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Standard mechanical energy analyses do not correlate with muscle work in cycling

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Abstract

The goal of this study was to assess the utility of experimental methods to quantify mechanical energy expenditure (MEE) in human movement. To achieve this goal, a theoretical model of steady-state cycling driven by individual muscle actuators was used to produce two distinct pedal simulations. The simulations yielded the same pedaling rate and power output, but one reduced the MEE by avoiding eccentric muscle contractions. Contractile element force and length change in the individual muscles was used to quantify the total positive and negative work produced by the muscles. Three methods using external measurements were applied to the simulated movement. The three methods to quantify MEE were based on: (1) segment kinematic measurements, (2) work done by total joint powers and (3) intercompensated joint powers, i.e. negative work from one joint is transferred to an adjacent joint where energy is being generated (positive work) via biarticular muscles.

The results showed that none of the MEE analyses were correlated to the MEE of the individual muscles, with errors reaching 40%. Errors were mainly attributed to the inability of the MEE methods to account for co-contractions of antagonistic muscle groups. This phenomenon occurred primarily when one muscle generated force during activation while the antagonist generated force during deactivation.

Keywords: Cycling; Mechanical Energy; Simulation

1. Introduction

Many experimental techniques have been developed to quantify the energy expenditure of movement. These techniques can be classified as: (1) metabolic energy, (2) external work, (3) kinematic methods and (4) kinetic methods, with the last three derived from fundamental mechanics.

The first approach, *metabolic energy*, reflects the metabolic energy cost of movement by measuring the rate of oxygen uptake ($\dot{V}O_2$). Although measuring $\dot{V}O_2$ during a given movement has utility, its application to quantify energy expenditure is limited to sub-maximal steady-state activities where the primary energy system utilized is aerobic. Further, $\dot{V}O_2$ is a global measure of the energetic cost for the entire body making it difficult to isolate the cost associated with different components within the system such as individual muscles or mechanical parameters.

The second approach, *external work*, attempts to relate the work performed against the environment to metabolic cost. In some activities, the amount of work required to overcome external forces is clearly dependent on the movement pattern. For example, aerodynamic forces in cycling and skating, and drag forces in swimming all depend on orientations and velocities of the body segments and can be determined experimentally. For these activities, the measured external work is one indicator of the economy of movement but gives no indication of the energy expenditure inside the system. A clear example of problems associated with this method is seen in the analysis of walking which has negligible external work, although there is energy required to perform the task. In other applications, external work is nearly constant and part of the task specification (e.g. ergometer cycling at 90 RPM and 260 W as in this paper) and therefore not suitable as a measure of economy.

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The third approach, *kinematic methods*, is based on the theory that changes in kinetic and potential energy require work. The kinematic approach computes mechanical work based on changes in kinetic and potential energies of the body center of mass and/or the individual body segments recorded from external kinematic data. The instantaneous energy of an n-segment body is equal to (Winter, 1979):

$$E_B(t) = \sum_{i=1}^{n} \mathrm{PE}(i, t) + \sum_{i=1}^{n} \mathrm{TKE}(i, t) + \sum_{i=1}^{n} \mathrm{RKE}(i, t) \quad (1)$$

where $E_\mathrm{B}(t)$ is the instantaneous energy of an n segment body, $\mathrm{PE}(i, t)$, the potential energy of ith segment at time t, $\mathrm{TKE}(i, t)$, the translational kinetic energy of ith segment at time t, $\mathrm{RKE}(i, t)$ the Rotational kinetic energy of ith segment at time t.

The changes in the energy system are then used to estimate the mechanical power and when summed over the entire crank cycle yield the mechanical work. This definition of mechanical work was termed 'internal', that is, the energy required to move the segments (Winter, 1979):

$$W_{\mathrm{Internal}} = \int_0^T \left| \frac{\mathrm{d}E_B(t)}{\mathrm{d}t} \right| \mathrm{d}t \quad (2)$$

where T is the duration of the movement. When significant external work against the environment exists (e.g. cycling), the amount of external work has been added to the kinematic estimate to determine total work (Winter, 1979). This type of analysis has been applied to walking (e.g. Wells and Evans, 1987), running (e.g. Cavagna and Kaneko, 1977) and cycling (e.g. Widrick et al., 1992).

The fourth approach, *kinetic methods*, is based on inverse dynamics analysis and uses the work performed by hypothetical joint torque actuators as a measure of the energetic cost of the movement. This approach has been applied to cycling (e.g. Ingen Schenau et al., 1990; Kautz et al., 1994) to assess the muscular mechanical energy expenditure (MMEE) as the sum of the positive and negative work done by the n-intersegmental joint torques [Equation (3)].

$$\mathrm{MMEE} = \int_0^T \left(\sum_{j=1}^{n} |P_j| \right) \mathrm{d}t. \quad (3)$$

Recognizing the importance of quantifying energy expenditure in human movement, the goal of this paper was to examine the utility of the kinematic and kinetic methods by applying them to data obtained from a computer simulation of cycling. The advantage of using a theoretical model of cycling is the direct access to the complete time history of individual muscle contractile element data, thus allowing the estimation of the true muscle mechanical energy expenditure which cannot be performed in human subjects. The specific objective of this study was to test the premise that these two widely used methods, based on external quantities, were adequate to quantify the mechanical energy expenditure (MEE) during steady-state cycling.

2. Methods

2.1. Simulation model

A planar two-legged bicycle-rider model was developed in a previous study (Neptune and Hull, 1996) using SIMM (MusculoGraphics, Inc., Evanston, IL) and will be reviewed briefly here. The model was driven by fourteen individual musculotendon actuators per leg with first-order activation dynamics, and musculoskeletal geometry and parameters based on the work of Delp et al. (1990). The force generating capacity of each muscle was based on a Hill-type model governed by the muscles' force–length–velocity characteristics (Zajac, 1989).

The dynamical equations-of-motion for the bicycle-rider system were derived using SD/FAST (Symbolic Dynamics, Inc., Mountain View, CA), and a forward dynamic simulation was produced by Dynamics Pipeline (MusculoGraphics, Inc., Evanston, IL).

Two distinctly different simulations were generated by using optimization to find the muscle stimulation patterns which replicated the essential features of pedaling data collected from eight subjects. The first simulation was produced by using a performance criterion which solved the 'tracking' problem by minimizing the differences between experimental and model data in the general form of:

$$J_1 = \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{(Y_{ij} - \hat{Y}_{ij})^2}{\mathrm{SD}_{ij}^2} \quad (4)$$

where Y_{ij} are the experimentally measured data, $\hat{Y}_{ij}$ are the model data, SD_{ij} are the inter-subject standard deviations, n is number of data points and m is number of variables evaluated. The tracking quantities Y_{ij} included the horizontal and vertical pedal force components, pedal angle, hip, knee and ankle intersegmental joint moments and crank torque. This criterion has been shown in a previous study to produce steady-state pedaling simulations replicating experimental kinetic and kinematic data (Neptune and Hull, 1996).

The second simulation was produced by minimizing the performance criterion J_1 and a weighted term which included the sum of average negative muscle power in the form of:

$$J_2 = \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{(Y_{ij} - \hat{Y}_{ij})^2}{SD_{ij}^2} + W \cdot \sum_{j=1}^{p} |\overline{\mathrm{NP}}_j| \quad (5)$$

where $\overline{\mathrm{NP}}_j$ is the average negative power of muscle j over the crank cycle, and W is the weighting factor. W was arbitrarily chosen to weigh the negative muscle power (in watts) five times more than the sum of relative tracking errors. This performance criterion was formulated to produce a simulation with a substantially different energetic cost from the first simulation by reducing the amount of negative muscle work while still producing realistic pedaling mechanics.

Simulations were performed over four revolutions to assure that initial start-up transients had decayed. Each performance criterion was evaluated during the fourth revolution when the simulation had reached its steady-state and was considered to be independent of the initial conditions. A final time constraint was enforced to assure the simulation pedaled at an average pedaling rate of 90 ± 2 RPM.

The optimization algorithm was formulated to find the muscle stimulation patterns (muscle stimulation onset, offset and magnitude) which minimized the performance criterion, subject to the system state vector, state variable constraints, and control bounds while satisfying the pedaling rate constraint. The optimal control problem was solved by converting the optimal control formulation into a parameter optimization problem (Pandy et al., 1992) using a simulated annealing algorithm (Goffe et al., 1994).

2.2. Experimental data

To provide data for the tracking problem, both kinetic and kinematic data were collected from eight male competitive cyclists (height $\bar{x} = 1.79 \pm 0.07$ m; mass $\bar{x} = 68.8 \pm 7.6$ kg). Informed consent was obtained before the experiment. The subjects rode a conventional road racing bicycle adjusted to match their own bicycle's geometry at 90 RPM and a workrate of 260 W. Intersegmental moments were computed using a standard inverse dynamics technique (e.g. Hull and Jorge, 1985). The rider was modeled as a five-bar linkage in plane motion. The equations of motion for each link were solved using inverse dynamics, starting with the foot and proceeding through each link to the hip. The anthropometric estimates of each segment's mass and center of gravity were defined based on Dempster (1955). Moments of inertia were computed by the data presented in Wittsett (1963) which were personalized to each subject based on Dapena (1978).

The intersegmental joint centers were determined using a Motion Analysis system (Motion Analysis Corp., Santa Rosa, CA) from reflective markers located over the right anterior-superior iliac spine (ASIS), greater trochanter, lateral epicondyle, lateral malleolus, pedal spindle and crank spindle. The hip joint center was located relative to the marker over the ASIS (Neptune and Hull, 1995). The crank arm and pedal kinematics were measured with optical encoders and the pedal force data were measured with a pedal dynamometer described by Newmiller et al. (1988). All derivatives to determine segment or angle velocity and acceleration were calculated by fitting a quintic spline to the position or angle data and differentiating the resulting equations.

All tracking quantities were computed on a cycle-by-cycle basis, averaged across cycles for each subject and then averaged across subjects. Further details about the data collection and processing can be found in Neptune and Hull (1996).

2.3. Energy analysis

Four different MEE models were examined. The first model (MEE_1) was based on the kinematic method as:

$$\mathrm{MEE}_1 = \sum_{i=1}^{360} |\mathrm{ME}_i - \mathrm{ME}_{i-1}| + W_{\mathrm{ext}} \tag{6}$$

where ME_i was the total mechanical energy of the system at crank angle i and W_{ext} was the external work load. Models MEE_2 and MEE_3 were based on the work of Kautz et al. (1994) as:

$$\mathrm{MEE}_2 = \int_0^T \sum_{j=1}^{6} |P_j| \mathrm{d}t \tag{7}$$

$$\mathrm{MEE}_3 = \int_0^T \sum_{k=1}^{2} (P_{Ik}) \mathrm{d}t \tag{8}$$

where P_j were the hip, knee and ankle joint powers of both legs and P_{Ik} were intercompensated joint powers (i.e. power absorbed at one joint is transferred to an adjacent joint where it is liberated as positive power via biarticular muscles) determined by a decision algorithm developed by Kautz et al. (1994). The decision algorithm represents the upper limit for work savings by allowing intercompensations between the ankle plantarflexor and knee flexor torques (due to the biarticular gastrocnemius) and the hip and knee torques (due to the hamstring muscles, sartorius and rectus femoris). The decision algorithm examined the sign of the appropriate joint torque combinations and computed the total joint power assuming intercompensation between joints crossed by a biarticular muscle. Note that there is no term for the work due to the hip joint force because the model assumed the hip joint was fixed.

The fourth model (MEE_4) summed the muscle contractile element work across all muscles for both legs as:

$$\mathrm{MEE}_4 = \int_0^T \sum_{l=1}^{28} |P_l| \mathrm{d}t \tag{9}$$

where P_l is the power of muscle l. Note that muscle power was computed from the force and velocity of the contractile element so it does not include storage and release of elastic energy in the muscle which might otherwise lead to an overestimation of MEE. Also, no attempt was made to weight the relative cost of positive and negative muscle work. To assess how well the three experimental methods (models MEE_1–MEE_3) estimated the muscle mechanical energy expenditure (MEE_4), relative differences were computed.

3. Results

The calculated work by either the individual muscle or joint moment powers done on the mechanical system equaled the total work done against the environment by the pedal reaction force (Table 1). Performance criteria J_1 and J_2 yielded simulations with average pedaling rates of 90.5 and 87.5 RPM and workloads of 179.9 and 179.7 J, respectively. The individual joint moment powers generated by the two pedaling simulations were consistent with those produced by the subjects (Fig. 1). The net hip moment produced power during the downstroke (0° to 180°) and mid-upstroke (225° to 315°) while the ankle moment produced power during the late downstroke (90° to 180°). The net knee moment produced power during the early downstroke and right after bottom-dead-center (BDC) while absorbing a substantial amount of power during the late downstroke. The net crank power produced by both simulations were almost always within one standard deviation of the subject's data.

Individual muscles primarily generated power when they were in a position to shorten (Fig. 2). Substantial negative power was observed for the psoas and vasti muscle groups during the downstroke and after BDC, respectively, when the muscles were deactivating and lengthening (Fig. 2). Simulation J_1 had 30.5 J more

Table 1
Simulation results for performance criteria J_1 and J_2

	Simulation J_1	Simulation J_2
Average power output (W)	272.0	262.7
Average work (J)	179.9	179.7
Average pedaling rate (RPM)	90.5	87.5
Kinematic method		
MEE_1 (J)	233.0	233.3
Difference from MEE_4 (%)	24.2	5.2
Kinetic methods		
Positive work (J)	207.0	205.7
Negative work (J)	−27.1	−26.0
Net work (J)	179.9	179.7
MEE_2 (J)	234.1	231.7
Difference from MEE_4 (%)	23.8	5.9
MEE_3 (J)	185.1	185.1
Difference from MEE_4 (%)	39.8	24.8
Muscle fibers		
Positive work (J)	243.6	212.9
Negative work (J)	−63.7	−33.2
Net work (J)	179.9	179.7
MEE_4 (J)	307.3	246.1

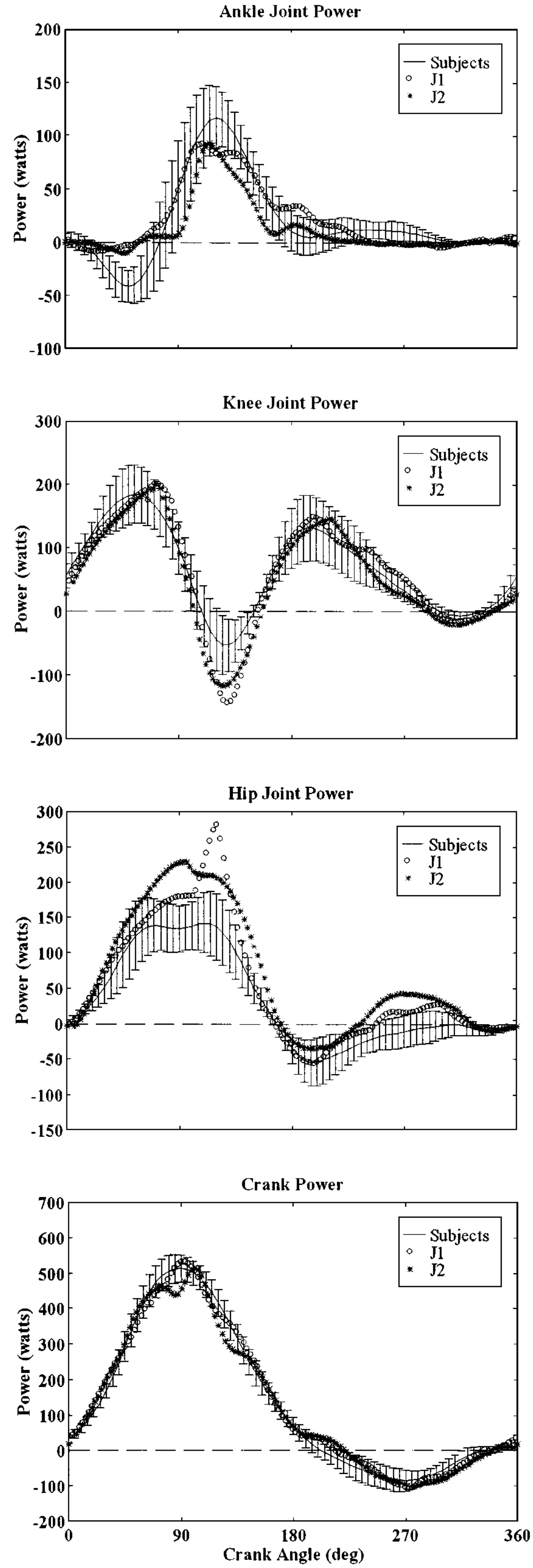

Fig. 1. Right leg joint powers: (a) hip, (b) knee, (c) ankle and (d) net crank power.

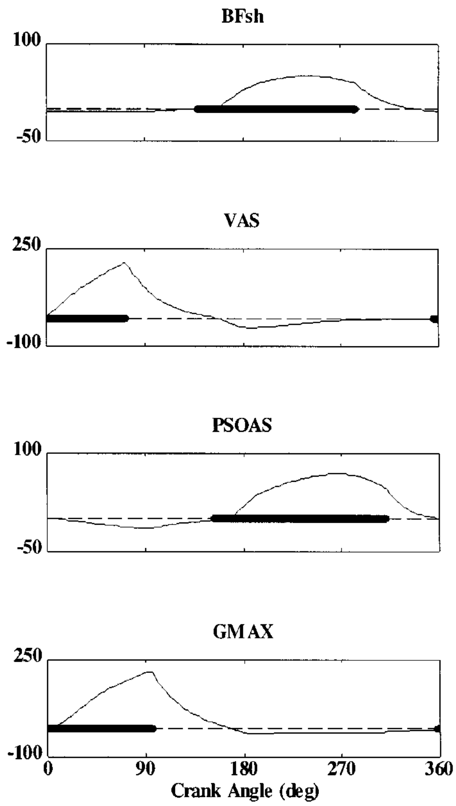

Fig. 2. Individual muscle powers for simulation J_2. Positive muscle power occurred when the muscle was shortening. The solid horizontal line indicates muscle stimulation timing. Fourteen muscles per leg were included in the model and further combined into muscle sets, with each muscle within each set receiving the same excitation signal. The muscles presented above are the single-joint muscles PSOAS (iliacus, psoas), GMAX (gluteus maximus, adductor magnus), BFsh (biceps femoris short head) and VAS (3-component vastus). The crank angle is 0° at top-dead-center and positive in the clockwise direction.

negative muscle work than simulation J_2 (Table 1). The increase in negative work caused J_1 to yield a higher MEE_4 (307.3 J) than J_2 (246.1 J) for the same external workload. Although simulation J_1 had 30.5 J more negative work than simulation J_2, the joint moment powers were within two standard deviations of the subjects (Fig. 1).

A comparison of the experimental MEE estimates showed that the methods produced similar results for both simulations, although simulation J_2 had nearly 50% less negative work. Model MEE_1 yielded identical results for simulation J_1 (233.0 J) and J_2 (233.3 J) while model MEE_2 yielded similar magnitudes, 234.1 J and 231.7 J for simulations J_1 and J_2, respectively. MEE_2 was the only method that hinted the pedaling simulation produced by minimizing J_2 was more efficient. Model MEE_3, which accounts for intercompensations by biarticular muscles, substantially reduced the energy expenditure and yielded identical results of 185.1 J for both simulations. But this reduction in energy expenditure produced the highest differences from MEE_4 (Table 1). In all cases, the experimental methods underestimated the energy expenditure of the individual muscles with differences ranging from 5% to 40% (Table 1). Therefore, the premise that the externally based energy methods could predict the muscle MEE was found to be invalid.

4. Discussion

The goal of this study was to evaluate the utility of experimental MEE models to estimate muscle energy expenditure in human movement. To achieve this goal, a musculoskeletal model of cycling was used to evaluate three models previously used in the literature. Two distinctly different simulations of steady-state pedaling were produced using a dynamic optimization framework to determine the muscle stimulation patterns with two different performance criteria, one that solved the 'tracking' problem, and the other that solved the same tracking problem while reducing the amount of negative muscle power. The tracking criterion used in this study had previously been shown to produce simulations replicating experimental pedaling data (Neptune and Hull, 1996). Although the utility of the cost function including the negative muscle power has not been previously established in the literature, this cost function was used primarily to produce a simulation with substantially less negative muscle work to illustrate the ability of the various MEE models to identify differences in energy cost between the two movements.

The two simulations produced by criteria J_1 and J_2 reproduced the important features of the subject's pedaling data (Fig. 1). A comparison between the subject's joint moment and net crank power curves and the simulation data showed that it is not unreasonable to assume that a subject could produce the pedaling trajectories of both simulations.

The results showed that the three experimental methods greatly underestimated the MEE of the muscle fibers. The muscle power profiles revealed that muscles can absorb considerable power when the net joint power is positive (i.e. the direction of the torque and movement are the same). For example, after BDC, the vasti muscle group is absorbing power while it is deactivating when the net joint power is still positive (Fig. 2). At the same time, the biceps femoris short head is generating positive power, possibly to overcome the power loss due to the vasti. A similar co-contraction of antagonistic muscles is observed between the psoas and gluteus maximus muscles during the downstroke (0° to 180°). The psoas is lengthening while deactivating thus generating substantial negative muscle power which is absorbed by the shortening gluteus maximus. Since the kinetic and kinematic methods rely on net joint moments and segment kinematics respectively, the amount of muscle co-contractions cannot be uniquely quantified. Co-contractions are inevitable in fast human movements due to the activation dynamics associated with muscle force development (i.e. the delay in muscle force rise and decay) and the need for movement control (e.g. to prevent knee hyperextension).

The results of this study agree with previous research which have tried to correlate MEE estimates based on external measurements with metabolic energy expenditure. Martin et al. (1993) found no correlation between $\dot{V}O_2$ and models MEE_1 and MEE_2 during walking and running while Foerster et al. (1995) found no correlation between $\dot{V}O_2$and the same models for different above-knee prostheses. Similarly, Hull et al. (1992) showed that a bicycle chainring design which reduced internal work (MEE_1) did not correspond to a decrease in $\dot{V}O_2$. Although the chainring design reduced the cost associated with changes in the mechanical energy of the system, it appears the new design may have caused the subjects to use an inefficient coordination strategy which increased the amount of negative work or muscle co-contractions. Kautz et al. (1994) performed a MEE analysis using models MEE_2 and MEE_3 and found no correlation between internal work (MEE_1) and these MEE measures. Their study clearly demonstrated that internal work is not a valid measure of the energy associated with moving the limbs. They showed that decreases in kinetic and potential energy of the limbs can do work on the environment to overcome the external load without requiring muscle work. Their results combined with the results of this study clearly shows that the internal work method is theoretically flawed and should not be used in cycling analyses.

Several studies have used kinetic methods in an attempt to identify possible energy savings by accounting for energy transfers between joints via biarticular muscles (e.g. Broker and Gregor, 1994; Kautz et al., 1994). This situation can arise when mechanical energy from one joint where energy is being absorbed (negative work) is transferred to an adjacent joint where energy is being generated (positive work) via biarticular muscles. Although these MEE models have sound theoretical foundations, the results herein indicate that these reductions in MEE may introduce even greater errors in the analysis, up to 40% as seen in this study. The more conservative model (MEE_2) which used completely compensated muscle sources (i.e. all negative work absorbed at a joint is degraded to heat and cannot be returned to the system) still underestimated the muscle MEE up to 24%.

Although the MEE_2 model produced smaller errors than the intercompensated MEE_3 model, examination of the individual joint powers showed that the assumption of intercompensated joint powers more accurately reflects the mechanics of the movement. The largest negative joint power occurs at the knee near 135° (Fig. 1). Although not all the individual muscle powers are presented in Fig. 2, no muscles produce significant negative power in this region which coincides with the peak hip joint power (Fig. 1). These results suggest that the assumption of intercompensation is valid for this situation in which the biarticular hamstring and gastrocnemius muscles transfer power from the knee joint to the hip and ankle joints, respectively. Although MEE_2 provided a better estimate for the MEE, the improvement comes at the expense of adding negative work at a time when the muscles are not actually producing negative work (e.g. negative knee power near 135°). Adding negative work when muscles are not producing it to make up for underestimates elsewhere in the crank cycle is not based on sound fundamental mechanics. Thus it appears MEE_3 is a more accurate measure from a mechanics perspective, but is limited by its inability to account for co-contractions of antagonistic muscles. These results indicate that kinetic based methods, while less flawed than the internal work method from a theoretical perspective, ultimately cannot quantify the energetic cost of cycling, nor provide the insight necessary to further our understanding of muscle coordination.

In summary, the errors reported in this study clearly indicate the inability of MEE methods based on external measurements to explicitly quantify the energy transfers between segments and to account for co-contractions of antagonistic muscles during movement. These methods are also limited by their inability to quantify the contribution of elastic energy storage to positive and negative work and to determine how much power is required to accelerate and decelerate the limb segments. The combination of these limitations have led to significant errors in the computation of mechanical efficiencies (Williams and Cavanagh, 1983).

This study has illustrated the limitations with quantifying MEE based on external measurements and demonstrated the utility of forward dynamic musculoskeletal models combined with experimental data to address such questions. Movement simulation analyses which include individual muscle actuators provide muscle force data which removes the limitations associated with the external methods and have the potential to vastly improve our understanding of muscle function and coordination principles.

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References

Broker, J.P., Gregor, R.J., 1994. Mechanical energy management in cycling: source relations and energy expenditure. Medical Science and Sports Exercise 26, 64–74.

Cavagna, G.A., Kaneko, M., 1977. Mechanical work and efficiency in level walking and running. Journal of Physiology (London) 268, 647–681.
Dapena, J., 1978. A method to determine angular momentum of the human body about three orthogonal axes passing through the center of gravity. Journal of Biomechanics 11, 251–256.
Delp, S.L., Loan, J.P., Hoy, M.G., Zajac, F.E., Topp, E.L., Rosen, J.M., 1990. An interactive graphics-based model of the lower extremity to study orthopaedic surgical procedures. IEEE Transactions of Biomedical Engineering 37, 757–767.
Dempster, W.T., 1955. Space requirements of the seated operator. WADC Technical Report 55–159. Wright-Patterson Air Force Base, Dayton, OH.
Foerster, S.A., Bagley, A.M., Mote, C.D. Jr., Skinner, H.B., 1995. The prediction of metabolic energy expenditure during gait from mechanical energy of the limb: a preliminary study. Journal of Rehabilitation Research Development 32, 128–134.
Goffe, W.L., Ferrier, G.D., Rogers, J., 1994. Global optimization of statistical functions with simulated annealing. Journal of Econometrics 60, 65–99.
Hull, M.L., Jorge, M., 1985. A method for biomechanical analysis of bicycle pedalling. Journal of Biomechanics 18, 631–644.
Hull, M.L., Williams, M., Williams, K., Kautz, S., 1992. Physiological response to cycling with both circular and noncircular chainrings. Medical Science and Sports Exercise 24, 1114–1122.
Ingen Schenau, G.J. van, Woensel, W.W. van, Boots, P.J., Snackers, R.W., De Groot, G., 1990. Determination and interpretation of mechanical power in human movement: application to ergometer cycling. European Journal of Applied Physiology 61, 11–19.
Kautz, S.A., Hull, M.L., Neptune, R.R., 1994. A comparison of muscular mechanical energy expenditure and internal work in cycling. Journal of Biomechanics 27, 1459–1467.
Martin, P.E., Heise, G.D., Morgan, D.W., 1993. Interrelationships between mechanical power, energy transfers, and walking and running economy. Medical Science Sports Exercise 25, 508–515.
Neptune, R.R., Hull, M.L., 1995. Accuracy assessment of methods for determining hip movement in seated cycling. Journal of Biomechanics 28, 423–437.
Neptune, R.R., Hull, M.L., 1998. An evaluation of performance criteria for simulation of submaximal steady-state cycling using a forward dynamic model. Journal of Biomechanical Engineering, in press.
Newmiller, J., Hull, M.L., Zajac, F.E., 1988. A mechanically decoupled two force component bicycle pedal dynamometer. Journal of Biomechanics 21, 375–386.
Pandy, M.G., Anderson, F.C., Hull, D.G., 1992. A parameter optimization approach for the optimal control of large-scale musculoskeletal systems. Journal of Biomechanical Engineering 114, 450–460.
Wells, R., Evans, N., 1987. Functions and recruitment patterns of one- and two-joint muscles under isometric and walking conditions. Human Movement Science 6, 349–372.
Whittsett, D.R., 1963. Some dynamic response characteristics of weightless man. AMRL Technical Documentary Report 63–18. Wright Air Force Base, Dayton, OH.
Widrick, J.J., Freedson, P.S., Hamill, J., 1992. Effect of internal work on the calculation of optimal pedaling rates. Medical Science and Sports Exercise 24, 376–382.
Williams, K.R., Cavanagh, P.R., 1983. A model for the calculation of mechanical power during distance running. Journal of Biomechanics 16, 115–128.
Winter, D.A., 1979. A new definition of mechanical work done in human movement. Journal of Applied Physiology 46, 79–83.
Zajac, F.E., 1989. Muscle and tendon: properties, models, scaling, and application to biomechanics and motor control. Critical Review of Biomedical Engineering 17, 359–411.