Biomechanical Determinants of Pedaling Energetics: Internal and External Work Are Not Independent

Steven A. Kautz¹ and Richard R. Neptune²

¹Rehabilitation Research and Development Center, Veterans Affairs Palo Alto Health Care System, CA;

KAUTZ, S.A., and R.R. NEPTUNE. Biomechanical determinants of pedaling energetics: Internal and external work are not independent. Exerc. Sport Sci. Rev., Vol. 30, No. 4, pp. 159–165, 2002. Simulation analyses of pedaling demonstrate how individual muscle forces act to accelerate and decelerate the leg segments and the crank to perform external work. The work done by the muscles to accelerate the leg segments ultimately drives the pedals because when the legs decelerate, their mechanical energy is used to overcome the external load. Keywords: locomotion, simulation, mechanical power, muscles, coordination

INTRODUCTION

Understanding muscle function and the mechanical work performed by muscles during human locomotion has been very challenging, in part because of the difficulty in measuring muscle forces in vivo and in part because of complex musculoskeletal system dynamics. Thus, muscle function has usually been indirectly inferred from correlations between anatomical classification, inverse dynamics analyses, and EMG analyses; although these correlations cannot causally relate muscle excitation to task performance during locomotion. However, recent simulation and experimental studies using pedaling as the locomotor paradigm have illustrated how muscle forces generate and redistribute energy between the leg segments and the external environment (e.g., ergometer resistive load at the crank). A finding of particular importance for estimating the mechanical work performed by muscles is that muscle force generation can cause significant energy transfer between the legs and the environment at given instants during the pedaling cycle such that the external power (rate of doing work against the environment) can

Address for correspondence: Steven A. Kautz, Dept. of Physical Therapy, Box 100154, University of Florida HSC, Gainesville, FL 32610-0154 (E-mail: skautz@hp.ufl.edu). Steven A. Kautz is now at the Brain Rehabilitation Research Center, Malcolm Randall VA Medical Center, Gainesville, FL and Brooks Center for Rehabilitation Studies and Department of Physical Therapy, University of Florida.

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0091-6631/3004/159–165 Exercise and Sport Sciences Reviews Copyright © 2002 by the American College of Sports Medicine instantaneously exceed the mechanical muscle power (8). Often, researchers have assumed that the total mechanical work done by muscles includes two independent quantities: "external work" (work done overcoming external resistance, which can be accurately measured) and "internal work" (putative work done accelerating and decelerating the leg segments, which cannot be directly measured) (15). Although there are some physiological data consistent with (but not necessarily supporting) this intuitively appealing concept of internal work as an independent cost necessary for moving the legs (2), biomechanical studies have discredited internal work as an accurate reflection of such a cost (8,9).

The purpose of this review is to summarize the results of recent simulation analyses that demonstrate how muscles perform external work both directly and indirectly during pedaling. Indirectly, muscles do work to accelerate the leg segments, and when the segments are decelerated, muscles transfer the associated energy to the crank to perform external work. We propose that the acceleration and deceleration of the legs arises from the redistribution of segmental energy by muscle forces, and that these energy changes are an integral part of performing the external work such that there is no additional independent mechanical cost associated with the changes. Any additional mechanical cost associated with moving the legs will be the result of negative muscle work, which can be completely independent of "internal work." In addition, the redistribution of segmental energy by muscle forces provides a conceptual framework for understanding how muscles function to coordinate the pedaling task, which allows us to address several misconceptions in the literature related to pedaling biomechanics and energetics. Because

²Department of Mechanical Engineering, University of Texas at Austin

acceleration and deceleration of the legs is integral to all forms of legged locomotion, the observed mechanisms of redistribution of segmental energy by muscle forces are likely to play important roles in other locomotor forms.

The Metabolic Cost to Move the Legs

As pedaling rate increases beyond approximately 60 rpm during constant mechanical power pedaling, the metabolic cost (oxygen consumption) also increases (3), leading many investigators to assume that the increased metabolic cost is associated with muscles increasing their mechanical work output to move the legs faster (i.e., internal work) (2). Experimental studies have attempted to estimate the metabolic cost of moving the legs at a given pedaling rate by either directly measuring unloaded pedaling (3) or by establishing a relationship between external mechanical power and metabolic cost. Franscescato et al. (2) established such a relationship and estimated the metabolic cost of moving the legs (i.e., internal work) by linearly extrapolating the metabolic cost to the zero value of external mechanical power (y intercept) (Fig. 1). An important assumption is that this cost is constant for a given pedaling rate, regardless of the external workload. However, we will show below that the cost of moving the legs is not independent of the external workload.

Independent Mechanical Work to Move the Legs—A Flawed Concept?

Although metabolic cost increases when pedaling faster, the increased cost in and of itself does not directly support the internal work concept. The metabolic cost of generating muscle force is expected to change even if no extra mechanical work is required at higher pedaling rates, due to the

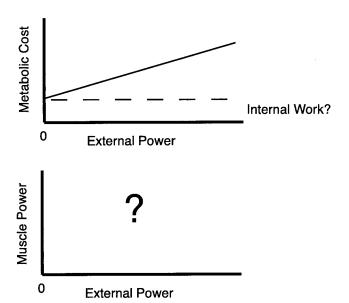


Figure 1. Conceptual illustration of the physiological basis for postulating a mechanical cost to moving the legs that is independent of external workload. Experimental studies have established a significant linear relationship between external mechanical power and metabolic cost. It has been assumed that linear extrapolation to the *y*-intercept yields the metabolic equivalent of internal work (the purported mechanical cost of only moving the legs at this pedaling rate). However, the relationship between muscle power and external power has not been established.

physiological force-velocity-activation relationships of skeletal muscle. Thus, whether more mechanical work is actually done at higher pedaling rates remains to be demonstrated. To date, studies that have attempted to quantify total mechanical work in pedaling have used internal work to quantify "extra" mechanical work, without establishing a relationship between the two quantities (14).

Winter (15) developed a formal equation for internal work that was proposed to be generally applicable to all human locomotor tasks by accounting for the positive and negative energy exchanges among the body segments and movements of the segments relative to the body's center of mass. The increases and decreases in the total body energy were explicitly attributed to positive and negative muscle work, respectively (15).

Since then, experimental and theoretical analyses of mechanical work (1), as well as more detailed simulation-based analyses (9), have all demonstrated that the amount of mechanical work done by muscles during locomotor movements is incorrectly estimated when internal work is considered as an independent cost. Other pedaling studies have used specially designed noncircular chainrings that force cyclists to produce three distinct levels of internal work by increasing the variation in the total mechanical energy of the leg segments for a given external workload and average pedaling rate. Although the chainrings caused internal work to increase by 60%, neither oxygen consumption (5) nor the mechanical work done by the joint moments (8) exhibited related increases.

The sum of internal and external work cannot estimate the total mechanical work done by muscles during pedaling because decreases in total leg energy are not always due to negative work by muscles as assumed (15). Experimental pedaling studies have found that decreases in the total energy of the leg were more coincident with power being transferred from the leg to the crank to overcome the external workload than with power being absorbed by the joint moments (negative joint power being analogous to negative muscle work) (6,8). If the decreases in total leg energy are mostly due to the transfer of energy between the leg and crank, which assists in overcoming the external resistance, then changes in the total mechanical energy of the legs (i.e., sum of the kinetic and potential energy of the segments in both legs) cannot be directly related to an independent cost of moving the legs as predicted by the internal work hypothesis (15).

The Redistribution of Segmental Energy by Muscle Forces Results in External Work

In contrast to the internal work hypothesis, our hypothesis is that the redistribution of segmental energy by muscle forces, including the transfer of energy between the leg and crank, intimately links changes in the total mechanical energy of the legs to external work production. Support for our hypothesis is provided by forward dynamics simulations of pedaling that replicate the kinematics, kinetics, and EMGs to analyze the effect each muscle has on the energetics of the legs and crank (9,10). Forward dynamics simulations are ideal for testing both our hypothesis on redistribution of segmental energy performing external work and the internal work hypothesis because dynamics simulations can establish precise

causal relationships between individual muscle forces and the accelerations and energetics of all segments including the legs and crank (e.g., see (10)). Below, we use data from Neptune et al. (10) to demonstrate how muscle forces accelerate and decelerate the legs, and the how energy can be transferred from the legs to the crank to overcome the external workload. The simulation is of pedaling at 60 rpm and 140 W.

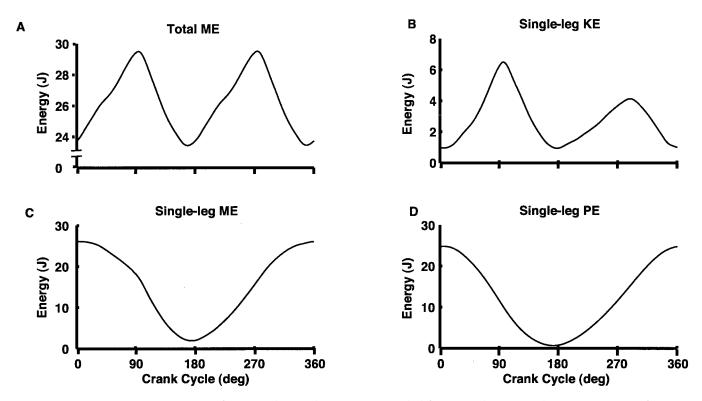
During pedaling, the total mechanical energy of the legs increases during the beginning of the downstroke of each leg, reaches a maximum slightly after 90°, decreases until near 180°, and then repeats this pattern when the opposite leg starts its downstroke (Fig. 2A). The position of the maximum and minimum peak values coincides with the position of the corresponding peaks in the single-leg kinetic energy (KE) (Fig. 2B), which reaches a larger peak in the downstroke than in the upstroke. The single-leg potential energy (PE) varies sinusoidally with its maximum near top-dead-center and minimum near bottom-dead-center (Fig. 2D). The total single-leg energy decreases steadily from 0 to 90°, as the KE increase partially offsets the PE decrease, and then decreases more rapidly until reaching a minimum just before 180° (Fig. 2C). Because the total potential energy is nearly constant when the net effect of the sinusoidal variations in each leg is considered (because each leg's PE is 180° out of phase from the other), the decrease in combined total energy of the legs is primarily due to the decreases in KE of each leg.

The internal work hypothesis (15) predicts that muscle action is responsible for both the increases (from positive muscle work) and decreases (from negative muscle work) in total leg energy. The mechanical power associated with these

increases and decreases can be visualized by plotting the rate of change in total mechanical energy of the two legs (i.e., mechanical power = dME/dt, Fig. 3). Because power is the rate of doing work, the area under a power curve during a given region is by definition proportional to the work done during that region. Therefore, the area under the crank power curve represents the external work done to overcome the external workload (Fig. 3, area under dashed line) and the area under the net muscle power curve represents the work done by muscles (Fig. 3, area under solid line).

Energy conservation principles require that when the total energy of the legs is increasing, more muscle work is done on the legs than the legs are doing on the crank to overcome the external workload. Furthermore, the difference between these values must be equal to the energy increase associated with the leg acceleration. From approximately 350 to 95°, the muscles generated more power than was delivered to the crank (Fig. 3, solid line is greater than dashed line), thus the legs accelerated and their energy increased (Fig. 3, dME/dt is positive).

Energy conservation principles also require that when total leg energy is decreasing, more work is done to overcome the external workload than the muscles do, and the difference must be equal to the energy decrease associated with the leg deceleration. From 95 to 170°, even though muscles are actually performing significant positive work and little negative work, the energy of the legs decreased (Fig. 3, dME/dt is negative) as more energy was delivered to the crank than was generated by the muscles (Fig. 3, dashed line is greater than solid line). Thus, there is the nonintuitive result that the



A. The total mechanical energy of both legs (total ME). B. Kinetic energy (KE) for one leg (single-leg KE). C. The total energy for one leg (single-leg ME). D. Potential energy for one leg (single-leg PE). The peak values of total ME coincide with the peaks in the single-leg KE.

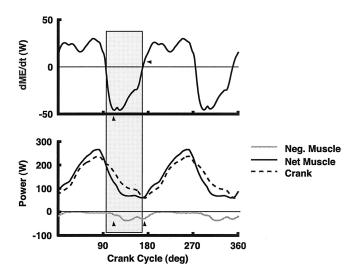


Figure 3. Power flow associated with both legs during pedaling. We calculated the time derivative of the total ME in Figure 2 to determine the instantaneous power associated with the total energy changes (dME/dt). Negative values of dME/dt during the downstroke (shaded box) represent leg energy decreases that account for more power being delivered to the crank than generated by the muscles (dashed line > solid line) because energy lost by the legs is transferred to the crank. Net muscle power refers to the sum of the positive and negative powers of all muscles. Note that negative muscle power is not closely matched to dME/dt (e.g., compare curves at arrows).

external work done in this region exceeded the net work done by muscles.

In our hypothesis, the production of external work when segmental energy is redistributed by muscle force explains the nonintuitive result that external work exceeds the net muscle work for a large region of the crank cycle. We assume that concentric muscle action can, in addition to performing external work directly, also decelerate the leg segments causing the inertia of the leg segments to perform additional external work. This occurs when the mechanical energy of the decelerating legs is transferred to the crank. Therefore, the net muscle work need not exceed the external workload. This contradicts the internal work hypothesis, which assumes that muscles must do an amount of positive muscle work equal to the external work, but that an amount of negative muscle work equal to the energy decrease in the legs reduces the net muscle work.

Differentiating between our hypothesis and the internal work hypothesis requires a fundamental understanding of how muscles function to decrease the total energy of the legs and to perform external work. The individual muscle contributions to the energy flow between the legs and the crank (Fig. 4) have been previously reported (10). Just as the total power delivered to the crank can exceed the total instantaneous power of all muscles, the power delivered to the crank as the result of an individual muscle force can be substantially greater than the power produced by that muscle (e.g., soleus (SOL) and gastrocnemius (GAS)). In this case, the muscle force is acting to redistribute energy between the legs and crank (i.e., decelerating the legs and transferring that energy to the crank). In addition, some muscles (e.g., vasti, VAS) function to provide power directly to the crank (i.e., accelerate the crank directly by contributing to the pedal reaction

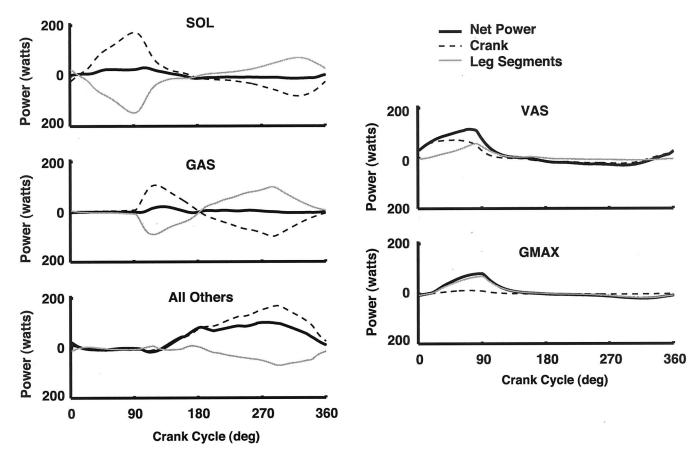
force) whereas others (e.g., gluteus maximus, GMAX) provide power directly to the leg segments (i.e., accelerating the legs directly with little power going directly the crank).

Analysis of the individual muscle contributions to the energy flow between the leg and the crank reveals that the most powerful deceleration of the leg segments is caused by SOL and gastrocnemius GAS (Fig. 4 SOL, GAS: area under dashed line <0). By acting to decelerate the leg, SOL and GAS transfer energy from the leg to the crank, which allows them to deliver much more energy to the crank than they produce (Figure 4 SOL, GAS: area under dotted line ≥ area under solid line during 0 to 180°). The deceleration induced by the plantar flexors is the primary mechanism for the energy transfer from the leg to the crank. By acting to stiffen the ankle joint, both SOL and GAS act to accelerate the foot powerfully into plantar flexion, which creates a pedal reaction force that accelerates the crank (Fig. 5, tangential crank force by SOL and GAS >0) and corresponding intersegmental joint forces that decelerate the thigh and shank (Fig. 5). The deceleration of the thigh and shank is the result of SOL and GAS acting to accelerate the ankle upward and the hip and knee into flexion (because of the resulting reduction in the ankle-to-hip distance) (12). Because the hip and knee are extending then, the induced hip and knee flexion acceleration acts to slow extension and decrease the energy of both the thigh and shank (Fig. 5). This energy from the thigh and shank is transferred to the crank through the force applied to the pedal and external work is done. It should be noted that these energy changes are the result of concentric muscle activity by GAS and SOL (Fig. 4, net muscle power is positive), with no significant energy dissipation by eccentric activity in the other muscles (Fig. 4). These results clearly show that the decrease in leg energy (resulting from concentric muscle work) is not independent of the external work done, but instead is a biomechanical mechanism necessary to produce that external work.

Revisiting the Concept of an Independent Mechanical Work Necessary to Move the Legs

In summary, although the energy increases and decreases of the leg are the result of redistribution of segmental energy by muscle forces, the energy decrease resulting from the deceleration of the legs can and does generate a pedal force tangential to the crank, and thus does positive external work. Evidence of this dynamical phenomenon is provided by the demonstration that the external power at the crank can instantaneously exceed the total power produced by the muscles during this period of deceleration (Fig. 3B).

Analysis of individual muscle contributions demonstrates why the internal work hypothesis improperly estimates total work. Above, we showed that a unit of work (J) could be done during the beginning of the downstroke (e.g., 70°) by one muscle (e.g., GMAX), that the energy of the leg segments can increase by that J, and then, via transfer by other muscles (e.g., SOL), that same J can be delivered to the crank when the legs are decelerating (e.g., 110°). Although the muscle that is transferring that same J from the legs to the crank (e.g., SOL) may be acting isometrically (no mechanical work performed), it is often simultaneously performing an additional amount of concentric work (all of which is



Mechanical power produced by GAS, SOL, VAS, GMAX, and the combined action of all other muscles included in the model (mostly flexors, which explains why most of their power is produced in the upstroke). The net musculotendon power (solid line) is the sum of the power delivered to the legs and the crank. Positive (negative) power represents generation (absorption). SOL and GAS transfer much more power from the legs to the crank than they generate (dashed > solid line).

delivered directly to the crank in addition to the J being transferred). Thus, through the coordination of multiple muscles, a J of work performed by one muscle to accelerate the legs is subsequently recovered as 1 J of external work as the legs decelerate. However, in this same scenario, the internal work method would estimate that a J of work (positive internal work) was performed at 70° to accelerate the segments, a second J of work (negative internal work) was performed at 110° to decelerate the segments, and a third J of work (positive external work) was done by muscles directly at 110°. Thus, the internal work hypothesis would estimate that 3 J of work were required from muscles when, in reality, only 1 J is required to perform that 1 I of external work.

Therefore, the internal work hypothesis is invalid as a direct measure of the mechanical energy cost of moving the legs in pedaling. Furthermore, because its entire theoretical basis depends on the relationship between changes in total energy and mechanical work done to accelerate and decelerate the legs, its use is inappropriate for scientific investigations of pedaling even in the unlikely event that it can provide a useful indirect measure of the metabolic cost. Instead, the energetics of pedaling need to be interpreted with respect to the cost of producing muscle force.

Although the description of energetics above might seem to imply that no extra mechanical work beyond the external

workload is performed during pedaling, the physiological properties of muscle suggest that additional mechanical work is performed, even though it cannot be quantified by internal work. Specifically, the activation and deactivation dynamics associated with muscle force development should result in negative muscle work (Fig. 3B) because muscles cannot activate and deactivate instantaneously (9). Because the average muscle power for a crank cycle must equal the average external power (assuming no dissipation except that by muscles), the negative muscle power (Fig. 3B) must be overcome with equal amounts of additional positive muscle power beyond the external power. However, it is extremely important to note that this negative muscle work due to muscle deactivation (i.e., negative area under solid line for VAS, Fig. 4) is not coincident with the decreases in total mechanical energy (Fig. 2A), and thus it is not even indirectly being measured in the internal work calculation.

Consequences for Pedaling Biomechanics

The redistribution of segmental energy by muscle forces provides a conceptual framework for understanding how muscles function to coordinate the pedaling task, which allows us to address several misconceptions in the literature related to pedaling energetics.

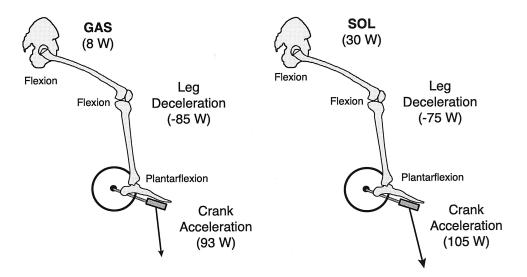


Figure 5. Schematic illustration of the mechanism by which concentric plantar flexor activity transfers energy from the leg to the crank at the point of maximum instantaneous total energy decrease (110°). Their action accelerates the foot powerfully into plantar flexion, which accelerates the hip and knee into flexion and decreases the energy of leg because it is extending. A pedal reaction force results that accelerates the crank and the decreased lea energy is transferred to the crank. Thus, the decrease in leg energy is a fundamental aspect of producing external work during pedaling.

The cost of "unloaded" pedaling is not related to the cost of moving the legs in pedaling

Studies have sought to estimate the cost of moving the legs by measuring metabolic cost during unloaded pedaling (i.e., against no workload) (3). However, it is clear that if the legs are moving similarly to normal pedaling during unloaded pedaling, then the energy of the legs will have to be decreased by negative muscle work because the energy can no longer be decreased by doing external work. Because this is completely dissimilar to normal pedaling, the metabolic cost is likely to be completely dissimilar as well. By the same logic, metabolic costs associated with "internal" work at low workloads may not be consistent with costs at higher workloads, such that linear models are inappropriate for relationships between metabolic cost and workload.

"Effective" crank force is a misnomer

Studies have also sought to explain differences in the cost of pedaling with the concept of more "effective" crank force generation (e.g., (11)). The tangential crank force is the only component that directly acts to propel the crank and contributes to external work, and the terms "effective" and "ineffective" have been used to describe pedal forces that are oriented tangential and radial to the crank (Fig. 6), respectively. However, it is erroneous to conclude that the radial component should be minimized to decrease energy expenditure or cycle faster because a significant amount of the radial component results from nonmuscular (i.e., gravity, coriolis, and centripetal forces) contributions (7). In addition, our demonstration above that the pedal force is dominated by energy transferred to the crank by the plantar flexors reveals that the muscular contributions to the crank force are likely to have significant radial components as well. Specifically, the orientation of the pedal force due to the plantar flexors at any instant in the crank cycle is uniquely determined by geometry of the leg, with changes in the magnitude of muscle excitation merely changing the magnitude of this pedal force. Thus, there is a limited ability to control the orientation of this crank force. As a result, the net action of muscles will almost assuredly produce a radial component of the crank force and eliminating it would require the recruitment of additional muscles to offset the radial component, causing the metabolic cost to go up.

Negative crank power during the upstroke does not imply additional mechanical cost

Previous work has partially refuted this misconception by showing that more muscular effort would be required to eliminate the negative torque (counter-torque) during the upstroke than would be required to pedal normally with a counter-torque (13). Note that the counter-torque represents work done on the upstroke leg, not necessarily that the leg is pushing down. In fact, during the upstroke muscles do significant positive work (e.g., Fig. 4, all other muscles) and little negative work. Most of this negative work would still exist even if counter-torque were eliminated by other concentric muscles (e.g., hip flexors) because it is associated with the deactivation of muscles such as VAS (Fig. 4). Because little dissipation of energy by negative muscle work occurs, the work associated with the counter-torque results in increased energy of the leg as the potential and kinetic energies increase as the leg is pushed upwards (Fig. 2B and 2D). Thus, counter-torque can simply represent the transfer of energy from the downstroke leg to the upstroke leg. Although there is nothing inherently inefficient about the presence of counter-torque, it would represent poor performance if sig-

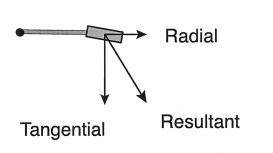


Figure 6. Definition of the radial and tangential components of the resultant force applied to the pedal.

nificant negative work were done by the muscles of the upstroke leg.

Alternative crank/pedal mechanisms do not improve efficiency by manipulating mechanical work

The mechanical design of an improved transmission system in pedaling has a long history in the cycling literature with the ultimate goal of making either a faster or more efficient bicycle. Several of these devices have been designed in an attempt to improve efficiency by manipulating mechanical work, either by decreasing internal work (5) or by increasing the effective moment arm during the downstroke (to increase the crank torque from the pedal reaction force) and decreasing it during the upstroke (to decrease the counter-torque) (4). As we have shown above, decreasing the internal work has no predictable or useful effect on the mechanical work cost of pedaling. Also, as noted above, the counter-torque need not be the result of negative work performed by muscles. If, for the sake of theoretical argument, we assume that muscle coordination can be timed perfectly so that the negative work done by individual muscles is negligible, then the total mechanical work done by all muscles must be exactly equal to the total external workload (assuming no transmission losses in the drive system). Because it is impossible to get more work out of the system than is put into it, a pedal-crank mechanism could not result in more work from the same pedal (or muscle) forces in this case. To do so would mean a perpetual motion machine, with energy created by the mechanism. Thus, a pedal-crank mechanism design should be based on the force-length-velocity-activation relationships of skeletal muscle and their influence on the cost of generating muscle force, not only on the mechanical work performed.

SUMMARY

Muscles generate forces that redistribute segmental energy between the leg segments and the crank. They accelerate and decelerate the legs resulting in an increase and decrease of their total energy. Increases in the mechanical energy of the legs are predominantly due to work done by the uniarticular hip and knee extensor muscles. Decreases in mechanical energy of the legs are predominantly the result of concentric activity by the ankle plantar flexors that acts to transfer energy from the legs to the crank to overcome the external resistance with no significant dissipation by eccentric muscle activity. Thus, changes in the mechanical energy of the legs are neither independent of external work production nor predictive of the mechanical work required to move the legs; instead they are an integral part of producing the external work. Therefore, the internal work hypothesis is inappropriate for scientific investigations of pedaling. Furthermore, by showing how muscles accelerate and decelerate the legs, and how they transfer energy from the legs to the crank, it was possible to address several misconceptions in the literature related to pedaling biomechanics and energetics. Our analysis of pedaling also suggests that internal work measures will be similarly flawed in other locomotor tasks where muscles cause significant external work to be done by the deceleration of the body segments (e.g., cross-country skiing, swimming).

Acknowledgments

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