Stability and Multi-scale Analysis for PDEs

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Book of Abstracts

Paul Blochas, University of Texas at Austin

Uniform convective stability for advection-reaction-diffusion equations in the inviscid limit

In this talk, I will present results on the study of the stability in time of families $(\underline{U}_\epsilon)_{0<\epsilon<\epsilon_0}$ of traveling waves solutions to

$$\partial_t u + \partial_x(f(u)) = g(u) + \epsilon \partial_x^2 u$$

that approximate either a given Riemann shock or a smooth wave. The aim is to show some uniform asymptotic orbital stability result of these waves under some conditions that guarantee the asympotic orbital stability of the corresponding hyperbolic wave, as proved in a previous work of L. Garnaux and L. M. Rodrigues. We will present results in weighted topologies that allow to stabilize waves which are unstable in unweighted topologies. This work is an extension of a previous joint work with Professor Miguel Rodrigues (University of Rennes, Rennes, France).

José A. Carrillo, University of Oxford

The Landau equation as a gradient flow and PIC methods for inhomogeneous collisional plasmas

The Landau equation is one of the most important partial differential equations in kinetic theory. It gives a description of colliding particles in plasma physics, and it can be formally derived as a limit of the Boltzmann equation when grazing collisions are dominant. The purpose of this talk is to propose a new perspective inspired from gradient flows for weak solutions of the spatially homogeneous Landau equation, which is in analogy with the relationship of the heat equation and the 2-Wasserstein metric gradient flow of the Boltzmann entropy. From the analytical viewpoint, we use the theory of metric measure spaces for the Landau equation by introducing a bespoke Landau distance. We show for a regularized version of the Landau equation that we can construct gradient flow solutions, curves of maximal slope, via the corresponding variational scheme. The main results obtained for the Landau equation shows that H-?solutions with certain apriori bounds are equivalent to gradient flow solutions of the Landau equation, and that the grazing collision limit from Boltzmann to Landau can be obtained via a Gamma-?convergence approach. From the numerical viewpoint, we aim at using this interpretation to derive a structure preserving deterministic particle method to solve efficiently the spatially homogeneous and inhomogenous Landau equation. This talk is based on a summary of works in collaboration with R. Bailo, M. Delgadino, L. Desvillettes, J. Hu, L. Wang, and J. Wu.

Geng Chen, University of Kansas

Some recent progress on compressible Euler equations

Compressible Euler equations are a typical system of hyperbolic conservation laws, whose solution forms shock waves in general. It is well known that global BV solutions of system of hyperbolic conservation laws exist, when one considers small BV initial data. We will discuss some recent progress on Euler equations including the L^2 stability and inviscid limit from Navier-Stokes equations, global smooth solution and singularity formation for radially symmetric solutions.

Gui-Qiang Chen, University of Oxford

Some Inverse Problems for Shock Wave Control for the Euler Equations

Controlling shock waves is crucial in applications in various fields of science and engineering, including aerodynamics, aerospace engineering, explosion mitigation, and shock tube experiments. In this talk, we present some inverse problems and related stability issues for controlling multi-dimensional steady shock waves and discuss some recent progress in controlling both the leading shock waves generated by wedges/conical bodies and associated fluid flows by designing the boundary geometries of the wedges/conical bodies with desired pressure distributions and/or leading shock locations through the inverse problems. Some further perspectives and open problems in this direction will also be addressed.

Ming Chen, University of Pittsburgh

Orbital stability for internal waves

I will discuss the nonlinear stability of capillary-gravity water waves propagating along the interface dividing two immiscible fluid layers of finite depth – a situation that can be described as a vortex sheet. The motion in both regions is governed by the incompressible and irrotational Euler equations, with the density of each fluid being constant but distinct. We prove that for supercritical surface tension, all known small-amplitude localized waves are (conditionally) orbitally stable in the natural energy space. Moreover, the trivial solution is shown to be conditionally stable when the Bond and Froude numbers lie in a certain unbounded parameter region. The results are obtained by applying a recently developed variant of the Grillakis-Shatah-Strauss method, and the key part of the analysis consists of computing the spectrum of the linearized augmented Hamiltonian at a shear flow or small-amplitude wave. This is joint work with S. Walsh.

Thomas Chen, University of Texas at Austin

Explicit construction of global L^2 minimizers and geometry of Deep Learning networks

In this talk, we present some recent joint work with Patricia Munoz Ewald (UT Austin) in which we provide an explicit construction of global minimizers of the L^2 cost function in underparametrized Deep Learning networks, in the context of supervised learning and ReLU activation functions; no use of gradient descent is made here. Moreover, the arbitrariness of choice of a Riemannian structure is used to construct an adapted gradient descent flow for the overparametrized case which exhibits a uniform exponential convergence rate, under a rank condition; links of this construction with sub-Riemannian geometry are pointed out.

Constantine Dafermos, Brown University

The Zero Relaxation Limit in BV

The lecture will survey progress and open problems in a research program aiming at the construction of BV solutions to hyperbolic systems of balance laws with a stiff source modeling relaxation, and the passage to the zero relaxation limit.

Mikhail Feldman, University of Wisconsin - Madison

Self-similar solutions to two-dimensional Riemann problems and transport equations

In this talk, we discuss two-dimensional Riemann problems involving transonic shocks. Examples include regular shock reflection, Prandtl reflection, and four-shocks Riemann problem. We first review the results on existence, regularity and properties of global self-similar solutions in the framework of potential flow equation. An open problem is to extend these results to compressible Euler system, i.e. to understand the effects of vorticity. We show that for the isentropic Euler system, solutions have low regularity, specifically velocity and density do not belong to the Sobolev space H^1 in self-similar coordinates. We further discuss existence, uniqueness and stability of renormalized solutions to the transport equation for vorticity in the resulting low regularity setting.

Irene M. Gamba, University of Texas at Austin

Existence of global weak solutions to quasilinear theory for electrostatic plasmas

The quasilinear theory has been widely used to describe the resonant particle-wave interaction in plasmas as a model reduction from Vlasov coupled to Poisson or Maxwell models for weak turbulence regimes. The electrostatic case, i.e. a model originating from Vlasov-Poisson system, is the most fundamental and representative one to understand the role of the Lorentzian forces. We prove the existence of global weak solutions for electrostatic plasmas of the full resonance system in low dimension. I?ll discuss the first what we have completed so far with Kun Huang and work in progress with Kun Huang and William Porteous for higher regularity.

Maria Gualdani, University of Texas at Austin

Recent development for the Landau-Coulomb equation

The Landau equation, introduced by Lev Landau in 1936, is a modification of the Boltzmann equation to specific applications in plasma physics, and describes the interactions and collisions among charged particles in a plasma. The mathematical investigation of the Landau equation has been active for several decades, with researchers exploring various aspects of its behavior and properties and has culminated in the recent global well-posedness result by Guillen and Silvestre for smooth initial data. In this talk I will present the first global well-posedness theory for the Landau-Coulomb with rough initial data and show that, even for non-smooth configurations, the equation has a very strong regularization effect thanks to the diffusion dominating the reaction at any time. I will also show a particular example of blow-down mechanism based on separation of scales.

Tian Jing, University of Michigan

Two-phase problems in magnetohydrodynamics

We consider the motion of two immiscible fluids in a bounded domain. The fluids are incompressible, conductive and viscous, and a magnetic field exists in the space. The surface tension between two fluids will be considered. Different types of solutions and their existence results will be discussed.

Moon-Jin Kang, KAIST

Well-posedness of small BV solutions to isentropic Euler from Navier-Stokes

The Cauchy problem for compressible Euler system from inviscid limit of Navier-Stokes remains completely open, as a challenging issue in fluid dynamics. This talk gives a resolution for this problem in the 1D isentropic case. We will show the global well-posedness of entropy solutions with small BV initial data in the class of inviscid limits from the associate Navier-Stokes. More precisely, any small BV entropy solutions are inviscid limits from Navier-Stokes. Those are unique and stable among inviscid limits from Navier-Stokes. The proof is based on the three main methodologies: the modified front tracking algorithm; the a-contraction with shifts; the method of compensated compactness. This is a joint work with Geng Chen (U. Kansas) and Alexis Vasseur (UT-Austin).

Nataša Pavlović, University of Texas at Austin

On the effective dynamics of Bose-Fermi mixtures

Investigating degenerate mixtures of bosons and fermions is an extremely active area of research in experimental physics for constructing and understanding novel quantum bound states such as those in superconductors, superfluids, and supersolids. These ultra-cold Bose-Fermi mixtures are fundamentally different from degenerate gases with only bosons or fermions. They not only show an enriched phase diagram, but also a fundamental instability due to energetic considerations coming from the Pauli exclusion principle. Inspired by this activity in the physics community, recently we started exploring the mathematical theory of Bose-Fermi mixtures. One of the main challenges is understanding the physical scales of the system that allow for suitable analysis. We will describe how we overcame this challenge by identifying a novel scaling regime in which the fermion distribution behaves semi-clasically, but the boson field remains quantum-mechanical. In this regime, the bosons are much lighter and more numerous than the fermions. The talk is based on the joint work with Esteban Cárdenas and Joseph Miller.

Mikhail Perepelitsa, University of Houston

PDE models for bacterial motion

In this presentation, we will explore various partial differential equation (PDE) models describing bacterial motion. These models represent a large number of self-propelled rods interacting by aligning their long axes and periodically reversing their direction of motion. Specifically, we will focus on one model where the rate of reversal is significantly higher than the rate of alignment, and show how to construct weak solutions using the method of kinetic averaging.

Edriss Titi, Texas A&M University

Mathematical Analysis of Atmospheric and Oceanic Dynamics Models: Cloud Formation and Sea-ice Models

In this talk we will present rigorous analytical results concerning global regularity, in the viscous case, and finite-time singularity, in the inviscid case, for oceanic and atmospheric dynamics models. Moreover, we will also provide a rigorous justification of the derivation of the Primitive Equations of planetary scale oceanic dynamics from the three-dimensional Navier-Stokes equations as the vanishing limit of the small aspect ratio of the depth to horizontal width. In addition, we will also show the global well-posedeness of the coupled three-dimensional viscous Primitive Equations with a micro-physics phase change moisture model for cloud formation. Eventually, we will also present short-time well-posedness of solutions to the Hibler's sea-ice model.

Monica Torres, Purdue University

Extended divergence-measure fields and Cauchy fluxes

We extend the balance law from classical continuum physics to the case where the production on any open set is measured with a Radon measure, and the associated Cauchy flux is bounded by a Radon measure concentrated on the boundary of the set. We prove that there exists an extended divergence-measure field (i.e.; a vector-valued Radon measure whose distributional divergence is also a Radon measure) such that the Cauchy flux can be recovered through the field, locally on almost every open set, and globally for every open set. We establish a one-to-one correspondence between Cauchy fluxes and extended divergence-measure fields. Our results generalize the previous formulations of the Cauchy flux that generate vector fields in L^p, while the classical Cauchy?s theorem corresponds to continuous vector fields. This is a joint work with Gui-Qiang Chen (University of Oxford) and Christopher Irving (Technical University Dortmund).

Dehua Wang, University of Pittsburgh

Elastic effects on vortex sheets and vanishing viscosity

Elasticity is important in continuum mechanics with a wide range of applications and is challenging in analysis. In this talk we shall first review some basic mathematical results and then discuss some special elastic effects in elastic fluids. The first elastic effect is the stabilizing effect of elasticity on the vortex sheets in compressible elastic flows. Some recent results on linear and nonlinear stability of compressible vortex sheets will be presented. The second effect is on the vanishing viscosity process of compressible viscoelastic flows on the half plane under the no-slip boundary condition. It is well-known that for the corresponding inviscid limit of the compressible Navier-Stokes equations with the no-slip boundary condition, one does not expect uniform energy estimates of solutions due to the appearance of strong boundary layers. Our results show that the deformation tensor can prevent the formation of strong boundary layers. The talk is based on the recent results with several collaborators.

Cheng Yu, University of Florida

Vanishing viscosity limit of the Navier-Stokes equations to the Euler equations under the Kolmogorov hypothesis

In this talk, I will discuss the vanishing viscosity limit of the Navier-Stokes equations in the context of global weak solutions satisfying the Kolmogorov hypothesis. This limit yields a weak solution to the Euler equations, ensuring energy conservation for all $t > 0$. This talk is based on a joint work with Wenjun Wang.

Ewelina Zatorska, University of Warwick

Recent developments for the one-dimensional compressible Euler system with local and non-local interactions and dissipation terms

will summarise our recent findings on the existence of regular and weak solutions for the compressible Euler equations with nonlocal interaction terms including attraction-repulsion and alignment. In particular, I will present the relative entropy structure based on the two-velocity reformulation of the system, the viscous approximation, and the long-time behaviour of solutions.

Datong Zhou, Penn State University

Mean-field limits of systems of biological neurons with generic heterogeneous connections

We investigate the mean-field limits of large-scale networks of interacting biological neurons. The neurons are represented by the so-called integrate-and-fire models that follow the membrane potential of each neuron and captures individual spikes. However we do not assume any prior structure on the graph of interactions but consider instead any connection weights that obey certain generic mean-field scalings. In the dense scaling, we are able to achieve the mean-field limit in a spatially extended space, through a graphon limit that tracks the role of individual neurons in the network. When the networks are potentially sparse, we extend the concept of extended graphons, introduced in Jabin-Poyato-Soler, by introducing a novel notion of discrete observables that intentionally avoids the graph limit argument and gives direct large-scale description of the networks. This is a joint work with P.-E. Jabin and V. Schmutz.