Optimization Algorithm Performance in Determining Optimal Controls in Human Movement Analyses

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The objective of this study was to evaluate the performance of different multivariate optimization algorithms by solving a "tracking" problem using a forward dynamic model of pedaling. The tracking problem was defined as solving for the muscle controls (muscle stimulation onset, offset, and magnitude) that minimized the error between experimentally collected kinetic and kinematic data and the simulation results of pedaling at 90 rpm and 250 W. Three different algorithms were evaluated: a downhill simplex method, a gradient-based sequential quadratic programming algorithm, and a simulated annealing global optimization routine. The results showed that the simulated annealing algorithm performed far superior to the conventional routines by converging more rapidly and avoiding local minima.

Introduction

The use of optimal control theory in computer simulations of human movement has increased tremendously in recent years. Applications have ranged from gait (e.g., Davy and Audu, 1987) and cycling (e.g., Raasch et al., 1997) to head (e.g., Hannaford and Stark, 1987) and arm movements (e.g., Gonzalez et al., 1993). One advantage of this type of analysis is the direct access to the muscle forces required to accomplish the desired motor task or performance criteria. These results combined with experimental data can provide increased understanding of muscle function and movement control principles.

Fundamental to the success of solving the optimal control problem is the algorithm used to solve for the controls. Most studies have converted the optimal control problem into a parameter optimization problem (e.g., Pandy et al., 1992) and used various algorithms to solve for the parameters. These algorithms have ranged from a simple downhill simplex method (e.g., Bogert and Soest, 1993) to more sophisticated gradient-based methods (e.g., Pandy et al., 1992). These algorithms are computationally efficient for functions that are smooth and continuous with very few local minima. But large dimensional functions in human movement analyses are often plagued by many nonlinear ridges, valleys, and local minima, which can result in slow convergence or convergence to local minima. Global optimization routines have been developed to overcome these difficulties and have been applied to other problems ranging from computer and circuit design (Kirkpatrick et al., 1983) to finance (Ingber et al., 1991). To date, these methods have not been applied to human movement problems, primarily because these algorithms can be computationally intensive since they search the entire solution space. But as computer speeds increase, the utility of these algorithms looks more promising and may improve the current methods used in movement analyses.

Therefore, the objective of this study was to evaluate the performance of different optimization algorithms including a global optimization routine by solving a "tracking" problem in cycling using a forward dynamic model. The tracking problem was defined as solving for the control parameters that minimized the error between experimentally collected kinetic and kinematic data and the simulation results of pedaling at 90 rpm and 250 W. Three different algorithms were evaluated: a downhill simplex method, a gradient-based sequential quadratic programming algorithm, and a simulated annealing global optimization routine.

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Methods

Bicycle-Rider Model. A two-legged forward dynamic musculoskeletal model was developed in a previous study (Neptune and Hull, 1998) using SIMM (MusculoGraphics, Inc., Evanston, IL) and will be reviewed briefly here. Each leg consisted of three rigid-body segments (thigh, shank, and foot) with the hip joint center fixed and the foot rigidly attached to the pedal. The model was driven by 14 individual musculotendon actuators with first-order activation dynamics and musculoskeletal geometry and parameters based on the work of Delp et al. (1990). The 14 muscles were further combined into nine muscle sets, with muscles within each set receiving the same stimulation level. The muscle stimulations were modeled as block patterns defined by a duration and magnitude. The stimulation patterns for the right and left leg were considered symmetric and 180 deg out-of-phase. The force-generating capacity of each muscle was based on a Hill-type model governed by the muscles' force–length–velocity characteristics (Zajac, 1989). The crank-load dynamics were modeled by an equivalent inertial and resistive torque applied about the center of the crank arm (Fregly, 1993) to yield an average power output of 250 W at 90 rpm.

The dynamic equations-of-motion for the bicycle–rider system were derived using SD/FAST (Symbolic Dynamics, Inc., Mountain View, CA), and a forward dynamic simulation was produced using the Dynamics Pipeline (MusculoGraphics, Inc., Evanston, IL).

The objective function was formulated to solve the tracking problem by minimizing the differences between experimental and simulation pedaling data in the general form of:

$$J = \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{(Y_{ij} - \hat{Y}_{ij})^2}{SD_{ij}^2} \tag{1}$$

where Y_i are the experimentally measured data, $\hat{Y}_i$ are the model data, SD_{ij} are the intersubject standard deviations, n is

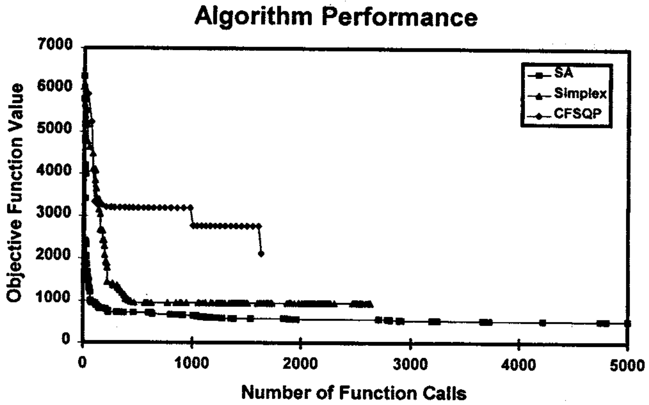

Fig. 1 Algorithm performance. Each algorithm was set up identically with the same initial conditions and executed for a maximum of 5000 function calls. Both the CFSQP and SIMPLEX routines converged on a local minimum before the maximum number of function calls.

number of data points, and m is number of variables evaluated. The specific variables tracked were the horizontal and vertical pedal force components, pedal angle, crank torque, and hip, knee, and ankle intersegmental joint moments. This criterion was shown in a previous study to produce steady-state pedaling simulations replicating experimental kinetic and kinematic data (Neptune and Hull, 1998). Simulations were performed over four revolutions to assure that initial start-up transients had decayed. The objective function was not evaluated until the fourth revolution when the simulation had reached steady state and was considered to be independent of the initial conditions. A final time constraint was enforced to assure the simulation pedaled at an average pedaling rate of 90 ± 2 rpm.

Experimental Data. To provide data for the tracking problem, both kinetic and kinematic data were collected from ten male competitive cyclists (height $\bar{x}$ = 1.81 ± 0.04 m; mass $\bar{x}$ = 76.5 ± 3.4 kg; age $\bar{x}$ = 29.6 ± 4.1 yr). Informed consent was obtained before the experiment. The subjects rode a conventional racing bicycle adjusted to match their own bicycle's geometry at 90 rpm and work rate of 250 W. Pedal force, crank, and pedal angular displacement and video data were all collected simultaneously. Intersegmental joint moments were computed using a standard inverse dynamics approach.

Optimization Algorithms. The optimization algorithms were formulated identically to find the muscle stimulation patterns that minimized the objective function (Eq. (1)), subject to the system state vector, state variable constraints, pedaling rate constraint, and control bounds over the time interval [0, t_{final}]. The muscle controls were allowed to vary between the bounds of 0 and 1, which are defined as zero and maximum stimulation, respectively. The optimal control problem was solved by converting the optimal control formulation into a parameter optimization problem (Pandy et al., 1992). The parameters optimized were the stimulation onset, offset, and magnitude of the nine muscle groups yielding 27 variables. The stimulation patterns were optimized using three different algorithms, a sequential quadratic programming method (CFSQP, Lawrence et al., 1997), a downhill simplex method (Nelder and Mead, 1965), and a simulated annealing algorithm (Goffe et al., 1994). Briefly, the CFSQP algorithm determines the optimal direction and step length to decrease the objective function using a gradient-based line search. Since the gradients were not known analytically, they were computed by finite differences. The simplex method (SIMPLEX) seeks to find the minimum objective function by moving from high to low function values without the computation of gradients by evaluating the function at $N + 1$ vertices of an N-dimensional volume (defined by the N parameters being optimized) at each step. The vertex with the worst (highest) function value is replaced by a new guess based on the other vertices. The set of vertices thus expands, contracts, and moves until it has converged on a local minimum. An implementation of this algorithm can be found in Press et al. (1992).

The simulated annealing algorithm (SA) is based upon Monte Carlo methods in statistical analyses. The algorithm performs a random global search and avoids local optima by probabilistically accepting nonoptimal steps within the solution space. The probability of accepting nonoptimal steps depends on a "temperature," which decreases as the algorithm converges on the most promising region with a user-defined temperature schedule. In this study, a rapid temperature reduction schedule or "quenching" technique was utilized.

The algorithms were all started with the same initial guess and executed for a maximum of 5000 function evaluations. The initial guess was generated from a previous optimization that minimized the variations in the crank angular velocity to produce a smooth pedaling simulation.

Results

The SA algorithm was the most successful in minimizing the total tracking error between the simulation and subjects' data (Table 1). The objective function was reduced from 6631 to 522 while the SIMPLEX and CFSQP algorithms only reduced the error to 950 and 2126, respectively (Fig. 1). These errors were apparent in both the root-mean-square errors (Table 1) and the pedal reaction force plots (Fig. 2). The pedaling simulation produced by the SA algorithm reproduced the subjects' data usually within ±1 SD in all of the measured or computed kinetic and kinematic quantities.

The SA algorithm continued to reduce the objective function up to the maximum number of function calls, while both the SIMPLEX method and CFSQP algorithm converged on local optima well before the maximum number of function calls (Fig. 1). In addition, the SA algorithm initially reduced the value of the objective function more rapidly than the other two routines.

Discussion

Theoretical analyses of human movement using musculoskeletal models and simulations have become a fundamental part of biomechanics and motor control research. But the controls required to accomplish the desired motor task are usually difficult to measure experimentally or solve analytically. Therefore, researchers have applied optimization techniques to solve such problems and the algorithms employed have varied throughout the literature. Although the specific algorithms have varied, they have been primarily gradient-based or downhill-type routines. To improve optimization performance, studies have examined various optimization routines (e.g., Audu and Davy, 1988), objective functions (e.g., Buchanan and Shreeve, 1996), and computer architectures (Anderson et al., 1995), but to date,

Table 1 The individual kinetic and kinematic quantities root-mean-square errors between the experimental and simulation results.

	Optimization Method		
Quantity	CFSQP	Simplex	SA
Pedal Angle	13.01	10.94	8.45
Pedal Force (Fx)	18.11	10.47	5.51
Pedal Force (Fy)	28.48	15.58	8.62
Crank Torque	17.36	11.65	7.37
Hip Moment	11.04	8.32	5.72
Knee Moment	12.22	7.20	6.14
Ankle Moment	15.65	14.88	14.85
Total RMS Error	115.87	79.04	56.64

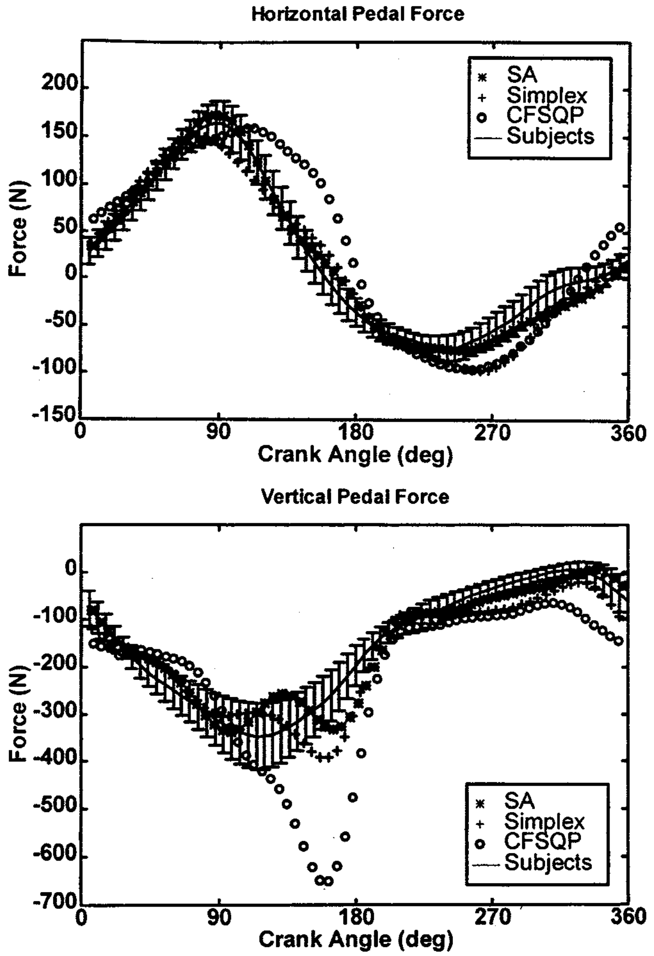

Fig. 2 Horizontal and vertical pedal force components. The error bars represent ±1 SD of the subjects' average data.

no study has directly examined the performance of different optimization algorithms including global routines to solve these types of problems. Therefore, the objective of this study was to evaluate the performance of three different algorithms and to assess the potential advantages and disadvantages of each.

In most human movement optimal control analyses, the objective function of interest can have many nonlinear valleys, ridges, and plateau regions that the optimization algorithm will have to overcome to find the global minimum. The objective function used in this study (Eq. (1)) is quadratic in nature, but the problem constraints make the function surface highly nonlinear with nonconvex regions. An example of the type of nonlinear function surfaces that may be expected in these type of analyses is illustrated in Fig. 3. Figure 3 was generated by systematically varying the stimulation onset and offset timing of one muscle while keeping its magnitude constant and computing the corresponding objective function value (Eq. (1)). The other muscle stimulation patterns were the optimized controls generated by the SA algorithm. Substantially more complex surfaces can be expected when the other control variables are varied simultaneously or the initial guess is far from the optimal solution.

The results of this study showed that the SA algorithm was more robust than the other two routines in overcoming such complex surfaces. Not only did the SA algorithm converge more rapidly, but it avoided the local minima that the conventional algorithms converged on (Fig. 1). Although the SA results were not presented beyond the specified maximum number of function calls, the SA algorithm continued to reduce the objective function to improve the pedaling simulation further.

The superior performance of the SA algorithm is the result of several advantages it has over the other two routines. First, SA starts by exploring the entire solution space and then converges on the most promising region while moving in both uphill and downhill directions, thus allowing it to overcome local optima. SA algorithms have been shown to be robust in effectively handling such regions (Ingber, 1993) and are well suited to solve these types of nonlinear optimization problems. Second, by initially exploring the entire solution space, the algorithm is insensitive to the initial guess of the control parameters. The results of both the SIMPLEX and CFSQP algorithms are very sensitive to the initial guess in nonconvex optimization problems. Therefore, these algorithms require multiple restarts with different initial guesses. Although not presented in this study, restarts were performed by systematically perturbing the initial guess and performing the optimizations again. The results were conceptually the same, the SIMPLEX and CFSQP algorithms both converged on local minima while SA continued to reduce the objective function until the maximum number of function calls. Third, SA does not require the function to be smooth or even continuous, which is a fundamental requirement for gradient-based algorithms and thus allows the user to include constraint violation penalties in the objective function such as the pedaling rate constraint enforced in this study. Therefore, the SA algorithm can effectively handle cost functions with severe nonlinearities, discontinuities, and arbitrary boundary conditions and constraints (Desai and Patil, 1996). These advantages have allowed SA algorithms to repeatedly outperform many conventional gradient-based and other global optimization routines across a variety of standard test problems (Corana et al., 1987; Goffe et al., 1994; Ingber, 1993).

One of the most important features of the SA algorithm is the statistical guarantee to find the global minimum if an appropriate temperature schedule is used (Goffe et al., 1994). But from a practical point of view, this is computationally exhaustive with large-dimension control vectors and extensive simulation times. For the pedaling simulations in this study, typical execution time for one simulation or function call required 46 seconds CPU time on a Silicon Graphics R10000 workstation (Silicon Graphics, Inc., Mountain View, CA). Because of this inherent limitation, the SA algorithm is most effective using a quenching schedule. But the faster reduction of the temperature schedule nullifies the guarantee to find the global optimum, which also cannot be guaranteed using the other methods examined in this study. The SA algorithm has been used in other

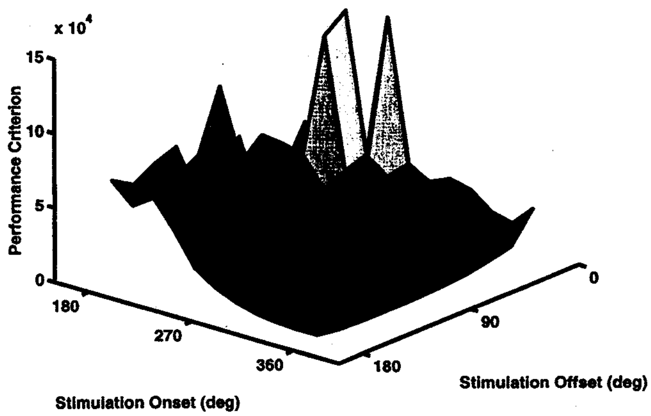

Fig. 3 Objective function surface. The surface was generated by varying the stimulation onset and offset timing of one muscle. The other muscle controls were those found by the SA algorithm and were held constant. The stimulation onset was varied between 180 and 360 deg while the offset was varied between 0 and 180 deg.

studies (e.g., Neptune and Hull, 1998) with restarts using a different initial seed in the random number generator to fine-tune the solution and give confidence in the results.

A well-known disadvantage of global optimization routines that search the entire solution space is that there is often an increased execution time. But the results of this study showed that not only was the SA algorithm more robust in decreasing the objective function, but it also reduced it more rapidly. Other studies have shown for a variety of test problems that the SA execution time is often comparable to conventional algorithms when multiple restarts are employed to test different initial guesses (e.g., Goffe et al., 1994). These results may be different for other optimization problems. As available computer speeds increase, the feasibility of employing global optimization routines to solve widespread problems will also increase.

The simplex method has previously been shown to be very effective in handling discontinuous and nonlinear functions (Nelder and Mead, 1965), but the robustness of the algorithm comes with the expense of slow convergence. Although the SIMPLEX method initially converged at the same rate as the CFSQP algorithm, it still converged more slowly than the SA algorithm (Fig. 1) and eventually suffered from the same inability of the gradient-based method to escape from local optima (Fig. 1).

In the situation where the objective function is known in advance to be smooth with few local optima, then a gradient-based method may be most effective. The advantage of gradient-based methods is that they are very efficient at converging quickly on local optima. This characteristic combined with the robustness of the SA algorithm and its ability to get out of local optima has recently led to the development of an algorithm that combines the best characteristics of each method (Desai and Patil, 1996). Although this algorithm was not applied in this study, the method was shown to compare well with other more sophisticated simulated annealing algorithms.

In conclusion, it must be clearly stated that no one algorithm is likely to be the most effective method for solving all optimal control problems in movement analyses. The results presented here for finding the controls during pedaling may not be representative of algorithm performance in other problems, but clearly illustrate the utility of the SA algorithm to solve complex problems and the potential errors that may occur using conventional methods. It is desired that this paper illustrate alternative methods available and inspire careful consideration before a particular algorithm is selected.

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