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The University of Texas at Austin
Population Wellbeing Initiative

The long-run relationship between per capita incomes and population size*

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Abstract

The relationship between human population sizes and per capita income has been long debated. Two competing forces feature prominently in these discussions. On the one hand, a larger population means that limited natural resources must be shared among more people. On the other hand, more people means more innovation and faster technological progress, other things equal. We study a model that features both of these channels. A calibration suggests that, in the long-run, the relationship between population and income per-capita is positive.

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1 Introduction

Later this century the global population is expected to peak and then begin declining. This prompts one of the oldest open questions in economics: should we expect smaller populations to generate better per capita outcomes?

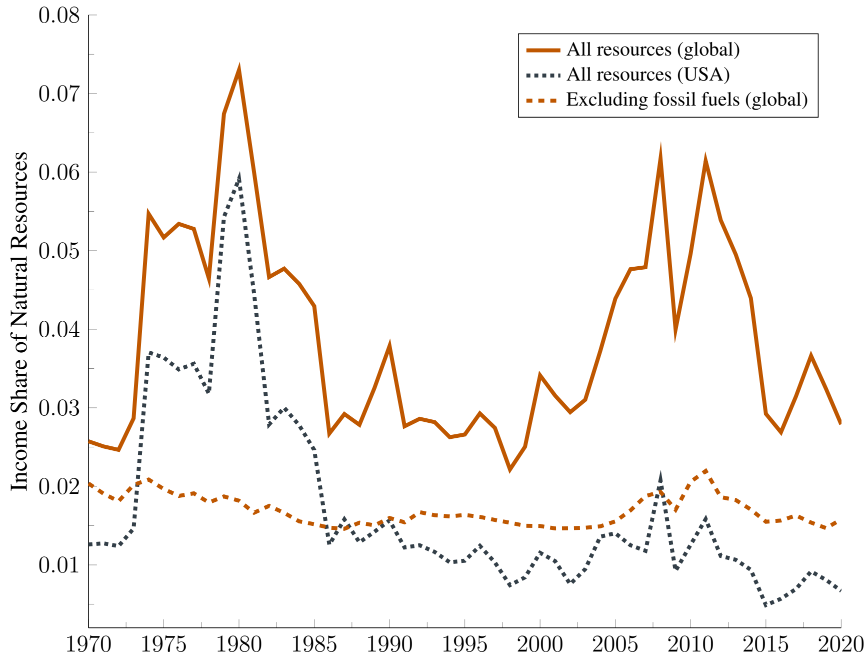
There are, of course, many dimensions to this question. Here, we focus on one: the long-run relationship between population levels and per capita incomes. The classic Malthusian concern is that larger populations strain our productive natural resources and spread the benefits of ecosystem services more widely. This suggests a negative relationship between population and per-capita incomes. More recently, economists have come to formalize important channels by which a larger population could have competing benefits. In particular, people produce infinitely shareable knowledge that increases everyone's productivity. If the number of ideas is increasing in the number of people, then larger populations will contribute to a more productive economy (Romer, 1990; Jones, 2005).

This paper compares the quantitative importance of these two channels using standard models from these respective sub-disciplines. In our framework, the Malthusian concern is captured by a production function that utilizes natural resources that are in fixed supply. The innovation channel is captured by a positive relationship between population size and productivity growth. One difficulty in comparing these two forces is that diluting environmental services across people has a persistent *level* effect on income, while innovation gains are modeled as *growth* effects. We overcome this with a minimal tweak to the standard semi-endogenous growth framework that delivers a long-run level relationship between population size and productivity. The model generates a steady-state level of per capita income that depends on population sizes directly.

Using this solution we can calibrate the long-run relationship between population levels and per-capita income based on previously-estimated parameters. The innovation process depends on a ratio of two parameters that has been recently studied in detail (Bloom et al., 2020; Peters, 2022; Ekerdt and Wu, 2023). The effect of diluting the fixed factor across more individuals is directly captured by the income share of natural resources, a feature anticipated by Weil and Wilde (2009). The overall effect of population on long-run income is determined by whether the income share of natural resources is larger or smaller than the innovation parameters.

The main finding is that the innovation effect dominates. Underlying this finding is the observation that the share of global income accruing to natural resources is small and has remained trendless over the past few decades, despite a doubling of the global population. Based on this observation, we calibrate a Cobb-Douglas production function, and find that the elasticity of output with respect to natural resources is too small to overturn the benefits of natural resources. This will remain true under any calibration that leaves the share of income earned by natural resources small in the long run.

Figure 1: Natural resource shares are small and non-increasing



Notes: Share of income paid to natural resources over time. (Solid) Global income share paid to all natural resources, following World Bank classification to include: (a) subsoil energy and minerals; (b) timber resources; (c) crop land; (d) pasture land. Details of each category are contained in the Appendix. (Dotted) This same income share, but using only US data to ensure trend or level not driven by unreliable global data. (Dashed) Excludes fossil fuels which drive the level and volatility of this series, but not its flat trend.

The baseline model we use to generate this result is intentionally simple. The point of this exercise is to draw out the implications of the most straightforward combination of these two forces once they have been modified to fit together and calibrated to existing data. This of course leaves other open questions: What if technological progress is biased against natural resources in the long-run? What if there are non-rival, rather than just rival, benefits of ecosystem services? We extend the model along these dimensions to ask whether the initial finding can be overturned. Extensions of the analytical model do not lead to a qualitatively different comparison; the positive population-income relationship is retained.

1.1 Relationship to existing literature

This paper sits closely to and draws from work studying potential agglomeration and innovation benefits coming from increased population sizes, density, or growth rates. Recent notable work on this topic includes Desmet et al. (2018); Peters and Walsh (2021); Jones (2022); Peters (2022); Gross and Klein (2023). Jones (2022) and Peters (2022) are especially relevant for our work. Peters (2022) shares our focus on population size differences and accounts for fixed factors in production. Our approach complements Peters (2022) by attempting to generalize in a parsimonious way what he learns from a local, well-identified case in immediately-post-war rural Germany. Jones (2022) instead shares our global perspective, and is similarly motivated by the projected end of population growth. But his study omits the possibility of environmental benefits as a countervailing benefit of a declining population, something that remains top-of-mind for many assessing the implications of different population sizes.

Methodologically, our approach has similarities with an earlier literature studying the transition from a Malthusian growth regime—with stagnant incomes and relatively small populations—to a modern growth regime with unprecedented growth in both incomes and populations (e.g., Kremer, 1993; Galor and Weil, 2000; Jones, 2001). These models similarly rely on a fixed factor and productivity that increases in the size of the population. A difference in this paper is our forward-looking focus, which leads us to take a few noteworthy modeling departures. Most consequentially, we study different long-run population *levels*. This has the advantage of allowing us to avoid modeling joint decisions over fertility and human capital investments (e.g., Galor and Weil, 2000): If populations stabilize at *any* level, fertility decisions must converge to one child per adult-lifetime, and hence common per capita child-raising costs.[1]

Finally, a few recent papers share our forward-looking focus as well as an interest in long-run population levels (as opposed to growth rates) and consider environmental consequences. Henderson et al. (2022) compares the negative effects of population growth and climate change in a model where each reduces natural resources per person. Dasgupta (2019) and Córdoba et al. (2022) study the optimal population level in the face of a finite resource. However, none of these papers allows for a positive relationship between population and technology. Instead, the models assume a Malthusian environment with decreasing returns (the question of optimality arises when

[1] This is in contrast to a balanced growth path (BGP) with slower or faster population growth: on a BGP with faster population growth the parent:child relationship is permanently worse than one with slower population growth, perhaps resulting in lower per capita human capital investment. This mechanism is at the core of Unified Growth Theory (Galor and Weil, 2000; Galor, 2011) and can lead to different takeaways than standard semi-engodegenous models without this explicit human capital decision.

the planner values the number of existences, not just per capita outcomes). Pindyck (2022) characterizes sustainable consumption paths conditional on a given relationship between per capita productivity and population. Our objective is complementary to these papers, studying whether (and at what population sizes) a trade-off exists between population size and per capita income. Peretto and Valente (2015) is likewise closely related, adding a finite resource to a Schumpeterian model of growth and studying long-run solutions with stable populations. While sharing much of our motivation regarding non-perpetual population growth/decline, they instead focus on modeling fertility with the explicit objective of highlighting under what conditions a stable solution exists (as opposed to endogenous explosion/collapse of populations). They say less about the question of whether per capita incomes would in fact be higher or lower for larger or smaller long-run population levels, a question that will be of relevance should governments want to arrest an endogenously occurring dynamic of persistent population decline.

2 Model

Time is continuous and indexed $t \in \mathbb{R}_{++}$. In each period aggregate GDP is given by

$$Y_t = A_t F(N_t, K_t, E_t) \tag{1}$$

where $A_t > 0$ is total factor productivity, N_t is the global labor force, K_t is physical capital and E_t are the natural resources used in production.

We assume that the production function, F, has constant returns to scale. This means that, for a given technology level, doubling the amount of labor, capital, and natural resources would double aggregate output. Given that natural resources are fixed, this feature of the model creates scope for a negative relationship between per-capita GDP and population: if we double population (and even capital) without increasing natural resources, then GDP will less-than-double. Consequently (as we doubled population), GDP per-capita will decline.

The possibility of a positive relationship between population and per-capita GDP is introduced through the process of technological progress. The technology parameter, A_t, consists of ideas, which are produced by people and can be used freely and indefinitely to improve the production process. Because more people generate more ideas, productivity is increasing in the number of people.

Our focus in this paper is on the long-run relationship between GDP per-capita and population.

In particular, we restrict attention to trajectories in which long-run population levels are fixed at $N_t = N$ for some $N > 0$. We show that, under some additional assumptions, output per-capita converges to a constant as well:

$$y = \lim_{t\to\infty} \frac{Y_t}{N_t}$$

Our investigation concerns the quantitative relationship between long-run income per-capita, y, and the long-run population level, N.

2.1 Natural resources

We distinguish between three types of natural resources, respectively representing (i) exhaustible and non-renewable resources (e.g., fossil fuels); (ii) exhaustible, but renewable resources (e.g., timber, fish stocks); and (iii) non-exhaustible, but finite, resources (e.g., land, minerals, solar energy). These combine in some generalized function to create the aggregate E_t used in production.

$$E_t = g(e_{1,t}, e_{2,t}, e_{3,t}) \tag{2}$$

To understand the conditions under which E_t converges to a long-run steady state, consider each of these components in turn. First, note that, in the long-run, exhaustible and non-renewable resources will be exhausted (and not renewed), and therefore

$$\lim_{t\to\infty} e_{1,t} = \bar{e}_1 = 0$$

(throughout, we use upper-bars to indicate steady state variables). Appendix A discusses how the population level affects the rate at which non-renewable natural resources are exhausted. It turns out that, in standard settings, it does not.

The second category – exhaustible but renewable natural resources – is more complicated. For example, it is possible to generate a long-run cycle in which resources are periodically exhausted, regenerated, and exhausted again; in this case, $e_{2,t}$ will not converge to a steady state. However, for tractability, we will restrict our focus to steady-state solutions where the level of $e_{2,t}$ is constant. Appendix B details the problem of renewable resource management in long-run steady states. The relevant and intuitive takeaway is that the level drawn each period depends only on the regeneration rate of the natural resources, and is therefore independent of population size.

The third category, $e_{3,t}$, consists of natural resources that are non-exhaustible but in fixed supply. By definition, the level of $e_{3,t}$ is the same in each period. For example, there is some total amount of lithium on the planet that can be embodied within products at any given time, and that amount is not reduced by its use.[2] Thus, there exists some $\bar{e}_3$ such that $e_{3,t} = \bar{e}_3$ for all t.

When each type of natural resource converges to a long-run steady state, we have that

$$\bar{E} = g(0, \bar{e}_2, \bar{e}_3) \tag{3}$$

To reiterate the relevant takeaway, this formulation does not depend on the population size. Constraints imposed by nature are aggregate, not per-capita, constraints. Therefore, we have a negative relationship between population and *per-capita* natural resource availability. Holding fixed A, this would imply a negative relationship between population and per-capita income:

$$y = \frac{Y}{N} = AF\Big(1, \frac{K}{N}, \frac{\bar{E}}{N}\Big) \tag{4}$$

(where the first equality is a definition and the second equality follows from the assumption that F has constant returns to scale).

Climate change. One might wonder to what extent this framework captures the concern of climate change. In one way, it does. The atmosphere's capacity to hold greenhouse gases is an exhaustible resource (category e_1). While there is no discrete physical point when we "run out" of greenhouse gasses we can emit into the atmosphere, the risks of climate change have led us to (attempt to) self-impose such constraints. So, in the limit, no greenhouse gases can be emitted.[3] In fact, this is a more realistic description of the reason that fossil fuel use will be zero in a steady-state; levels of global warming will be catastrophic if humanity exhausts the supply of accessible fossil fuels. A related issue is that global temperatures themselves represent a non-rival good; this is omitted by our formulation that focuses only on rival environmental goods. Section 4.2 discusses an extension of the model where environmental health produces such non-rival benefits. The main results are unaffected by such an extension.

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[2]Note that this refers to the stock in use, not the newly mined value in t; in the long-run these minerals will be recycled between products if higher value applications become available.

[3]Technically, over long time horizons, greenhouse gases are naturally cycled out of the atmosphere. From this vantage the problem instead looks more like a renewable resource. However, with the extremely slow speed at which the atmosphere "regenerates" (i.e., cycles out CO_2, making room for more) it would remain true that a steady-state solution coincides with near-zero emissions of greenhouse gases.

2.2 Technology

The literature on endogenous economic growth highlights a positive relationship between population size and productivity (Jones and Romer, 2010). Larger populations generate more goods, some of which are non-rival. An increase in non-rival goods increases per capita variables, since they can be used/consumed by everyone, regardless of population size. Drawing on the semi-endogenous growth literature (Jones, 1995, 2022), we focus on the non-rival good that is knowledge and assume that the law of motion for total factor productivity is

$$\frac{\dot{A}}{A} = \alpha N^{\lambda} A^{-\beta} - \delta_A \tag{5}$$

where $\alpha, \lambda, \beta, \delta_A > 0$. To interpret this expression, consider first the case in which $\lambda = 1$ and $\beta = 0$. In this case, the accumulation of knowledge is proportional to the size of the population. This captures a simplistic model in which each person adds a constant amount to the stock of knowledge. The parameter δ_A governs the rate at which the stock of knowledge depreciates: unless people invest in knowledge preservation, ideas get forgotten or go unused.

When $\beta > 0$, we have a negative relationship between the rate of productivity growth and the level of productivity. If the stock of knowledge is already large, new ideas become harder to find. A larger β implies more decreasing returns to innovation due to this channel.

The parameter λ governs the extent to which population size affects the amount of innovation in each period. Specifying $\lambda > 1$ captures a situation in which innovation benefits from collaboration among more people. In contrast, $\lambda < 1$ captures a situation in which there are diminishing returns to R&D efforts. For example, if innovation happens through sequential discoveries, there may be diminishing returns to having more people work on discovering the same thing in the same period.

When population is constant at N, Equation 5 implies a steady-state for A.[4]

$$\bar{A}(N) = \left(\frac{\alpha N^{\lambda}}{\delta_A}\right)^{\frac{1}{\beta}} \tag{6}$$

Conceptually, this steady state might be thought of as the stock of knowledge that is sufficiently large that to even maintain, organize, and employ it commands all people-hours in this sector.[5] As the stock of knowledge gets unwieldy, the challenge is making use of existing knowledge, not

[4] Simply set $\dot{A} = 0$ and solve for A.

[5] This could be due to the sheer breadth of knowledge society acquires or the increased domain-expertise necessary to even contribute to organizing and preserving knowledge.

generating new ideas.

This interpretation is not crucial: any arbitrarily small δ_A generates this expression. We do not take a stance on whether this is a quantitatively significant force. Of course, the value of δ_A matters for the level of long-run per incomes. But it can be easily seen—by taking logs of Equation 6—that the relevant elasticity between N and A relies only on the ratio of λ to β. This conceptual introduction of δ_A is the minimal modification we make to the standard innovation equation that generates the analytical solution for the long-run relationship between populations and per capita incomes we study.

3 Main result

The goal of the current exercise is to determine the sign of the elasticity of GDP per-capita, y, with respect to population, N. Note that GDP per-capita is given by

$$\bar{y}(N) = \frac{\bar{Y}(N)}{N} = \frac{\bar{A}(N)F(N,K,E)}{N} \tag{7}$$

We can derive the elasticity in a few simple steps.

$$\begin{aligned}
\frac{\partial ln(\bar{y})}{\partial ln(N)} =& \frac{\partial ln(\bar{A})}{\partial ln(N)} + \frac{\partial ln(F(N,K,E))}{\partial ln(N)} + \frac{\partial ln(F(N,K,E))}{\partial ln(K)}\frac{\partial ln(K)}{\partial ln(N)} - \frac{\partial \ln(N)}{\partial \ln(N)} \\
=& \frac{\lambda}{\beta} + \frac{\frac{\partial F(N,K,\bar{E})}{\partial N}N}{F(N,K,\bar{E})} + \frac{\frac{\partial F(N,K,\bar{E})}{\partial K}K}{F(N,K,\bar{E})}\frac{\partial ln(K)}{\partial ln(N)} - 1 \\
=& \frac{\lambda}{\beta} + \phi_N + \phi_K\frac{\partial ln(K)}{\partial ln(N)} - 1
\end{aligned}$$

The first equality merely takes advantage of the additive nature of elasticities between the multiplicative A, F, and N (in the denominator). There is no term corresponding to a natural resource elasticity because the derivative of $\bar{E}$ with respect to N is zero in steady state. The second equality subs in what we know about how A responds to N from Equation 6 and replaces the other log-derivatives with more useful terms, which, in the third equality are replaced by ϕs. These ϕs have an economically meaningfully interpretation; they represent the share of income paid to the respective factors in competitive markets. To see this, note that $\bar{A}\frac{\partial F(N,K,\bar{E})}{\partial N}$ is the marginal product of labor. Thus, the term $\bar{A}\frac{\partial F(N,K,\bar{E})}{\partial N}\bar{N}$ is the aggregate payments to labor, and the ratio $(\frac{\partial F(N,K,\bar{E})}{\partial N}\bar{N})/F(N,K,\bar{E}) = (\bar{A}\frac{\partial F(N,K,\bar{E})}{\partial N}\bar{N})/(\bar{A}F(N,K,\bar{E}))$ is the share of these payments in output.

Now consider that under standard isoelastic utility functions, the savings rate s will be constant across different steady-state levels of output. Therefore, $\bar{K} = \frac{sY}{\delta_K}$, which implies that the elasticity of K with respect to N is equal to the elasticity of Y with respect to N. Equivalently $\frac{\partial lnK}{\partial lnN} = \frac{\partial lnY}{\partial lnN} = \frac{\partial lny}{\partial lnN} + 1$. Substituting this in we get:

$$\begin{aligned}\frac{\partial lny}{\partial lnN} =& \frac{\lambda}{\beta} + \phi_N + \phi_K \left[\frac{\partial lny}{\partial lnN} + 1\right] - 1 \\ =& \frac{\lambda}{\beta} + \phi_K \frac{\partial lny}{\partial lnN} - \phi_E \\ =& \frac{\frac{\lambda}{\beta} - \phi_E}{1 - \phi_K}.\end{aligned} \tag{8}$$

Moving from the first line to the second recognizes that under our CRS function $1-\phi_N-\phi_K = \phi_E$. The share of income going to E is the share of income not going to capital and labor.[6] The second equality comes from subtracting the remaining elasticity term and dividing out the $(1 - \phi_K)$.

The sign of this elasticity is determined by which of two terms—λ/β or ϕ_E—is larger. The inclusion of capital only serves to amplify the elasticity.[7] The first term represents how much more knowledge can be accumulated and productively used in the steady-state of this economy from a 1% increase in N. This is governed by the ratio of the intra-period returns to research effort, λ, and the degree to which knowledge becomes more difficult to accumulate as A increases, β. Recent work by Bloom et al. (2020) directly targets this parameter and estimates that $\frac{\lambda}{\beta} \in (0.2, 0.5)$ for the aggregate economy. Related analyses generalizing the analytical assumptions of Bloom et al. (2020) imply lower values of β, and hence larger values for $\frac{\lambda}{\beta}$ (Ekerdt and Wu, 2023). Peters (2022) leverages quasi-random population assignments after World War II and estimates a closely related term to be about 0.5.

Turning to ϕ_E, consider again Figure 1 which plots the time-series for this value. Our construction of ϕ_E is detailed in Appendix C, but we note here that we follow Caselli and Feyrer (2007) and Monge-Naranjo et al. (2019) who themselves follow the World Bank's *Changing Wealth of Na-*

[6]We recognize that in a world with increasing returns, all factors cannot in fact be paid their marginal product. In spite of this issue, it is common to retain this assumption when studying factor shares in other contexts. We follow that convention here. Formally, our model employs a learning-by-doing assumption rather than an endogenous choice to develop a new idea, so it is consistent within our framework to assign ideas zero income despite a positive marginal product. This would result in the rival factors earning their marginal products.

[7]The intuition for this is that the change in K will respond to the change in the marginal product of capital. If the innovation benefits exceed the natural resource costs, the marginal product of capital rises. In the opposite parametric case, the marginal product of capital falls when N rises, compounding the effect in the negative direction.

tions reports. Subsoil energy and minerals, timber resources and all agricultural land are included as the economically relevant natural resources earning non-trivial rents. The levels of ϕ_E in Figure 1 correspond closely to the values others in this literature report and use (see also e.g., Weil and Wilde, 2009; Hassler et al., 2021).

The value of ϕ_E that is relevant for Equation 8 is the long-run value it converges to in steady state. Historical values will, in general, be only partially informative about the long-run value. The special case where observed values of ϕ_E will be directly relevant without further assumptions is the case where the input share of E in the long-run is similar the input shares observed in our sample period. For example—because we have assumed that A is Hicks-neutral—if each of N, K, and E remain at their 2019 levels indefinitely, then the 2019 value of ϕ_E will be its long-run value. In this case the main result follows immediately: observed levels of ϕ_E are well below estimates of λ/β, so the elasticity of interest will be positive. However, if we expect factor input shares to change over the long run, ϕ_E may change as well.

To say something about this more general case, we will assume that $F()$ is a constant-elasticity of substitution production function of the following form:

$$Y = A\Big[ag(N,K)^{\rho} + (1-a)E^{\rho}\Big]^{\frac{1}{\rho}} \tag{9}$$

Here, $g(N, K)$ is a constant-returns to scale production function that combines capital and labor; $a \in (0, 1)$ is a parameter that governs the "importance" of capital and labor relative to natural resources; and ρ governs the elasticity of substitution between $g(N, K)$ and natural resources. We follow Hassler et al. (2021) by separating out E to focus on the elasticity of substitution for the input of interest. The case $\rho \to 0$ corresponds to the Cobb-Douglas production function, with $(1 - a)$ being the income share of natural resources.

Given this functional form, it holds that

$$\frac{\phi_E}{1-\phi_E} = \frac{\frac{\partial Y}{E}E}{\frac{\partial Y}{g(N,K)}g(N,K)} = \frac{E^{\rho}}{(g(N,K))^{\rho}} = \left(\frac{E}{(g(N,K))}\right)^{\rho}$$

Taking logs, we have that

$$\ln\left(\frac{\phi_E}{1-\phi_E}\right) = \rho \ln\left(\frac{E}{(g(N,K))}\right)$$

Using Δ to denote a change over time ($\Delta x = x_t - x_0$ for some variable x), it must hold that

$$\Delta \ln \left(\frac{\phi_E}{1 - \phi_E} \right) = \rho \Delta \ln \left(\frac{E}{(g(N, K))} \right) \tag{10}$$

This equation allows us to easily summarize nine relevant cases, depicted in Table 1. The middle row and middle column represent the cases where the observed values for ϕ_E are a good approximation for its long-run value. These are cases where the input shares remain roughly unchanged (middle row) or the function is Cobb-Douglas in E (middle column).

Table 1: Sign of population-income relationship as a function of parameters

	$\rho < 0$	$\rho \approx 0$	$\rho > 0$
$\bar{E}/g(\bar{N}, \bar{K})$ smaller than observed	$-$	$+$	$+$
$\bar{E}/g(\bar{N}, \bar{K})$ in observed range	$+$	$+$	$+$
$\bar{E}/g(\bar{N}, \bar{K})$ larger than observed	$+$	$+$	$-$

Notes: Cases for the long-run relationship between population and per capita income in a CES production function where the relative inputs evolve over time. Row definitions are imprecise: for "smaller" and "larger" implicitly we mean divergences from observed ratios large enough to plausibly flip the sign of interest (which will depend on ρ).

Outside of those cases, there are two (top right, bottom left) where the income share of natural resources falls in the long-run and two (top left, bottom right) where the income share grows. The income share of natural resources is already small, so the qualitative takeaways if it shrinks will be unchanged. The more interesting cases are the top left—where inputs are complementary and the input share of E grows in the long run—and the bottom right—where inputs are substitutable and the input share of E shrinks in the long run. These are cases where the current low values of ϕ_E may be misleading; we therefore need to distinguish which of these appears most likely based on the evolution of these respective terms over the medium term.

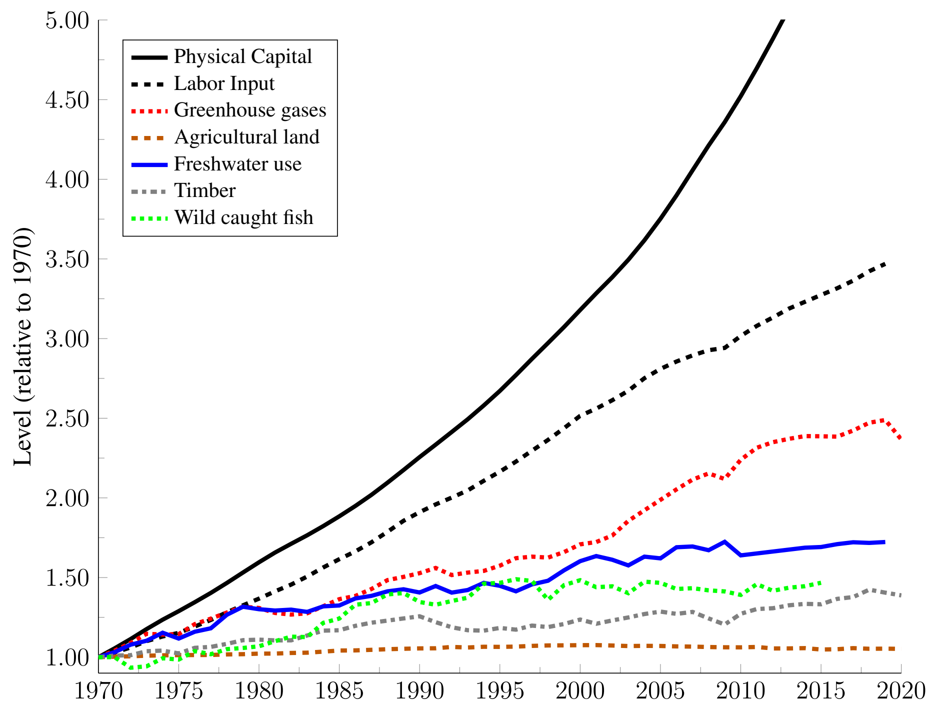
There is little we can say about the future of long-run input shares. Many observers believe that climate change or unsustainable withdrawal of resources implies that long-run levels of E will decline (see e.g., Dasgupta, 2021; Henderson et al., 2022). Additionally, if TFP grows substantially in the long-run, that will be a force that pushes up the capital stock, which reduces the input share of E. On the other hand, global low-fertility may imply that the future population sizes are much smaller than today (see e.g., Jones, 2022; Geruso and Spears, 2023). This would directly reduce N and indirectly reduce K. Which of these forces will dominate is a question of forecasts that are beyond the scope of this paper.

We can instead make progress by calibrating ρ. Equation 10 implies a value for ρ once the growth in income shares is compared to growth in factor inputs. Figure 1 displays no pervasive trend in the income share of natural resources. Based on this, we set

$$\Delta \ln \left(\frac{\phi_E}{1 - \phi_E} \right) = 0 \tag{11}$$

To calibrate ρ based on (10), it is also necessary to establish that $\Delta \ln \left(\frac{E}{g(N,K)} \right) \neq 0$ (otherwise, the parameter ρ is unidentified—in any constant returns to scale production function, when inputs increase by the same proportion then income shares remain unchanged).

Figure 2: Natural resource use does not tightly track human capital growth



Notes: Levels of resource inputs since 1970. Phyisical capital and the labor input are taken from the Penn World Tables 10.01 where labor input is an aggregate of hours provided and a human capital index (Feenstra et al., 2023). Natural resources are proxied by a range of indicators with well-documented use. Greenhouse gases are from aggregate fossil fuel use; agricultural land is the sum of crop and grazing land; freshwater measures agricultural, industrial and domestic use; timber is measured as roundwood (the pre-production measure of wood retrieved from forests); wild caught fish measures all fish production not from aquaculture.

Figure 2 plots the growth of N, K and various components of E since 1970. Both N and K have grown significantly faster than any of the E components plotted. Based on this figure, we conclude that

$$\Delta \ln(E) < \Delta \ln(N) < \Delta \ln(K)$$

As g is a constant-returns to scale production function, it holds that

$$\Delta \ln(E) < \Delta \ln(N) \leq \Delta \ln(g(N,K)) \leq \Delta ln(K)$$

and hence

$$\Delta \ln\left(\frac{E}{g(N,K)}\right) = \Delta \ln(E) - \Delta \ln(g(N,K)) \neq 0 \tag{12}$$

By (10), (11) and (12) it follows that $\rho = 0$, and hence

$$Y = (g(N,K))^a E^{1-a} \text{ and } \phi_E = 1 - a$$

In particular, the income share ϕ_E is independent of factor inputs. We can thus calibrate the long-run values of ϕ_E based on the current income share of natural resources.

Returning to Equation 8, we can combine the range of estimates for ϕ_E with the estimates from the innovation literature to put quantitative bounds on the elasticity of GDP per-capita with respect to population. Given a range of (0.2,0.5) for $\frac{\lambda}{\beta}$, a range of (0.02,0.08) for ϕ_E, and a range of (0.3,0.4) for ϕ_K, we have that

$$0.17 = \frac{0.2 - 0.08}{0.7} \leq \frac{\frac{\lambda}{\beta} - \phi_E}{1 - \phi_K} \leq \frac{0.5 - 0.02}{0.6} = 0.8.$$

Hence, combinations of off-the-shelf model ingredients and existing empirical estimates suggest that the elasticity of long-run income per-capita with respect to population is positive.

As documented in Table 1, if the CES function is not Cobb-Douglas ϕ_E may grow in the long-run and flip the sign of this relationship. Existing aggregate data makes it difficult to definitively rule out these cases. However, the central estimates for λ/β are nearly an order of magnitude larger than the central estimates of ϕ_E; growth in ϕ_E would need to be substantial to reverse the sign of this relationship.

4 Extensions

Here we consider two extensions to the model that could plausibly strengthen the Malthusian channel. First, the possibility of factor-augmenting technological change—in particular, technological progress biased away from natural resources—and second, non-rival *benefits* of ecosystem services, such that there is an additional channel by which the environment contributes to income.

4.1 Factor-Augmenting Technical Change

A more realistic version of this model might have factor-augmenting technological change. For example, endogenous directed technological change (e.g., Boserup, 1965; Acemoglu et al., 2012) may be an important mechanism for pinning down the quantitative elasticities. Indeed, in a series of recent papers Hassler et al. (2021, 2023) argue that this is a necessary mechanism to explain patterns related to fossil fuel use. In particular, Hassler et al. (2021) demonstrate that high degrees of complementarity over short time horizons can be consistent with flat long-run factor shares if technological change is E-augmenting (whether exogenously or endogenously). Our omission of this possibility has been for conservatism: the main result demonstrates that even without directing technological change towards increasingly tight resource constraints, the marginal relationship between population and per capita income is estimated to be positive. However, it is straightforward to generalize the main finding to an arbitrary degree of factor-augmenting technological improvements, including cases where non-directed change results in entirely N-augmenting improvements.

To show this, we must generalize the original production function one degree further. Let A be the level of TFP that is relevant for human-provided inputs N and K; A_E is the level of TFP relevant for environmental inputs.

$$Y = F(AN, AK, A_E E) \Rightarrow$$
$$y = \frac{AF\Big(N, K, a_E E\Big)}{N}, \quad \text{where } a_e \equiv \frac{A_E}{A}$$

The corresponding elasticity between population and per capita income becomes the following, where again ϕ_E equals the share of income going to the fixed factor.

$$\frac{\partial ln(\bar{y})}{\partial ln(N)} = \frac{\frac{\partial ln(\bar{A})}{\partial ln(N)} - \left(1 - \frac{\partial ln(a_E)}{\partial ln(N)}\right)\phi_E}{1 - \phi_K} \tag{13}$$

The relationship is intuitive and informative. If the elasticity of a_E with respect to N is one, then it's as if there has been no change in the fixed factor and the Malthusian channel drops from the expression: increasing N by 1% decreases $\frac{E}{N}$ by 1% (by construction), but increases a_E by 1% (by assumption), leaving the product of these terms unchanged. In the original case of Hicks-neutral technological change, a_E is unchanging, the elasticity equals zero, and this collapses to the baseline case in Equation 8.

In the case where technological progress operates disproportionately on N, K, a_E will decrease. This compounds the losses from $\frac{E}{N}$ decreasing. However, there is a limit on how severe the log-decline in a_E can be. This fraction can only decrease in proportion to the increase in A (since this is the denominator of a_E). Formally, $\frac{\partial ln(a_E)}{\partial ln(N)} \geq -\frac{\partial ln(\bar{A})}{\partial ln(N)}$. In this extreme case where *all* technological progress is N-,K-augmenting, we can write the numerator of (13) as:

$$(1 - \phi_E)\frac{\partial ln(\bar{A})}{\partial ln(N)} - \phi_E \tag{14}$$

The productivity-elasticity is now mitigated by $\phi_E\%$ before being compared to ϕ_E. Under the baseline values for these parameters this will not be quantitatively meaningful. Rather than comparing roughly 0.3 (λ/β) to 0.04 (ϕ_E), the comparison would be 0.29 to 0.04. In short, variants of factor-augmenting technical change can not themselves overturn the results, even under the most pessimistic version of this assumption.

4.2 Non-rival benefits from natural resources

Alongside the rival ecosystem services that earn profits—and are conceptually captured in our income-share terms—there may also be non-rival benefits of nature that increase with its stock. Consider a generalized production function building from Dasgupta (2021).[8]

$$Y = AB^{\xi}F(N, K, \bar{E}) \tag{15}$$

[8] See Dasgupta (2021) Chapter 4*.

Here B captures the general health (or stock) of the nature, what Dasgupta (2021) calls the biosphere. It performs regenerative services—such as filtering H_20 throughout the water cycle—and helps promote innovation and learning—such as plants in the Amazon that provide the ideas for new pharmaceuticals. If a broader view of Y beyond measured GDP is taken, B might be considered to contribute to Y via non-use values; everyone can simultaneously enjoy the mere existence of bio- and scenic-diversity.

What we have called E throughout the paper is the flow of ecosystem services being drawn from B. Previously the stock of B was only indirectly useful (in that it determines how much E can be drawn) so was left undiscussed. These non-rival benefits introduce a channel by which the stock itself is directly beneficial. This indirect channel changes the optimal level of E, as the flow of E will determine the level of B (see Appendix B). Previously, it would have been optimal to erode the biosphere until it reached its peak growth rate.[9] When B enters directly we have a competing incentive to keep B larger than where it reaches its regenerative peak.

However, the withdrawal rate in steady state remains independent of the population level. We therefore have that, in the long-run, the value of B is independent of population, as long as the rate of E is unaffected by population. Our analysis therefore carries through.

This discussion highlights a potential objection to the analysis: we assume a "sustainable" long-run. That is, we assume the long-run path is not one where natural resources are continuously eroded until (presumably) human extinction. If the size of the global population influences the probability that we successfully coordinate on a sustainable path, that may be a source of value omitted in this framing. This is an open and interesting question beyond the scope of this paper. Smaller populations may find coordination easier because there are fewer independent actors, but may find it harder if they are poorer or less technologically/institutionally capable (from the decline in non-rival goods of a smaller population). One feature of our analysis that makes this concern less pressing is that we have restricted our focus to marginal changes in population. The probability that we find ourselves on a margin where a small $\varepsilon\%$ increase in population makes the difference between extinction or not will be correspondingly small.

[9]For example, in an ocean with fish populations at their carrying capacity, there is no net-regeneration since populations cannot grow. Doing some fishing that reduces the stock in fact increases the rate of growth, and hence what can be drawn in a steady state.

5 Conclusion

The human population is projected to stop growing—and perhaps begin shrinking—during the lifetimes of children alive today. Whether it would be better to stabilize at higher population sizes depends on many things: transition dynamics, non-market goods, etc. This analysis focuses on one important aspect of this question: the effect on per-capita income.

Using a model that captures the most frequently discussed competing forces—that (i) nature imposes aggregate constraints and (ii) larger populations produce more non-rival goods—a simple analytical relationship between population size and long-run per-capita incomes arises. Calibrating the relevant parameters suggests this is relationship is positive. Insofar as non-rival goods and fixed natural resources are the primary channels by which populations influence per capita incomes, this result provides reason to believe that a future that stabilizes at a larger global population will be richer per capita than a world that stabilizes at a smaller population.

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Appendix

A Exhaustible resources and population size

Some observers are worried about the stock of exhaustible resources rather than the long-run withdrawal of renewable resources. Fossil fuels are foremost among these concerns. Exhaustible resources amount to a classic "cake-eating" problem where we can denote the initial stock as $\mathbb{E}$. For analytical simplicity, and consistent with the flat trend in factor shares here and in Hassler et al. (2021), assume a Cobb-Douglas specification for this exercise. A planner maximizes per capita income that is subject to diminishing returns through a CRRA parameter σ.

$$\max \sum_{t=0}^{\infty} \delta^t \frac{1}{1-\sigma} y_t^{1-\sigma} \tag{A1}$$

$$\text{with } \; y_t = A_t \left(\frac{E_t}{N}\right)^{1-a} \tag{A2}$$

$$\text{and } \; \sum_{t=0}^{\infty} E_t = \mathbb{E} \tag{A3}$$

Here the discount rate is denoted δ, which now matters as we are focused on environments without a steady state level of consumption to analyze. We have already established that $A \to \left(\frac{\alpha N^{\lambda}}{\delta_A}\right)^{\frac{1}{\beta}}$ independent of the environmental side of the model. For simplicity then assume A reaches this level reasonably quickly and is therefore fixed for the long-run study of this model. With a fixed A, N it can be easily shown that the solution is characterized by $\frac{E_t}{E_{t+1}} = \delta^{\frac{1}{1+\sigma a - a}}$. The level of environmental withdrawals remains independent of the population size.

Therefore, since withdrawal levels are independent of N, each period the effect of a 1% increase in N is a 1% decline in $\frac{E}{N}$ and a $\lambda \; \beta\%$ increase in A. This is the same as in the long-run steady state with a constant E, despite this being a case where long-run per capita incomes converge to zero (as A hits an upper limit, but E converges to zero).

B Details of renewable resource problem

As noted in the main text, Dasgupta (2021) argues that the entire biosphere, B, can be roughly conceptualized as renewable resource problem. If nature is undisturbed by human activity, most resources will regenerate.

Steady-state solutions to such problems are characterized by withdrawals of constant ecosystem services, $\bar{E}$, exactly equal to the amount of regeneration the renewable resource produces (which will be a function of its stock). Formally, assume that the biosphere has a regeneration function, $R(B)$, as in Equation A4, taken from Dasgupta (2021).

$$R(B) = rB\left[1 - \frac{B}{K}\right]\left[\frac{B-T}{K}\right] \tag{A4}$$

B is the stock of biosphere as it relates to human production/consumption and $R(B)$ is the amount of regeneration. K is the carrying capacity on this renewable resource—where the natural world would converge with minimal human interference. T is a "tipping point"—should we degrade the environment below T, regeneration becomes negative and the system collapses towards $B = 0$. r is the rate of regeneration in the absence of a tipping point or carrying capacity. The law of motion on B is then governed by the difference between regeneration, $R(B)$, and the amount of ecosystem services drawn for human production/consumption, E.

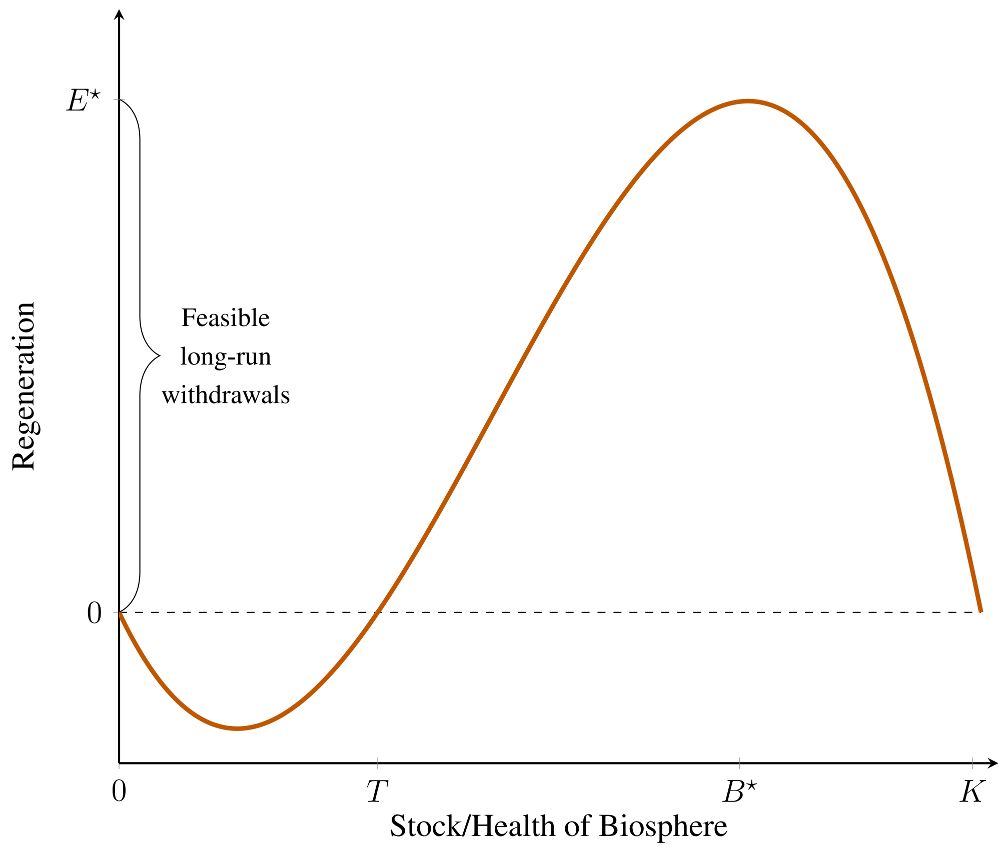
$$\dot{B}(B) = R(B) - E \tag{A5}$$

In a steady-state solution $\bar{E} = R(\bar{B})$. Notice that any level of $B > T$ can be consistent with a steady-state outcome. According to Equation A5, the steady-state level of B pins down the level of $\bar{E}$. In this model, there is a unique $B^\star$ that optimizes the regeneration rate and hence the value of $\bar{E}$.[10] Income is increasing in E, so the optimal solution in the baseline model is to manage B such that the peak of regeneration is reached. In the model of Section 4.2, there is an additional benefit from a higher level of B. Generally, this will increase $B^\star$ beyond the level that maximizes only E.

Nothing about this solution depends on the population size. The population size has no effect on regeneration rates, conditional on E, and it would be similarly inefficient for any population

[10]Notice that it is *not* the maximal B. Once B is at it's carrying capacity there is not net growth to drawn down, by definition.

Figure A1: Regeneration rate with tipping point



size to not manage B at the level that maximizes $R(B)$. Large and small populations alike face the challenge of intertemporal externalities. We abstract from the underlying details of this resource management problem and import a steady state solution, $\bar{E}$, of this independent subproblem into the aggregate production function.

C Data Appendix

C.1 Resource rents and useage

There exists public estimates on natural resource shares from the World Bank and the US Department of Agriculture. This leaves us only with the task of aggregating existing estimates. Figures on rents as a percent of GDP for timber/forests, minerals, coal, oil, and natural gas come from the "Adjusted Net Savings" dataset (updated 9/23/2022) from the World Bank's 2021 "Changing Wealth of Nations" report. These figures are provided on an annual basis and beginning in 1970. The dataset has missing years at the country-level but makes estimates at the global level in each year.

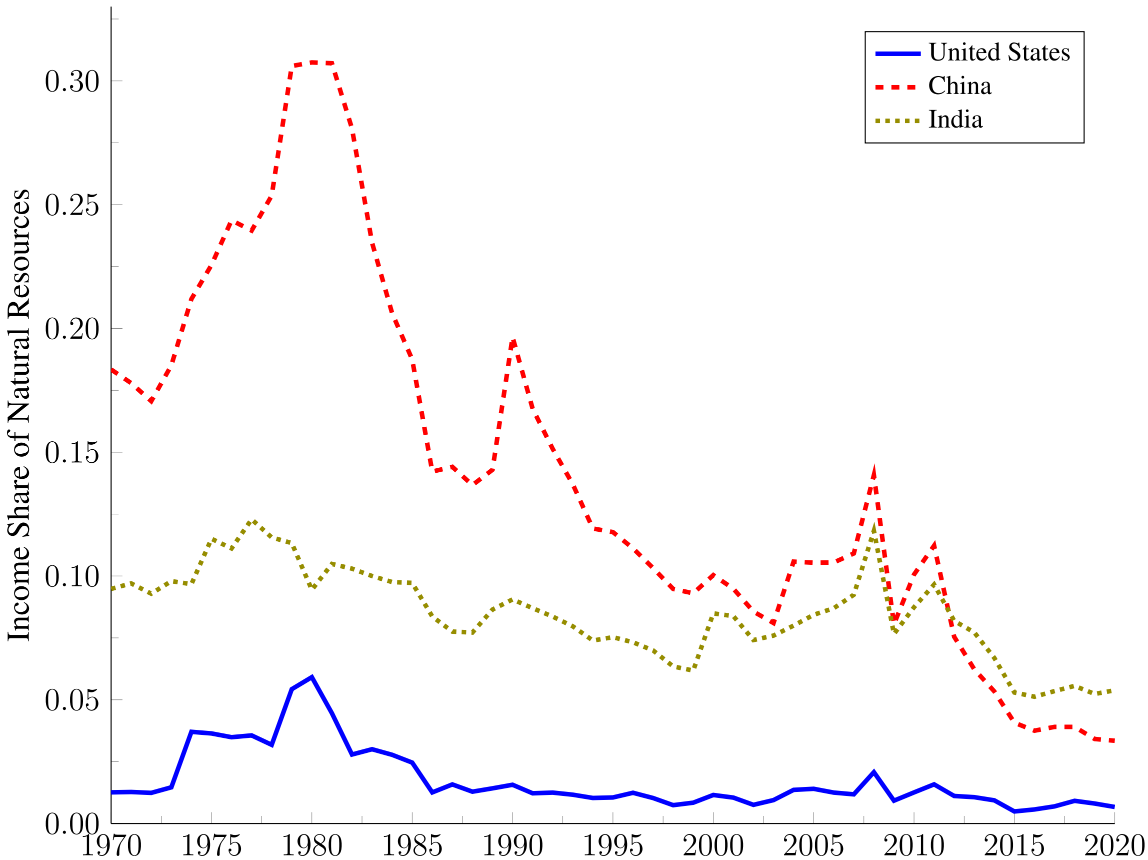
To determine the rents paid to agricultural land—inclusive of both crop and pasture applications—three data series are employed. First, FAOSTAT's "Value of Agricultural Production" (updated 11/15/2022) provides the total value of agricultural output in units of "Gross Production Value (constant 2014-2016 thousand US$)". We drop all animal products from the dataset except for cattle, sheep and goat products because the cost share due to land rents for animal products other than these 3 are small enough to be justifiably ignored. We then sum the total value of agricultural output for each country and at the global level.

Second, we need the share of total agricultural cost that is paid to agricultural land in each country and at the global level. The USDA's "International Agricultural Productivity" (updated 10/7/2022) can be leveraged here. Estimates of factor shares within agricultural production are provided for every decade from 1961-2020. Total agricultural revenues equal total agricultural costs inclusive of implicit land and capital rents. Therefore we can multiply the decadal factor shares for land by total annual agricultural revenue to get the total rents paid to agricultural land in each location-year.

Finally, we use the World Bank's GDP dataset (updated in constant 2015 US$ to be in the same units as agricultural land rents). We divide agricultural land rents by GDP to get the percent of GDP paid to all agricultural land. We then simply combine the World Bank rent estimates for timber and subsoil minerals with the agricultural land rents to get the total percent of GDP paid to the recorded natural resources.

Estimates of the factor share of agricultural land (in agricultural production) are between 20-30%, which are consistent with estimates exceeding 20% for land's factor share in agriculturally based societies (Weil and Wilde, 2009). Agricultural output is now a small share of global produc-

Figure A2: Different countries have similar long-run trend



Notes: Income share of resources by country. Both across and within countries levels of economic development (e.g., human and physical capital accumulation) the share of income going to natural resources shrinks. This suggests natural resources have been complements over the domain of economic development observed through 2020.

tion, partially giving rise to the small estimates in Figure 1.

Another reason for these low values is that we omit urban land from "land's" share of GDP. Urban land values are clearly tied to man-made structures and the people living on or near it. Put differently, humanity could choose to make more urban land, so it is fundamentally not a fixed factor.[11] It would be a mistake to use the high value commanded by urban land as a reflection of natural resources becoming scarce. If anything, this seems to be evidence for a desire (directly or indirectly) to have *more* nearby people.

In terms of resource use in Figure 2 we take data from various sources. Timber use is measured in pre-processed ("roundwood") and comes from the FAO's Forestry Production and Trade

[11] If the resources necessary to build cities were becoming scarce that would of course matter. But this is already reflected in non-urban land prices, mineral rents, and to some extent timber values that are captured in our methodology.

Database. The rest of the variables come from ourworldindata.org, which aggregates and hosts data from other sources.

C.2 Labor input and physical capital definitions

This data is taken with minimal modifications directly from the Penn World Tables v.10.01 (Feenstra et al., 2015, 2023). Physical capital is the variable *rnna* which is the real (in 2017 USD) national accounts recordings of physical capital (aggregated to the global level, of course). The labor input is the product of number of people employed (*emp*) $\times$ hours worked per person (*avh*) $\times$ a human capital index constructed from data on schooling (*hc*).

Not all countries have these data available for all years. Therefore, before aggregating across countries we restrict the sample to countries with non-missing data from all variables between 1970-2019. This ensures that the growth over the period in our global approximation is a (weighted) average growth rate across countries with reliable data, rather than a feature of more countries having the relevant data over time.