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COVARIANCE MATCHING FILTER FOR IMU ERROR ESTIMATION

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In this paper, an on-line adaptive accelerometer calibration algorithm is presented. The accelerometer is corrupted with an exponentially correlated random bias and white Gaussian noise. Assuming the availability of noisy position and velocity measurements, the estimates of position, velocity, bias as well as the accelerometer's noise characteristics are estimated. These results are made possible through the application of use a covariance matching adaptive filter recently established by the authors. Numerical simulations are performed to evaluate the performance of the proposed calibration algorithm and its effectiveness subject to noisy accelerometer measurements.

INTRODUCTION

Inertial measurement units (IMUs) consisting of accelerometers and angular-rate gyroscopes are extensively used in conjunction with GPS,[1] visual-INS,[2] and magnetometers[3,4] for navigation and tracking applications. Accelerometers measurements are usually used to propagate the dynamics, while the position and velocity measurements are used as external measurements to achieve better navigation accuracy. The performance of these estimation algorithms is limited by the accuracy of the calibrated sensor bias, scale factors, and non-orthogonality, as well as the knowledge of the statistics of the residual calibration error. The bias or drift in accelerometer measurements is known to be a major contributor to the navigation accuracy.[5] Hence, many estimation algorithms have been developed to compensate for the sensor bias.[6] However, these algorithms assume that the velocity random walk or the statistics of the additive white noise in the accelerometer is completely known. Filter divergence has been studied for a Kalman filter with an incorrect covariance matrix.[7–9] In this paper, a Covariance Matching Kalman Filter (CMKF) that was recently established by the authors[10] is applied to adaptively estimate the power spectral density of the velocity random walk.

Previous methods of sensor error estimation include artificially inflating the process noise covariance heuristically and augmenting the state vector with the bias. Techniques which involve decoupling the state and bias estimation have also been presented. Filters treating bias as an error term rather than a state component have also been formulated.[11] A survey of attitude estimation methods presents additional methods for estimating sensor biases.[12] Nonlinear complementary filters for estimating attitude and gyroscope bias have also been developed.[2,13] In some filters, a constant random bias model was used while in some others, an exponentially correlated random bias model

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was used. Another approach is to calibrate the accelerometers using the modified Allan variance method.[14] While most works concentrate on estimating the bias, this paper uses the measurements to simultaneously estimate both the bias and the covariance of the velocity random walk.

Unlike for biases, the estimation of covariances cannot be achieved by state space augmentation, rather adaptive filtering is used instead to estimate both the state and parameters of the model.[15] Innovation-based Sage-Husa adaptive filters and their various modifications are one approach for adaptable filtering with GPS/INS applications.[16] The formulation presented here is derived from recent formulations of adaptive filters,[10,17] where a stacked measurement equation is formulated and the state and covariance estimators are subsequently derived. An example problem is then numerically simulated to test the validity of the algorithm. Certain limitations and proposed future work are discussed as part of concluding remarks.

PROBLEM FORMULATION

Dynamics Model

Consider a simplified two-axis linear model for position and velocity as state components of a two-axis accelerometer sensor. The algorithm presented here is intended for calibration on a planar table of two axes. Hence, the z-axis (vertical axis) is not considered. However, the same results can be readily extended to include all 3 axes and the effects of gravity. Thus, we have

$$\ddot{r}_x = a_x \tag{1}$$

$$\ddot{r}_y = a_y \tag{2}$$

where in r_x and r_y are scalar positions and a_x and a_y are the true accelerations in the x and y directions of the sensor's center of mass respectively. The quantities b_x and b_y are the accelerometer biases in the x and y measurement channels modeled as exponentially correlated random variables:

$$\dot{b}_x(t) = -\frac{b_x(t)}{\tau_x} + w_x(t) \tag{3}$$

$$\dot{b}_y(t) = -\frac{b_y(t)}{\tau_y} + w_y(t) \tag{4}$$

wherein, τ_x and τ_y are the time constants for the decay rates for the respective bias terms and $w_x(t)$ and $w_y(t)$ are zero-mean white processes with power spectral densities $S_{b_x} = \frac{\sigma^2_{ss,x}}{2\tau_x}$ and $S_{b_y} = \frac{\sigma^2_{ss,y}}{2\tau_y}$ respectively, where $\sigma^2_{ss,x}$ and $\sigma^2_{ss,y}$ are the steady-state variances of the two exponentially correlated random biases. Writing the state dynamical equations as a continuous-time linear equation

$$\underbrace{\frac{d}{dt}\begin{bmatrix} r_x \\ \dot{r}_x \\ b_x \\ r_y \\ \dot{r}_y \\ b_y \end{bmatrix}}_{\triangleq \dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_y} \end{bmatrix}}_{\triangleq A} \underbrace{\begin{bmatrix} r_x \\ \dot{r}_x \\ b_x \\ r_y \\ \dot{r}_y \\ b_y \end{bmatrix}}_{\triangleq \boldsymbol{x}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\triangleq B} \underbrace{\begin{bmatrix} a_x \\ a_y \end{bmatrix}}_{\boldsymbol{a}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\triangleq \Lambda} \underbrace{\begin{bmatrix} w_x \\ w_y \end{bmatrix}}_{\triangleq \boldsymbol{w}} \tag{5}$$

In order to express the foregoing continuous-time dynamics to a discrete-time difference equation, we assume that the sample time T is small enough such that acceleration $\boldsymbol{a}(t) \approx \boldsymbol{a}_k, \forall t \in [t_k, t_k{+}T]$

$$\boldsymbol{x}_{k+1} = F\boldsymbol{x}_k + G\boldsymbol{a}_k + \Lambda\boldsymbol{w}_k \tag{6}$$

Here,

$$F = e^{AT} = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-\frac{T}{\tau_x}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\frac{T}{\tau_y}} \end{bmatrix}, \text{ and } G = \int\limits_0^T e^{At}Bdt = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \\ 0 & 0 \end{bmatrix}$$

The zero-mean white sequence $\boldsymbol{w}_k$ has a covariance matrix

$$Q_b = \begin{bmatrix} (1 - e^{-\frac{2T}{\tau_x}})\sigma_{ss,x}^2 & 0 \\ 0 & (1 - e^{-\frac{2T}{\tau_y}})\sigma_{ss,y}^2 \end{bmatrix} \tag{7}$$

Measurement Model

The system is assumed to have position and velocity measurements through external sensors with additive white Gaussian noise.

$$\boldsymbol{z}_k = \begin{bmatrix} r_{x_k} \\ \dot{r}_{x_k} \\ r_{y_k} \\ \dot{r}_{y_k} \end{bmatrix} + \begin{bmatrix} v_{r_x} \\ v_{v_x} \\ v_{r_y} \\ v_{v_y} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{H} \boldsymbol{x}_k + \boldsymbol{\eta}_k \tag{8}$$

Along with that, the acceleration measurement $\tilde{\boldsymbol{a}}(t)$ corrupted by bias and noise is given by

$$\tilde{\boldsymbol{a}}(t) = \boldsymbol{a}(t) + \boldsymbol{b}(t) + \boldsymbol{v}(t) \tag{9}$$

where the zero mean white process $\boldsymbol{v}(t)$ is known as the velocity random walk (VRW). Most accelerometers do not measure output acceleration directly, rather they accumulate the measurement into a $\Delta\tilde{\boldsymbol{v}}_k$, the actual measurement produced by the sensor is

$$\Delta\tilde{\boldsymbol{v}}_k = \int_{t_{k-1}}^{t_k} \tilde{\boldsymbol{a}}(t)dt = \int_{t_{k-1}}^{t_k} \boldsymbol{a}(t)dt + \int_{t_{k-1}}^{t_k} \boldsymbol{b}(t)dt + \int_{t_{k-1}}^{t_k} \boldsymbol{v}(t)dt \tag{10}$$

and the true change in velocity is given by

$$\Delta\boldsymbol{v}_k = \int_{t_{k-1}}^{t_k} \boldsymbol{a}(t)dt \tag{11}$$

Assuming the sample time of the accelerometer measurements is much smaller than the time constant of the bias, the measurement is given by

$$\Delta\tilde{\boldsymbol{v}}_k = \Delta\boldsymbol{v}_k + \boldsymbol{b}_kT + \boldsymbol{v}_k \tag{12}$$

where $\boldsymbol{v}_k$ is a zero-mean white sequence with covariance matrix given by $\begin{bmatrix} S_{v_x}T & 0 \\ 0 & S_{v_y}T \end{bmatrix}$. The measurement noise $\boldsymbol{\eta}_k$ is assumed to have a known constant diagonal covariance of R. The initial state estimate, the measurement noise and the process noise are assumed to be uncorrelated with each other.

FILTER FORMULATION

Propagation

The acceleration measurement accumulated as $\Delta\tilde{\boldsymbol{v}}_k$ is a known exogenous input to the system. Therefore, the velocity estimate propagation equation is given by the following equation.

$$\hat{r}_{x_{k+1|k}} = \hat{r}_{x_k} + \dot{\hat{r}}_{x_k}T + \frac{1}{2}(\Delta\tilde{v}_{x_k} - \hat{b}_{x_k}T)T \tag{13}$$

$$\dot{\hat{r}}_{x_{k+1|k}} = \dot{\hat{r}}_{x_k} + \Delta\tilde{v}_{x_k} - \hat{b}_{x_k}T \tag{14}$$

$$\hat{r}_{y_{k+1|k}} = \hat{r}_{y_k} + \dot{\hat{r}}_{y_k}T + \frac{1}{2}(\Delta\tilde{v}_{y_k} - \hat{b}_{y_k}T)T \tag{15}$$

$$\dot{\hat{r}}_{y_{k+1|k}} = \dot{\hat{r}}_{y_k} + \Delta\tilde{v}_{y_k} - \hat{b}_{y_k}T \tag{16}$$

wherein, $\Delta\tilde{\boldsymbol{v}}_k = \begin{bmatrix} \Delta\tilde{v}_{x_k} \\ \Delta\tilde{v}_{y_k} \end{bmatrix}$ is the acceleration measurement and $\hat{r}_{x_k}$ is the x position estimate at time-step k. The state estimate propagation equation is given as follows.

$$\hat{\boldsymbol{x}}_{k+1|k} = \underbrace{\begin{bmatrix} 1 & T & -\frac{T^2}{2} & 0 & 0 & 0 \\ 0 & 1 & -T & 0 & 0 & 0 \\ 0 & 0 & e^{-\frac{T}{\tau_x}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & -\frac{T^2}{2} \\ 0 & 0 & 0 & 0 & 1 & -T \\ 0 & 0 & 0 & 0 & 0 & e^{-\frac{T}{\tau_y}} \end{bmatrix}}_{\triangleq \hat{F}} \hat{\boldsymbol{x}}_k + \underbrace{\begin{bmatrix} \frac{T}{2} & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & \frac{T}{2} \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\triangleq \hat{G}} \Delta\tilde{\boldsymbol{v}}_k \tag{17}$$

Also, using Eq. (12) in Eq. (6), we get the following truth propagation equation.

$$\boldsymbol{x}_{k+1} = F\boldsymbol{x}_k + \hat{G}\Delta\boldsymbol{v}_k + \Lambda\boldsymbol{w}_k \tag{18}$$

Note that $(F - \hat{F})\boldsymbol{x}_k = \hat{G}T\begin{bmatrix} b_{x_k} \\ b_{y_k} \end{bmatrix} = \hat{G}T\boldsymbol{b}_k$. The state error covariance $P_{k+1|k} = E[(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k})(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k})^T]$ is obtained by:

$$\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k} = F\boldsymbol{x}_k - \hat{F}\hat{\boldsymbol{x}}_k + \hat{G}(\Delta\boldsymbol{v}_k - \Delta\tilde{\boldsymbol{v}}_k) + \Lambda\boldsymbol{w}_k \tag{19}$$

Using Eq. (12), we get

$$\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k} = F\boldsymbol{x}_k - \hat{F}\hat{\boldsymbol{x}}_k - \hat{G}(b_kT + \boldsymbol{v}_k) + \Lambda\boldsymbol{w}_k \tag{20}$$

$$\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k} = \hat{F}(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k) + (F - \hat{F})\boldsymbol{x}_k - \hat{G}T\boldsymbol{b}_k - \hat{G}\boldsymbol{v}_k + \Lambda\boldsymbol{w}_k \tag{21}$$

$$\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k} = \hat{F}(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k) - \hat{G}\boldsymbol{v}_k + \Lambda\boldsymbol{w}_k \tag{22}$$

$$P_{k+1|k} = \hat{F}P_k\hat{F}^T + \hat{Q}_a + \Lambda Q_b\Lambda^T \tag{23}$$

wherein, the matrix $\hat{Q}_a$ is the estimate of the covariance matrix of $\hat{G}\boldsymbol{v}_k$ and has the true value:

$$Q_a = \begin{bmatrix} \frac{T^3}{3}S_{v_x} & \frac{T^2}{2}S_{v_x} & 0 & 0 & 0 & 0 \\ \frac{T^2}{2}S_{v_x} & TS_{v_x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{T^3}{3}S_{v_y} & \frac{T^2}{2}S_{v_y} & 0 \\ 0 & 0 & 0 & \frac{T^2}{2}S_{v_y} & TS_{v_y} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The unknown parameters to be estimated in the adaptable filter are the VRW power spectral densities S_{v_x} and S_{v_y}.

Measurement Update

The Kalman Filter measurement update equation is as follows.

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}_{k+1|k} + K_{k+1}(\boldsymbol{z}_{k+1} - H\hat{\boldsymbol{x}}_{k+1|k}) \tag{24}$$

$$K_{k+1} = P_{k+1|k}H^T(HP_{k+1|k}H^T + R)^{-1} \tag{25}$$

$$\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k+1} = (I - K_{k+1}H)(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k}) - K_{k+1}\boldsymbol{v}_{k+1} \tag{26}$$

$$P_{k+1} = (I - K_{k+1}H)P_{k+1|k}(I - K_{k+1}H)^T + K_{k+1}RK_{k+1}^T \tag{27}$$

Covariance Estimator

The $(\hat{F}, H)$ pair stated respectively in Eq. (17) and Eq. (8) can be readily verified to be is completely observable. Following the formulation in our previous work,[10] the measurements are stacked in time to obtain the following equation.

$$\boldsymbol{x}_k = \hat{F}\boldsymbol{x}_{k-1} + \hat{G}\Delta\tilde{\boldsymbol{v}}_{k-1} - \boldsymbol{v}_{k-1} + \Lambda\boldsymbol{w}_{k-1} \tag{28}$$

$$\begin{bmatrix}\boldsymbol{z}_k \\ \boldsymbol{z}_{k-1}\end{bmatrix} = \underbrace{\begin{bmatrix}H\hat{F} \\ H\end{bmatrix}}_{\triangleq M_o}\boldsymbol{x}_{k-1} - \underbrace{\begin{bmatrix}H \\ \boldsymbol{0}\end{bmatrix}}_{\triangleq M_v}\boldsymbol{v}_{k-1} + \underbrace{\begin{bmatrix}H\hat{G} \\ \boldsymbol{0}\end{bmatrix}}_{\triangleq M_a}\Delta\tilde{\boldsymbol{v}}_{k-1} + \underbrace{\begin{bmatrix}H\Lambda \\ \boldsymbol{0}\end{bmatrix}}_{\triangleq M_w}\boldsymbol{w}_{k-1} + \underbrace{\begin{bmatrix}\boldsymbol{\eta}_k \\ \boldsymbol{\eta}_{k-1}\end{bmatrix}}_{\triangleq E_k} \tag{29}$$

$$\underbrace{\begin{bmatrix}\boldsymbol{z}_k \\ \boldsymbol{z}_{k-1}\end{bmatrix} - M_a\Delta\tilde{\boldsymbol{v}}_{k-1}}_{\triangleq \mathcal{Y}_k} = M_o\boldsymbol{x}_{k-1} + M_w\boldsymbol{w}_{k-1} - M_v\boldsymbol{v}_{k-1} + E_k \tag{30}$$

$$\mathcal{Y}_k = M_o\boldsymbol{x}_{k-1} + M_w\boldsymbol{w}_{k-1} - M_v\boldsymbol{v}_{k-1} + E_k \tag{31}$$

$$\mathcal{Y}_{k-1} = M_o\boldsymbol{x}_{k-2} + M_w\boldsymbol{w}_{k-2} - M_v\boldsymbol{v}_{k-2} + E_{k-1} \tag{32}$$

The matrix M_o is full column rank as a result of observability of the system given in Eqs. (18) and (8). Eliminating the state from Eq. (31), we get the following equation.

$$\underbrace{M_o^\dagger\mathcal{Y}_k - \hat{F}M_o^\dagger\mathcal{Y}_{k-1} - \hat{G}\Delta\tilde{\boldsymbol{v}}_{k-2}}_{\triangleq \mathcal{Z}_k} = \underbrace{\boldsymbol{w}_{k-2} + M_o^\dagger M_w\boldsymbol{w}_{k-1} - \hat{F}M_o^\dagger M_w\boldsymbol{w}_{k-2}}_{\triangleq \mathcal{W}_k} + \underbrace{\hat{F}M_o^\dagger M_v\boldsymbol{v}_{a_{k-2}} - \boldsymbol{v}_{k-2} - M_o^\dagger M_v\boldsymbol{v}_{k-1}}_{\triangleq \mathcal{V}_k} + \underbrace{M_o^\dagger E_{k+1} - \hat{F}M_o^\dagger E_k}_{\triangleq \mathcal{E}_k} \tag{33}$$

Note that $\mathcal{W}_k$, $\mathcal{V}_k$, and $\mathcal{E}_k$ are zero mean, uncorrelated and their covariance matrices are constant in time. Hence, the covariance of $\mathcal{Z}_k$ is also time invariant and is given by the following.

$$Cov(\mathcal{Z}_k) = Cov(\mathcal{W}_k) + Cov(\mathcal{V}_k) + Cov(\mathcal{E}_k) \tag{34}$$

The left hand side of Eq. (35) can be calculated on-line using the measurements up to time-step k. The right hand side is a strictly stationary time series with the process and measurement noises as inputs. The covariance of $\boldsymbol{v}_k$ can be estimated using the following equation.

$$Cov(\mathcal{Z})_k - Cov(\mathcal{W}) - Cov(\mathcal{E}) = A_1 \hat{Q}_{a_k} A_1^T + A_2 \hat{Q}_{a_k} A_2^T \tag{35}$$

wherein, $Cov(\mathcal{Z})_k$ is the sample covariance of $\mathcal{Z}$, A_1, A_2 are the coefficients of the noise terms $\boldsymbol{w}_{k-1}, \boldsymbol{w}_{k-2}$, and $\hat{Q}_{a_k}$ is the estimate of Q_a at the k^{th} time-step. Using our previous results,[10] it can be proved that the estimate $\hat{Q}_{a_k}$ converges to Q_a in probability. Note that here, a check has to be performed for the positive definiteness of $\hat{Q}_{a_k}$ to be used in the filter. Therefore, the most recent positive definite value of $\hat{Q}_{a_k}$ is used in the filter.

SIMULATIONS

As a simulated example, consider a calibration procedure for the accelerometer placed on top of a flat table revolving around the center of the table. The accelerometer is assumed to have no angular velocity about the center of the table. The acceleration profile in the x and y directions is given in Fig. (1). Eqs. (17) and (8) are used as the dynamics and the measurement model. Assume that

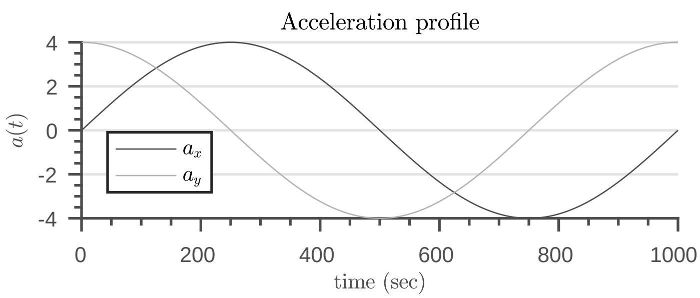

Figure 1. Acceleration of the accelerometer when spinning on the table top

$T = 0.1$ sec, $\tau_x = 3600$ sec, and $\tau_y = 3600$ sec. The true noise covariance matrices mentioned above have the following values.

$$S_{v_x} = 10^{-6}\ m^2/s^3,\ S_{v_x} = 10^{-6}\ m^2/s^3,\ Q_{b_x} = 5.55 \times 10^{-11}\ m^2/s^4,\ Q_{b_y} = 5.55 \times 10^{-11}\ m^2/s^4$$

$$R = \begin{bmatrix} 10^{-8}m^2 & 0 & 0 & 0 \\ 0 & 10^{-6}m^2/s^2 & 0 & 0 \\ 0 & 0 & 10^{-8}m^2 & 0 \\ 0 & 0 & 0 & 10^{-6}m^2/s^2 \end{bmatrix}$$

Here, the initial estimate is taken to be $\hat{S}_{v_{x0}} = 2 \times 10^{-6}\ m^2/s^3$ and $\hat{S}_{v_{y0}} = 2 \times 10^{-6}\ m^2/s^3$. The values mentioned above are for the chosen value of T and differ with varying the frequency of observation. However, the convergence of the algorithm is guaranteed regardless of this frequency.

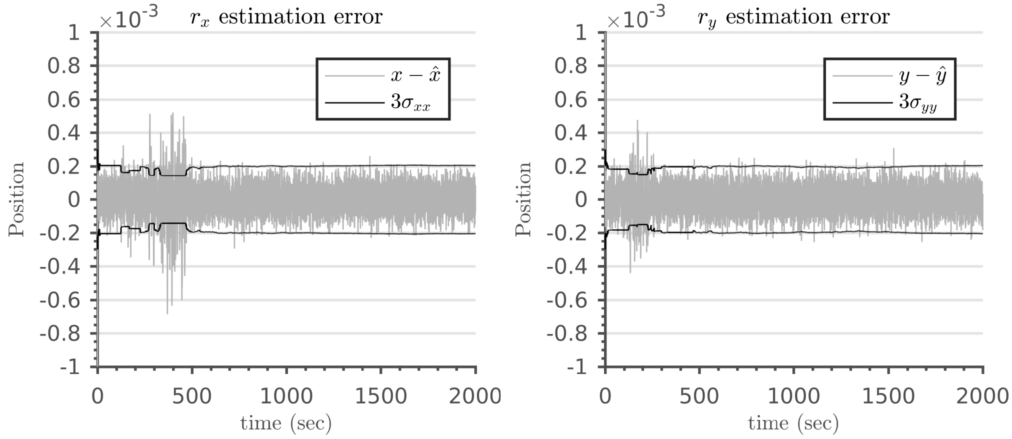

Figure 2. x **and** y **position estimation error** (m) **vs. time along with the** 3σ **bounds.**

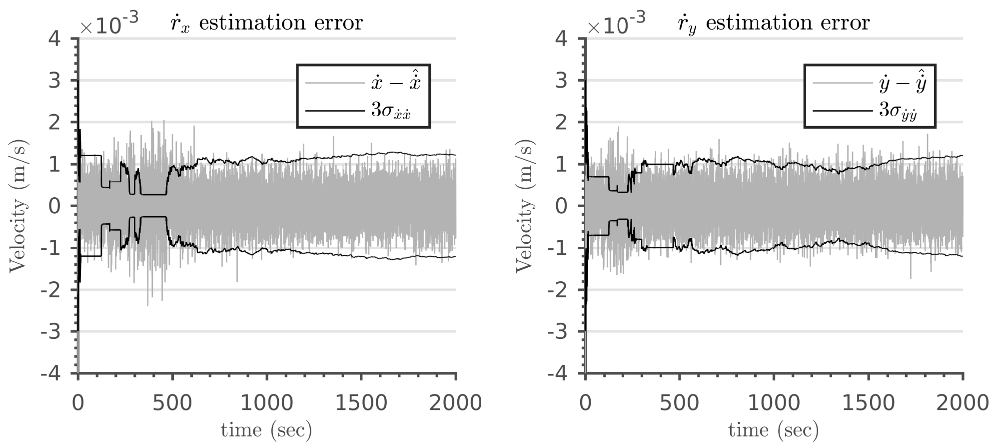

Figure 3. x **and** y **velocity estimation error** (m/s) **vs. time along with the** 3σ **bounds.**

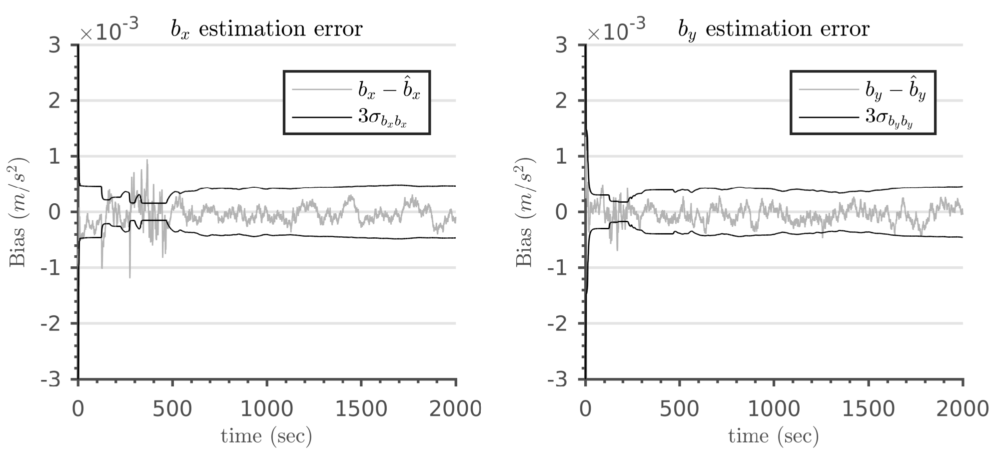

Figure 4. x **and** y **bias estimation error** (m/s^2) **vs. time along with the** 3σ **bounds.**

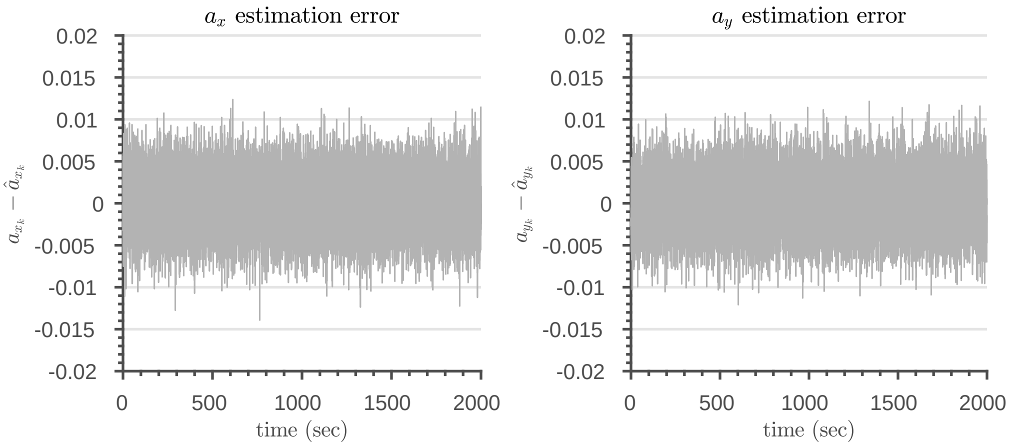

Figure 5. Acceleration estimation error in the x and y directions.

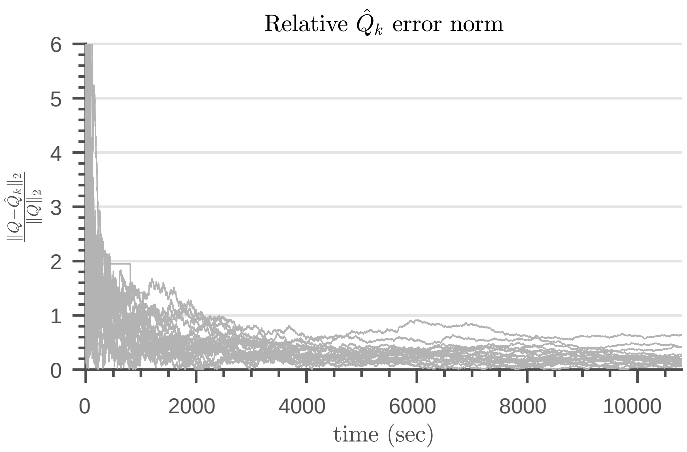

Figure 6. Norm of the error in the estimate $\hat{Q}_{a_k}$ and Q vs. time for 20 simulations. The constant regions of the profile are the times when the measurements resulted in $\hat{Q}_{a_k}$ that was not positive definite and hence, the most recent positive definite $\hat{Q}_{a_k}$ was used to propagate.

Figs. (2), (3) and (4) show the state estimation error of the system stays within their respective 3σ bounds for a single simulation. The regions where the error is outside the 3σ bound shows that the state error covariance propagated using the estimated $\hat{Q}_{a_k}$ is incorrect in the transients but converges to the true value once $\hat{Q}_{a_k}$ converges. Fig. (5) shows the estimation error of acceleration. Fig. (6) shows that the error norm of the estimate $\hat{Q}_{a_k}$ for 20 simulations.

CONCLUSION AND FUTURE WORK

A recent Covariance Matching Kalman Filter (CMKF) is used to calibrate the accelerometer bias using noisy accelerometer measurements while simultaneously estimating the velocity random walk of the accelerometer. The bias is assumed to follow an exponentially correlated random process with an zero-mean additive noise having a known time constant and steady-state variance. A numerical simulation example demonstrates convergence of all the quantities to be estimated. Future work includes deriving a filter for attitude dynamics using gyroscope measurements for rigid-body rotational dynamics.

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