

# THICKNESS AND D33 EFFECTS ON THE ENERGY CONVERSION AND ACTUATION OF PIEZOELECTRIC UNIMORPHS

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**Summary** Piezoelectric unimorph is composed of a piezo-layer (with top and bottom electrodes) uniformly laminated on an inactive flexible substrate. Because of their simple construction and flexibility, unimorphs are widely used as flexible sensors and actuators. Here we report a comprehensive theoretical framework to investigate the effects of the film-to-substrate thickness ratio on voltage, charge, and energy outputs when the unimorph is subjected to eight different boundary/loading conditions. For not so thin unimorphs, there is non-zero normal stress in the thickness direction ( $\sigma_3$ ), in which case  $d_{33}$  can play a significant role. Non-monotonic voltage and energy generation versus thickness ratio relations have been found in some cases and optimum thickness ratio for unimorph generator can be predicted. When the unimorph is actuated by voltage applied across the piezo-layer, non-monotonic actuated deflection versus thickness ratio relation is also found.

## INTRODUCTION

Piezoelectric materials have found wide applications in energy harvesting as well as actuation because of their unique combination of mechanical form factors and energy transduction capabilities. Existing unimorph models [1-3] are mostly limited to cantilever configurations and the  $d_{33}$  contribution has been neglected because  $\sigma_3$  is always assumed to be zero. However, unimorphs can also operate under the boundary conditions of pure bending or simple-simple support and can be subjected to variety types of load or displacement excitations. Moreover,  $\sigma_3$  is non-zero for thick unimorphs and may have significant contributions to the outputs. To find a remedy for aforementioned deficiencies, we derive closed-form solutions for unimorphs subjected to eight different boundary/loading conditions [4]. To validate our theory, finite element modeling (FEM) was performed using COMSOL Multiphysics. Non-monotonic electrical outputs versus thickness ratio curves are found for several boundary value problems (BVP). To resolve the discrepancy between theoretical and FEM results for some scenarios, the  $d_{33}$  effect had to be taken into consideration. Simple models to determine average  $\sigma_3$  along beam length has been proposed and turn out to be very effective in accounting for the  $d_{33}$  contribution.

## METHOD AND RESULTS

Suppose the piezoelectric layer and the substrate have thicknesses of  $h_1$  and  $h_2$  and Young's moduli of  $Y_1$  and  $Y_2$ , as labeled in Fig. 1(a), calculating the voltage, charge, and power output of such a unimorph first requires the determination of the neutral axis and the effective second moment of inertia of the bilayer. The distance from the neutral axis to the bottom surface of the substrate is represented by  $\Delta h_1$ , as labeled in Fig. 1(a), where  $\Delta$  has been given by [5]

$$\Delta = \frac{1 + 2\Sigma\eta + \Sigma\eta^2}{2\eta(1 + \Sigma\eta)} \quad (1)$$

where  $\Sigma = \bar{Y}_1/\bar{Y}_2$  is the film-to-substrate modulus ratio with  $\bar{Y} = Y/(1 - \nu^2)$  being the plane strain modulus,  $Y$  being the Young's modulus,  $\nu$  being the Poisson's ratio, and  $\eta = h_1/h_2$  is the film-to-substrate thickness ratio.

The effective second moment of inertia of the bilayer is given by [4]

$$I = h_2^3 \left\{ \Sigma \left[ \eta(\Delta\eta - 1)^2 - \eta^2(\Delta\eta - 1) + \frac{\eta^3}{3} \right] + \Delta\eta(\Delta\eta - 1) + \frac{1}{3} \right\} = h_2^3 \bar{I} \quad (2)$$

where  $\bar{I}$  represents the non-dimensional second moment of inertia and hence the bending stiffness of the unimorph can be written as  $\bar{Y}_2 I$ . For given piezoelectric and substrate materials (i.e.,  $\Sigma$  fixed), the only dimensionless variable in the problem is the thickness ratio  $\eta$ , whose effect is the focus of this study.

Based on Euler- Bernoulli beam theory, the bending induced normal stress in  $x$  direction and its vertical distribution of the unimorph is given by  $\sigma_1 = (z - \Delta h_1)\Sigma M/I$ . When the beam is thick, non-zero  $\sigma_3$  may also exist. By the linear piezoelectric effect [6], deformation induced polarization density is proportional to stress through the piezoelectric coefficient, i.e.  $P_i = d_{ij}\sigma_j$  ( $i = 1, 2, 3$ ,  $j = 1, 2, \dots, 6$ ), therefore  $z$  direction polarization is given by

$$P_3 = d_{31}\sigma_1 + d_{33}\sigma_3 \quad (3)$$

Since charges are only collected from the top and bottom surface electrodes of the piezo-layer, only top ( $z = h_1 + h_2$ ) and bottom ( $z = h_2$ ) surface polarization density will be considered to calculate the total amount of charges. Therefore surface charge density can be calculated as the total charge from the surface polarization density divided by the surface area:

$$\rho = \frac{Q}{S} = \frac{- \int_S a_n \cdot P_3 dS}{S}. \quad (4)$$

where  $a_n$  stands for the surface normal vector and  $S$  stands for the overall surface area. Subsequently, the voltage derived through the definition of capacitance,  $V=Q/C$ , where  $C = \varepsilon_p' L/h_1$ , with  $\varepsilon_p'$  being the effective permittivity of the piezoelectric material [2]. Lastly, the generated energy density can also be calculated by  $U=\rho V/2$ . Following this procedure, the unimorph electric outputs versus thickness ratio has been analysed for different boundary/loading conditions. As an example, Figs. 1(b)-(e) offers the results of simply supported unimorph subjected to central point load. The analytical results (curves) show excellent agreement with finite element modelling (FEM) results (markers) if the effect of  $d_{33}$  (or  $\sigma_3$ ) is considered. Non-monotonic outputs also suggest that optimal thickness ratio exists.

Thickness ratio also affects the actuated displacement of a unimorph subjected to applied electric field  $E = V_0/h_1$  across the thickness of the piezo-layer [7]. As a result, a uniform stress  $\sigma_1 = d_{31} \bar{Y}_1 V_0/h_1$  is generated in the piezo-layer, while stress in the substrate remains zero. The resultant moment will therefore bend the unimorph and the actuated displacement is again found to be non-monotonic with respect to the thickness ratio, as displayed in Figs. 1(f) & (g).

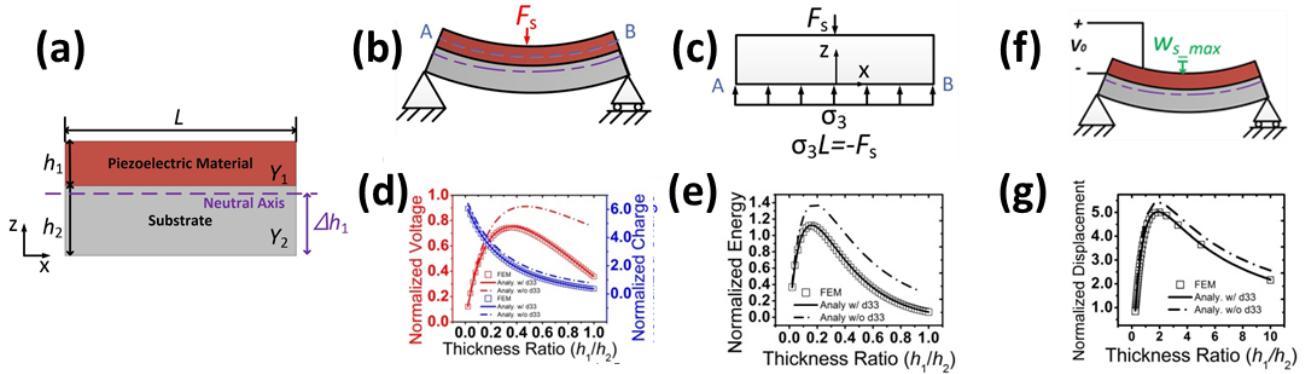


Fig. 1 (a) Illustration of basic variables of the unimorph. (b) A schematic for the simply supported unimorph subjected to central point load. (c) Free body diagram used to calculate average  $\sigma_3$ . (d, e) The analytical and FEM results of normalized voltage, charge density, and energy density as functions of the thickness ratio. (f) A schematic of a simply supported unimorph actuator subjected to constant electric potential. (g) The analytical and FEM results of normalized actuated displacement of the unimorph as a function of the thickness ratio.

## CONCLUSIONS

We investigate the electromechanical behaviors of flexible unimorph power generators and actuators. Analytical and numerical models are built to unveil the effects of piezo-layer-to-substrate thickness ratio and piezoelectric material constants on energy conversion under eight different boundary/loading conditions. Our theory reveals that when the unimorph is subjected to displacement-controlled loading conditions, the charge, voltage, and energy outputs are monotonic functions of the thickness ratio whereas when the unimorph is subjected to load-controlled conditions, optimal thickness ratios for maximum voltage and energy outputs exist. Our linear piezoelectric theory has been fully validated by FEM. We have also found that except pure bending conditions, all cantilever and simply supported unimorphs should care about the  $d_{33}$  (i.e.  $\sigma_3$ ) contribution when the unimorph length is not much larger than the thickness. A simplified average stress model is proven effective in accounting for the  $d_{33}$  effect.  $d_{33}$  effect may also change the outputs of displacement controlled problems from monotonic to non-monotonic. The effects of elastic mismatch and thickness-to-length ratio have been discussed and analytical solutions for unimorph based actuators are also offered. This work provides a comprehensive and accurate solution for the design and optimization of unimorph based power generators and actuators.

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