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FAULT DETECTION AND ISOLATION STRATEGY FOR REDUNDANT INERTIAL MEASUREMENT UNITS

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Aerospace vehicle are often required to be two-fault tolerant in order to be man-rated. This paper presents a two-fault tolerant fault detection and isolation algorithm for a set of four redundant inertial measurement units (IMUs). The paper derives the fault detection thresholds. The algorithm tests the IMU data after it is processed by an infinite impulse response filter. Two tests are performed; the first applies a low cut-off frequency filter to the data in order to detect biases and slowly growing biases. The second test is performed with a high cut-off frequency filter in order to detect off-nominal abrupt changes in IMU errors.

INTRODUCTION

The goal of this paper is to present a fault detection and isolation (FDI) strategy to determine failures in one of a set of redundant inertial measurement units (IMUs). Two IMUs are needed to detect a single failure, three to isolate it. The scope is different from much of the work existing in the literature involving FDI in a single redundant IMU. The proposed algorithm is useful for most man-rated aerospace vehicles (aircraft or spacecraft) that often require a two-fault tolerant architecture.

This paper contributes a detailed derivation of how IMU errors are affected by processing IMU errors through an infinite impulse response (IIR) filter, the need for the IIR filter is also explained. Another contribution of this work is a novel fault detection decision logic. Finally this paper presents in clear, coherent, and concise manner the design of a man-rated IMU FDI algorithm and justifies all design decisions made.

FAULT VECTOR AND PARITY SPACE

In this section we briefly review the work by Sturza [1] and we make minor modifications to apply it to our problem. Assume we have m redundant measurements $\tilde{y}_i$, we model each as given by the

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sum of the true value of the measurement y_i and the measurement error δy_i. For the time being assume the measurement is a linear function of the n dimensional state $\mathbf{x}$, i.e. $\mathbf{y} = \mathbf{H}\mathbf{x}$. Following Sturza's derivation we define the generalized inverse $\mathbf{H}^*$ and the parity matrix $\mathbf{P}$

$$\mathbf{H}^* = (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}} \tag{1}$$

$$\mathbf{P} = null(\mathbf{H}^{\mathrm{T}})^{\mathrm{T}} \tag{2}$$

The parity matrix, $\mathbf{P}$, has rank, $m - n$, dimensions of $m - n \times m$, is unitary, and has the following property:

$$\mathbf{P}\mathbf{H} = \mathbf{0}$$

Matrix $\mathbf{A}$ maps the measurement space into the state space and the parity space, and is given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}^* \\ \mathbf{P} \end{bmatrix}$$

The inverse map is easily obtained since

$$\mathbf{A}^{-1} = \begin{bmatrix} \mathbf{H} & \mathbf{P}^{\mathrm{T}} \end{bmatrix}$$

The parity vector $\mathbf{p}$ is defined as $\mathbf{p} = \mathbf{P}\tilde{\mathbf{y}}$. The fault vector $\boldsymbol{\epsilon}$ is defined as the parity vector transformed back to the measurement space

$$\boldsymbol{\epsilon} = \mathbf{A}^{-1} \begin{bmatrix} 0 \\ \mathbf{p} \end{bmatrix} = \mathbf{P}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{y}} = \mathbf{S}\tilde{\mathbf{y}}$$

The fault vector has the useful property of being independent of the state, therefore

$$\mathrm{E}\{\boldsymbol{\epsilon}\} = \mathbf{S}\,\mathrm{E}\{\delta\mathbf{y}\}$$

and

$$\mathrm{E}\left\{\boldsymbol{\epsilon}\boldsymbol{\epsilon}^{\mathrm{T}}\right\} = \mathbf{S}\,\mathrm{E}\left\{\delta\mathbf{y}\delta\mathbf{y}^{\mathrm{T}}\right\}\mathbf{S}^{\mathrm{T}}$$

Finally, notice that no singular value decomposition is needed since null space computation is unnecessary, this is due to the following property: $\mathbf{S} = \mathbf{I} - \mathbf{H}\mathbf{H}^*$.

Assume $m = 4$, $n = 1$ and the state is measured directly, i.e.

$$\tilde{\mathbf{y}} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \tilde{y}_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} x + \begin{bmatrix} \delta y_1 \\ \delta y_2 \\ \delta y_3 \\ \delta y_4 \end{bmatrix} = \mathbf{H}x + \delta\mathbf{y} \tag{3}$$

for this case matrix $\mathbf{S}$ is simply given by

$$\mathbf{S} = \frac{1}{4} \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \tag{4}$$

IMU FAULT DETECTION AND ISOLATION

The scope is to detect and isolate a failure in redundant IMU boxes, these checks are done at the box level, that is: redundancy is not sought by skewing the IMUs and checking the measurements at the axis level. There are two reasons for this choice. First is that internal IMU misalignments are very small, much smaller than mounting errors, therefore the navigation system is more accurate when navigating the IMU rather than the CG in vehicle-fixed coordinates such as the body frame. By matching measurements from different IMUs to feed the navigation system, these measurements will be affected by misalignments between the IMUs, therefore the solution will be less accurate. We will use either all the information from an IMU or none.

It would still be possible to do IMU FDI by skewing the boxes and trying to detect failures in the single axes rather the labeling the entire unit as failed. The problem with this approach is that man-rated systems normally utilize navigation grade IMUs which run internally at a very high rate, several times higher than the output provided. The output is then down-sampled and compensated for coning and sculling, which means the measurement in each axis is a function of the measurements in the other two axis.

These two reasons led us to the following IMU FDI design choice: four IMUs are onboard the vehicle, IMU FDI checks the four sets of measurements for consistency, if one set of measurements does not match the other three the IMU is labeled as suspect and failed if the difference persists. If another discrepancy arises between the remaining three boxes the fault can still be detected and isolated. A third fault can only be detected.

Traditionally fault detection and identification is done separately [2], [3]. For example the Space Shuttle checks every possible pair of measurements against each other; if they differ by more than a threshold the fault is detected. A separate identification algorithm determines which IMU has failed based on the detections from all the IMU combinations.

At the heart of the fault detection and identification is an algorithm which compares the fault vector with the statistical noise level in that vector. Sturza states that a common choice for the fault detection decision variable is the square magnitude of the fault vector. Assuming the errors are Gaussian, the magnitude of the fault vector will have a χ^2 distribution with $m-n$ degrees of freedom. The probability associated with the χ^2 distribution then gives us the likelihood of false alarm and of misdetection. If the vector exceeds the noise level by a prescribed amount we know that with a certain probability, the apparent fault is not just due to an unlikely set of random noise values. The fault vector and noise are viewed in null space which is free of the actual quantity being measured so it cannot be driving the fault vector. The null space has fewer dimensions than the measurement space so the faults in individual measurements projected into this null space cannot all be mutually orthogonal. This fact contributes to the challenge of correctly identifying the fault especially if its vector in null space is nearly parallel to that of another fault.

Both the Space Shuttle algorithm and the FDI approach just described have a clear separation between fault detection and fault identification. A different approach is taken here, which makes the fault identification trivial and combined with the fault detection algorithm.

We have four measurements $\tilde{y}_i$, $i = 1, 2, 3, 4$, we model each as given by the sum of the true value of the measurement y and the measurement error δy_i. The errors are assumed independent and identically distributed with zero mean and variance σ_y^2. We then do the following operation:

$$\epsilon_1 = (3\tilde{y}_1 - \tilde{y}_2 - \tilde{y}_3 - \tilde{y}_4)/\sqrt{12} = (3\delta y_1 - \delta y_2 - \delta y_3 - \delta y_4)/\sqrt{12}$$

under normal circumstances ϵ_1 is zero mean and has variance σ_y^2. If ϵ_1 does not match its predicted standard deviation we know a measurement is suspect. We similarly calculate

$$
\begin{aligned}
\epsilon_2 &= (3\tilde{y}_2 - \tilde{y}_1 - \tilde{y}_3 - \tilde{y}_4)/\sqrt{12} = (3\delta y_2 - \delta y_1 - \delta y_3 - \delta y_4)/\sqrt{12} \\
\epsilon_3 &= (3\tilde{y}_3 - \tilde{y}_1 - \tilde{y}_2 - \tilde{y}_4)/\sqrt{12} = (3\delta y_3 - \delta y_1 - \delta y_2 - \delta y_4)/\sqrt{12} \\
\epsilon_4 &= (3\tilde{y}_4 - \tilde{y}_1 - \tilde{y}_2 - \tilde{y}_3)/\sqrt{12} = (3\delta y_4 - \delta y_1 - \delta y_2 - \delta y_3)/\sqrt{12}
\end{aligned}
$$

under normal circumstances ϵ_2, ϵ_3, and ϵ_4 are also zero mean with variance σ_y^2. If a single fault occurs the IMU associated with the highest value of ϵ_i is the culprit. Therefore the proposed algorithm consists in checking all four ϵ_i against a user defined threshold, if the any of them exceed the threshold, then a fault is detected and the identification occurs by simply determining which component has the largest absolute value.

In order to reduce the probability of misdetection and false alarm, FDI does not permanently fail a sensor because of a single threshold exceedance, rather a time window is used. The size of the time window is dictated by a user-defined parameter N_{test}, every exceedance out of the last N_{test} is recorded. If a sensor has accrued at least N_{prob} exceedances out of the last N_{test} then the sensor is marked as Probationary. If a sensor has accrued at least N_{fail} exceedances out of the last N_{test} then the sensor is marked as Failed. Otherwise the sensors status is Nominal.

In order to implement the algorithms it is necessary to calculate the value of σ_y^2. This is addressed in the next section.

IMU ERRORS

Navigation-grade IMUs usually measure integrated angular velocities $\boldsymbol{\omega}$ and integrated non-gravitational acceleration $\mathbf{a}$. We denote with $\tilde{\mathbf{y}}$ the general measured quantity (either angular velocity or acceleration) and with $\tilde{\mathbf{x}}$ the integrated measurement (either angle or velocity). We have that the total cumulative IMU measurement is given by

$$
\tilde{\mathbf{x}}(t) = \boldsymbol{\eta}_{ro} + \int_0^t \tilde{\mathbf{y}}(t)dt \tag{5}
$$

where $\boldsymbol{\eta}_{ro}$ is the readout noise, and $\tilde{\mathbf{y}}$ is given by

$$
\tilde{\mathbf{y}} = \Big(\mathbf{I} + \mathbf{S} + \mathbf{M}\Big)\mathbf{y} + \mathbf{Q}\begin{bmatrix} y_1^2 \\ y_2^2 \\ y_3^2 \end{bmatrix} + \mathbf{A}\begin{bmatrix} |y_1| \\ |y_2| \\ |y_3| \end{bmatrix} + \mathbf{b} + \boldsymbol{\eta} \tag{6}
$$

where $\mathbf{y}$ is the true value of acceleration or angular velocity, $\mathbf{I}$ is the identity matrix, $\mathbf{S}$ is a diagonal matrix of scale factor errors, $\mathbf{M}$ is the matrix with misalignment and non-orthogonality errors, $\mathbf{Q}$ is the matrix of quadratic errors, $\mathbf{A}$ is the matrix of scale factor asymmetry, $\mathbf{b}$ is a bias, and $\boldsymbol{\eta}$ is white noise. All lower case bold quantities are three dimensional vectors, upper case bold are 3×3 matrices. Rather than producing an accumulated delta-v and delta-theta, IMUs often provide many incremental samples

$$
\Delta\tilde{\mathbf{x}}_k = \tilde{\mathbf{x}}(t_k) - \tilde{\mathbf{x}}(t_{k-1}) = \boldsymbol{\eta}_{ro,k} - \boldsymbol{\eta}_{ro,k-1} + \int_{t_{k-1}}^{t_k} \tilde{\mathbf{y}}(t)dt \tag{7}
$$

The measurement error $\delta\mathbf{y}$ is given by

$$\delta\mathbf{y} = \tilde{\mathbf{y}} - \mathbf{y} = \Big(\mathbf{S} + \mathbf{M}\Big)\mathbf{y} + \mathbf{Q}\begin{bmatrix} y_1^2 \\ y_2^2 \\ y_3^2 \end{bmatrix} + \mathbf{A}\begin{bmatrix} |y_1| \\ |y_2| \\ |y_3| \end{bmatrix} + \mathbf{b} + \boldsymbol{\eta} \tag{8}$$

with the following definitions:

$$\mathbf{S} = [\mathbf{s}\diagdown] \tag{9}$$

$$\mathbf{M} = [\mathbf{m}\times] + [\mathbf{n}*] \tag{10}$$

$$\mathbf{Q} = [\mathbf{q}\diagdown] \tag{11}$$

$$\mathbf{A} = [\mathbf{a}\diagdown] \tag{12}$$

$$[\mathbf{m}\times] = \begin{bmatrix} 0 & -m_3 & m_2 \\ m_3 & 0 & -m_1 \\ -m_2 & m_1 & 0 \end{bmatrix} \tag{13}$$

$$[\mathbf{n}*] = \begin{bmatrix} 0 & n_3 & n_2 \\ n_3 & 0 & n_1 \\ n_2 & n_1 & 0 \end{bmatrix} \tag{14}$$

$$[\mathbf{y}\diagdown] = \begin{bmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{bmatrix} \tag{15}$$

$$|\mathbf{y}| = \begin{bmatrix} |y_1| \\ |y_2| \\ |y_3| \end{bmatrix} \tag{16}$$

we have that

$$\delta\mathbf{y} = [\mathbf{y}\diagdown]\mathbf{s} - [\mathbf{y}\times]\mathbf{m} + [\mathbf{y}*]\mathbf{n} + [\mathbf{y}\diagdown]^2\mathbf{q} + [|\mathbf{y}|\diagdown]\mathbf{a} + \mathbf{b} + \boldsymbol{\eta} + (\boldsymbol{\eta}_{ro,k} - \boldsymbol{\eta}_{ro,k-1})/\Delta t \tag{17}$$

where Δt is the time over which $\tilde{\mathbf{y}}$ is calculated, i.e. $\tilde{\mathbf{y}} = \Delta\tilde{\mathbf{x}}/\Delta t$. Assuming all errors are uncorrelated from each other and that all error covariance matrices are of the form $\sigma^2\mathbf{I}$ we have that the 3×3 covariance matrix of the measurement error is given by

$$\mathbf{R}_y = \sigma_s^2[\mathbf{y}\diagdown]^2 - \sigma_m^2[\mathbf{y}\times]^2 + \sigma_n^2[\mathbf{y}*]^2 + \sigma_q^2[\mathbf{y}\diagdown]^4 + \sigma_a^2[\mathbf{y}\diagdown]^2 + \left[\sigma_b^2 + (\sigma_\eta^2/\Delta t) + (2\sigma_{ro}^2/\Delta t^2)\right]\mathbf{I} \tag{18}$$

σ_η^2 is the spectral density of the random walk $\boldsymbol{\eta}$ [4]. The low frequency errors (either constants or slowly varying errors) are unaffected by the choice of Δt, the random walk is scaled by the inverse of Δt and the readout error is scaled by the square of the inverse of Δt. Therefore larger Δt will make the low frequency errors dominant while shorter time intervals will exacerbate the contribution of the noises.

Notice that some authors prefer to define the non-orthogonality matrix as

$$[\mathbf{n}*] = \begin{bmatrix} 0 & n_3 & n_2 \\ 0 & 0 & n_1 \\ 0 & 0 & 0 \end{bmatrix}$$

in which case the factor multiplying σ_n^2 would be different.

AVERAGING AND FILTERING

As mentioned in the previous section, different choices of Δt will enable detecting different error sources. The control loop is usually fed by filtered IMU outputs with a small time constant of the filter in order to suppress high frequency noise. The navigation solution on the other hand, integrates IMU data in between measurement updates, hence is more sensitive to higher than nominal biases, especially when external measurements are not available (e.g. blackouts). An especially important error to be detected are slowly growing biases, as they can potentially make the navigation solution slowly diverge without being detected by internal measurement residual checks. Therefore we propose to test data on two different intervals, a longer one meant to test the data going to navigation and a shorter one, meant to test the data going to controls.

A possible approach is to accumulate N samples (N being a different number for the control and for nav) each spanning a time interval Δt_i, i.e.

$$\mathbf{y}_i = \frac{1}{N} \sum_{k=i-N}^{i} \frac{\Delta \mathbf{x}_k}{\Delta t_i}$$

with this choice in the error equations Eq. (18) and Eq. (37) we will have $\Delta t = N \Delta t_i$. This choice however, would require to store the last N incremental outputs of the IMU, or at least accumulate them and store them as lower frequency data.

For the remainder of this section we define the input $\mathbf{u}_k$ as

$$\mathbf{u}_k = \frac{\Delta \mathbf{x}_k}{\Delta t_i}$$

Infinite Impulse Response Filter

The general second order IIR filter used has the form

$$\mathbf{y}_i = \alpha_1 \mathbf{y}_{i-1} + \alpha_2 \mathbf{y}_{i-2} + \beta_0 \mathbf{u}_i + \beta_1 \mathbf{u}_{i-1} + \beta_2 \mathbf{u}_{i-2}$$

as long as $\alpha_1^2 + 4\alpha_2 \neq 0$ this recursion equation can be written in diagonal state-space form

$$\mathbf{z}_{k+1} = \mathbf{A}\mathbf{z}_k + \mathbf{B}\mathbf{u}_k = \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix} \mathbf{z}_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{u}_k \tag{19}$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{z}_k + \mathbf{D}\mathbf{u}_k = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \mathbf{z}_k + \beta_0 \mathbf{u}_k \tag{20}$$

where $p_i = (\alpha_1 \pm \sqrt{\alpha_1^2 + 4\alpha_2})/2$ and

$$c_i = \frac{\beta_0 p_i^2 + \beta_1 p_i + \beta_2}{p_i - p_j} \qquad j \neq i$$

We have that $\alpha_2 = -p_1 p_2$ and $\alpha_1 = p_1 + p_2$. The solution of the difference equation is given by

$$\mathbf{y}_k = \mathbf{D}\mathbf{u}_k + \mathbf{C} \sum_{j=0}^{k-1} \mathbf{A}^{k-j-1} \mathbf{B}\mathbf{u}_j = \beta_0 \mathbf{u}_k + \sum_{j=0}^{k-1} (c_1 p_1^{k-j-1} + c_2 p_2^{k-j-1}) \mathbf{u}_j \tag{21}$$

As long as both poles have magnitude less than one the series is convergent. In order for the coefficients to add to one the following condition must be satisfied

$$1 - \alpha_1 - \alpha_2 = \beta_0 + \beta_1 + \beta_2$$

or equivalently

$$\beta_0 + \frac{c_1}{1-p_1} + \frac{c_2}{1-p_2} = 1.$$

Notice that a first order filter can be retrieved by setting $\alpha_2 = \beta_2 = 0$. In this case one pole is zero as is its corresponding coefficient c_i.

From Eq. (21) it follows immediately that the contribution of the random walk to the total error in Eq. (18) is given by

$$\left(\beta_0^2 + \sum_{j=0}^{k-1}(c_1 p_1^{k-j-1} + c_2 p_2^{k-j-1})^2\right)\frac{\sigma_\eta^2}{\Delta t_i} \to \left(\beta_0^2 + \frac{c_1^2}{1-p_1^2} + \frac{c_2^2}{1-p_2^2} + 2\frac{c_1 c_2}{1-p_1 p_2}\right)\frac{\sigma_\eta^2}{\Delta t_i} = \frac{\sigma_\eta^2}{\Delta t_\eta} \tag{22}$$

where the right hand side is the sum of the series as $k \to \infty$.

To calculate the contribution of the readout noise we notice that

$$\begin{aligned}\mathbf{y}_k &= \beta_0\frac{\mathbf{x}_k}{\Delta t} - \beta_0\frac{\mathbf{x}_{k-1}}{\Delta t} + \sum_{j=0}^{k-1}(c_1 p_1^{k-j-1} + c_2 p_1^{k-j-1})\frac{\mathbf{x}_j}{\Delta t} - \sum_{j=-1}^{k-2}(c_1 p_1^{k-j-2} + c_2 p_2^{k-j-2})\frac{\mathbf{x}_j}{\Delta t} \\ &= \beta_0\frac{\mathbf{x}_k}{\Delta t} + (c_1 + c_2 - \beta_0)\frac{\mathbf{x}_{k-1}}{\Delta t} + \sum_{j=0}^{k-2}\left\{c_1(p_1-1)p_1^{k-j-2} + c_2(p_2-1)p_2^{k-j-2}\right\}\frac{\mathbf{x}_j}{\Delta t} \end{aligned} \tag{23}$$

therefore the contribution of the readout noise to the total error in Eq. (18) is given by

$$\left\{\beta_0^2 + (c_1+c_2-\beta_0)^2 + c_1^2\frac{1-p_1}{1+p_1} + c_2^2\frac{1-p_2}{1+p_2} + 2c_1c_2\frac{(p_1-1)(p_2-1)}{1-p_1p_2}\right\}\frac{\sigma_{ro}^2}{\Delta t_i^2} = \frac{\sigma_\eta^2}{\Delta t_{ro}^2} \tag{24}$$

First Order Filters

Two simple cases are worth investigating. The simplest first order filter is given by

$$\mathbf{y}_i = \alpha\mathbf{y}_{i-1} + \beta\mathbf{u}_i = \beta\sum_{k=0}^{i-1}\alpha^k\mathbf{u}_{i-k} \tag{25}$$

for $0 \le \alpha \le 1$ the series α^k converges to $1/(1-\alpha)$, in order for this filter to produce an exponentially weighted average all the weights must sum to one, i.e. we must choose $\beta = 1-\alpha$. This choice implies that all bias errors remain unchanged using this filter or using a regular average as in Eq. (18) and Eq. (37). The random walk contribution to the total error in Eq. (18) is given by

$$\beta^2\sum_{k=0}^{i-1}\alpha^{2k}\frac{\sigma_\eta^2}{\Delta t_i} = (1-\alpha)^2\sum_{k=0}^{i-1}\alpha^{2k}\frac{\sigma_\eta^2}{\Delta t_i} \to \frac{(1-\alpha)^2}{1-\alpha^2}\frac{\sigma_\eta^2}{\Delta t_i} = \frac{1-\alpha}{1+\alpha}\frac{\sigma_\eta^2}{\Delta t_i}$$

Set $\alpha = (\Delta t_\alpha - \Delta t_i)/(\Delta t_\alpha + \Delta t_i)$, noticing that $\beta = (1-\alpha) = 2\Delta t_i/(\Delta t_\alpha + \Delta t_i)$ and $(1+\alpha) = 2\Delta t_\alpha/(\Delta t_\alpha + \Delta t_i)$ the random walk contribution can be expressed as

$$\frac{1-\alpha}{1+\alpha}\frac{\sigma_\eta^2}{\Delta t_i} = \frac{2\Delta t_i}{2\Delta t_\alpha}\frac{\sigma_\eta^2}{\Delta t_i} = \frac{\sigma_{ro}^2}{\Delta t_\alpha}$$

hence using this first order filter we need to replace $\Delta t \to \Delta t_\alpha$ in the random walk contribution of Eq. (18) and Eq. (37).

Eq. (25) can be rewritten as

$$\begin{aligned}\mathbf{y}_i &= (1-\alpha)\sum_{k=0}^{i-1}\alpha^k \frac{\mathbf{x}_{i-k}-\mathbf{x}_{i-k-1}}{\Delta t_i} \\ &= (1-\alpha)\left[\frac{\mathbf{x}_i}{\Delta t_i} - \alpha^i \frac{\mathbf{x}_0}{\Delta t_i}\right] - (1-\alpha)^2 \sum_{k=1}^{i-1}\alpha^{k-1}\frac{\mathbf{x}_{i-k}}{\Delta t_i}\end{aligned}$$

the contribution of the readout noise to the total error in Eq. (18) is therefore given by

$$(1-\alpha)^2\frac{\sigma_{ro}^2}{\Delta t_i^2} + \frac{(1-\alpha)^4}{1-\alpha}\frac{\sigma_{ro}^2}{\Delta t_i^2} = (1-\alpha)^2(2-\alpha)\frac{\sigma_{ro}^2}{\Delta t_i^2} = \frac{4(\Delta t_\alpha + 3\Delta t_i)}{(\Delta t_\alpha + \Delta t_i)^3}\sigma_{ro}^2$$

In the common case where $\Delta t_\alpha \gg \Delta t_i$ we have that

$$\frac{4(\Delta t_\alpha + 3\Delta t_i)}{(\Delta t_\alpha + \Delta t_i)^3}\sigma_{ro}^2 \simeq 4\frac{\sigma_{ro}^2}{\Delta t_\alpha^2}$$

Another simple first order filter is given by

$$\mathbf{y}_i = \alpha\mathbf{y}_{i-1} + \beta\mathbf{u}_i + \beta\mathbf{u}_{i-1} = \beta\mathbf{u}_i + \beta(\alpha+1)\sum_{k=1}^{i-1}\alpha^{k-1}\mathbf{u}_{i-k}$$

in order for the weights to asymptotically add to one the following condition is necessary:

$$2\beta = 1-\alpha. \tag{26}$$

The random walk contribution is given by

$$\left(\beta^2 + \beta^2(1+\alpha)^2\sum_{k=0}^{i-2}\alpha^{2k}\right)\frac{\sigma_\eta^2}{\Delta t_i} \to \left(\beta^2 + \beta^2\frac{1+\alpha}{1-\alpha}\right)\frac{\sigma_\eta^2}{\Delta t_i} = \beta^2\frac{2}{1-\alpha}\frac{\sigma_\eta^2}{\Delta t_i}$$

Substituting Eq. (26) we obtain

$$\beta^2\frac{2}{1-\alpha}\frac{\sigma_\eta^2}{\Delta t_i} = \beta\frac{\sigma_\eta^2}{\Delta t_i} = \frac{\sigma_\eta^2}{\Delta t_\beta}$$

where $\Delta t_\beta = \Delta t_i/\beta$.

The readout noise contribution is derived from

$$\mathbf{y}_i = \beta\frac{\mathbf{x}_i}{\Delta t_i} + \beta\alpha\frac{\mathbf{x}_{i-1}}{\Delta t_i} - \beta(\alpha+1)\alpha^{i-2}\mathbf{x}_0 + \beta(\alpha^2-1)\sum_{k=2}^{i-1}\alpha^{k-2}\frac{\mathbf{x}_{i-k}}{\Delta t_i}$$

that results in a contribution given by

$$2\beta^2\frac{\sigma_{ro}^2}{\Delta t_i^2} = 2\frac{\sigma_{ro}^2}{\Delta t_\beta^2}$$

this form of the first order filter is particularly simple because it has the exact same structure as the conventional average and the only substitution is $\Delta t_\beta \rightarrow \Delta t$. Therefore the same effect of a moving average filter is obtained without the need to store a lot of past data. Once again the filter has the form

$$\mathbf{y}_i = \alpha \mathbf{y}_{i-1} + \beta \mathbf{u}_i + \beta \mathbf{u}_{i-1}$$

with $\beta = \Delta t_i / \Delta t_\beta$ and $\alpha = 1 - 2\beta = (\Delta t_\beta - 2\Delta t_i)/\Delta t_\beta$

THREE ADDITIONAL ERROR SOURCES

Three additional sources of error are lever arm correction inaccuracies due to the IMUs not being co-located, time-tag differences, and alignment mounting errors of the boxes.

The IMU outputs are not perfectly synchronized, therefore when comparing the measurements an additional error source is due to this error discrepancy. Let's define a common time t_0 and the time of the measurement from the i-th IMU as t_i, then the error due to time discrepancy is given by

$$\delta \mathbf{y}_{td,i} = \mathbf{y}_i - \mathbf{y}_0 \simeq \dot{\mathbf{y}}_i (t_i - t_0) = \dot{\mathbf{y}}_i \, \delta t_i \tag{27}$$

The alignment mounting errors $\delta \boldsymbol{\alpha}_i$ behave in a similar manner as the internal misalignment errors, therefore we have that

$$\delta \mathbf{y}_{align,i} = -[\mathbf{y}_i \times] \delta \boldsymbol{\alpha}_i \tag{28}$$

In order to compare the acceleration of the redundant IMUs it is necessary to bring all the measured accelerations to a common location $\mathbf{r}_0$. The acceleration $\mathbf{a}_{0,i}$ at the common location is calculated from the acceleration $\mathbf{a}_i$ of the i-th IMU and its location $\mathbf{r}_i$.

$$\mathbf{a}_{0,i} = \mathbf{a}_i - \left\{ [\boldsymbol{\omega} \times]^2 + [\dot{\boldsymbol{\omega}} \times] \right\} (\mathbf{r}_i - \mathbf{r}_0) \tag{29}$$

Therefore the error due to the uncertainty in the IMU location, $\delta \mathbf{r}_i$ is given by

$$\delta \mathbf{a}_{loc,i} = - \left\{ [\boldsymbol{\omega} \times]^2 + [\dot{\boldsymbol{\omega}} \times] \right\} \delta \mathbf{r}_i \tag{30}$$

The total error associated with an IMU measurement is given by the sum of the contributions in Eq. (17) and the three errors above

$$\delta \mathbf{y}_{tot} = \delta \mathbf{y} + \delta \mathbf{y}_{td} + \delta \mathbf{y}_{align} + \delta \mathbf{y}_{loc} \tag{31}$$

where the location error applies to accelerometers only. The 3×3 covariance matrix is given by

$$\begin{aligned} \mathbf{R}_y = {} & \sigma_s^2 [\mathbf{y} \diagdown]^2 - \sigma_m^2 [\mathbf{y} \times]^2 + \sigma_n^2 [\mathbf{y} *]^2 + \sigma_q^2 [\mathbf{y} \diagdown]^4 + \sigma_a^2 [\mathbf{y} \diagdown]^2 + \left[\sigma_b^2 + (\sigma_\eta^2 / \Delta t_\eta) + (2\sigma_{ro}^2 / \Delta t_{ro}^2) \right] \mathbf{I} \\ & (\dot{\mathbf{y}} \dot{\mathbf{y}}^{\mathrm{T}}) \sigma_t^2 - \sigma_\alpha^2 [\mathbf{y}_i \times]^2 + \left\{ [\boldsymbol{\omega} \times]^2 + [\dot{\boldsymbol{\omega}} \times] \right\} \left\{ [\boldsymbol{\omega} \times]^2 + [\dot{\boldsymbol{\omega}} \times] \right\}^{\mathrm{T}} \sigma_r^2 \end{aligned} \tag{32}$$

FAULT DETECTION AND ISOLATION STRATEGY

From Eq. (32) it is clear that to calculate the covariance of the IMU measurement errors covariance it is necessary to use the IMU measurement itself. This creates a problem because in order to check if the measurement is good we need its covariance and in order to calculate the covariance we need to use the measurement. It is assumed that only one fault at the time occurs, this assumption could be relaxed by testing the three possible combinations of three IMUs (i.e. 1,2, and 3; 1,2, and 4; 2, 3, and 4) rather than testing all four at once. Under this alternative strategy if only one IMU fails then two of the three checks will fail. If two IMUs fail simultaneously then checking them by pairs is necessary to isolate the fault. This alternative strategy makes the FDI algorithm considerably more complex, in addition the chances of more than one IMU soft failures at the same time are quite small, therefore an algorithmic assumption (and possibly limitation) is that only one failure at the time will occur.

Since only one IMU failure can occur at one given time the IMU measurement error covariance is calculated using the IMU measurement that is closest to the average of all IMU measurements. It is assumed that the faulty IMU will be the outlier, therefore by choosing the middle one for covariance computations we are guaranteed not to pick the faulty one. Once four IMUs are tested we have that

$$y_{avg} = 0.25(\|\mathbf{y}_1\| + \|\mathbf{y}_2\| + \|\mathbf{y}_3\| + \mathbf{y}_4\|) \tag{33}$$

and the IMU used to calculate the error covariance in Eq. (32) is the closest one to y_{avg}, i.e.

$$\min_i (\|\mathbf{y}_i\| - y_{avg})^2. \tag{34}$$

After the first IMU failure the same procedure is followed with the remaining three IMUs. The IMU used to calculate the covariance is the mid-value. After a second failure we can still perform a test to detect a third failure; this failure however cannot be isolated. Because it is not possible to calculate the mid value of only two IMUs the test will need to be performed twice, once calculating the covariance with each IMU; if either of the tests does not pass, a failure is declared [5].

When no prior failure occurs four IMU measurements are tested. There are three possibilities of how to test these measurements. The first option is to compare the vector measurement directly. After expressing the measurements in the same coordinate system the following quantities are calculated together with the covariance in Eq. (32).

$$\begin{aligned}
\boldsymbol{\epsilon}_1 &= (3\mathbf{y}_1 - \mathbf{y}_2 - \mathbf{y}_3 - \mathbf{y}_4)\sqrt{12} \\
\boldsymbol{\epsilon}_2 &= (3\mathbf{y}_2 - \mathbf{y}_1 - \mathbf{y}_3 - \mathbf{y}_4)\sqrt{12} \\
\boldsymbol{\epsilon}_3 &= (3\mathbf{y}_3 - \mathbf{y}_1 - \mathbf{y}_2 - \mathbf{y}_4)\sqrt{12} \\
\boldsymbol{\epsilon}_4 &= (3\mathbf{y}_4 - \mathbf{y}_1 - \mathbf{y}_2 - \mathbf{y}_3)\sqrt{12}
\end{aligned}$$

We can then set thresholds to detect failures based on the following positive, scalar quantities

$$\boldsymbol{\epsilon}_i^{\mathrm{T}} \mathbf{R}_y^{-1} \boldsymbol{\epsilon}_i \tag{35}$$

As previously mentioned, testing the vector measurements necessitates that they are expressed in a common coordinate system and therefore mounting errors between the IMUs will be part of the error making the errors larger and hence making other faults harder to detect. This is a good strategy if mounting errors or internal misalignments need to be included in the FDI tests.

If mounting errors and internal misalignments are not part of the classes of faults to be tested by FDI, the second option is to test the measurements' magnitudes. The error of $\|\tilde{\mathbf{y}}\|$ can be approximated to first order as $\left(\mathbf{y}^{\mathrm{T}}\delta\mathbf{y}/\|\mathbf{y}\|\right)$. Therefore, ignoring higher order contributions we have that

$$\begin{aligned}
\sigma^2_{\|\mathbf{y}\|} =& \big(\mathbf{y}^{\mathrm{T}}\mathbf{R}_y\,\mathbf{y}\big)/\|\mathbf{y}\|^2 && (36)\\
=& (\sigma_s^2+\sigma_a^2)(y_1^4+y_2^4+y_3^4)/\|\mathbf{y}\|^2 + 2\sigma_n^2(y_1^2y_2^2+y_1^2y_3^2+y_2^2y_3^2)/\|\mathbf{y}\|^2 + \\
&+\sigma_q^2(y_1^6+y_2^6+y_3^6)/\|\mathbf{y}\|^2+\sigma_b^2+\sigma_\eta^2/\Delta t_\eta + 2\sigma_{ro}^2/\Delta t_{ro}^2+\sigma_t^2(\dot{\mathbf{y}}^{\mathrm{T}}\mathbf{y})^2/\|\mathbf{y}\|^2+ \\
&+\sigma_r^2\mathbf{y}^{\mathrm{T}}\left([\boldsymbol{\omega}\times]^2+[\dot{\boldsymbol{\omega}}\times]\right)^{\mathrm{T}}\left([\boldsymbol{\omega}\times]^2+[\dot{\boldsymbol{\omega}}\times]\right)\mathbf{y}/\|\mathbf{y}\|^2 && (37)
\end{aligned}$$

The contribution of the internal misalignments and mounting errors are absent from Eq. (37). We perform the tests on

$$|\epsilon_i|/\sigma_{\|\mathbf{y}\|} \tag{38}$$

where

$$\epsilon_1 = (3\|\mathbf{y}_1\| - \|\mathbf{y}_2\| - \|\mathbf{y}_3\| - \|\mathbf{y}_4\|)\sqrt{12} \tag{39}$$

$$\epsilon_2 = (3\|\mathbf{y}_2\| - \|\mathbf{y}_1\| - \|\mathbf{y}_3\| - \|\mathbf{y}_4\|)\sqrt{12} \tag{40}$$

$$\epsilon_3 = (3\|\mathbf{y}_3\| - \|\mathbf{y}_1\| - \|\mathbf{y}_2\| - \|\mathbf{y}_4\|)\sqrt{12} \tag{41}$$

$$\epsilon_4 = (3\|\mathbf{y}_4| \ - \|\mathbf{y}_1\| - \|\mathbf{y}_2\| - \|\mathbf{y}_3\|)\sqrt{12} \tag{42}$$

The third option is to test $\|\mathbf{y}_i\|^2$ rather than $\|\mathbf{y}_i\|$ this saves computing a few square roots.

We will proceed with option number two.

Table 1 contains the FDI algorithm for both gyros and accelerometers.

Table 1. IMU FDI Algorithm

if four non-failed IMUs **then**
  calculate the four ϵ_i as in Eqs. (39)-(42)
  determine the mid-value out of $\mathbf{y}_i$ using Eqs. (33) and (34)
  use the mid-value of $\mathbf{y}_i$ to calculate the error variance as in Eq. (37)
end if
if three non-failed IMUs **then**
  calculate the three ϵ_i using the unfailed measurements as $\epsilon_i = (2\|\mathbf{y}_i\| - \|\mathbf{y}_j\| - \|\mathbf{y}_k\|)\sqrt{6}$
  determine the mid-value of the unfailed $\mathbf{y}_i$
  use the mid-value of $\mathbf{y}_i$ to calculate the error variance as in Eq. (37)
end if
if two non-failed IMUs **then**
  calculate $\epsilon = (\|\mathbf{y}_i\| - \|\mathbf{y}_j\|)\sqrt{2}$ using the unfailed measurements
  calculate the error variance $\sigma^2_{\|\mathbf{y}_i\|}$ as in Eq. (37) for both unfailed measurements
  utilize the smaller of the two variances in the following checks
end if
if $|\epsilon_i|/\sigma_{\|\mathbf{y}\|} > THRESHOLD$ **then**
  set the exceeding-threshold flag corresponding to the max value of ϵ_i to one
  if more than N_{prob} exceedances out of N_{test} **then**
    declared suspect the IMU corresponding to the unsuccessful test
  end if
  if more than N_{fail} exceedances out of N_{test} **then**
    declare failed the IMU corresponding to the unsuccessful test
  end if
end if

SIMULATION RESULTS

The following simulation results implement the above algorithm on a vehicle with a set of four redundant IMUs. One IMU is forced to fail 40sec into the simulation by introducing a small growing bias in the gyro signal. As the bias increases the low cut-off frequency gyro test begins to show suspect measurements as shown in Figure 1. Figure 2 shows the status flag for IMU4 ultimately switching to 2 indicating a failure has occurred. The fault vectors for each test are shown in Figure 3, large values near the end of the simulations are due to the vehicle landing.

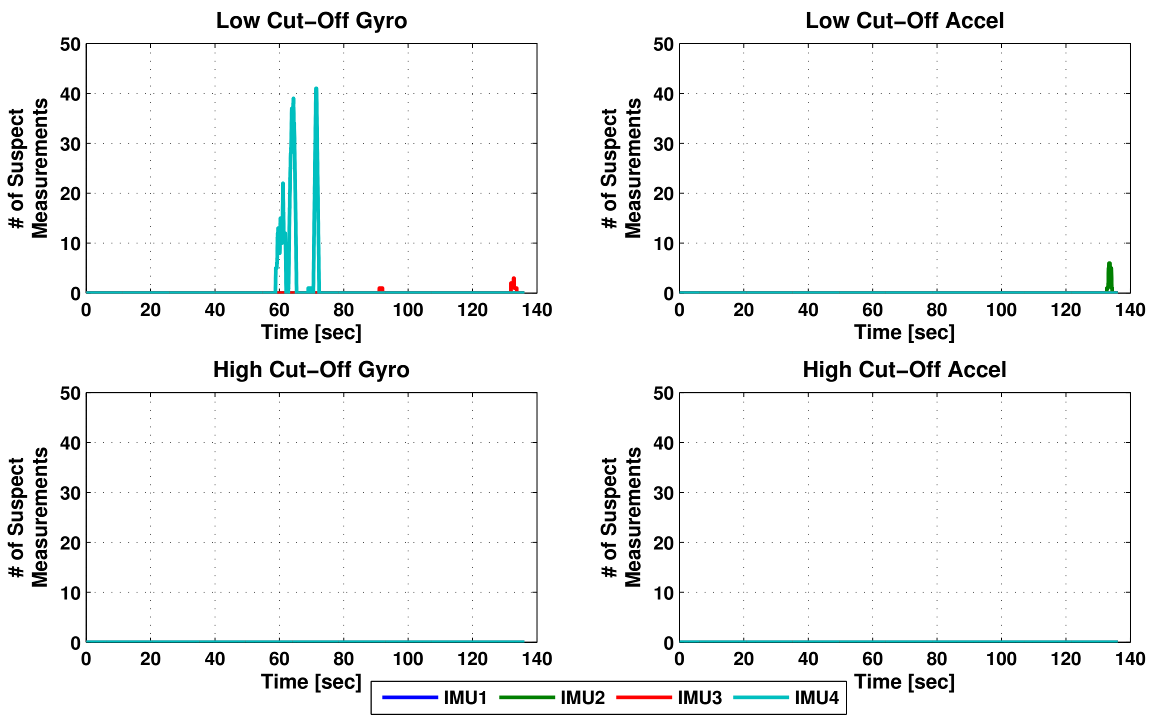

Figure 1. # of Suspect Measurements

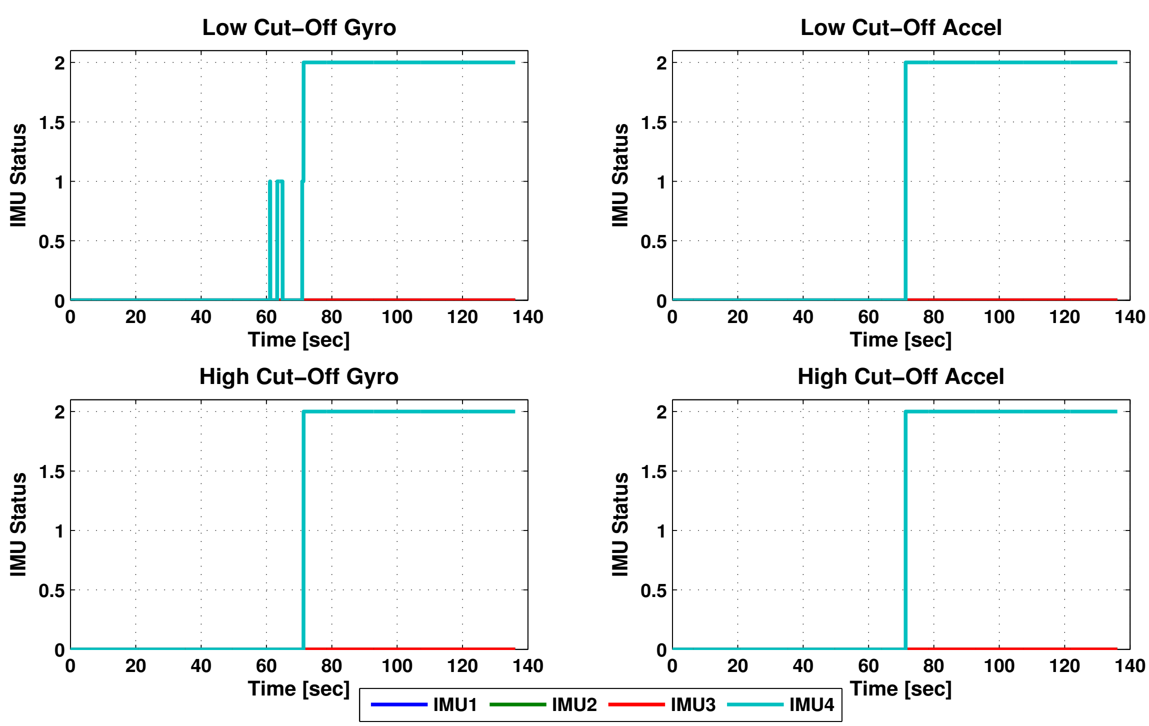

Figure 2. IMU Status

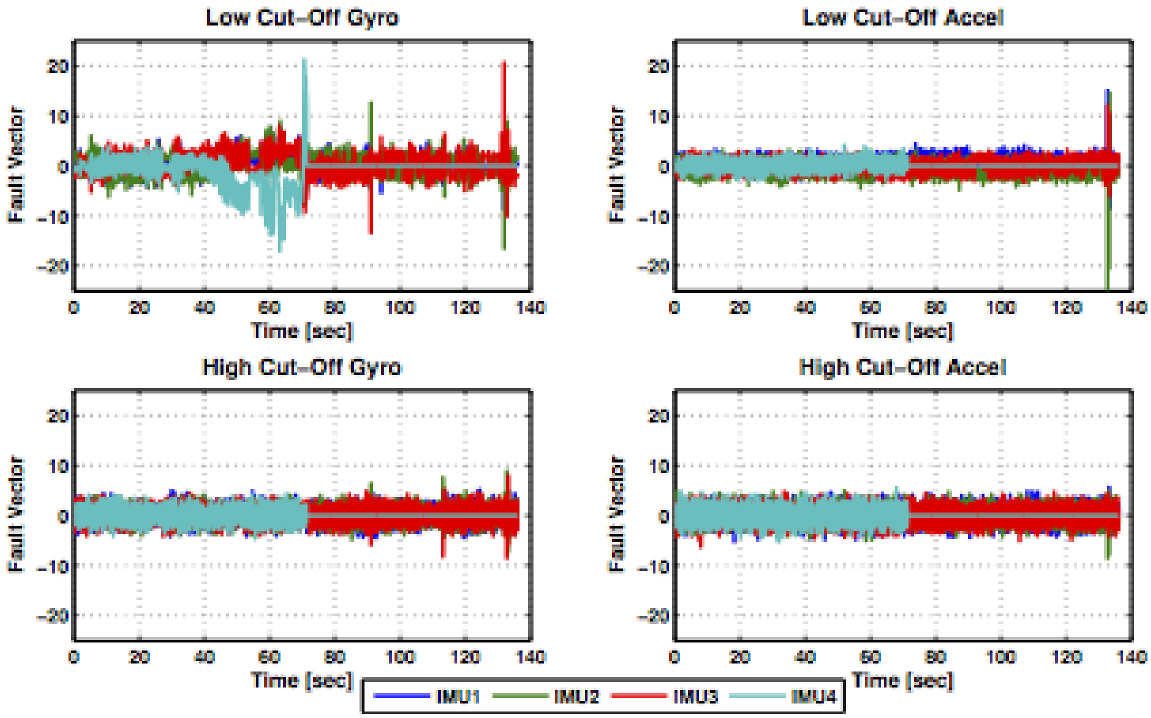

Figure 3. Fault Vector

CONCLUSIONS

An IMU fault detection and isolation strategy is introduced to support autonomous man-rated vehicles. The need of infinite impulse response (IIR) filters is explained and a detailed derivation show the effect these IIR filters have on the measurement errors. Dynamic thresholds are calculated from the filtered IMU measurement errors. A strategy to declare a fault is outlined based on a moving window approach. The number of threshold violations over the last N_t measurements is stored. The number of allowed violations is a design parameter that is determined based on the desired probabilities of misdirection and false alarm. A numerical simulation is used to validate the proposed approach.

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