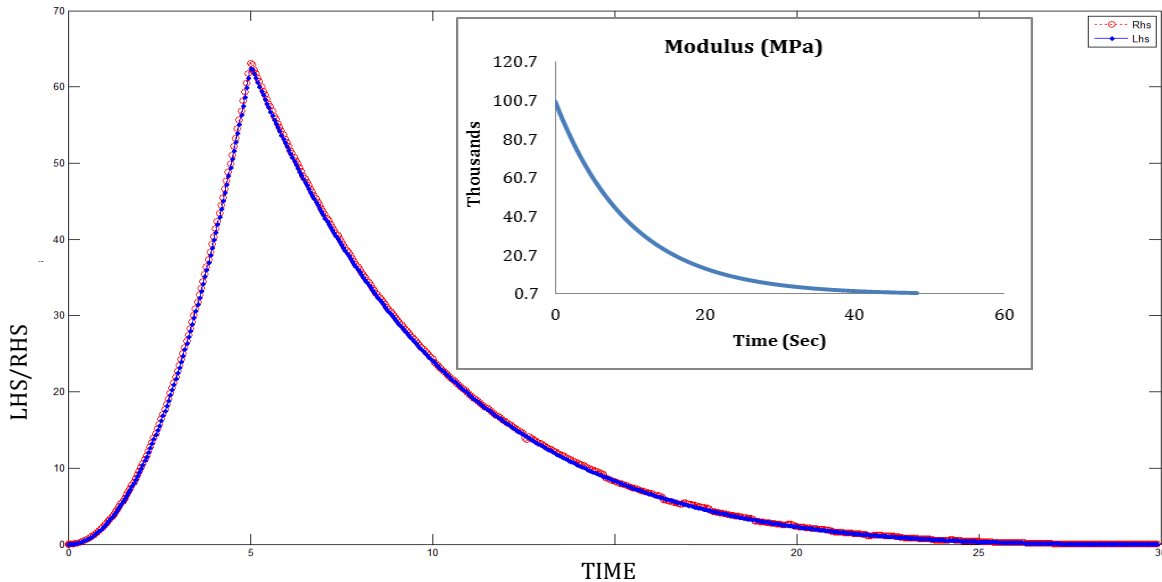
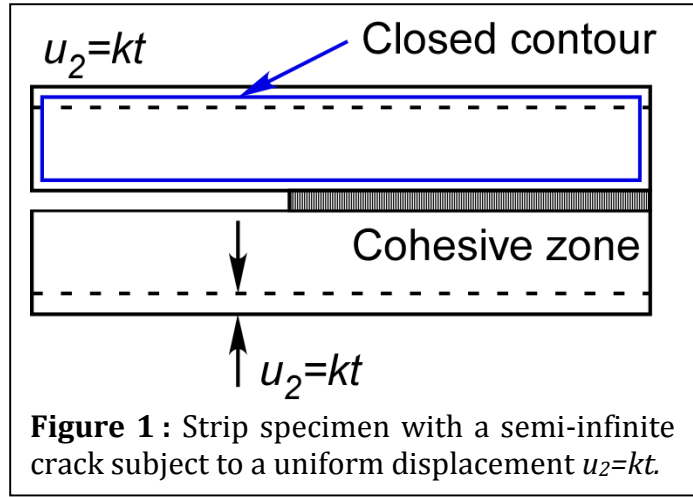
Chebyshev orthogonal polynomials  $\phi_i$  are used in test functions in equation (8) in the form  $T = a_i \phi_i$  and  $u = b_i \phi_i$ . This allows a system of equations

$$a_n \int \phi^n u_i' dx + a_{n-1} \int \phi^{n-1} u_i' dx + \dots a_0 \int u_i' dx + b_n \int f_i \phi'^n dx + b_{n-1} \int f_i \phi'^{n-1} dx + \dots b_0 \int f_i \phi'^0 dx = \left[ \oint W^{int} dy \right] \quad (9)$$

to be obtained for the  $a_i, b_i$  and therefore the traction and tide-separation.

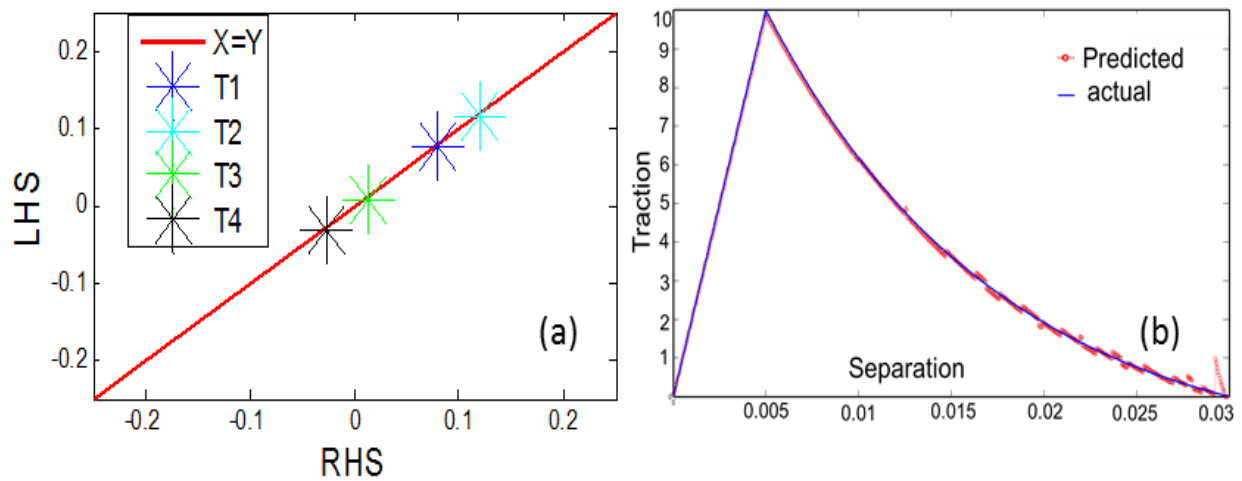
## Results and analysis

Owing to the theoretical nature of this work at this time, numerical experiments were carried out using ABAQUS and other custom built finite element codes first to validate equations (5) and (8) and eventually use (9) to determine the input traction-separation relation. The problem considered is one that is being considered for the experimental phase of this work; a clamped strip with a semi-infinite crack (Fig. 1). The relaxation modulus assigned to the strip specimen (see insert to Fig. 2) was chosen to mimic polyvinyl acetate's (PVAc) three orders of magnitude drop in modulus in a finite time. A linear-exponential decay type traction-separation relation represents the fracture characteristics of the material along the crack plane or interface. The boundary value problem was solved by ABAQUS. The closed contour chosen to evaluate equation (5) is marked blue in Figure 1 and the terms  $\oint T_i \frac{d\tilde{u}_i}{dx} ds$  and  $\oint \tilde{W} dy$  are marked as LHS and RHS and plotted with time as shown in figure 2.



**Figure 2:** Plot of LHS/RHS in (5) to validate  $\tilde{J}$ . The insert represents the relaxation modulus of PVAc.

As shown in Figure 2, the fact that the RHS and LHS are the same validates the claim that  $\tilde{J}$  for viscoelastic media. Furthermore, in order to validate equation (8), a test problem was constructed with its geometry coinciding with the contour chosen to evaluate  $\tilde{J}$  in which a series of Chebyshev polynomials of first kind were applied in place of the actual traction distribution. For each polynomial, the quantity  $\int T_i \tilde{u}_1' dx + \int f_1 \tilde{U}'_i dx$  (LHS) and quantity  $[\oint W^{int} dy]$  (RHS) were computed and plotted as shown in Figure 3 below:



**Figure 3:** (a) LHS vs RHS are plotted to validate equation (8) and (b) a comparison between input and extracted traction-separation relation.

As demonstrated in Figure 3a, the close correlation in the computed RHS and LHS quantities for each of the four traction functions that were selected indicate that calculating traction-tilde displacement distribution is relatively error free. The next step was to see if the extracted traction-separation relation tracked the input one. Figure 3b demonstrates that this was indeed the case.

### Conclusions

A viable scheme based on elastic-viscoelastic correspondence principles for extracting rate dependent traction-separation relations in viscoelastic media has been validated. This sets the stage for experiments to extract traction-separation relations in healed PVAc specimens.

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