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Authors

Johan E. Gustafsson

Dean Spears

Stéphane Zuber

The University of Texas at Austin
Population Wellbeing Initiative

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Johan E. Gustafsson,* Dean Spears,† and Stéphane Zuber‡

Abstract

The expectation of a sum of utilities is a core criterion for evaluating policies and social welfare under variable population and social risk. Our contribution is to show that a previously unrecognized combination of weak assumptions yields general versions of this criterion, both in fixed-population and in variable-population settings. We show that two dimensions of weak dominance (over risk and individuals) characterize a social welfare function with two dimensions of additive separability. So social expected utility emerges merely from social statewise dominance (given other axioms). Moreover, additive utilitarianism, in the variable-population setting, arises from a new, weak form of individual stochastic dominance with two attractive properties: It only applies to lives certain to exist (so it does not compare life against non-existence), and it avoids prominent egalitarian objections to utilitarianism by only applying if certain correlations are preserved. Our result provides a foundation for evaluating climate change, growth, and depopulation.

Keywords: Social risk, variable population, utilitarianism.

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*University of Texas at Austin, USA. University of York, UK. IFFS, Sweden. E-mail: *johan.eric.gustafsson@gmail.com*.

†University of Texas at Austin, USA. IZA, Germany. IFFS, Sweden. GPI, Oxford University, UK. r.i.c.e., India. E-mail: *dspears@utexas.edu*.

‡Paris School of Economics (CNRS) and Centre d'Économie de la Sorbonne, France. E-mail: *Stephane.Zuber@univ-paris1.fr*.

1 Introduction

Policies that change future mortality rates (like climate mitigation) or change future fertility rates (like public education) not only change the quality of lives in the future but also who will live in the future. Humanity's global population has quadrupled over the past hundred years and is projected to peak then shrink within the lifetime of children born today — with uncertain consequences.[1] Hence, to evaluate economic policies, we need to assess both social risk and variable population. A standard principle for economic policy evaluation is Expected Total Utilitarianism, which maximizes the expected value of the sum of individuals' transformed lifetime well-being.[2] Despite the prominent use in public economics of both additive utilitarianism and expectation-taking under risk, these methods continue to be questioned in welfare economics, in part because existing axiomatic justifications make arguably strong assumptions (Fleurbaey, 2010; Golosov et al., 2007).

We provide a new axiomatic path to Expected Total Utilitarianism. Our result builds upon a new combination of weak assumptions that yields additive separability in the dimensions of states of the worlds and individuals.[3] Results are obtained both in the fixed-population setting of Harsanyi's foundational ag-

[1]For two recent overviews of economists' emerging understanding and open questions about low fertility, see Kearney et al. 2022 and Doepke et al. 2022. For possible macroeconomic implications of depopulation, see Jones 2022.

[2]For simplicity, we use the (strictly speaking) improper label "Expected Total Utilitarianism"throughout the text. What we have in mind is looking at the expected value of a sum of transformed utilities (where, in the variable case, the transformation implicitly includes setting a critical-level of utilities such that adding an individual to a population is good if and only if her utility is above that level). In a fixed-population case, and a different framework, Grant et al. (2010) named this approach *Generalized Utilitarianism.* In a variable-population framework, Blackorby et al. (1998) introduced this criterion under the label *Expected-Utility Critical-Level Generalized Utilitarianism.* Spears and Zuber (2023) used the label *Expected Critical-Level Generalized Utilitarianism.* A more proper label would be *Generalized Expectational Total Utilitarianism*

[3]An active current theoretical literature in welfare economics has been exploring additive separability over two dimensions, including Fleurbaey 2009, Mongin and Pivato 2015, McCarthy et al. 2020, Spears and Zuber 2023, and Li et al. 2023. We discuss related prior literature in more depth in our conclusion section.

gregation theorem for utilitarianism (Harsanyi, 1955), and in a variable-population setting. By comparing these two theorems, we show that the variable-population setting allows us to avoid contentious assumptions that the fixed-population setting requires to characterize Expected Total Utilitarianism.

We introduce a new, weakened version of individual stochastic dominance. Using it, our variable-population characterization offers several striking advantages over prior characterizations of Expected Total Utilitarianism in the literature:

- **Dominance axioms, not expectation-taking.** Our approach does not assume Expected Utility Theory either for society or for individuals. Instead, we show that two dimensions of weak dominance (one over risky states and the other over individuals) characterize a social welfare function with two dimensions of additive separability. So social expected utility emerges merely from social statewise dominance (in the context of our other axioms).

- **Additivity from individual stochastic dominance.** Moreover, generalized utilitarianism arises merely from our version of individual stochastic dominance, which is *only assumed to compare lives certain to exist*; this is crucial because it means that our core variable-population axiom does not compare life against non-existence.

- **Individual expected utility without assuming individual completeness.** Finally, without assuming complete individual preferences, we derive that the social order respects individual-level expected utility.

The new axiom that we introduce, called *Correlated Stochastic Dominance for Sure Individuals*, is at the core of our new results. To explain the axiom, we present two examples. The first example distinguishes Expected Total Utilitarianism from approaches to population ethics that are not additively separable. The second example distinguishes Expected Total Utilitarianism from fixed-population egalitarian criteria.

For the first example, consider Table 1, where columns are individuals, rows are equiprobable states, and for the purposes of this example, Ω represents an individual's non-existence in a state.[4]

Table 1: First motivating example: The only affected person is sure to exist in both prospects and is stochastically better-off in g^*

	prospect f^*		prospect g^*	
state	Ann	Bob	Ann	Bob
s_1	1	1	1	9
s_2	Ω	7	Ω	1

Our approach distinguishes between individuals who are sure to exist in any state and individuals who may or may not exist, depending on which state is realized. In Table 1's comparison of f^* and g^*, Ann is not sure to exist in either prospect. But she is altogether unaffected by the social choice of f^* or g^*: her utility conditional-on-existence does not differ between her prospects, nor her probability of existence, nor even the state in which she exists. Bob is the only person in Table 1 who is sure to exist. Given the equal probability of the two states, prospect g^* stochastically dominates prospect f^* for him. Our new principle says that, in this situation, g^* is better than f^*.

The starting point for our new principle is an intuition of *Individual Stochastic Dominance*: A prospect is better if it can be shown, without comparing existence to non-existence, to be better for somebody sure to exist and worse for nobody, where "better for" merely means in the sense of stochastic dominance, which is an incomplete ranking. In this case, that's g^*. Notice, however, that looking at the expected value of average utility, or the expected value of minimal utility, would each imply choosing f^* over g^* in Table 1. And so would a non-expectation-taking rule that evaluates outcomes according to the sum of utilities

[4]This example was first introduced in the philosophy literature, as a counterexample against using the expected value of average utility, by Gustafsson and Spears (2022); their informal paper does not include any characterization results. Gustafsson and Spears emphasize that their counterexample uses only *positive* lifetime utilities, so unlike other classic objections to Average Utilitarianism and to other non-separable approaches to population ethics, their counterexample does not depend upon a meaningful zero for utility nor upon the plausible existence of lives not worth living.

but then uses maximin for social risk, choosing the prospect with the highest least-socially-valuable state.[5] Therefore, the example in Table 1 differentiates each of these alternative approaches to variable-population social choice under risk from Expected Total Utilitarianism.

Table 2: Second motivating example: Equal individual prospects can yield unequal outcomes

	prospect f^{**}		prospect g^{**}	
state	Ann	Bob	Ann	Bob
s_1	1	1	1	0
s_2	0	0	0	1

Principles for social choice that evaluate each individual's *ex-ante* prospect — like Individual Stochastic Dominance does — are controversial in the normative literature, because of the implications for *ex-post* fairness (Myerson 1981; Broome 1991, p. 185; Parfit n.d., ch. 1; 1995, p. 17). So our main result uses a weaker axiom than Individual Stochastic Dominance.

To see why, consider Table 2, which is originally due to Myerson (1981). Continue to assume that the two states are equally probable. So both individuals face the same individual prospect in f^{**} and g^{**}. Accordingly, Individual Stochastic Dominance implies that those two prospects must be indifferent. But several authors have argued that society may prefer prospect f^{**} because it does not imply any inequality *ex post* (Fleurbaey, 2010). Our novel variable-population individual dominance axiom, however, avoids this implication and avoids the direct conflict with egalitarianism in Table 2. That is because our new principle — namely Correlated Stochastic Dominance for Sure Individuals — weakens stochastic dominance to only apply if states can be permuted in the same way for all sure-to-exist individuals. Therefore, this principle is consistent with egalitarian (or any other) choice in Table 2.

Our paper demonstrates that the axioms that can characterize Expected To-

[5] That is, $\min_s \left(\sum_i u_{is}\right)$. There is literature in decision theory proposing to maximize the minimal expected utility using a set of probability, in particular Gilboa and Schmeidler 1989.

tal Utilitarianism in a variable-population setting (which is the realistic setting for our variable-population world) are meaningfully weaker than the axioms that can do so in a fixed-population setting. Principally, this is because the variable-population setting can use Correlated Stochastic Dominance for Sure Individuals instead of Individual Stochastic Dominance, which the fixed-population setting requires. Additionally, the fixed-population setting requires an axiom (called *Compensation*) that can be dropped in our variable-population setting. Our paper demonstrates this by first proving a representation theorem in a fixed-population setting (our Theorem 1) and then proving a representation theorem in a variable-population setting (our Theorem 2). In this way, we apply the insight of Blackorby and Donaldson (1984), who showed that variable population provides an axiomatic path to utilitarianism that is distinct from Harsanyi's path using social risk (Harsanyi, 1955). More broadly, we demonstrate what can be achieved by combining Blackorby and Donaldson's approach with Harsanyi's result. Our result shows that Harsanyi's assumptions can be considerably weakened in a variable-population setting — which is also the setting that is pragmatically needed for economic policy assessments.

After presenting our main results, we show that they can be reinterpreted to apply to further economic settings and questions with two dimensions of value. Macroeconomists, for one example, canonically use time-separable social objective functions that add a value for each time period, which in turn is found by adding a period utility for each individual: $\sum_t \alpha_t \sum_i u_{it}$ for individuals i and times t. Although our main result considers risky states and individuals, our result can also be applied to time periods and individuals. So our results offer a new axiomatic justification for this practice in the macroeconomic literature. For example, we provide a microfoundation for Klenow, et al.'s (2022) recent growth accounting model that incorporates population growth while assuming a Total Utilitarian perspective: They conclude that, even though per-person living standards have improved radically, over the past decades population growth has accounted for even more of the vast improvements in aggregate well-being. Climate economics, such as Nordhaus' (2017) DICE and RICE models, also uses a version of this functional form for optimizing macroeconomic climate policy. We provide a new foundation for this standard tool in normative macroeconomics.

In another application, we show that our formal result can be reinterpreted to propose that a prudent individual making risky decisions about a life of unknown length and per-period utility should maximize the expected sum of per-period utility over time, $\sum_s \alpha_s \sum_t u_{st}$ for risky states s and time periods t. This application would exclude, for example, risk aversion over the length of the person's life.

2 Framework

Let $\mathbb{N}$ denote the set of positive integers, $\mathcal{N}$ the set of non-empty finite subsets of $\mathbb{N}$, $\mathbb{R}$ the set of real numbers, and $\mathbb{R}_{++}$ the set of positive real numbers. For a set D and any $n \in \mathbb{N}$, D^n is the n-fold Cartesian product of D. Also, for any two sets D and E, D^E denotes the set of mappings from E into D.

The set of *potential* individuals who may or may not exist is I. We will consider two cases: a fixed-population case where $I = \{1, \ldots, n\}$ for some finite n and individuals always exist; and a variable-population case where $I = \mathbb{N}$ and only a finite non-empty subset of individuals exist in any realized outcome. In the variable-population case, because $I = \mathbb{N}$, $\mathcal{N}$ is the set of all possible realized populations of individuals. That is, in any outcome, a population N exists: $N \in \mathcal{N}$ in the variable-population case and $N = I$ in the fixed-population case.

We consider a welfarist framework where the only information necessary for social decisions is the utility levels of people alive in a certain state of affairs. An outcome's welfare information is given by $u = (u_i)_{i \in N} \in \mathbb{R}^N$, where N is the population, and $u_i \in \mathbb{R}$ is, for each existing individual i, the lifetime utility experienced by i.

Although we represent lifetime utilities with real numbers, we do not require, as a property of the setting, that they be measured on a ratio or even interval scale. We require only that lifetime utilities be ordered and have the cardinality of the continuum, so that our use of $\mathbb{R}$ can be interpreted as a representation of this order; that they have a topology that will allow us to use an axiom of continuity in lifetime utilities for fixed, sure populations; and that we can meaningfully use an axiom that assumes the existence of a critical level for indifferently adding a life in at least some egalitarian situations.

We denote U the set of outcomes; the exact definition will be different in the fixed-population and variable-population cases. For each u, we denote $N(u)$ the set of individuals alive in u, and $n(u)$ the number of individuals alive in u. For two any outcomes u and v such that $N(u) \cap N(v) = \emptyset$ (that is, u and v are distributions of utility for two disjoint populations), we denote uv the outcome where the two populations are merged. Formally, it is the outcome w such that $N(w) = N(u) \cup N(v)$, $w_i = u_i$ for all $i \in N(u)$, and $w_j = u_j$ for all $j \in N(v)$.

We assume that it is not always known for sure what the final utility vector will be nor what set of individuals will exist. For simplicity we assume that there is a fixed, finite set of states of the world $S = \{1, \ldots, m\}$, with $m \geq 2$, where all states are equally probable so that each state has probability $\frac{1}{m}$. A "supplementary material" appendix discusses the extension to the more general case. Note that we are in a framework where probabilities are given and/or individuals have the same beliefs (an "objective" probability framework).

A social prospect f is a mapping from S to U. For $s \in S$, $f(s)$ is therefore the outcome induced by the prospect f in state s. We let $F = U^S$ be the set of functions from S to U. For an outcome $u \in U$, we abuse notation and also denote u the sure social prospect f such that $f(s) = u$ in all $s \in S$.

For any outcome $u \in U$, whenever $i \in N(u)$, $u_i \in \mathbb{R}$ denotes the utility of individual i. For a prospect $f \in F$, whenever $i \in N\big(f(s)\big)$, $f_i(s)$ denotes the utility of individual i in state of the world $s \in S$. For an individual $i \in I$ and a social prospect $f \in F$, we let $S_i(f) = \{s \in S | i \in N(f(s))\}$ be the set of states of the world where individual i exists.[6]

In the fixed-population case, we have $S_i(f) = S$ for all individuals and prospects. But this may not be case in the variable-population case: In that

[6]Notice that, like essentially all research in the traditions of Harsanyi (1955) and of Blackorby et al. (2005) — such as Broome 1991, Fleurbaey 2010, McCarthy et al. 2020, Spears and Zuber 2023, and Li et al. 2023 — our domain assumes that personal identity is such that it is meaningful to talk about the same person existing in different risky states of the world.

case a potential individual may not exist in some (or any) state of the world. In our variable-population case, we call any individual i, under any social prospect f such that $S_i(f) = S$, "necessary" under f, because if f is chosen then i certainly exists. Because we assume the complete domain $F = U^S$, our variable-population domain includes prospects with both necessary and non-necessary individuals.[7]

The task of our paper is to characterize a social preorder $\succsim$ on F. That $\succsim$ is a preorder means that it is a reflexive and transitive binary relation. In particular, the preorder $\succsim$ is not directly assumed by our axioms to be complete on F, although both of our theorems derive completeness on F from the combination of our axioms. Throughout the paper we assume completeness only on sure prospects, as stated in our first axiom:

Completeness for Sure Prospects For all $u, v \in U$, either $u \succsim v$, or $v \succsim u$, or both.

Completeness for Sure Prospects is not contentious within the literature for same-population cases. It is more contentious in the philosophical population-ethics literature. Completeness, in that case, would hold that populations with different sizes are always comparable. Some authors argue that variable-population completeness may not hold because we do not know the critical level for adding an additional life.[8] But approaches with incompleteness are typically subject to time-consistency problems or money pump arguments (Hammond, 1988; Gustafsson, 2022). Variable-population incompleteness would also have deeply unattractive practical and normative implications, such as that climate mitigation policy is not preferable to large global temperature increases, because different sets of people would exist.[9] Ordinary economic analysis and policy-making routinely (if

[7]Our axiom Correlated Stochastic Dominance for Sure Individuals distinguishes between necessary and non-necessary people, in a comparison of two social prospects. We do not model time explicitly, but some readers may find that it aids their intuition to interpret necessary individuals as those who are already alive (such as you and us), although this interpretation is not required by our formal framework.

[8]See the literature on using a wide range for the critical level: Blackorby et al. 1996, Rabinowicz 2009, and Gustafsson 2020.

[9]This observation is an application of Parfit's (1984, p. 362) *Depletion case* in the philo-

implicitly) assume that outcomes with different populations can be compared; we follow that tradition.

3 Fixed-population results

We assume in this section that the population is a fixed set of n individuals, $I = \{1, \ldots, n\}$ with $n \geq 3$. The set U of outcomes is $U = \mathbb{R}^I$. In a slight abuse of notation, it will sometimes be useful to consider subsets of I, which we will call N in this section, and to consider the utility distribution of the subpopulation within N that is an element of $\mathbb{R}^N$.

Our first results are based on two dominance principles, one for society and one for individuals. In our social dominance principle, the notation $f(s) \succsim g(s)$ means "if $\succsim$ faced a binary choice between the outcome of f in s occurring for sure (that is, in every state), or the outcome of g in s occurring for sure, then $\succsim$ would prefer the former to the latter."

Social Statewise Dominance For all $f, g \in F$, if $f(s) \succsim g(s)$ for all $s \in S$, then $f \succsim g$. If in addition there exists $s \in S$ such that $f(s) \succ g(s)$, then $f \succ g$.

Social Statewise Dominance is a very weak rationality principle for social decision making. It means that if we are sure that a social prospect would be better than another under any state, then we should prefer it.

For individuals, we will require a property slightly stronger than statewise dominance, namely stochastic dominance.[10]

sophical population-ethics literature.

[10]The definition of stochastic dominance used here may not be the most familiar one. Typically, in our framework we would say that individual prospect f_i first-order stochastically dominates individual prospect g_i if for all $z \in \mathbb{R}$ $|\{s \in S : f_i(s) \leq z\}| \leq |\{s \in S : g_i(s) \leq z\}|$ (with a strict dominance if the inequality is strict for some $z \in \mathbb{R}$). It can easily be verified that our definition is equivalent. We use this formulation because it is similar to that in our Correlated Stochastic Dominance for Sure Individuals principle below.

Individual Stochastic Dominance For all $f, g \in F$, if for each $i \in I$ there exists a bijection $\pi_i : S \to S$ such that $f_i(\pi_i(s)) \geq g_i(s)$ for all $s \in S$, then $f \succsim g$. If in addition there exists $j \in I$ and $s' \in S$ such that $f_j(\pi_j(s')) > g_j(s')$, then $f \succ g$.

Individual Stochastic Dominance can be interpreted as a weak *ex-ante* Pareto principle: If a prospect is better than another for all individuals (in the sense of stochastic dominance), then it is also socially better. In that sense, it is in the lineage of Harsanyi's foundational result on social aggregation under risk (Harsanyi, 1955). Note, however, that Individual Stochastic Dominance is weaker than the usual *ex-ante* principles for two reasons: because it is compatible with non-expected utility assessments of individual prospects, and because it only uses an incomplete ranking of individual prospects. An interpretation is that the social ranking needs not always respect individual preferences, but instead only respects a part of individual preferences: that part that is compatible with stochastic dominance (assuming that individual preferences at least respect this principle).[11]

Recall the conflict between Individual Stochastic Dominance Dominance and the egalitarian intuition behind Table 2. This conflict emerges, in fact, from the Anteriority axiom, which is weaker than Individual Stochastic Dominance and which says that the social preorder only depends on which prospect each individual faces, that is:

Anteriority For all $f, g \in F$, if for each $i \in I$ there exists a bijection $\pi_i : S \to S$ such that $f_i(\pi_i(s)) = g_i(s)$ for all $s \in S$, then $f \sim g$.

McCarthy et al. (2020) have argued that Anteriority expresses a weak sense in which the social preorder is *ex-ante*. So our characterization results can be seen as attractive to people endorsing a weak *ex-ante* view, or as additional arguments for people who resist that view.

[11] Many non-expected utility models of choice are compatible with first-order stochastic dominance in the context of risk. For instance, Chew and Epstein (1989) studied extensions of the rank-dependent expected utility model and conditions to obtain first-order stochastic dominance for a large class of models. Also, Tversky and Kahneman (1992) developed the cumulative prospect theory model to ensure compatibility with first-order stochastic dominance.

The first step is to show that those two dominance principles, together with Completeness for Sure Prospects, imply the following separability property for sure prospects:

Separability for Sure Prospects For any $N \subset I$, for any $u, v \in \mathbb{R}^N$ and $w, \hat{w} \in \mathbb{R}^{I \setminus N}$, $uw \succsim vw$ if and only if $u\hat{w} \succsim v\hat{w}$.

Lemma 1 *If the social ordering $\succsim$ satisfies Completeness for Sure Prospects, Social Statewise Dominance and Individual Stochastic Dominance, then it satisfies Separability for Sure Prospects.*

Proof. The proof is by contradiction. Assume that $N \subset I$, $u, v \in \mathbb{R}^N$ and $w, \hat{w} \in \mathbb{R}^{I \setminus N}$ are such that $uw \succsim vw$ but $v\hat{w} \succ u\hat{w}$. Consider the three following prospects f, g, and h (where each row gives the vector of utilities in a specific state of the world):

state	f	g	h
1	uw	vw	uw
2	$u\hat{w}$	$u\hat{w}$	$v\hat{w}$
3	uw	uw	uw
$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	uw	uw	uw

By Social Statewise Dominance, given that $uw \succsim vw$, we must have $f \succsim g$. By Social Statewise Dominance, given that $v\hat{w} \succ u\hat{w}$, we must have $h \succ f$. Hence, by transitivity, we should have $h \succ g$. But it is the case that for all $i \in N$ $g_i(1) = h_i(2)$, $g_i(2) = h_i(1)$, and $g_i(s) = h_i(s)$ for all $s \in \{3, \dots, m\}$, while $g_j(s') = h_j(s')$ for all $j \in (I \subset N)$ and $s' \in S$. So, Individual Stochastic Dominance requires $g \sim h$, a contradiction. Completeness for Sure Prospects implies that, if we do not have $v\hat{w} \succ u\hat{w}$, we must have $u\hat{w} \succsim v\hat{w}$. ■

Note that for this first result, we do not need the full force of Individual Stochastic Dominance, but only Anteriority.

Lemma 1 is already a big step towards additive separability, because we now have a strong separability condition. But to obtain our fixed-population main result, we need two additional technical properties.

Continuity For all $u \in U$, the sets $\{v \in U | u \succsim v\}$ and $\{v \in U | v \succsim u\}$ are closed.

Compensation For any $u, v \in U$ and $i \in I$, there exists $z \in \mathbb{R}$ such that, if $w \in U$ is defined by $w_i = z$ and $w_j = v_j$ for all $j \neq i$, then $u \sim w$.

Compensation means that we can compensate losses or gains of all but one individuals by adjusting the welfare level of the last individual.[12] Although Compensation may intuitively appear utilitarian, it is consistent with views that are sensitive to distribution, such as equally-distributed-equivalent egalitarianism $\left(\phi^{-1}\left(\frac{1}{n}\sum_i \phi(u_i)\right)\right)$ and rank-discounted generalized utilitarianism $\left(\sum_{[r]} \beta^r \phi(u_r)\right.$, where $[r]$ indicates rank from worst-off$\left.\right)$, if ϕ is an unbounded increasing transformation.

Theorem 1 *The following statements are equivalent:*

1. *The social preorder $\succsim$ satisfies Completeness for Sure Prospects, Social Statewise Dominance, Individual Stochastic Dominance, Continuity and Compensation.*

2. *$\succsim$ is a complete social order and there exist continuous, increasing and unbounded functions $\phi_i : \mathbb{R} \to \mathbb{R}$ such that:*

$$f \succsim g \Longleftrightarrow \sum_{s \in S} \tfrac{1}{m} \sum_{i \in I} \phi_i\big(f_i(s)\big) \geq \sum_{s \in S} \tfrac{1}{m} \sum_{i \in I} \phi_i\big(g_i(s)\big).$$

An impartiality axiom would replace ϕ_i with a shared ϕ.

The proof is in the Appendix. It has two steps that we describe here informally. The first step is to derive additive separability within a state (or for

[12] Such a property is sometimes named Solvability in the literature (see for instance Pivato and Tchouante, 2023).

sure prospects). It relies on Separability for Sure Prospects, using Lemma 1, and on the theorem by Debreu (1960) on additive representations. The second step is to construct the across-state additivity of social expected utility. Informally, this is done by combining the use of the additive formula within a state and Stochastic Dominance for Sure Individuals to move the consequences of other states all into one state. This is illustrated by the following two-by-two example (which disregards ϕ for illustration), where columns are individuals, rows are two equiprobable risky states (s_1 and s_2), and x, y, z, and w are real lifetime utilities:

$$\begin{array}{c} s_1 \\ s_2 \end{array} \begin{bmatrix} x & y \\ w & z \end{bmatrix} \sim \begin{bmatrix} x+y & 0 \\ 0 & w+z \end{bmatrix} \sim \begin{bmatrix} x+y & w+z \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} x+y+w+z & 0 \\ 0 & 0 \end{bmatrix}.$$

The first equivalence uses Social Statewise Dominance and the additive structure within states. The second equivalence uses Stochastic Dominance for Sure Individuals. The third equivalence again uses Social Statewise Dominance and the additive structure within states.

Notice that we can derive Theorem 1 with even weaker principles. As explained before, Lemma 1 only requires Anteriority. Similarly, our full proof only requires Anteriority and a Pareto-like property that is implied by Individual Stochastic Dominance (this is detailed in the proof). So Theorem 1 can be reformulated using Anteriority and this Pareto-like property in place of Individual Stochastic Dominance. However, we present Individual Stochastic Dominance because it foreshadows our variable-population theorem.

Finally, notice that here or in our variable-population result, if ϕ is concave, then our criterion would be an instance of "prioritarianism," which is the name for an additively-separable social welfare function which gives priority to worse-off individuals. A Pigou-Dalton axiom for transfers of lifetime utility would be sufficient for this curvature of ϕ.

4 Variable-population results

Theorem 1 is a powerful weakening of the Harsanyi approach. But fixed-population utilitarianism leaves open the question of how to expand to variable-population questions — which are the real-world questions of much actual economic policy decision-making. Blackorby et al. (2005) detail many variable-population social welfare functions (such as Average Utilitarianism or Number-Dampened Utilitarianism) that simplify to fixed-population utilitarianism in fixed-population cases. This section shows how social and individual dominance further narrow down the possibilities for utilitarianism in a variable-population setting. We show that our axioms imply a specific family of generalized utilitarianisms for variable-population cases, namely, Expected Total Utilitarianism that corresponds in the sure case (absent any risk) to the well-known Critical-Level Generalized Utilitarianism (henceforth CLGU, Blackorby et al., 2005).

Here we take advantage of the variable-population setting — which has its own "existence independence" route to additive separability, due to Blackorby and Donaldson (1984) — to weaken our assumptions. In particular, Individual Stochastic Dominance, in the fixed-population setting, is inconsistent with making the egalitarian choice that $f^{**} \succ g^{**}$ in Table 2, but the weaker axiom we use here is consistent with that choice. Moreover, in the fixed-population case, we use the Compensation principle, but this may not be obviously appealing from some ethical viewpoints. By moving to the variable-population case, we will be able to instead use the principles of Anonymity and Critical Level for Egalitarian Expansion, specified below.

In this section, the set of *potential* individuals who may or may not exist is $I = \mathbb{N}$. In an outcome, only a non empty finite population $N \in \mathcal{N}$ exists. We thus define $U = \bigcup_{N \in \mathcal{N}} \mathbb{R}^N$ as the set of possible outcomes when at least one individual exists. For each population $N \in \mathcal{N}$, we also denote $U_N = \{u \in U | N(u) = N\}$ the set of outcomes such that the population is N.

We adopt six principles that are properties of the social preorder $\succsim$. Completeness for Sure Prospects and Social Statewise Dominance are the same as in the previous sections. We next have three principles that we expect to be uncontroversial in the economics literature.

Anonymity for Sure Outcomes For all $u, v \in U$, if $n(u) = n(v)$ and there exists a bijection $\pi : N(u) \to N(v)$ such that $u_i = v_{\pi(i)}$ for all $i \in N(u)$, then $u \sim v$.

Same-Population Continuity for Sure Outcomes For all $N \in \mathcal{N}$, for all $u \in U_N$, the sets $\{v \in U_N | u \succsim v\}$ and $\{v \in U_N | v \succsim u\}$ are closed.

Critical Level for Egalitarian Expansion There exists $c \in \mathbb{R}$ such that, for any $N \in \mathcal{N}$ and $j \in (I \setminus N)$, if u and $v \in U$ are defined by $N(u) = N$, $N(v) = N \cup \{j\}$, $v_i = u_i = c$ for all $i \in N$ and $v_j = c$, then $u \sim v$.

Notice that these three axioms — Anonymity and Same-Population Continuity and Critical Level for Egalitarian Expansion — each only apply to comparisons among sure outcomes (we suppress this in the title of the critical level axiom for brevity).

Critical Level for Egalitarian Expansion asserts that there is a wellbeing level such that a one-person expansion of a population in which everyone is at that level is indifferent.[13] This weak axiom would be acceptable to many diverse variable-population social welfare functions named and studied in the literature, including average utilitarianism, total utilitarianism, maximin (Blackorby et al., 2005, p. 176), critical-level leximin (Blackorby et al., 2005, p. 169), number-dampened generalized utilitarianism, (Blackorby et al., 2005, p. 172), and rank-discounted critical-level generalized utilitarianism (Asheim and Zuber, 2014, p. 632). It is important to notice what this axiom does not assume. It does not assume that the critical-level is fixed, whatever the pre-existing population: It only applies to egalitarian populations with each lifetime utility at the critical level. So, it does not assume that a critical level always exists for each population.

The heart of our variable-population characterization is our stochastic dominance axiom for individuals: Correlated Stochastic Dominance for Sure Individuals. This axiom formalizes the principle behind our motivating example in Table

[13]Fleurbaey and Zuber (2015) and Spears and Zuber (2023) call this axiom simply "Egalitarian Expansion," but we use this name to emphasize that it also serves the role of our critical level axiom in characterizing CLGU.

1. To adapt Individual Stochastic Dominance to the variable-population setting, we apply the principle only to individuals who are sure to exist — like Bob is in Table 1's motivating example. Additionally, this axiom only applies when states of the world where not-sure-to-exist individuals exist and their utilities conditional on existence are left unchanged.[14]

Correlated Stochastic Dominance for Sure Individuals For all $f, g \in F$, if:

(i) $S_i(f) = S_i(g)$ for all $i \in I$;

(ii) for all $j \in I$ such that $S_j(f) \notin \{\emptyset, S\}$, there exists $x_j \in \mathbb{R}$ such that $f_j(s) = g_j(s) = x_j$ for all $s \in S_j(f)$;

(iii) there exists a bijection $\sigma : S \to S$ such that for all $k \in I$ such that $S_k(f) = S$ and all $s \in S$, $f_k\big(\sigma(s)\big) \geq g_k(s)$;

then $f \succsim g$.

If, in addition, there exists $l \in I$ such that $S_l(f) = S$ and $s' \in S$ such that $f_l\big(\sigma(s')\big) > g_l(s')$, then $f \succ g$.

This axiom, Correlated Stochastic Dominance for Sure Individuals, has three important features:

- In condition (i), individuals exist in the same states of the world in the two social prospects f and g being compared, which equivalently means that in each state of the world the populations existing with f and g are the same. The principle does not speak to situations with different populations in a state of the world.

- In condition (ii), people who do not exist for sure either do not exist at all, or they do not bear any risk and exist with the same level of utility in f and g. People who do not necessarily exist, in the comparison between f and g, are altogether unaffected.

[14]We thank Marcus Pivato for suggesting this formulation of Correlated Stochastic Dominance for Sure Individuals.

- For people who are sure to exist, the condition (iii) entails that the individual prospect they face in f stochastically dominates the one they face in g. But, in fact, condition (iii) is weaker than individual stochastic dominance because the same permutation of states σ is used for all individuals.

Notice, then, that Correlated Stochastic Dominance for Sure Individuals requires that $g^* \succ f^*$ in the example from Table 1 but it permits any ranking of f^{**} and g^{**} in Table 2, including the non-utilitarian judgment that $f^{**} \succ g^{**}$. We cannot conclude from Correlated Stochastic Dominance for Sure Individuals that f^{**} and g^{**} in Table 2 are socially equivalent, because to obtain dominance we need to use different permutations of states for Ann and Bob. Yet we can conclude for Table 1 that $g^* \succ f^*$ because only Bob exists for sure, so we can permute the outcome for Bob in states 1 and 2. These examples, therefore, distinguish Correlated Stochastic Dominance for Sure Individuals from Anteriority, because Anteriority would immediately imply the utilitarian judgement that $f^{**} \sim g^{**}$.[15] Indeed, although the representation in Theorem 2 implies that $f^{**} \sim g^{**}$, no one axiom used in our variable-population theorem individually requires this.

Our first result is that the restricted social ordering to sure prospects must be a CLGU social ordering. Fundamentally, we achieve additive separability from our axioms because, in our variable population setting, Social Statewise Dominance and Correlated Stochastic Dominance for Sure Individuals are sufficient to obtain the Separability property discussed before. With Critical Level for Egalitarian Expansion, they also imply that there exist a fixed critical level. The proof in the Appendix provides the details on how these properties deliver the Critical Level Generalized Utilitarian social ordering.

[15] Anteriority, as written above, is not defined for variable-population cases. So consider, further, a variable population extension of Anteriority which holds that two prospects are equally good if each potential person faces the same individual distribution of the probability of non existence and the same distribution of utility levels conditional on existence (McCarthy et al., 2020). Such an Anteriority axiom would both hold that Table 1's $g^* \succ f^*$, like Correlated Stochastic Dominance for Sure Individuals, and that Table 2's $f^{**} \sim g^{**}$. Such an Anteriority axiom is thus stronger than Stochastic Dominance for Sure Individuals.

Proposition 1 *If $\succsim$ satisfies Completeness for Sure Prospects, Anonymity for Sure Outcomes, Same-Population Continuity for Sure Outcomes, Critical Level for Egalitarian Expansion, Social Statewise Dominance, and Correlated Stochastic Dominance for Sure Individuals, then there exists a continuous and increasing function $\phi : \mathbb{R} \to \mathbb{R}$ and a number $c \in \mathbb{R}$ such that for all $u, v \in U$, $u \succsim v$ if and only if $\sum_{i \in N(u)} \big[\phi(u_i) - \phi(c)\big] \geq \sum_{i \in N(v)} \big[\phi(v_i) - \phi(c)\big]$.*

We can then state our main result for this section:

Theorem 2 *The following statements are equivalent:*

1. *The social preorder $\succsim$ satisfies Completeness for Sure Prospects, Anonymity for Sure Outcomes, Same-Population Continuity for Sure Outcomes, Critical Level for Egalitarian Expansion, Social Statewise Dominance and Correlated Stochastic Dominance for Sure Individuals.*

2. *$\succsim$ is a complete social order and there exists a continuous and increasing function $\phi : \mathbb{R} \to \mathbb{R}$ and a number $c \in \mathbb{R}$ such that for all $f, g \in F$, $f \succsim g$ if and only if*

$$\sum_{s \in S} \tfrac{1}{m} \left[\sum_{i \in N(f(s))} \big[\phi(f_i(s)) - \phi(c)\big] \right] \geq \sum_{s \in S} \tfrac{1}{m} \left[\sum_{i \in N(g(s))} \big[\phi(g_i(s)) - \phi(c)\big] \right].$$

The basic approach is to use the within-state additivity of CLGU to construct the across-state additivity of social expected utility.[16] Informally, this is done first by using CLGU to have a separate set of individuals with welfare different from c in each state of the world. Then we use Correlated Stochastic Dominance for Sure Individuals to move the consequences of other states all into one state. Then we can apply CLGU to get an additive formula. Consider the following example for

[16] An alternative version of Theorem 2 would substitute Extended Replication Invariance (Blackorby et al., 2005, p. 165) and Intermediate Existence of Critical Levels (Blackorby et al., 2005, p. 160) instead of Critical Level for Egalitarian Expansion, characterizing the same social welfare function.

intuition of the proof. There are four possible individuals (in columns) and two equiprobable states (in rows); x, y, z, and w are real lifetime utilities:

$$\begin{array}{c} s_1 \\ s_2 \end{array} \begin{bmatrix} x & y & \Omega & \Omega \\ \Omega & w & z & \Omega \end{bmatrix} \sim \begin{bmatrix} x & y & c & c \\ \Omega & \Omega & w & z \end{bmatrix} \sim \begin{bmatrix} x & y & w & z \\ \Omega & \Omega & c & c \end{bmatrix}.$$

The first equivalence uses CLGU from Proposition 1 in each state of the world (and then Social Statewise Dominance). The second equivalence uses Correlated Stochastic Dominance for Sure Individuals (the last two individuals). We can then use the additive formula of CLGU applied to the first state of the last prospect. The full proof is presented in the Appendix.

Notice that Correlated Stochastic Dominance for Sure Individuals is independent of the other axioms of Theorem 2, because the other axioms are each consistent with Expected Average Utilitarianism, but Correlated Stochastic Dominance for Sure Individuals is not. The next logical weakening of Correlated Stochastic Dominance for Sure Individuals would be to weaken stochastic dominance to statewise dominance, but this would not be sufficient for Theorem 2, which suggests that Correlated Stochastic Dominance for Sure Individuals may be the weakest axiom that can narrow variable-population utilitarianism to Expected Total Utilitarianism.

In this section, contrary to the previous one, we have assumed Anonymity, which is a widely admitted principle of social ethics. However, Anonymity may not make sense for other applications of our result (as discussed below). In an intertemporal setting, it has been argued that individuals living in different generations perhaps should not be treated symmetrically because there are permissible reasons to discount future utility. The core of our line of arguments however does not depend on Anonymity. We show in the Supplementary Material (Section S.A) that we obtain the expected value of a non-symmetric, additively-separable function when we replace Anonymity with Compensation.

5 Further applications

In this section, we note that our formal results can be usefully reinterpreted if the dimensions and utility-bearers are understood in different ways.[17] We give an example for macroeconomics and another for individual rational choice. Where our main setting uses risky states and individuals as the two dimensions, our applications below use, first, time periods and individuals and, second, time periods and risky states.

5.1 Macroeconomic welfare accounting with time separability: Time periods and individuals

Macroeconomists typically use a social welfare function that is additively separable across time periods and sums individual time-period-specific utility within time periods. This practice has two important implications: that individual lifetime utility is also additively time-separable, and that the implied population ethics is totalist. For example, the climate-economy model of Nordhaus (2017), like other leading climate-policy models, maximizes a social objective function $\sum_t \alpha_t \sum_i u_{it}$, for individuals i and periods t experiencing flow utility u_{it} — or more precisely $\sum_t \alpha_t L_t \bar{u}_t$, where L_t is population size and $\bar{u}_t$ is average wellbeing in t. Particularly relevantly to our paper, Klenow et al. (2022) use this functional form (without time discount factors α_t) to conduct a growth accounting exercise that decomposes aggregate growth into population growth and improvements in per-person living standards.

These conventions invite the question: How can this social objective function be normatively justified? Our Theorem 2 provides a justification, if cells are reinterpreted as individual-by-time flows of utility, risky states are reinterpreted as discrete time periods (ignoring risk for this application), and potential individuals have lives composed of a variable number of time periods.

[17] Mongin and Pivato (2015) have made a similar observation, in surveying multiple applications of their own result about two-dimensional separability, although their result and applications are different. A similar discussion can also be found in Li et al. 2023.

- Social Statewise Dominance axiom would become Social Period-wise Dominance, holding that an intertemporal allocation f is better than another g if each time period of f would be better, if made permanent, than the corresponding time period of g, if made permanent.

- Correlated Stochastic Dominance for Sure Individuals would become Temporal dominance for fixed-longevity individuals, holding that an intertemporal allocation f is better than another g if

  - every person who only lives for some (but not all) populated time periods is unaffected by a choice between f and g, and

  - every person who lives throughout the entire span of populated time has a lifelong distribution of period well-being in f that dominates that person's lifelong distribution of well-being in g.

These, combined with the technical axioms, would yield additivity across and within time periods.[18] So this result can justify Klenow, et al.'s (2022) Total Utilitarian growth accounting, with the same sort of weak axioms that justify our result.[19] To be sure, various intuitions (including a taste for pattern goods such as flat or increasing utility profiles over time) might lead an economist to reject Temporal dominance for fixed-longevity individuals, but such economists would already have rejected macroeconomists' entire time-separable project. This is formally analogous to, in our original social risk setting, a concern for egalitarianism that might bring about a rejection of utilitarianism and our axioms that characterize it.

[18]Pure social time preference could be accommodated by period weights which would be analogous to probabilities in our interpretation. Note that Blackorby et al. (1995), in an early contribution to population ethics, also derive additive separability from lives born at different times, but consider only lifetime utilities, not period-specific utility flows.

[19]In fact, because they compare time periods with other time periods, without integrating over time and without time discounting, our Proposition 1 is sufficient to justify their approach.

5.2 Individual decision-making for a lifetime of risky length and per-period utility: Time periods and risky states

Consider an individual's rational choice over a risky temporal distribution of state-specific period flows of utility, u_{st}, where s are risky states and t are time periods when the individual may or may not be alive and, if so, would experience a flow utility. Reinterpreting our model of social risky choice as a model of individual risky choice, with i in our model now becoming periods t in a life of unknown length, results in the decision criterion that maximizes the expectation of the sum of period-specific utility flows over a lifetime: $\sum_s \frac{1}{m} \sum_t u_{st}$.

- Social Statewise Dominance axiom would become Individual statewise dominance, but its interpretation would otherwise be similar to the interpretation of statewise dominance in our main setting, holding that a risky intertemporal allocation f is better than another g if each state of f would be better, if received for certain, than the corresponding state of g, if received for certain.

- Correlated Stochastic Dominance for Sure Individuals would become Stochastic dominance for fixed-longevity outcomes, holding that a risky intertemporal allocation f is better than another g if

  - every time period in which the decision-maker is not certain to live is unaffected by a choice between f and g, and
  - every time period in which the decision-maker is certain to live has a period-specific lottery of well-being in f that dominates that period's lottery of well-being in g.

This would be a novel justification of individual-level expected utility and of evaluating lifetime utility as the sum of period utility flows. As in the macroeconomic interpretation, the axioms rule out certain pattern goods. So whether or not this application makes normative sense for a prudent decision-maker may depend upon your interpretation of personal identity over a lifetime and whether lifetime pattern goods make sense and are valuable.

As indicated above, we can also dispense with the Anonymity assumption that, in the present context, would mean treating all time periods in the same way. So, we can allow for time discounting in individual decisions. Our key result is about expectation-taking and having time-separable preferences.

6 Discussion and conclusion

6.1 Extension to more general probability distributions

Until now, we have assumed that we have a finite number m of states of the world, all of them having the same probability $1/m$ to occur. In the Supplementary Material (Section S.B), we show that the results very easily extend to a case with events whose probability of occurrence is a rational number.

The main intuition is as follows. Consider two social prospects f and g and let d be the least common denominator of the probability of the events generated by f and g. It means that we can divide each events into subevents of probability $1/d$. And that the two prospects can be seen as inducing consequences on d equiprobable states of the world. Because we can apply all of our results to spaces where each state of the world has the same probability $1/d$ to occur, this extends to cases where events have a rational probability.

There are additional steps due to the fact that we must relate f and g to acts that induce the same partition into d equiprobable events. This relation is done by appropriately adapting the two key axioms of Individual Stochastic Dominance and Correlated Stochastic Dominance for Sure Individuals. All other axioms straightforwardly adapts to the more general framework.

6.2 Related literature

Our paper joins a recent literature that has characterized objective functions with two dimensions of value. A theme of this literature is that separability in one dimension creates pressure for separability in another. None of these papers connect axioms as weak as ours to a conclusion as strong as ours.

Harsanyi's (1955) aggregation theorem is recognized as a foundation of utilitarian welfare economics, which is widely used throughout macroeconomics and public economics. As Fleurbaey (2009) summarized, Harsanyi showed that "in the presence of risk, weighted utilitarianism is the only criterion that satisfies the ex-ante Pareto principle and can be written as the expected value of social welfare," where *ex-ante* Pareto, in Harsanyi's case, meant assuming complete individual expected utilities. Harsanyi's result has received much attention and has been weakened in several directions. Fleurbaey (2009), in a founding contribution to this recent literature, weakens Harsanyi's assumptions in a setting of fixed-population social risk. Fleurbaey uses a weak dominance axiom like ours for social risk, but maintains an assumption of expectation-taking for individual *ex-ante* Pareto. In an uncertainty framework à la Savage, without objective probabilities, Mongin and Pivato (2015) obtained the Harsanyi's result with assumptions akin to statewise dominance for the social ordering and *ex-ante* Pareto for individuals, without assuming that individuals maximize an expected utility. A similar result is obtained by Zuber (2016) in an uncertainty framework à la Anscombe–Aumann. Li et al. (2023) recall the generality of this result that applies also to the context of risk and time or time and individuals as explained above. One way that all of these axiomatizations are stronger than ours is in requiring an individual order, where our axiom for individuals requires only dominance; also we do not assume a complete social ordering of all prospects.

Another contribution is the paper by McCarthy et al. (2020). They consider a framework with objective probabilities and use the property of Anteriority, which is related to our properties of Individual Stochastic Dominance and Stochastic Dominance for Sure Individuals. They obtain a "quasi-utilitarian" result with axioms that are similar to ours. But it must be clarified that their result is not exactly utilitarian in the sense that we use here. What they get is that the society should evaluate social prospects as if one of the individuals in the society was facing an average prospect, in the sense that she faces the prospect of each individual with equal probability. To clarify the difference, assume that individuals assess prospects only on the basis of first order stochastic dominance (to be consistent with our axioms). Consider a society with two individuals and two prospects: in one prospect the two individuals get utility 1/2 for sure, in the other prospect one individual gets utility 1 for sure and the other gets 0 for sure. McCarthy et al. (2020) require that we assess these prospects like an individual would do if she compared a sure outcome of 1/2 with the lottery of having 0 or 1 with equal probability. Given that the individual uses first order stochastic dominance, these two prospects cannot be compared. On the contrary, our approach can compare them and will prefer the former to the latter if $\phi(1/2) > \frac{1}{2}\phi(0) + \frac{1}{2}\phi(1)$ — for instance when ϕ is concave. McCarthy et al. (2020) could obtain this result by further assuming that individuals maximize an expected utility — which we do not assume.

Harsanyi (1955) only considered a fixed-population case. We show that the axioms leading to Harsanyi's result can be significantly weakened in a variable-population setting. There exist other extensions of Harsanyi's to the variable population framework. A founding result is by Blackorby et al. (1998) but they assume social expected utility as well as some utility independence for unconcerned individuals (or individual-level expected utility). Other, more recent, papers do combine the logic of two dimensions with variable population. Spears and Zuber (2023), for example, extend Harsanyi's result to variable population, but maintain an assumption of social expected utility throughout. McCarthy et al. (2020), which we mentioned above, is a recent contribution with wide mathematical generality, including the variable-population case. Their variable-population results differ from ours in assuming an axiom that they call Omega Independence that contains a comparison of existence in a risky outcome to non-existence. As explained above, they also do not get a generalized utilitarian criterion in the sense that we use here. Finally, Thomas (2022) offers an overview of the relationship between separability and additivity for the philosophical population-ethics literature. Thomas makes use of the Anteriority axiom that we have discussed.

Any axiomatization of a social welfare function can be read as an argument for that approach or as a warning of what the approach entails, depending upon one's perspective. To a reader who shares the interpretation that the axioms of Theorem 2 are weak and normatively attractive, our result raises the theoretical cost of departing either from additively-separable utilitarianism or from social expectation-taking. Because, as we have shown, these axioms are weaker in a variable-population setting than in a fixed-population setting, the theoretical cost of departing from additively-separable utilitarianism or from social expectation-taking is greater in a variable-population setting than in a fixed-population setting. As the large changes over time in the size of the human population have shown, the relevant economic world is such a variable-population setting.

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A Proof of Theorem 1

Proof. It is straightforward to check that statement 2. implies statement 1. Then, the proof has two steps.

Step 1: an additive representation of $\succsim$ for sure prospects. By Completeness for Sure Prospects, we know that $\succsim$ is a complete pre-order for sure prospects. By Lemma 1, $\succsim$ satisfies Separability for Sure Prospects. By definition of $\succsim$ and Individual stochastic dominance, it is easily shown that $\succsim$ satisfies the following *Pareto-like property* (mentioned in the text on page 13):

for any $u, v \in \mathbb{R}^I$ if $u \geq v$ and $u \neq v$, then $u \succ v$,

where $\geq$ means at least as good for each person. Hence, by the well-known result of Debreu (1960), there exist continuous and increasing functions $\phi_i : \mathbb{R} \to \mathbb{R}$

such that, for all $u, v \in U$,

$$u \succsim v \Longleftrightarrow \sum_{i \in I} \phi_i(u_i) \geq \sum_{i \in I} \phi_i(v_i).$$

Without loss of generality, we can normalize the ϕ_i functions so that $\phi_i(0) = 0$ and $\sum_{i \in I} \phi_i(1) = 1$.

Let us show that each ϕ_i is unbounded. Assume by contradiction that ϕ_i is bounded above for some $i \in I$ (the reasoning is similar for the case where ϕ_i would be bounded below). Given that ϕ_i is increasing, it means that there exists $B \in \mathbb{R}$ such that, for any $\varepsilon > 0$ there exists $K \in \mathbb{R}$ such that, for any $z \geq K$, $0 < K - \phi_i(z) < \varepsilon$. As a consequence, for any $\varepsilon > 0$ there exists $K \in \mathbb{R}$ such that, for any $z \geq K$ and any $x \in \mathbb{R}$, $\phi_i(x) - \phi_i(z) < \varepsilon$. Now consider any $u, v \in U$ such that $u_j > v_j$ for some $j \neq i$ and $u_k = v_k$ for all $k \neq i, j$. Let $\varepsilon = \phi_j(u_j) - \phi_j(v_j) > 0$. So, by the reasoning above, there exists $z \in \mathbb{R}$ such that $\phi_i(x) - \phi_i(z) < \varepsilon$ for all $x \in \mathbb{R}$. Assume that $u_i = z$. By compensation, there exists $\tilde{z} \in \mathbb{R}$ such that, if $w \in U$ is defined by $w_j = v_j$ for all $j \neq i$ and $w_i = \tilde{z}$ then $u \sim w$. But, by the representation above, this would imply:

$$\phi_i(z) + \phi(u_j) = \phi_i(\tilde{z}) + \phi(v_j),$$

and therefore $\phi_i(z) - \phi_i(\tilde{z}) = \varepsilon$, which is impossible. Hence, it cannot be the case that ϕ_i is bounded above (nor bounded below by a similar reasoning).

Step 2: an expected utility representation. Consider any $f, g \in F$. Let us first construct $\hat{f}^{(1)}, \hat{g}^{(1)} \in F$ in the following way, using Compensation: $\hat{f}_1^{(1)}(1) = z^{(1)}$ and $\hat{g}_1^{(1)}(1) = \bar{z}^{(1)}$ while $\hat{f}_i^{(1)}(1) = \hat{g}_i^{(1)}(1) = 0$ for all $i \neq 1$, where $z^{(1)}$ and $\bar{z}^{(1)} \in \mathbb{R}$ are such that $\hat{f}^{(1)}(s) \sim f(s)$ and $\hat{g}^{(1)}(s) \sim g(s)$ (we know that such $z^{(1)}$ and $\bar{z}^{(1)}$ exist by Compensation). For each $s > 1$, $\hat{f}^{(1)}(s) = f(s)$ and $\hat{g}^{(1)}(s) = g(s)$ so that $\hat{f}^{(1)} \sim f$ and $\hat{g}^{(1)} \sim g$ by Social Statewise Dominance. By Step 1, it is the case that $\phi_1\left(\hat{f}_1^{(1)}(1)\right) = \sum_{i \in I} \phi_i(f_i(1))$ and $\phi_1\left(\hat{g}_1^{(1)}(1)\right) = \sum_{i \in I} \phi_i(g_i(1))$.

The next move is to construct two sequences of prospects $\hat{f}^{(1)}, \ldots, \hat{f}^{(m)}$ and $\hat{g}^{(1)}, \ldots, \hat{g}^{(m)}$ with the following properties:

- $\phi_1\Big(\hat{f}_1^{(k)}(1)\Big) = \sum_{s=1}^k \sum_{i\in I} \phi_i(f_i(s))$, $\phi_1\Big(\hat{g}_1^{(k)}(1)\Big) = \sum_{s=1}^k \sum_{i\in I} \phi_i(g_i(s))$, and $\hat{f}_i^{(k)}(1) = \hat{g}_i^{(k)}(1) = 0$ for all $i > 1$;

- $\hat{f}_i^{(k)}(s) = \hat{g}_i^{(k)}(s) = 0$ for all $i \in I$ and $2 \leq s \leq k$;

- $\hat{f}^{(k)}(s) = f(s)$ and $\hat{g}^{(k)}(s) = g(s)$ for all $s > k$;

- $\hat{f}^{(k+1)} \sim \hat{f}^{(k)}$ and $\hat{g}^{(k+1)} \sim \hat{g}^{(k)}$ for all $k = 1, \ldots, m-1$.

Let us show that the construction is possible by recursion. Notice that all the properties (except the last) are already satisfied by $\hat{f}^{(1)}$ and $\hat{g}^{(1)}$. Let $k \in \{1, \ldots, m-1\}$ and assume that we have constructed $\hat{f}^{(k)}$. Let us show that we can construct $\hat{f}^{(k+1)}$ with the desired properties so that $\hat{f}^{(k+1)} \sim \hat{f}^{(k)}$ (the proof is similar for $\hat{g}^{(1)}, \ldots, \hat{g}^{(m-1)}$, and thus not repeated).

By Compensation, there exists a number $\tilde{z}^{(k+1)} \in \mathbb{R}$ such that, if we define $\tilde{u}^{(k+1)} \in U$ by $\tilde{u}_2^{(k+1)} = \tilde{z}^{(k+1)}$ and $\tilde{u}_j^{(k+1)} = 0$ for all $j \in I \setminus \{2\}$, it is the case that $\tilde{u}^{(k+1)} \sim \hat{f}^{(k)}(k+1)$. By construction and step 1, it is the case that

$$\phi_2\left(\tilde{u}_2^{(k+1)}\right) = \sum_{i\in I} \phi_i\left(\hat{f}_i^{(k)}(k+1)\right) = \sum_{i\in I} \phi_i\left(f_i(k+1)\right). \tag{1}$$

Define $\tilde{f}^{(k+1)}$ by $\tilde{f}^{(k+1)}(k+1) = \tilde{u}^{(k+1)}$ and $\tilde{f}^{(k+1)}(s) = \hat{f}^{(k)}(s)$ for all $s \neq k+1$. Social Statewise Dominance gives $\tilde{f}^{(k+1)} \sim \hat{f}^{(k)}$. Next construct $\bar{f}^{(k+1)}$ in the following way: $\bar{f}_i^{(k+1)}(s) = \tilde{f}_i^{(k+1)}(s)$ for all $s \in S$ and $i \neq 2$; $\bar{f}_2^{(k+1)}(1) = \tilde{f}_2^{(k+1)}(k+1)$, $\bar{f}_2^{(k+1)}(k+1) = 0$, while $\bar{f}_2^{(k+1)}(s) = \tilde{f}_2^{(k+1)}(s)$ for all $s \neq 1, k+1$. Individual 2 faces the same individual prospect in $\bar{f}^{(k+1)}$ and $\tilde{f}^{(k+1)}$, while all other individuals are not affected. By Individual stochastic dominance, $\bar{f}^{(k+1)} \sim \tilde{f}^{(k+1)}$, and by transitivity $\bar{f}^{(k+1)} \sim \hat{f}^{(k)}$.

The prospect $\bar{f}^{(k+1)}$ is such that $\bar{f}_1^{(k+1)}(1) = \hat{f}_1^{(k)}(1)$, $\bar{f}_2^{(k+1)}(1) = \tilde{f}_2^{(k+1)}(k+1) = \tilde{u}_2^{(k+1)}$ and $\bar{f}_i^{(k+1)}(1) = 0$ for all $i > 2$. By Compensation, there exists a number $z^{(k+1)} \in \mathbb{R}$ such that, if we define $\bar{u}^{(k+1)} \in U$ by $\bar{u}_1^{(k+1)} = z^{(k+1)}$ and $\bar{u}_j^{(k+1)} = 0$ for all $j \in I \setminus \{1\}$, it is the case that $\bar{u}^{(k+1)} \sim \bar{f}^{(k+1)}(1)$. By construction and step 1, it is also the case that

$$\phi_1\left(\bar{u}_1^{(k+1)}\right) = \sum_{i\in I}\phi_i\left(\bar{f}^{(k+1)}(1)\right) = \phi_1\left(\hat{f}_1^{(k)}(1)\right) + \phi_2\left(\tilde{u}_2^{(k+1)}\right) = \sum_{s=1}^{k+1}\sum_{i\in I}\phi_i(f_i(s)).$$

(Recall that $\phi_1\left(\hat{f}_1^{(k)}(1)\right) = \sum_{s=1}^{k}\sum_{i\in I}\phi_i(f_i(s))$ and $\phi_2\left(\tilde{u}_2^{(k+1)}\right) = \sum_{i\in I}\phi_i\left(f_i(k+1)\right)$ – see Equation (1)). It suffices to define $\hat{f}^{(k+1)}$ by $\hat{f}^{(k+1)}(1) = \bar{u}^{(k+1)}$ and $\hat{f}^{(k+1)}(s) = \bar{f}^{(k)}(s)$ for all $s > 1$ to obtain $\bar{f}^{(k+1)} \sim \hat{f}^{(k+1)}$ by Social Statewise Dominance. By transitivity, $\hat{f}^{(k)} \sim \hat{f}^{(k+1)}$. It can be checked that $\hat{f}^{(k+1)}$ has all the aforementioned features. Figure 1 describes the step between $\hat{f}^{(k)}$ and $\hat{f}^{(k+1)}$.

Figure 1: Construction of prospect $\hat{f}^{(k+1)}$ for $k \geq 2$

	$\hat{f}^{(k)}$				$\tilde{f}^{(k+1)}$			
	indiviuals				individuals			
state	1	2	3	$\cdots$	1	2	3	$\cdots$
1	$\hat{f}_1^{(k)}(1)$	0	0	$\cdots$	$\hat{f}_1^{(k)}(1)$	0	0	$\cdots$
2	0	0	0	$\cdots$	0	0	0	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	0	0	0	$\cdots$	0	0	0	$\cdots$
$k+1$	$f_1(k+1)$	$f_2(k+1)$	$f_3(k+1)$	$\cdots$	0	$\tilde{f}_2^{(k+1)}(k+1)$	0	$\cdots$
$k+2$	$f_1(k+2)$	$f_2(k+2)$	$f_3(k+2)$	$\cdots$	$f_1(k+2)$	$f_2(k+2)$	$f_3(k+2)$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

	$\bar{f}^{(k+1)}$				$\hat{f}^{(k+1)}$			
	indiviuals				individuals			
state	1	2	3	$\cdots$	1	2	3	$\cdots$
1	$\hat{f}_1^{(k)}(1)$	$\tilde{f}_2^{(k+1)}(k+1)$	0	$\cdots$	$\hat{f}_1^{(k+1)}(1)$	0	0	$\cdots$
2	0	0	0	$\cdots$	0	0	0	$\cdots$
k	0	0	0	$\cdots$	0	0	0	$\cdots$
$k+1$	0	0	0	$\cdots$	0	0	0	$\cdots$
$k+2$	$f_1(k+2)$	$f_2(k+2)$	$f_3(k+2)$	$\cdots$	$f_1(k+2)$	$f_2(k+2)$	$f_3(k+2)$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

By our construction and transitivity, we have $f \sim \hat{f}^{(m)}$ and $g \sim \hat{g}^{(m)}$. But

$\hat{f}^{(m)}$ and $\hat{g}^{(m)}$ are such that $\hat{f}_i^{(m)}(s) = \hat{g}_i^{(m)}(s) = 0$ for all $i \in I$ and $s > 1$. By Social Statewise Dominance and Completeness for Sure Prospects, we know that $\hat{f}^{(m)} \succsim \hat{g}^{(m)} \iff \hat{f}^{(m)}(1) \succsim \hat{g}^{(m)}(1)$. By transitivity, we also have $f \succsim g \iff \hat{f}^{(m)}(1) \succsim \hat{g}^{(m)}(1)$.

Using Step 1 and the definition of $\hat{f}^{(m)}$ and $\hat{g}^{(m)}$ we get:

$$\begin{aligned} f \succsim g &\iff \hat{f}^{(m)}(1) \succsim \hat{g}^{(m)}(1) \\ &\iff \sum_{i \in I} \phi_i \left(\hat{f}_i^{(m)}(1) \right) \geq \sum_{i \in I} \phi_i \left(\hat{g}_i^{(m)}(1) \right) \\ &\iff \sum_{s \in S} \sum_{i \in I} \phi_i \left(f_i(s) \right) \geq \sum_{s \in S} \sum_{i \in I} \phi_i \left(g_i(s) \right) \\ &\iff \sum_{s \in S} \tfrac{1}{m} \sum_{i \in I} \phi_i \left(f_i(s) \right) \geq \sum_{s \in S} \tfrac{1}{m} \sum_{i \in I} \phi_i \left(g_i(s) \right). \end{aligned}$$

■

B Proof of Proposition 1

Proof. The proof has three steps.

Step 1: The social ordering $\succsim$ satisfies Separability for Sure Prospects. We first show that if $\succsim$ satisfies Completeness for Sure Prospects, Social Statewise Dominance and Correlated Stochastic Dominance for Sure Individuals, then it also satisfies Separability for Sure Prospects, stated below for the variable population case:

Separability for Sure Prospects For all $u, v, w, w' \in U$ such that $N(u) = N(v)$ and $N(u) \cap N(w) = N(u) \cap N(w') = \emptyset$, $uw \succsim vw$ if and only if $uw' \succsim vw'$.

Let us first show that this is the case when $N(w) \cap N(w') = \emptyset$. The proof is by contradiction and is similar to that of Lemma 1. It is obtained by considering the three prospects:

state	f	g	h
1	uw	vw	uw
2	uw'	uw'	vw'
3	u	u	u
$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	u	u	u

Assume for contradiction that $uw \succsim vw$ but $vw' \succ uw'$. By Social Statewise Dominance, and given that $uw \succsim vw$, $f \succsim g$. Similarly, given that $vw' \succ uw'$, $h \succ f$. So, by transitivity, $h \succ g$. But this violates Correlated Stochastic Dominance for Sure Individuals (The necessary people are those in $N(u)$). By Completeness for Sure Prospects, we must have $uw' \succsim vw'$.

Now, if it is not the case $N(w) \cap N(w') = \emptyset$, it suffices to take $\hat{w} \in U$ such that $N(\hat{w}) \cap N(w) = N(\hat{w}) \cap N(w') = \emptyset$. By the reasoning above, $uw \succsim vw \iff u\hat{w} \succsim v\hat{w} \iff uw' \succsim vw'$. Hence Separability for Sure Prospects must hold.

Step 2: A fixed Critical Level. We show that if $\succsim$ satisfies Completeness for Sure Prospects, Social Statewise Dominance, Correlated Stochastic Dominance for Sure Individuals, and Critical Level for Egalitarian Expansion, then it also satisfies Fixed Critical Level:

Fixed Critical Level There exists $c \in \mathbb{R}$ such that, for any $u \in U$ and $j \in (I \setminus N(u))$, if $v \in U$ is defined by $N(v) = N(u) \cup \{j\}$, $v_i = u_i$ for all $i \in N(u)$ and $v_j = c$, then $u \sim v$.

By Critical Level for Egalitarian Expansion, we already know that exists $c \in \mathbb{R}$ such that, if $\tilde{u} \in U$ is such that $N(\tilde{u}) = N(u)$ and $\tilde{u}_i = c$ for all $i \in N(u)$, and $\tilde{v} \in U$ is such that $N(\tilde{v}) = \{j\}$ and $\tilde{v}_i = c$, then $\tilde{u} \sim \tilde{u}\tilde{v}$. We want to prove that $u \sim u\tilde{v}$. The proof is obtained by considering the four prospects:

state	f	g	h	h'
1	u	$\tilde{u}$	$\tilde{u}\tilde{v}$	$u\tilde{v}$
2	$\tilde{u}\tilde{v}$	$u\tilde{v}$	$u\tilde{v}$	$\tilde{u}\tilde{v}$
3	$\tilde{u}$	$\tilde{u}$	$\tilde{u}\tilde{v}$	$\tilde{u}\tilde{v}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$\tilde{u}$	$\tilde{u}$	$\tilde{u}\tilde{v}$	$\tilde{u}\tilde{v}$

By Correlated Stochastic Dominance for Sure Individuals, $f \sim g$ (people in $N(u) = N(\tilde{u})$ are necessary). But given that $\tilde{u} \sim \tilde{u}\tilde{v}$, we have $g \sim h$ by Social Statewise Dominance. Then, $h \sim h'$ by Correlated Stochastic Dominance for Sure Individuals (all individuals are necessary). By transitivity, $f \sim h'$. But, given Completeness for Sure Prospects, this is possible only if $u \sim u\tilde{v}$ (otherwise we have a violation of Social Statewise Dominance, given that $\tilde{u} \sim \tilde{u}\tilde{v}$ in states $s \geq 3$).

Step 3: A characterization of Critical Level Generalized Utilitarian. Let us define formally the Same-Population Pareto for Sure Outcomes property.

Same-Population Pareto for Sure Outcomes For any $N \subset I$, for any $u, v \in \mathbb{R}^N$, if $u \geq v$ and $u \neq v$, then $u \succ v$.

In this Step, we prove the following result:

Proposition 2 *If $\succsim$ satisfies Completeness for Sure Prospects, Same-Population Continuity for Sure Outcomes, Same-Population Pareto for Sure Outcomes, Separability for Sure Prospects, and Fixed Critical Level, then there exists continuous and increasing functions $\phi_i : \mathbb{R} \to \mathbb{R}$ (one for each individual $i \in I$) and a number $c \in \mathbb{R}$ such that for all $u, v \in U$, $u \succsim v$ if and only if $\sum_{i \in N(u)} \left[\phi_i(u_i) - \phi_i(c)\right] \geq \sum_{i \in N(v)} \left[\phi_i(v_i) - \phi_i(c)\right]$.*

For $n \in \mathbb{N}$ with $n \geq 3$, denote $I_n = \{1, \ldots, n\}$ the set of the n first individuals and $\succsim_n$ the restriction of $\succsim$ to U_{I_n}. The relation $\succsim_n$ is a continuous complete preorder (by Completeness for Sure Prospects and Same-Population Continuity for Sure Outcomes) that satisfies Separability (for Sure Prospects and Same-Population) and Pareto for Sure Outcomes, which corresponds to the Pareto-like property defined in the proof of Theorem 1. So like in the proof of Theorem 1 (Step 1), we can show that there exist continuous and increasing functions $\phi_i^n : \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in U_{I_n}$,

$$u \succsim_n v \Longleftrightarrow \sum_{i \in I_n} \phi_i^n(u_i) \geq \sum_{i \in I_n} \phi_i^n(v_i).$$

Functions ϕ_i^n are unique up to a positive affine transformation (see Debreu, 1960). Without loss of generality, we can normalize the ϕ_i^n functions so that $\phi_i^n(0) = 0$ for all $i \in I_n$ and $\phi_1^n(1) = 1$ to obtain a unique representation.

Now consider any $u, v \in U_{I_n}$ and define $\tilde{u}, \tilde{v} \in U_{I_{n+1}}$ by $\tilde{u}_i = u_i$ and $\tilde{v}_i = v_i$ for all $i \in I_n$ and $\tilde{u}_{n+1} = \tilde{v}_{n+1} = c$, where c is the level in the Fixed Critical Level axiom. By Fixed Critical Level and transitivity, $u \succsim v \Longleftrightarrow \tilde{u} \succsim \tilde{v}$, which by the result above implies the equivalences:

$$u \succsim_n v \Longleftrightarrow \sum_{i \in I_{n+1}} \phi_i^{n+1}(\tilde{u}_i) \geq \sum_{i \in I_{n+1}} \phi_i^{n+1}(\tilde{v}_i) \Longleftrightarrow \sum_{i \in I_n} \phi_i^{n+1}(u_i) \geq \sum_{i \in I_n} \phi_i^{n+1}(v_i).$$

Given the unique representation of functions ϕ_i^n under the normalization $\phi_i^n(0) = 0$ for all $i \in I_n$ and $\phi_1^n(1) = 1$, we must have $\phi_i^n = \phi_i^{n+1}$ for all $i \in I_n$. By induction, we obtain that $\phi_n^m = \phi_n^n$ for all $m \geq n$ and $n \in \mathbb{N}$ with $n \geq 3$.

Define $\phi_1 = \phi_1^3$, $\phi_2 = \phi_2^3$ and $\phi_i = \phi_i^i$ for all $i \geq 3$. We can conclude that for any $n \in \mathbb{N}$, and any $u, v \in U_{I_n}$:[20]

$$\sum_{i \in I_n} \phi_i(u_i) \geq \sum_{i \in I_n} \phi_i(v_i).$$

Last consider any $u, v \in U$. Let $m = \max\Big\{\max\{i \in N(u)\}, \max\{j \in N(v)\}\Big\}$. Define $\tilde{u}, \tilde{v} \in U_{I_m}$ by $\tilde{u}_i = u_i$ for all $i \in N(u)$, $\tilde{v}_j = v_j$ for all $j \in N(v)$, $\tilde{u}_k = c$ for

[20]For $n \geq 3$, this results from the reasoning above. For $n = 1$, given that ϕ_1^3 is increasing, it is clear that for all $u, v \in U_{I_1}$, $u \succsim v \Longleftrightarrow u_1 \geq v_1 \Longleftrightarrow \phi_1^3(u_1) \geq \phi_1^3(v_1)$. For $n = 2$, we can use the argument built on Fixed Critical Level above — adding person 3 at level c — to show that, for any $u, v \in U_{I_2}$, $u \succsim v \Longleftrightarrow \sum_{i \in I_2} \phi_i^3(u_i) \geq \sum_{i \in I_2} \phi_i^3(v_i)$.

all $k \in (I_m \setminus N(u))$, and $\tilde{v}_l = c$ for all $l \in (I_m \setminus N(v))$. By Fixed Critical Level and transitivity, $u \succsim v \iff \tilde{u} \succsim \tilde{v}$, which by the representation result above implies the equivalences:[21]

$$\begin{aligned}
u \succsim v \iff & \sum_{i \in I_m} \phi_i(\tilde{u}_i) \geq \sum_{j \in I_m} \phi_j(\tilde{v}_j) \\
\iff & \sum_{i \in N(u)} \phi_i(u_i) + \sum_{k \in (I_m \setminus N(u))} \phi_k(c) \geq \sum_{j \in N(u)} \phi_j(v_j) + \sum_{l \in (I_m \setminus N(u))} \phi_l(c) \\
\iff & \sum_{i \in N(u)} \big[\phi_i(u_i) - \phi_i(c)\big] \geq \sum_{i \in N(v)} \big[\phi_i(v_i) - \phi_i(c)\big].
\end{aligned}$$

Step 4: Conclusion. Assume that $\succsim$ satisfies Completeness for Sure Prospects, Anonymity for Sure Outcomes, Same-Population Continuity for Sure Outcomes, Critical Level for Egalitarian Expansion, Social Statewise Dominance, and Correlated Stochastic Dominance for Sure Individuals. By Step 1, it implies that $\succsim$ satisfies Separability for Sure Prospects. By Step 2, it implies that $\succsim$ satisfies Fixed Critical Level. Like in the proof of Theorem 1, Social Statewise Dominance and Correlated Stochastic Dominance for Sure Individuals imply Same-Population Pareto for Sure Outcomes. Therefore, by Step 3, we know that there exists continuous and increasing functions $\phi_i : \mathbb{R} \to \mathbb{R}$ (one for each individual $i \in I$) and a number $c \in \mathbb{R}$ such that for all $u, v \in U$, $u \succsim v$ if and only if $\sum_{i \in N(u)} \big[\phi_i(u_i) - \phi_i(c)\big] \geq \sum_{i \in N(v)} \big[\phi_i(v_i) - \phi_i(c)\big]$. By Anonymity for Sure Outcomes, all the functions ϕ_i must be identical. ■

C Proof of Theorem 2

Proof. It is straightforward to check that Expected Total Utilitarianism satisfies all of our six principles.

Let us show that the six principles imply Expected Total Utilitarianism. By Proposition 1, we know that there exists a continuous and increasing function $\phi : \mathbb{R} \to \mathbb{R}$ and a number $c \in \mathbb{R}$ such that for all $u, v \in U$, $u \succsim v$ if and only if

[21] Between the second and third line, we subtract $\sum_{i \in I_m} \phi_i(c)$ from both sides of the inequality.

$\sum_{i\in N(u)}\big[\phi(u_i)-\phi(c)\big]\geq\sum_{i\in N(v)}\big[\phi(v_i)-\phi(c)\big]$.

Consider any social prospect $f\in F$. Let us construct the social prospect $\tilde f\in F$ with the following properties:

- There exists a collection of state-indexed populations $N^1,\ldots,N^m$ such that: (i) $|N^s|=|N(f(s))|$ for all $s\in S$; (ii) $N^{s'}\cap N^s=\emptyset$ for all $s'\neq s$;
- $N\left(\tilde f(1)\right)=\bigcup_{s'\in S}N^{s'}$;
- There exist bijections $\sigma^s:N^s\to N\big(f(s)\big)$ such that $\tilde f_i(1)=f_{\sigma^s(i)}(s)$ for all $i\in N^s$;
- When $s\in\{2,\ldots,m\}$, $N\left(\tilde f(s)\right)=N^m$ and $\tilde f_i(s)=c$ for all $i\in N^m$.

Social prospect $\tilde f$ is a prospect where all utility levels of all states of the world have been moved to state 1 (by creating new people), and all individuals have level c or do not exist in other states of the world. We want to show that $f\sim\tilde f$. Notice that, by the definition of $\tilde f$:

$$\begin{aligned}\sum_{i\in N(\tilde f(1))}\left[\phi\left(\tilde f_i(1)\right)-\phi(c)\right] &= \sum_{s\in S}\sum_{j\in N^s}\left[\phi\left(\tilde f_j(1)\right)-\phi(c)\right]\\ &= \sum_{s\in S}\sum_{j\in N^s}\Big[\phi\big(f_{\sigma^s(j)}(s)\big)-\phi(c)\Big]\\ &= \sum_{s\in S}\sum_{k\in N(f(s))}\Big[\phi\left(f_k(s)\right)-\phi(c)\Big].\end{aligned}$$

To show that $f\sim\tilde f$, let us construct two sequences of social prospects $(\hat f^{(1)},\ldots,\hat f^{(m)})$ and $(\tilde f^{(1)},\ldots,\tilde f^{(m)})$ in the following way.

We have $\hat f^{(1)}=\tilde f^{(1)}$, defined as follows: $N\left(\tilde f^{(1)}(1)\right)=\bigcup_{s'\in S}N^{s'}$, $\tilde f_i^{(1)}(1)=f_{\sigma^1(i)}(1)$ for all $i\in N^1$, and $\tilde f_j^{(1)}(1)=c$ for all $j\in\left(N\left(\tilde f^{(1)}(1)\right)\setminus N^1\right)$; for all $s\geq 2$, $N\left(\tilde f^{(1)}(s)\right)=N^s$ and $\tilde f_i^{(1)}(s)=f_{\sigma^1(i)}(s)$ for all $i\in N^s$.

For any $k\in\{2,\ldots,m\}$:

- $N\left(\hat f^{(k)}(1)\right)=N\left(\tilde f^{(k)}(1)\right)=\bigcup_{s'=1}^{k}N^{s'}$; $\hat f_i^{(k)}(1)=\tilde f_i^{(k)}(1)=f_{\sigma^s(i)}(s)$ for all $i\in N^s$ and $s<k$; $\hat f_j^{(k)}(1)=c$ and $\tilde f_j^{(k)}(1)=f_{\sigma^k(j)}(k)$ for all $j\in N^k$;

- For all $1 < s < k$, $N\left(\hat{f}^{(k)}(s)\right) = N\left(\tilde{f}^{(k)}(s)\right) = N^k$, and $\hat{f}_i^{(k)}(s) = \tilde{f}_i^{(k)}(s) = c$ for all $i \in N^k$;
- $N\left(\hat{f}^{(k)}(k)\right) = N\left(\tilde{f}^{(k)}(k)\right) = N^k$, $\hat{f}_i^{(k)}(k) = f_{\sigma^k(i)}(k)$ and $\tilde{f}_i^{(k)}(k) = c$ for all $i \in N^k$;
- For all $s > k$, $N\left(\hat{f}^{(k)}(s)\right) = N\left(\tilde{f}^{(k)}(s)\right) = N^k \cup N^s$, $\hat{f}_i^{(k)}(s) = \tilde{f}_i^{(k)}(s) = c$ for all $i \in N^k$, and $\hat{f}_j^{(k)}(s) = \tilde{f}_j^{(k)}(s) = f_{\sigma^s(j)}(s)$ for all $j \in N^s$.

Figure 2: Construction of prospects $\hat{f}^{(k)}$ and $\tilde{f}^{(k)}$ for $k \geq 2$. Like in the main text, Ω denotes non-existence, here applied to a group of persons.

$\hat{f}^{(k)}$

	Populations						
state	N^1	$\cdots$	N^{k-1}	N^k	N^{k+1}	N^{k+2}	$\cdots$
1	$(f_i(1))_{i\in N^1}$	$\cdots$	$(f_i(k-1))_{i\in N^{k-1}}$	c	Ω	Ω	$\cdots$
2	Ω	$\cdots$	Ω	c	Ω	Ω	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$k-1$	Ω	$\cdots$	Ω	c	Ω	Ω	$\cdots$
k	Ω	$\cdots$	Ω	$(f_i(k))_{i\in N^k}$	Ω	Ω	$\cdots$
$k+1$	Ω	$\cdots$	Ω	c	$(f_i(k+1))_{i\in N^{k+1}}$	Ω	$\cdots$
$k+2$	Ω	$\cdots$	Ω	c	Ω	$(f_i(k+2))_{i\in N^{k+2}}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\tilde{f}^{(k)}$

	Populations						
state	N^1	$\cdots$	N^{k-1}	N^k	N^{k+1}	N^{k+2}	$\cdots$
1	$(f_i(1))_{i\in N^1}$	$\cdots$	$(f_i(k-1))_{i\in N^{k-1}}$	$(f_i(k))_{i\in N^k}$	Ω	Ω	$\cdots$
2	Ω	$\cdots$	Ω	c	Ω	Ω	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$k-1$	Ω	$\cdots$	Ω	c	Ω	Ω	$\cdots$
k	Ω	$\cdots$	Ω	c	Ω	Ω	$\cdots$
$k+1$	Ω	$\cdots$	Ω	c	$(f_i(k+1))_{i\in N^{k+1}}$	Ω	$\cdots$
$k+2$	Ω	$\cdots$	Ω	c	Ω	$(f_i(k+2))_{i\in N^{k+2}}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Figure 2 illustrates those social prospects.

By Social Statewise Dominance, $\tilde{f}^{(k)} \sim \hat{f}^{(k+1)}$ for any $k \in \{1, \ldots, m-1\}$; indeed, $\tilde{f}^{(k)}$ and $\hat{f}^{(k+1)}$ differ only in each state of the world (except state 1 where they are identical) by the existence of people with utility level c. By Proposition

1 and CLGU with critical-level c, they are thus equivalent in each state of the world. On the other hand, $\tilde{f}^{(k)} \sim \hat{f}^{(k)}$ for any $k \in S$ by Correlated Stochastic Dominance for Sure Individuals. Indeed, the necessary people are in N^k and their utility is permuted from state 1 to state k, so that they face the same prospect.

We thus obtain the chain of equivalences $\tilde{f}^1 \sim \hat{f}^{(2)} \sim \tilde{f}^{(2)} \sim \cdots \sim \tilde{f}^{(m-1)} \sim \hat{f}^{(m)} \sim \tilde{f}$. In addition, $f \sim \tilde{f}^{(1)}$ by Social Statewise Dominance (they differ only in each state of the world by the set of people with certain utility levels, and/or a number of people at critical-level c). So, by transitivity $f \sim \tilde{f}$.

Consider any f and $g \in F$. By the arguments above, there exist $\tilde{f}$ and $\tilde{g}$ such that (where ϕ and c are given by Proposition 1):

- $f \sim \tilde{f}$ and $g \sim \tilde{g}$;
- $\sum_{i \in N(\tilde{f}(1))} \left[\phi\left(\tilde{f}_i(1)\right) - \phi(c)\right] = \sum_{s \in S} \sum_{j \in N(f(s))} \left[\phi\left(f_j(s)\right) - \phi(c)\right]$;
- $\sum_{i \in N(\tilde{g}(1))} \left[\phi\left(\tilde{g}_i(1)\right) - \phi(c)\right] = \sum_{s \in S} \sum_{j \in N(g(s))} \left[\phi\left(g_j(s)\right) - \phi(c)\right]$; and
- for all $s \in \{2, \ldots, m\}$, $\tilde{f}_i(s) = c$ for all $i \in N(\tilde{f}(s))$ and $\tilde{g}_j(s) = c$ for all $j \in N(\tilde{g}(s))$.

By Proposition 1, $\tilde{f}(s) \sim \tilde{g}(s)$ for all $s \in \{2, \ldots, m\}$, so that, by Social Statewise Dominance and Completeness for Sure Prospects $\tilde{f} \succsim \tilde{g} \iff \tilde{f}(1) \succsim \tilde{g}(1)$. Gathering all the results, we obtain:

$$\begin{aligned}
f \succsim g &\iff \tilde{f} \succsim \tilde{g} \\
&\iff \tilde{f}(1) \succsim \tilde{g}(1) \\
&\iff \sum_{i \in N(\tilde{f}(1))} \left[\phi\left(\tilde{f}_i(1)\right) - \phi(c)\right] \geq \sum_{j \in N(\tilde{g}(1))} \left[\phi\left(\tilde{g}_j(1)\right) - \phi(c)\right] \\
&\iff \sum_{s \in S} \sum_{i \in N(f(s))} \left[\phi\left(f_i(s)\right) - \phi(c)\right] \geq \sum_{s \in S} \sum_{j \in N(g(s))} \left[\phi\left(g_j(s)\right) - \phi(c)\right] \\
&\iff \sum_{s \in S} \tfrac{1}{m} \left[\sum_{i \in N(f(s))} \left[\phi\left(f_i(s)\right) - \phi(c)\right]\right] \geq \sum_{s \in S} \tfrac{1}{m} \left[\sum_{j \in N(g(s))} \left[\phi\left(g_j(s)\right) - \phi(c)\right]\right].
\end{aligned}$$

■

References

Asheim, Geir B and Stéphane Zuber. 2014. "Escaping the repugnant conclusion: Rank-discounted utilitarianism with variable population." *Theoretical Economics*, 9(3): 629–650.

Blackorby, C. and D. Donaldson. 1984. "Social Criteria for Evaluating Population Change." *Journal of Public Economics*, 25: 13–33.

Blackorby, C., W. Bossert, and D. Donaldson. 1995. "Intertemporal Population ethics: Critical-level utilitarian principles." *Econometrica*, 63: 1303–1320.

——— 1996. "Quasi-Orderings and Population Ethics." *Soc Choice Welfare*, 13: 129–151.

——— 1998. "Uncertainty and Critical-Level Population Principles." *Journal of Population Economics*, 11: 1–20.

——— 2005. *Population Issues in Social Choice Theory, Welfare Economics, and Ethics*: Cambridge University Press.

Broome, J.. 1991. *Weighing Goods: Equality, Uncertainty and Time*, Oxford: Blackwell.

Chew, S.H. and L.G. Epstein. 1989. "A Unifying Approach to Axiomatic Non-expected Utility Theories." *Journal of Economic Theory*, 49: 207–240.

Debreu, G.. 1960. "Topological Methods in Cardinal Utility Theory." in *Mathematical Methods in the Social Sciences 1959*: 16–26: Stanford University Press.

Doepke, M., A. Hannusch, F. Kindermann, and M. Tertilt. 2022. "The Economics of Fertility: A New Era." Working Paper w29948, National Bureau of Economic Research.

Fleurbaey, M.. 2009. "Two Variants of Harsanyi's Aggregation Theorem." *Economics Letters*, 105: 300–302.

——— 2010. “Assessing Risky Social Situations.” *Journal of Political Economy*, 118: 649–680.

Fleurbaey, Marc and Stéphane Zuber. 2015. “Discounting, beyond utilitarianism.” *Economics*, 9(1): 20150012.

Gilboa, I. and D. Schmeidler. 1989. “Maxmin Expected Utility with Non-Unique Prior.” *J. Math. Econ.*, 18: 141–153.

Golosov, M., L.E. Jones, and M. Tertilt. 2007. “Efficiency with Endogenous Population Growth.” *Econometrica*, 75(4): 1039–1071.

Grant, Simon, Atsushi Kajii, Ben Polak, and Zvi Safra. 2010. “Generalized utilitarianism and Harsanyi’s impartial observer theorem.” *Econometrica*, 78(6): 1939–1971.

Gustafsson, J. E.. 2020. “Population Axiology and the Possibility of a Fourth Category of Absolute Value.” *Economics and Philosophy*, 36(1): 81–110.

——— 2022. *Money-Pump Arguments*, Cambridge: Cambridge University Press.

Hammond, P. J.. 1988. “Consequentialist Foundations for Expected Utility.” *Theory and Decision*, 25(1): 25–78.

Harsanyi, J. C.. 1955. “Cardinal Welfare, Individualistic Ethics and Interpersonal Comparisons of Utility.” *Journal of Political Economy*, 63: 309–321.

Jones, C. I. 2022. “The end of economic growth? Unintended consequences of a declining population.” *American Economic Review*, 112(11): 3489–3527.

Kearney, M.S., P.B. Levine, and L. Pardue. 2022. “The Puzzle of Falling US Birth Rates since the Great Recession.” *Journal of Economic Perspectives*, 36(1): 151–76.

Klenow, P., C. Jones, M. Bils, and M. Adhami. 2022. "Population and Welfare: The Greatest Good for the Greatest Number." Stanford University working paper.

Li, C., K.I.M. Rhodes, and P. Wakker. 2023. "The Deceptive Beauty of Monotonicity, and the Million-Dollar Question: Row-First or Column-First Aggregation?". Erasmus School of Economics working paper.

McCarthy, D., K. Mikkola, and T. Thomas. 2020. "Utilitarianism with and without Expected Utility." *Journal of Mathematical Economics*, 87: 77–113.

Mongin, P. and M. Pivato. 2015. "Ranking Multidimensional Alternatives and Uncertain Prospects." *Journal of Economic Theory*, 157: 146–171.

Myerson, R.B.. 1981. "Utilitarianism, Egalitarianism, and the Timing Effect in Social Choice Problems." *Econometrica*, 49(4): 883–897.

Nordhaus, W.D.. 2017. "Revisiting the Social Cost of Carbon." *Proceedings of the National Academy of Sciences*, 114(7): 1518–1523.

Parfit, D.. 1984. *Reasons and Persons*, Oxford: Clarendon Press.

——— 1995. *Equality or Priority?*, Lawrence: University of Kansas.

——— n.d.. "On Giving Priority to the Worse Off." unpublished manuscript.

Pivato, M. and E. F. Tchouante. 2023. "Bayesian Social Aggregation with Non-Archimedean Utilities and Probabilities." *Economic Theory*, forthcoming.

Rabinowicz, W.. 2009. "Broome and the Intuition of Neutrality." *Philosophical Issues*, 19(1): 389–411.

Spears, D. and S. Zuber. 2023. "Foundations of Utilitarianism under Risk and Variable Population." *Social Choice and Welfare*, 61: 101–129.

Thomas, T.. 2022. "Separability in Population Ethics." in *Oxford Handbook of Population Ethics*.

Tversky, A. and D. Kahneman. 1992. "Advances in Prospect Theory: Cumulative Representation of Uncertainty." *Journal of Risk and Uncertainty*, 5: 297–323.

Zuber, S.. 2016. "Harsanyi's Theorem without the Sure-Thing Principle: On the Consistent Aggregation of Monotonic Bernoullian and Archimedean Preferences." *Journal of Mathematical Economics*, 63: 78–83.

Supplementary materials

S.A Dropping Anonymity

Let us use the framework of Section 4. We try to characterize a social preorder $\succsim$ on $F = U^S$, where $U = \bigcup_{N \in \mathcal{N}} \mathbb{R}^N$. We will show that our result extends when we do not assume a symmetric treatment of individuals, i.e. we give up Anonymity. To do so, we need to replace Anonymity with the Compensation principle.

Here is the formulation of Compensation in the present context.

Compensation For any $u \in U$, $i \in N(u)$ and $v \in \mathbb{R}^{N(u)\setminus\{i\}}$, there exists $z \in \mathbb{R}$ such that, if $w \in U$ is defined by $v_i = z$ and $v_j = w_j$ for all $j \in (N(u) \setminus \{i\})$, then $u \sim v$.

We can prove the following Theorem:

Theorem S.A *The following statements are equivalent:*

1. *The social preorder $\succsim$ satisfies Completeness for Sure Prospects, Same-Population Continuity for Sure Outcomes, Compensation, Critical Level for Egalitarian Expansion, Social Statewise Dominance and Correlated Stochastic Dominance for Sure Individuals.*

2. *$\succsim$ is a complete social order and there exist continuous, increasing and unbounded functions $\phi_i : \mathbb{R} \to \mathbb{R}$ and a number $c \in \mathbb{R}$ such that for all $f, g \in F$, $f \succsim g$ if and only if*

$$\sum_{s\in S} \tfrac{1}{m} \left[\sum_{i \in N(f(s))} \left[\phi_i(f_i(s)) - \phi_i(c)\right] \right] \geq \sum_{s\in S} \tfrac{1}{m} \left[\sum_{j \in N(g(s))} \left[\phi_j(g_j(s)) - \phi_j(c)\right] \right].$$

The fact that Statement 2. implies Statement 1. is straightforward to check. Below we prove that Statement 2. implies Statement 1. The first step is the following Proposition:

Proposition S.A *If the social preorder $\succsim$ satisfies Completeness for Sure Prospects, Same-Population Continuity for Sure Outcomes, Social Statewise Dominance and Correlated Stochastic Dominance for Sure Individuals then there exist continuous, increasing and unbounded functions $\phi_i : \mathbb{R} \to \mathbb{R}$ such that for all $u, v \in U$, $u \succsim v$ if and only if*

$$\sum_{i \in N(u)} \big[\phi_i(u_i) - \phi_i(c)\big] \geq \sum_{i \in N(v)} \big[\phi_i(v_i) - \phi_i(c)\big].$$

Proof. Given that $\succsim$ satisfies Completeness for Sure Prospects, Same-Population Continuity for Sure Outcomes, Compensation, Critical Level for Egalitarian Expansion, Social Statewise Dominance and Correlated Stochastic Dominance for Sure Individuals, we can mimic the beginning of the proof of Proposition 2 to show that there exists continuous and increasing functions $\phi_i : \mathbb{R} \to \mathbb{R}$ (one for each individual $i \in I$) and a number $c \in \mathbb{R}$ such that for all $u, v \in U$, $u \succsim v$ if and only if $\sum_{i \in N(u)} \big[\phi_i(u_i) - \phi_i(c)\big] \geq \sum_{i \in N(v)} \big[\phi_i(v_i) - \phi_i(c)\big]$.

It is possible to show that each ϕ_i function is unbounded using the same reasoning as in the proof of Theorem 1 (end of Step 1 of the proof). ■

Let us now proceed with the proof of Theorem S.A. Consider any $f, g \in F$. Let $M = \big(\bigcup_{s \in S} N\big(f(s)\big)\big) \cup \big(\bigcup_{s \in S} N\big(g(s)\big)\big)$ be the set of individual who exist in at least one state of the world, in at least one of f or g. Let $\{i_1, \ldots, i_m\}$ a set of m distinct individuals (one individual per state of the world) who do not belong to M. By Fixed Critical Level (which is implied by Completeness for Sure Prospects, Social Statewise Dominance, Correlated Stochastic Dominance for Sure Individuals, and Critical Level for Egalitarian Expansion see Step 2 in the proof of Proposition 2), there exists $c \in \mathbb{R}$, such that, for any $u \in U$ and $i \notin N(u)$, if v is defined by $N(v) = N(u) \cup \{i\}$, $v_j = u_j$ for all $j \in N(u)$ and $v_i = c$, then $u \sim v$.

Let $f', g' \in F$ be defined as follows: for each $s \in S$, $N(f'(s)) = N(f(s)) \cup \{i_s\}$, $N(g'(s)) = N(g(s)) \cup \{i_s\}$, $f'_i(s) = f_i(s)$ for all $i \in N(f(s))$, $g'_j(s) = g_j(s)$ for all $j \in N(g(s))$, and $f'_{i_s}(s) = g'_{i_s}(s) = c$. Using Fixed Critical Level, we have $f(s) \sim f'(s)$ and $g(s) \sim g'(s)$ for each $s \in S$.

Next, by Compensation, for each $s \in S$, there exists $z_s^f \in \mathbb{R}$ such that, if $u \in U$ is defined by $N(u) = N(f'(s))$, $u_i = c$ for all $i \in N(f(s))$, and $u_{i_s} = z_s^f$, then $u \sim f'(s)$. By Proposition S.A, it must then be the case that:

$$\sum_{i \in N(f(s))} \phi_i(c) + \phi_{i_s}(z_s^f) = \sum_{i \in N(f(s))} \phi_i(f_i(s)) + \phi_{i_s}(c)$$

so that

$$\phi_{i_s}(z_s^f) - \phi_{i_s}(c) = \sum_{i \in N(f(s))} \big[\phi_i(f_i(s)) - \phi_i(c)\big].$$

Let $u' \in U$ be defined by $N(u') = \{i_s\}$, and $u'_{i_s} = z_s^f$. By Fixed Critical Level, $u' \sim u$, so that by transitivity $u' \sim f(s)$.

Similarly, for each $s \in S$, we can show that if $v' \in U$ be defined by $N(v') = \{i_s\}$, and $v'_{i_s} = z_s^g$, where

$$\phi_{i_s}(z_s^g) - \phi_{i_s}(c) = \sum_{i \in N(g(s))} \big[\phi_i(g_i(s)) - \phi_i(c)\big],$$

then $v' \sim g(s)$.

Let $f'', g'' \in F$ be defined as follows: for each $s \in S$, $N(f''(s)) = N(g''(s)) = \{i_s\}$, $f''_{i_s}(s) = z_s^f$, and $g''_{i_s}(s) = z_s^g$. We obtain that for each $s \in S$ $f''(s) \sim f(s)$ and $g''(s) \sim g(s)$. So, by Social Statewise Dominance, $f'' \sim f$ and $g'' \sim g$.

Let us construct two sequences of social prospects $(\hat{f}^{(2)}, \ldots, \hat{f}^{(m)})$ and $(\tilde{f}^{(2)}, \ldots, \tilde{f}^{(m)})$ in the following way. For any $k \in \{2, \ldots, m\}$:

- $N\left(\hat{f}^{(k)}(1)\right) = N\left(\tilde{f}^{(k)}(1)\right) = \{i_1, \ldots, i_k\}$; $\hat{f}^{(k)}_{i_s}(1) = \tilde{f}^{(k)}_{i_s}(1) = z_s^f$ for all $s < k$; $\hat{f}^{(1)}_{i_k}(1) = c$; $\tilde{f}^{(1)}_{i_k}(1) = z_k^f$;
- For all $1 < s < k$, $N\left(\hat{f}^{(k)}(s)\right) = N\left(\tilde{f}^{(k)}(s)\right) = \{i_k\}$, and $\hat{f}^{(k)}_{i_k}(s) = \tilde{f}^{(k)}_{i_k}(s) = c$;
- $N\left(\hat{f}^{(k)}(k)\right) = N\left(\tilde{f}^{(k)}(k)\right) = \{i_k\}$; $\hat{f}^{(k)}_{i_k}(1) = z_k^f$; $\tilde{f}^{(k)}_{i_k}(1) = c$;
- For all $s > k$, $N\left(\hat{f}^{(k)}(s)\right) = N\left(\tilde{f}^{(k)}(s)\right) = \{i_s\}$, and $\hat{f}^{(k)}_{i_s}(s) = \tilde{f}^{(k)}_{i_s}(s) = z_s^f$.

We have $f''(s) \sim \hat{f}^{(2)}(s)$ for each $s \in S$ by Fixed Critical Level, given that f'' and $\hat{f}^{(2)}$ only differ by the addition of individual i_2 at critical level c in each state of the world but state 2 (where they are the same). Thus, by Social Statewise Dominance, $f'' \sim \hat{f}^{(2)}$.

For any $k \in \{2, \ldots, m\}$, we have $\hat{f}^{(k)} \sim \tilde{f}^{(k)}$ by Correlated Stochastic Dominance for Sure Individuals, because i_k is the only individual existing for sure, and faces the same prospect (z_k^f in one state of the world, c in all other states).

Last, for each $k \in \{2, \ldots, m-1\}$, we have $\tilde{f}^{(k)} \sim \hat{f}^{(k+1)}$. Indeed, $\tilde{f}^{(k)}(s) \sim \hat{f}^{(k+1)}(s)$ in each state $s \in S$ by Fixed-Critical Level: we add i_{k+1} at utility level c (except in state $k+1$) and then remove i_k who was at that level c. Thus, by Social Statewise Dominance, $\tilde{f}^{(k)} \sim \hat{f}^{(k+1)}$.

In conclusion, by transitivity, $f \sim \tilde{f}^{(m)}$. Similarly, $g \sim \tilde{g}^{(m)}$ where $\tilde{g}^{(m)}$ is defined as follows:

- $N\left(\tilde{g}^{(m)}(1)\right) = \{i_1, \ldots, i_m\}$; $\hat{g}_{i_s}^{(m)}(1) = z_s^g$ for all $s \in S$;
- For all $1 < s$, $N\left(\tilde{g}^{(m)}(s)\right) = \{i_m\}$, and $\tilde{g}_{i_m}^{(m)}(s) = c$.

Thus, $f \succsim g \iff \tilde{f}^{(m)} \succsim \tilde{g}^{(m)}$. But $\tilde{f}^{(m)}(s) = \tilde{g}^{(m)}(s)$ for all $s > 1$. By Social Statewise Dominance and Proposition S.A, we obtain:

$$\begin{aligned}
f \succsim g &\iff \tilde{f}^{(m)} \succsim \tilde{g}^{(m)} \\
&\iff \tilde{f}^{(m)}(1) \succsim \tilde{g}^{(m)}(1) \\
&\iff \phi_{i_1}\left(\tilde{f}_{i_1}^{(m)}(1)\right) + \cdots + \phi_{i_m}\left(\tilde{f}_{i_m}^{(m)}(1)\right) \geq \phi_{i_1}\left(\tilde{g}_{i_1}^{(m)}(1)\right) + \cdots + \phi_{i_m}\left(\tilde{g}_{i_m}^{(m)}(1)\right) \\
&\iff \phi_{i_1}\left(z_{i_1}^f\right) + \cdots + \phi_{i_m}\left(z_{i_m}^f\right) \geq \phi_{i_1}\left(z_{i_1}^g\right) + \cdots + \phi_{i_m}\left(z_{i_m}^g\right).
\end{aligned}$$

But by definition, we have mentioned above that for all $s \in S$:

$$\phi_{i_s}(z_s^f) - \phi_{i_s}(c) = \sum_{i \in N(f(s))} \left[\phi_i(f_i(s)) - \phi_i(c)\right],$$

and

$$\phi_{i_s}(z_s^g) - \phi_{i_s}(c) = \sum_{j \in N(g(s))} \left[\phi_j(g_j(s)) - \phi_j(c)\right].$$

Therefore:

$$
\begin{aligned}
& f \succsim g \\
\iff & \sum_{s\in S}\left[\phi_{i_s}(c_{i_s}) + \sum_{i\in N(f(s))}\left[\phi_i\left(f_i(s)\right) - \phi_i(c)\right]\right] \geq \sum_{s\in S}\left[\phi_{i_s}(c_{i_s}) + \sum_{j\in N(g(s))}\left[\phi_j\left(g_j(s)\right) - \phi_j(c)\right]\right] \\
\iff & \sum_{s\in S}\tfrac{1}{m}\left[\sum_{i\in N(f(s))}\left[\phi_i\left(f_i(s)\right) - \phi_i(c)\right]\right] \geq \sum_{s\in S}\tfrac{1}{m}\left[\sum_{j\in N(g(s))}\left[\phi_j\left(g_j(s)\right) - \phi_j(c)\right]\right].
\end{aligned}
$$

S.B Extension to an infinite state space

Assume that there exists an infinite set of states of the world S, with typical element $s \in S$. We denote with Σ a σ-algebra over S, and by P a probability measure on the measurable space (S, Σ). We assume P to be given, i.e. that we are in a framework with 'objective' probability.

We make the following assumption on the measured space (S, Σ, P):

> For any event $E \in \Sigma$, $P(E)$ is a rational number. Furthermore, for any $m \in \mathbb{N}$, there exists a partition of S into m Σ-measurable events, $(E^1, \ldots, E^m)$ such that $P(E^k) = 1/m$ for all $k \in \{1, \ldots, m\}$.

Our assumption implies that for each number k we can find a partition of the state space into k equiprobable events. This is important because our proof in the main text applies to such cases.

We define social prospects as functions from S to U, which are assumed to be Σ-measurable. We actually focus only on simple prospects, that is social prospects such that there exists a finite partition $(E^1(f), \cdots, E^m(f))$ of S such that $f(s) = f(s')$ for all $s, s' \in E^k(f)$, all $k = 1, \cdots, m$, and each $E^k(f)$ is measurable. We let F be the set of all those simple and measurable prospects.

The properties of Completeness for Sure Prospects, Anonymity, Same population continuity, and Critical Level for Egalitarian Expansion all hold for sure prospects, so they do not need be to be adapted to the present framework. Social Statewise Dominance can still be formulated as before, because it is a state-by-state property, and so does not depend on the number of states of the world.

The only axioms that we need to adjust to the new framework are the stochastic dominance properties.

In the case of a fixed population, the formulation of Individual Stochastic Dominance must be adapted as follows:

Individual Stochastic Dominance For all $f, g \in F$, if for each individual $i \in I$ there exist $\ell_i \in \mathbb{N}$ and two partitions in Σ-measurable events $\{E_1, \ldots, E_{\ell_i}\}$ and $\{\tilde{E}_1, \ldots, \tilde{E}_{\ell_i}\}$ such that for all $r \in \{1, \ldots, \ell_i\}$, $P(E_r) = P(\tilde{E}_r)$ and $f_i(s) \geq g_i(s')$ for all $s \in E_r$ and $s' \in \tilde{E}_r$; then $f \succsim g$.

If in addition there exists $h \in I$ and $r' \in \{1, \ldots, \ell_h\}$ such that $f_h(s) > g_h(s')$ for all $s \in E_{r'}$ and $s' \in \tilde{E}_{r'}$ then $f \succ g$.

The definition is adapted to guarantee dominance on events with the same probability.

In the case of a variable population, the formulation of Correlated Stochastic Dominance for Sure Individuals must be adapted as follows:

Correlated Stochastic Dominance for Sure Individuals For all $f, g \in F$, if:

(i) $S_i(f) = S_i(g)$ for all $i \in I$;

(ii) for all $j \in I$ such that $S_j(f) \notin \{\emptyset, S\}$, there exists $x_j \in \mathbb{R}$ such that $f_j(s) = g_j(s) = x_j$ for all $s \in S_j(f)$;

(iii) there exists $\ell \in \mathbb{N}$ and two partitions in Σ-measurable events $\{E_1, \ldots, E_\ell\}$ and $\{\tilde{E}_1, \ldots, \tilde{E}_\ell\}$ such that, for all $k \in I$ such that $S_k(f) = S$ and for all $r \in \{1, \ldots, \ell\}$, $P(E_r) = P(\tilde{E}_r)$, $f_k(s) \geq g_k(s')$ for all $s \in E_r$ and $s' \in \tilde{E}_r$; then $f \succsim g$.

If in addition there exists $h \in I$ such that $S_h(f) = S$ and $r' \in \{1, \ldots, \ell\}$ such that $f_h(s) > g_h(s')$ for all $s \in E_{r'}$ and $s' \in \tilde{E}_{r'}$ then $f \succ g$.

Consider any $m \in \mathbb{N}$ and let $(E^1, \ldots, E^m)$ be the partition into m equiprobable and measurable events mentioned in our assumption above. Denote F^m the set of all prospects f such that for each $k \in \{1, \ldots, m\}$ we have $f(s) = f(s')$ for all $s, s' \in E^k$. Restricting our axioms to the set F^m, we clearly are formally in the same case as the one in the main text because each event is like an equiprobable state of the world where well-defined consequence occurs. So, we can apply all our results and deduce – for instance – that, if the social preorder $\succsim$ satisfies Completeness for Sure Prospects, Anonymity for Sure Outcomes, Same-Population Continuity for Sure Outcomes, Critical Level for Egalitarian Expansion, Social Statewise Dominance and Correlated Stochastic Dominance for Sure Individuals, then there exists a continuous and increasing function $\phi : \mathbb{R} \to \mathbb{R}$ and a number $c \in \mathbb{R}$ such that for all $f, g \in F^m$, $f \succsim g$ if and only if[2]

$$\sum_{s\in\{1,\dots,m\}} \tfrac{1}{m}\Bigg[\sum_{i\in N(f(E^s))} \big[\phi(f_i(E^s))-\phi(c)\big]\Bigg] \geq \sum_{s\in\{1,\dots,m\}} \tfrac{1}{m}\Bigg[\sum_{i\in N(g(E^s))} \big[\phi(g_i(E^s))-\phi(c)\big]\Bigg],$$

where, with an abuse of notation, $f(E^s) = f(t)$ where t is some $t \in E^s$ (which is well defined because $f(t') = f(t)$ for all $t \in E^s$); $f_i(E^s) = f_i(t)$ where t is some $t \in E^s$; and similar notation are used for g.

Consider any two prospects f and $g \in F$. Let $(E^1(f), \dots, E^m(f))$ and $(E^1(g), \dots, E^r(g))$ be the partitions generated by f and g. Given that probabilities of events are assumed to be rational numbers, there exists a least common denominator d such that for all $l = 1, \dots, m$ there exists $k_f^l \in \mathbb{N}$ such that $P(E^l(f)) = \frac{k_f^l}{d}$ and for all $l' = 1, \dots, r$ there exists $k_g^{l'} \in \mathbb{N}$ such that $P(E^{l'}(g)) = \frac{k_g^{l'}}{d}$. By definition, $\sum_{l=1}^m k_f^l = \sum_{l'=1}^r k_g^{l'} = d$. Recall that $(E^1, \dots, E^d)$ is a partition of the state space into m measurable and equiprobable events. Denote $K_f^p = \sum_{l=1}^p k_f^l$ and $K_g^{p'} = \sum_{l'=1}^{p'} k_f^{l'}$.Let us define $\tilde{f}, \tilde{g} \in F^d$ as follows:

- for each $p \in \{1, \dots, m\}$, $\tilde{E}^p(f) = \bigcup_{j=1+K_f^{p-1}}^{K_f^p} E^l$, and $\tilde{f}(s) = f(E^p(f))$ for any $s \in \tilde{E}^p(f)$.
- for each $p' \in \{1, \dots, r\}$, $\tilde{E}^{p'}(g) = \bigcup_{j=1+K_g^{p'-1}}^{K_g^{p'}} E^l$, and $\tilde{g}(s) = g(E^{p'}(g))$ for any $s \in \tilde{E}^{p'}(g)$.

[2]Formally, the function ϕ may depend on m, but we can link different representations for various values of m using Stochastic Dominance as explained below. Using the unicity of additive representations up to an increasing affine transformation, we can actually show that ϕ is independent of m.

So, what we do is to associate to each event $E^p(f)$ defined by f a collection $\tilde{E}^p(f)$ of k_f^p of equiprobable events and assume that the outcome in each state in this collection is the same as the common consequence obtained in prospect f on $E^p(f)$. By definition, $P(\tilde{E}^p(f)) = P(E^p(f)) = k_f^p/d$. Similarly, we associate to each event $E^{p'}(g)$ defined by f a collection $\tilde{E}^{p'}(g)$ of $k_g^{p'}$ of equiprobable events and assume that the outcome in each state in this collection is the same as the common consequence obtained in prospect g on $E^{p'}(g)$. What we will show next, using Stochastic Dominance, is that $f \sim \tilde{f}$ and $g \sim \tilde{g}$. Then we can conclude (because $\tilde{f}, \tilde{g} \in F^d$):

$$
\begin{aligned}
& f \succsim g \\
\iff & \tilde{f} \succsim \tilde{g} \\
\iff & \sum_{s \in \{1,\dots,d\}} \frac{1}{d}\Bigg[\sum_{i \in N(\tilde{f}(E^s))} \big[\phi(\tilde{f}_i(E^s)) - \phi(c)\big]\Bigg] \geq \sum_{s \in \{1,\dots,d\}} \frac{1}{m}\Bigg[\sum_{i \in N(\tilde{g}(E^s))} \big[\phi(\tilde{g}_i(E^s)) - \phi(c)\big]\Bigg] \\
\iff & \sum_{p=1}^{m} \sum_{s:E^s \subset \tilde{E}^p(f)} \frac{1}{d}\Bigg[\sum_{i \in N(\tilde{f}(E^s))} \big[\phi(\tilde{f}_i(E^s)) - \phi(c)\big]\Bigg] \geq \sum_{p'=1}^{r} \sum_{s:E^s \subset \tilde{E}^{p'}(g)} \frac{1}{d}\Bigg[\sum_{i \in N(\tilde{g}(E^s))} \big[\phi(\tilde{g}_i(E^s)) - \phi(c)\big] \\
\iff & \sum_{p=1}^{m} \frac{k_f^p}{d}\Bigg[\sum_{i \in N(f(E^p(f)))} \big[\phi(f_i(E^p(f))) - \phi(c)\big]\Bigg] \geq \sum_{p'=1}^{r} \frac{k_g^{p'}}{d}\Bigg[\sum_{i \in N(g(E^{p'}(g)))} \big[\phi(g_i(E^{p'}(g))) - \phi(c)\big]\Bigg] \\
\iff & \int_{s \in S} \Bigg[\sum_{i \in N(f(s))} \big[\phi_i\left(f_i(s)\right) - \phi_i(c_i)\big]\Bigg] dP(s) \geq \int_{s \in S} \Bigg[\sum_{j \in N(g(s))} \big[\phi_j\left(g_j(s)\right) - \phi_j(c_j)\big]\Bigg] dP(s).
\end{aligned}
$$

It only remains to show that $f \sim \tilde{f}$ and $g \sim \tilde{g}$. We only prove that $f \sim \tilde{f}$ in the variable population case (the proof that $g \sim \tilde{g}$, and the one for the same population case are similar). First remark that Proposition 1 still applies (it suffices to concentrate on prospects in F^2 to get existence independence and the rest of the proof follows). To show that $f \sim \tilde{f}$, let us denote $\bar{N} = \{i \in S | S_i(f) \neq \emptyset\}$ the set of individuals who exist in at least one state of the world, and introduce f' and f'' as follows:

- for all $s \in S$, $N(f') = \bar{N}$, $f'_i(s) = f_i(s)$ for all $i \in N(f(s))$ and $f'_j(s) = c$ for all $j \in (\bar{N} \setminus N(f(s))$;

- for all $s \in S$, $N(f'') = \bar{N}$, $f''_i(s) = \tilde{f}_i(s)$ for all $i \in N(\tilde{f}(s))$ and $f''_j(s) = c$ for all $j \in (\bar{N} \setminus N(\tilde{f}(s))$;

where c is the critical level in Proposition 1.

By Proposition 1, $f(s) \sim f'(s)$ and $\tilde{f}(s) \sim f''(s)$ for all $s \in S$. By Social Statewise Dominance, we obtain $f \sim f'$ and $\tilde{f} \sim f''$. Now, remark that the same population $\bar{N}$ exist in all states of the world in both f' and f''. Last, by definition, for any $p \in \{1, \ldots, m\}$, $f'(s) = f''(s')$ for all $s \in E^p(f)$ and $s \in E^p(\tilde{f})$. Therefore, by Correlated Stochastic Dominance for Sure Individuals, $f' \sim f''$.

As final remark, let us discuss how to deal with cases where probability are *not* rational numbers. Indeed, our assumption up to know was that the probability of each event defined by an act was rational. To include the more general case, we can introduce a more general property of probability continuity for prospects, which is as follows:

Probability Continuity for Prospects For all $f, g \in F$, if there exists a sequence of prospects $(f_n)_{n\in\mathbb{N}}$ such that $\lim_{n\to\mathbb{N}} P(s \in S | f_n(s) \neq f) = 0$ and $f_n \succsim g$ for all $n \in \mathbb{N}$, then $f \succsim g$.

The case of prospects that are not simple (that do not define a finite partition of the state space) would be more difficult to deal with.