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Share the Sugar.

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The University of Texas at Austin
Population Wellbeing Initiative

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Abstract

We provide a general argument against value incomparability, based on a new style of impossibility result. In particular, we show that, against plausible background assumptions, value incomparability creates an incompatibility between two very plausible principles for ranking lotteries: a weak "negative dominance" principle (to the effect that Lottery 1 can be better than Lottery 2 only if some possible outcome of Lottery 1 is better than some possible outcome of Lottery 2) and a weak form of *ex ante* Pareto (to the effect that, if Lottery 1 gives an unambiguously better prospect to some individuals than Lottery 2, and equally good prospects to everyone else, then Lottery 1 is better than Lottery 2). After spelling out our results, and the arguments based on them, we consider which principle the proponent of incomparability ought to reject.

1 Introduction

It often seems, when comparing two possible outcomes, that neither is better than the other, nor are they exactly equally good. Suppose that Patricia is deciding whether to pursue a career as an artist or a banker. Being an artist would give her an unlimited creative outlet and the freedom of being her own employer. Being a banker would give her financial stability and regular opportunities for international travel. Even setting aside her many uncertainties and imagining maximally specific ways that each of these lives might go, it seems that neither life would be *better* for Patricia overall—and that they would not be exactly equal, in the sense that a slight but noticeable improvement to one life (say, adding $1,000 in salary) would not be enough to make it better than the other. Similarly, from an impersonal point of view ("the point of view of the universe", or of social welfare), the outcome where Patricia becomes an artist could be neither better nor worse nor exactly as good as the outcome where she becomes a banker.[1]

Separate from this sort of incomparability between lives, many philosophers have claimed that a structurally similar phenomenon arises when comparing outcomes in which different individuals exist. Some have claimed that, from a personal point of view, existence is always incomparable with non-existence: no matter how good or bad an individual's life, we can never say that it is better, worse, or exactly as good for her as non-existence [Broome, 1999, p. 168]. Some have claimed that the same is true from an impersonal point of view: two possible worlds or outcomes that are identical except that some individual exists in one but not in the other are incomparable in terms of overall value, whatever life that individual lives in the world where they exist [Bader, 2022, 2023]. Others have claimed that this is true only of *some* lives: a life with the right mix of triumphs and tragedies may be incomparable with non-existence, from an impersonal point of view [Gustafsson, 2020, Thornley, 2022].

This paper presents a new argument against these and other forms of incomparability. Suppose that two possible lives, like *artist* and *banker*, are indeed incomparable. Let a denote a fully specified artist's life and b a fully specified banker's life, assumed to be incomparable, with a^+ a slightly improved version of the artist's life that is still incomparable with b. Now suppose we face a choice

[1]Philosophers who have argued that lives can be incomparable in this way include Raz [1985, 1986], Chang [2002], and Hedden and Muñoz [2023], among many others.

between the following two lotteries, based on the same fair coin flip, where our choice will affect only two individuals, P_1 and P_2:

(a) Lottery 1

	Heads	Tails
P_1	a	b
P_2	b	a

(b) Lottery 2

	Heads	Tails
P_1	a^+	b
P_2	a^+	b

There is, on the one hand, a strong argument that neither lottery is better than the other. First, if a and b are incomparable from a personal point of view, then it seems that two outcomes that differ only in that some individual enjoys life a in one and life b in the other should also be incomparable. Thus, (a, b) and (b, a) are both incomparable with (a, a) and (b, b), where the first and second terms in each ordered pair represent the lives of P_1 and P_2 respectively. Moreover, it is part and parcel of incomparability that it is preserved by small enough improvements, so (a^+, a^+), which is at most slightly better than (a, a), should likewise be incomparable with (a, b) and (b, a). We then find that *every* possible outcome of Lottery 1 (the Heads outcome and the Tails outcome) is incomparable with *every* possible outcome of Lottery 2. This suggests that neither lottery is strictly better than the other.

On the other hand, there is *also* a strong argument that Lottery 2 is better than Lottery 1: Lottery 1 gives both P_1 and P_2 a 50% chance of life a and a 50% chance of life b. Lottery 2 gives each of them a 50% chance of life a^+ and a 50% chance of life b. Since a^+ is better than a, Lottery 2 seems clearly more desirable from the point of view of each affected individual, and therefore better overall.

Taken together, these arguments amount to an apparent *reductio* of the supposition that a and b are incomparable. Similar arguments, as we will see, can be mounted against many other forms of incomparability in the ranking of social outcomes: between larger and smaller populations, between competing desiderata like total welfare, average welfare, and equality, between benefits conferred on different infinite populations, and more. The result will be a new, general argument for completeness of betterness, turning on considerations of a quite different character than the recent linguistic arguments of Dorr et al. [2021, 2023]. The central premises of this *reductio* are two dominance-like principles: first, that if no possible outcome of lottery L_i is better than any possible outcome of lottery L_j, then L_i is not better than L_j; and second, that if L_i is better for everyone affected than L_j (in a very strict sense, to be explained), then it is better overall. We will call the first principle *Negative Dominance* and the second principle *Personal Good*.

In §2, we formalize and motivate Negative Dominance, comparing it to the significantly stronger principle of "Recognition", which plays a central role in Caspar Hare's "opaque sweetening" puzzle [Hare, 2010]. In §3, we similarly motivate and explain Personal Good. In §4, we offer a more exact version of the above argument, showing that, against modest background assumptions, Negative Dominance and Personal Good rule out incompleteness in the personal ranking of lives, and by extension those forms of incompleteness in the impersonal ranking of outcomes that derive from incomparability between lives. We show how this argument differs from a *prima facie* related one (though developed for a different purpose) due to Nebel [2020]. §5 then presents a more general argument against incompleteness in the impersonal ranking of outcomes, likewise based on Negative Dominance and Personal Good. §6 takes the argument a step further by showing that Personal Good plus plausible auxiliary assumptions allows the ranking of a large class of lotteries to be reduced to the ranking of outcomes, so that if outcomes are completely ordered by impersonal value, this class of lotteries will be too. Finally, in §7, we consider whether either Negative Dominance or Personal Good can be plausibly rejected. While neither principle is sacrosanct, we think that the most natural move for those committed to incomparability is to reject Negative Dominance. This move is especially natural for those who understand evaluative incompleteness as indeterminacy between complete rankings. It has the cost, however, of ruling out the "deferentialist" position that has been endorsed by several prominent fans of incompleteness [Schoenfield, 2014, Bales et al., 2014, Doody, 2019a,b, 2021, 2023].

2 Negative Dominance

We will be interested in the implications of certain decision-theoretic principles in an ethical context, so we begin with an abstract decision-theoretic framework which we will subsequently give an ethical interpretation. At the most abstract level, we are interested in rankings of *lotteries* over *outcomes*. Here, *outcomes* can be understood as complete possible worlds, or as equivalence classes of equally good worlds, or as specifications of all evaluatively significant features of a world. A *lottery* is any probability distribution over outcomes. We will say that an outcome is *in* a lottery if the lottery assigns it positive probability. Except for one brief stretch in §7, we will limit our attention to lotteries with only finitely many possible outcomes in them. Finally, we will assume that lotteries can be compared in terms of *impersonal betterness*. Specifically, we will take as primitive an *at least as impersonally good as* relation on lotteries. If lottery L is at least as good as lottery L' but L' is not at least as good as L, we say that L is *strictly impersonally better* than L'. If they are each at least as good as each other, we say that they are *equally impersonally good*. If neither relation holds, we say that they are *impersonally incomparable*. The impersonal betterness relation on lotteries induces an impersonal betterness relation on outcomes: If the lottery that yields o with certainty is at least as good as the lottery that yields o' with certainty, we say that o is at least as impersonally good as o' (and likewise for other evaluative relations).

Our interest here will be in the ethics of population and distribution. In this context, we assume that the evaluatively significant features of an outcome are (i) which individuals exist in that outcome and (ii) how their lives go. Thus, we will identify outcomes with specifications of only these features. In particular, we will assume a countable set of possible individuals (individuals who exist in some possible world/outcome) and a set of possible *lives* (complete specifications of everything that matters, morally or prudentially, about an individual's life—e.g., the totality of their experiences). Importantly, the set of possible "lives" includes non-existence ("the empty life"). Given this background, we will understand outcomes as functions from the set of possible individuals to the set of possible lives, understood as specifying which individuals exist in a given outcome and what lives they lead. We will sometimes call such outcomes *populations*, when we want to emphasize the fact that they have less information in them than a possible world, but in general we will use the terms *outcome* and *population* interchangeably, and choose one or the other depending on which feature of these objects we wish to emphasize. Finally, alongside the *im*personal betterness relation introduced above, we will assume a *personal* betterness relation on lives.[2] As above, we take as primitive an *at least as (personally) good as* relation on lives. If life a is at least as good as life b but b is not at least as good as a, we say that a is *strictly (personally) better* than b. If they are each at least as good as each other, we say that they are *equally (personally) good*. If neither relation holds, we say that they are *(personally) incomparable*. (From now on, we will usually omit "personal" and "impersonal" where it is clear from context which relation is meant.)[3]

A betterness relation is *complete* if no two entities in its domain are incomparable—that is, for any two entities, either one is strictly better than the other, or they are equally good. Our interest in this paper will be in theories on which the impersonal betterness relation is incomplete. At some points we will also consider theories which hold that the personal betterness relation is incomplete as well.

Given these preliminaries, we can state the principle of Negative Dominance as follows:

Negative Dominance If for every outcome o_1 in lottery L_1 and every outcome

[2]We could define this relation also on lotteries over lives, but our principles and arguments will only make reference to personal betterness on lives, so we'll make do with that more minimal relation.

[3]This setup abstracts from a number of features which will not be important below. Most obviously we will be assuming that (the evaluatively significant aspects of) a life which is enjoyed by one individual could also be enjoyed by another. One might contest this on many grounds: perhaps, for instance, the particularity of friendships or familial relations might be thought to be evaluatively significant although an individual cannot be a friend of or a parent to themselves. A more perspicuous setting might take as primitive a set of worlds and individuals, and an "at least as (personally) good as" ranking of (lotteries over) relevant centered worlds, i.e. pairs of worlds and individuals. "Lives" could then be understood as equivalence classes of centered worlds with respect to the relation of *exactly equally as (personally) good as*. Populations/outcomes in our sense could be seen as equivalence classes of worlds under the equivalence relation which takes w and w' to be equivalent if and only if for all i, (w, i) is equally personally good as (w', i). Everything we do below could be done in a more specific setting like this one, but the extra detail would needlessly clutter the exposition.

> o_2 in lottery L_2, o_1 is not at least as good as o_2, then L_1 is not strictly better than L_2.

We can argue for this principle as follows. Suppose you are considering a lottery with objective chances, that you know these chances, and that your preferences coincide with the objective relation of betterness over the outcomes of those lotteries. Then consider the principle:

> If no epistemically possible outcome of L_1 is preferable to any epistemically possible outcome of L_2, then you are not required to choose L_1 over L_2.

The idea here is that, given the betterness facts about the outcomes (reflected in your preferences), there can't be a reason, justification, or rationale for choosing L_1 that would be stronger than or defeat your reasons, justifications, or rationales for choosing L_2.[4] Given that there is no reason in favor of L_1 that outweighs or defeats the reasons in favor of L_2, it can't be that you are *required* to choose L_1. But this means that L_1 can't be strictly better. If it were, then since we assumed that you knew all the betterness facts, and had preferences in line with them, you *would* be required to choose it. So it must be that L_1 is not strictly better in this case.

In his important paper "Take the Sugar", Caspar Hare proposes a principle, which he calls "Recognition", that shares some of the appeal of Negative Dominance, but (as we will now argue) is too strong in key cases.[5] Seeing the contrast between these two principles will bolster the case for Negative Dominance.

[4] For related discussion, see Hare [2022, Ch. 4].

[5] Our title is a play on Hare's, since we discuss a puzzle and a principle closely related to his, but in a context of multi-patient moral decision-making rather than individual prudential decision-making.

Hare works in a slightly different setting than ours, with two different primitive objects: states and prizes. In this setting, probabilities are defined on states, while betterness is defined in the first instance on prizes. *Gambles* are functions from states to prizes. If we think of outcomes as state-prize pairs, then gambles *induce* lotteries, since a probability distribution on states yields, via the gamble, a probability distribution on prizes-cum-states. But gambles have more structure than lotteries, allowing us to contrast a possible outcome of a given gamble with what *would have* happened (in the same state) if a different gamble had been chosen.[6]

The key idea in Hare's principle can be stated as:

Statewise Negative Dominance If for every state s, the prize that gamble L_1 yields in s is not at least as good as the prize that gamble L_2 yields in s, then L_1 is not strictly better than L_2.

If we understand outcomes as state-prize pairs, as above, then Statewise Negative Dominance is strictly stronger than Negative Dominance, so anyone who accepts the former (e.g. Schoenfield [2014], Bales et al. [2014], Doody [2023]) must also accept the latter.

But even those who reject Statewise Negative Dominance (like Hare himself, Bader [2018], and Rabinowicz [2021]) still have good reason to endorse Negative Dominance. In particular, as Bader [2018] has emphasized, Statewise Negative Dominance conflicts with another, more widely accepted dominance principle:[7]

[6]For those familiar with the distinction, Hare's framework is in this respect akin to Savage's, with the objects of choice understood as functions from states to prizes, while our framework is akin to von Neumann & Morgenstern's.

[7]Manzini and Mariotti [2008, 310] earlier independently presented a very similar argument for an incompatibility between Stochastic Dominance (their "Preference Sure Thing") and

Stochastic Dominance (i) If for every outcome o, L_1 has at least as great a probability as L_2 of yielding an outcome at least as impersonally good as o, then L_1 is at least as impersonally good as o. (ii) If in addition, for some o, L_1 has a strictly greater probability of yielding an outcome at least as impersonally good as o, then L_1 is strictly impersonally better than L_2.[8]

When condition (i) is satisfied, we say that L_1 *weakly stochastically dominates* L_2. When condition (ii) is also satisfies, we say that L_1 *strictly stochastically dominates* (or simply *stochastically dominates*) L_2.

Stochastic Dominance is extremely plausible, and among the most widely accepted principles in normative decision theory. (For an array of arguments in its favor, see Tarsney [2020].) It is compatible with a very wide range of attitudes toward risk, being satisfied not just by standard expected utility theory but also by more permissive alternatives like rank-dependent/risk-weighted expected utility [Quiggin, 1982, Buchak, 2013] and the version of weighted-linear utility theory recently defended by Bottomley and Williamson [forthcoming].[9] So the fact that Statewise Negative Dominance and Stochastic Dominance conflict is a strong argument against Statewise Negative Dominance. But no similar argument can be given against (plain) Negative Dominance, since it is consistent with Stochastic

their "Vague Sure Thing Principle" (VST). Their VST is even stronger than Statewise Negative Dominance, but the line of thought which leads to incompatibility is the same.

[8]Russell [forthcoming] shows that when the betterness ordering is incomplete (and in other, more outré contexts), this standard formulation of Stochastic Dominance gives the wrong results and must be revised. But the difference between the standard formulation and Russell's proposed revision won't matter here.

[9]To put the point more exactly, a classic result from Hadar and Russell [1969] and Hanoch and Levy [1969] implies that an expected utility maximizer will satisfy Stochastic Dominance with respect to a given ranking of outcomes if and only if her utility function is increasing with respect to that ranking, i.e., if and only if she prefers certainty of a better outcome to certainty of a worse outcome. The same thing turns out to be true of the alternatives to expected utility theory mentioned in the main text.

Dominance [Lederman, 2023b, Proposition 3.1].

3 Personal Good

How should the relation of personal betterness for individuals connect to the relation of impersonal betterness? The following articulates a weak constraint on this connection:

Pareto If the same individuals exist in outcomes o and o', and o is personally at least as good as o' for every individual and strictly personally better for at least one individual, then o is impersonally better than o'.

This principle only concerns *outcomes*; it doesn't tell us anything about lotteries in general.

How might it be extended to lotteries as well as outcomes? We will work with the following weak extension of Pareto:

Personal Good Suppose two lotteries L and L' are such that (1) every possible individual i has the same probability of existence in L as in L'. Then, (i) if for every individual i and life l, lottery L_1 gives i at least as great a probability of a life at least as personally good as l, compared to L_2, then L_1 is at least as impersonally good as L_2; and (ii) if in addition, for some i and l, L_1 gives i a strictly greater probability of a life at least as personally good as l, then L_1 is strictly impersonally better than L_2.[10]

This principle extends Pareto to lotteries, and to cases where different numbers

[10] Our arguments below will rely mainly on part (ii) of Personal Good. We will appeal to part (i) only in §6.

of people may exist in each outcome.[11] The extension is very modest in both respects. For the first, it only provides comparisons where each individual has a fixed *probability* of existence. This means that it is compatible with a wide range of views about how to value nonexistence—when a choice affects any individual's probability of existence, the principle remains silent. Personal Good also offers a modest extension in the way that it handles individual betterness for lotteries. In particular, it implicitly invokes a very weak sufficient condition for L_1 to be (weakly/strictly) better than L_2 from the perspective of an individual, namely, that the *personal lottery* (probability distribution over lives) she receives from L_1 (weakly/strictly) stochastically dominates the personal lottery she receives from L_2. As described above, Stochastic Dominance is a minimal and compelling principle for ranking lotteries, which is compatible with a very wide range of attitudes toward risk.

Personal Good should be especially compelling to those who take an "individualistic" view of population ethics, according to which our fundamental objects of concern should be the interests of each individual (rather than the impersonal value of the world as a whole), and so options should be compared in the first instance from the perspective of each affected individual.[12] From this perspective, it seems obvious that if L_1 is unambiguously better than L_2 from the perspective of some individuals, and equivalent from the perspective of everyone else, then it is better overall. Since Personal Good makes no commitments about the value of non-existence, it should also appeal to those who endorse "person-affecting" views in population ethics, which characteristically combine individualism with

[11] McCarthy et al. [2020] advocate a closely related principle which they call "Anteriority".

[12] In associating Pareto-like principles with "individualism", we are following Harsanyi [1955].

an unwillingness to compare existence and non-existence.

With that said, and though we ourselves find the principle attractive, there is some precedent for rejecting it. Some egalitarians, for instance, will reject Personal Good because, while it allows concern for *ex ante* equality (equality between personal lotteries), it rules out any independent concern for *ex post* equality (equality between individual lives in a given outcome). We return to this idea, and in general, the possibility of rejecting Personal Good in §7.

4 Incomparable lives

We now turn to the central arguments of the paper, showing that the combination of Negative Dominance and Personal Good rules out many forms of value incompleteness. We begin in this section by considering incompleteness in the personal betterness ranking of lives, along with those forms of incompleteness in the impersonal betterness ranking of outcomes that arise from it.

We first make an assumption that we think of as basically a convenience, without significant conceptual import:

Richness For any finite set of possible individuals and any function from those individuals to lives other than non-existence, there is an outcome where exactly those individuals exist, with exactly those lives.

Moreover, for any finite set of outcomes and any probability distribution defined on that set, there is a lottery which assigns those probabilities to those outcomes.

We are assuming (as we said earlier) that the set of possible individuals is countably infinite, and that the impersonal betterness relation is defined over the whole set of lotteries. We won't need anything like the full strength of Richness: all we need is that the domain of the betterness relation includes outcomes with the structure of our examples below.

Next, we make an assumption that connects incomparability in personal betterness to incomparability in impersonal betterness:

Incomparability Transmission Suppose that outcomes o and o' are identical except that one possible individual receives life a in o and life b in o', where a and b are incomparable in terms of personal value. Then o and o' are incomparable in terms of impersonal value.

As with Personal Good, this principle is a natural expression of "individualism" about impersonal betterness: We compare outcomes by comparing them from the vantage point of each individual, and when only one individual's fate differs between two outcomes, the impersonal betterness ordering should reflect that individual's personal betterness ordering. When the two lives that individual might receive are personally incomparable, their incomparability is "transmitted" to the outcomes in which they figure.

Finally, we will assume the following principle about the structure of incomparability on outcomes (which, to recall, we are treating as equivalent to lotteries which assign probability 1 to a single outcome).

Small Improvements Suppose that outcome o is impersonally incomparable with outcomes o' and o''. Then there is either

(i) an outcome o^+ which is strictly better than o for some individual, and equally good for everyone else, while still being impersonally incomparable with both o' and o'', or

(ii) an outcome o^- which is strictly worse than o for some individual, and equally good for everyone else, while still being impersonally incomparable with both o' and o''.

Insensitivity to small improvements or small worsenings is a defining feature of incomparability, by contrast to equal goodness. Thus it should not be controversial that, given incomparable o and o', there is either an o^+ that improves o and is still incomparable with o' or an o^- that worsens o and is still incomparable with o'. The above principle adds to this uncontroversial claim two further ideas. First, it adds that we can achieve this improvement (worsening) by improving (worsening) the life of some individual in o. This is slightly stronger, but we do not expect it to be particularly controversial. Second, it adds that when o is incomparable to *two* further outcomes o' and o'', there is a single improvement (or worsening) which preserves incomparability with *both* o' and o''. For instance, if an outcome where Patricia is an artist is incomparable with outcomes where she is a banker or a CIA operative, then we can either slightly improve or slightly worsen the first outcome (e.g., by slightly adjusting Patricia's income) while preserving incomparability. Like the first, this claim seems plausible to us, and holds in simple, natural models of value incomparability. In any case, as described below, our argument makes only limited use of Small Improvements, and does not require it to hold in full generality.

Now we can state our first result:

Proposition 4.1 (Individual-Level Incomparability)**.** *Given Richness, the princi-*

ples of Negative Dominance, Personal Good, Incomparability Transmission, and Small Improvements rule out incomparability in personal value.

This can be proved using a slight modification of the example from the introduction, illustrated in Table 2:

(a) Lottery 1

	Heads	Tails
P_1	a	b
P_2	b	a

(b) Lottery 2

	Heads	Tails
P_1	a^+	b
P_2	a	b

Table 2

Here again we are considering lotteries that affect just two possible individuals, P_1 and P_2. (We can assume that everyone else is assured of non-existence.) Given two personally incomparable lives a and b, Incomparability Transmission implies that the outcomes (a, b) and (b, a) are both incomparable with the outcomes (a, a) and (b, b) (where the first and second terms in each ordered pair represent the lives of P_1 and P_2 respectively). Small Improvements implies that we can find an outcome which is exactly as good for one individual and strictly better for the other than (a, a) while still being incomparable with both (a, b) and (b, a), or an outcome which is exactly as good for one individual and strictly worse for the other, while still being incomparable with both (a, b) and (b, a). Let's concentrate on the first case, and assume the improvement is to the first individual, giving this person the life a^+. (The other cases are very similar.) The outcome (a^+, a) is by assumption still incomparable with (a, b) and (b, a). Richness then guarantees that the domain of the impersonal betterness relation includes the two lotteries described in Table 2. Both possible outcomes of Lottery 2 are incomparable with both possible outcomes of Lottery 1, so Negative Dominance implies that Lottery 2 is not better than Lottery 1. But Lottery 2 gives P_1 a stochastically dominant personal lottery, and P_2 a stochastically equivalent lottery, so Personal Good implies that Lottery 2 is strictly better.[13]

This argument establishes, we think, that Negative Dominance and Personal Good are jointly incompatible with the existence of incomparable lives. To briefly consider the other ways out: Richness is only used to secure the existence of two lotteries with the structure of Lottery 1 and Lottery 2 in Table 2, which are hardly exotic. Incomparability Transmission is only used to secure the conclusion that, if a and b are personally incomparable, then (a, a) and (b, b) are impersonally incomparable with (a, b) and (b, a). And this claim seems very plausible in its own right, since if a and b are incomparable, what basis could we have for thinking that (a, b) or (b, a) are better or worse than (a, a) or (b, b)?[14] And finally, the application of Small Improvements should be uncontroversial: given that (a, a) is incomparable with (a, b) and (b, a), then it seems (in the spirit of insensitivity to small improvements) that it should be possible to slightly improve or worsen

[13] Lederman [2023a,b] presents negative results showing a conflict between Negative Dominance and "unidimensional" principles that are analogous to but much stronger than our Personal Good. Throughout, he uses an assumption that in our setting would amount to taking personal betterness to be complete. The present result shows that, if personal betterness fails to be complete, then much weaker assumptions already suffice to generate an inconsistency. Similarly Lederman [2023b, Proposition 3.1] provides a consistency result for a package which includes Richness, Negative Dominance, and Personal Good. Our result shows that if personal betterness (or, in his setting, the "unidimensional" order) can be incomplete, then this result breaks under plausible further assumptions.

[14] Given the possibility of small improvements, it is not plausible to claim that a pair of outcomes like (a, a) and (a, b) will in general be equally good.

In our particular example, one might claim that (a, a) is better than (a, b) on grounds of equality, or alternatively that (a, b) is better than (a, a) on grounds of diversity. But these claims would not seriously undermine our argument. If you think that more diverse outcomes are better, for instance, this only seems to strengthen our claim that neither outcome of Lottery 2 is better than either outcome of Lottery 1. If you think that more equal outcomes are better, we could simply change the example to sweeten an outcome in Lottery 1 rather than Lottery 2.

someone's life in (a, a) while maintaining that incomparability.[15] We think this principle is plausible in full generality, but, as with Richness, rejecting it in particular cases would not be enough to save the combination of Personal Good and Negative Dominance, since all our argument relies on is that there is *some* case with the structure of the one above.[16]

An important special case of the preceding result is where one of the incompa-

[15]One might think that (a^+, a) is better than (a, b) on the grounds that it is better for someone and worse for no one. But this reasoning, though intuitive, cannot be sound, since it would lead to cycles of betterness: for instance, given incomparable lives a and b, we would get that (a, a) is better than (a^-, b), which is better than (b, b^-), which is better than (b^-, a^+), which is better than (a, a).

There is a close analogy here with the *prima facie* plausible decision-theoretic principle that if one gamble yields a better outcome than another in some state of nature, and does not yield a worse outcome in any state of nature, then it is better overall. While this looks like an unobjectionable statewise dominance principle, it leads to cycles in the presence of incomparability [MacAskill, 2013]. The lesson is that Pareto (resp. statewise dominance) principles must require that the superior outcome (resp. gamble) is *at least as good for every individual* (resp. *in every state*), not just that it isn't worse for any individual (resp. in any state).

[16]Nebel [2020] also presents an opaque sweetening-inspired puzzle in a population-ethical context. His puzzle compares the outcomes (a, b) and (b^+, a^+), where a and a^+ are both incomparable with b and b^+. He points out that the latter outcome is intuitively better than the former, and that this verdict can be derived from Pareto, anonymity, and transitivity, but that this conflicts with the "Person-Affecting Restriction" (PAR), which claims that one outcome is better than another only if there is someone for whom it's better. Nebel sees this as a reason to give up PAR, and we agree. But giving up PAR does not resolve our puzzle, which uses only the much weaker and more plausible principle of Incomparability Transmission. (Unlike PAR, Incomparability Transmission is consistent with the other principles Nebel invokes, even in the presence of personal incomparability.) An alternative response to Nebel's argument might be to reject his anonymity assumption, for instance weakening it to "relative anonymity" (which asserts that applying the same permutation of individual welfare levels to a pair of outcomes does not affect the evaluative comparison between them). But since we don't rely on anonymity, this escape route likewise doesn't generate a response to our argument.

As Nebel does to Hare's argument, it's also possible to generate a structural analogue of our argument that inverts the roles of states and individuals. In this inverted argument, the analogue of Negative Dominance asserts that *if no individual's personal lottery under L_1 is at least as good as any individual's personal lottery under L_2, then L_1 is not strictly better than L_2*, while the analogue of Personal Good asserts that *if in at least one state, the outcome of L_1 is Suppes-Sen superior to the outcome of L_2 (i.e., Pareto superior to a permutation of that outcome), and in all other states, the outcomes of L_1 and L_2 are identical, then L_1 is better than L_2*. (To put these principles into conflict, just switch the rows and columns of Lottery 2 in Table 2, with a^+ in the top right. The argument will of course require auxiliary assumptions analogous to those used in Proposition 4.1.)

rable “lives” is nonexistence. Many philosophers accept the following principle:

Variable-Population Incompleteness There is some life a such that, whenever two outcomes o and o' are identical except that one possible individual does not exist in o and exists with life a in o', o and o' are incomparable in terms of impersonal value.

One motivation for this principle is the belief that existence and non-existence are incomparable in terms of *personal* value, coupled with a principle like Incomparability Transmission. This might motivate the view that populations of different size are *always* incomparable [Bader, 2022]. Alternatively, whatever one thinks about the personal value of nonexistence, one might accept Variable-Population Incompleteness to avoid undesirable conclusions about impersonal value. For instance, critical-range utilitarianism, which holds that adding an individual with slightly positive welfare to a population results in incomparability, can avoid both the Repugnant Conclusion [Parfit, 1984, §131, pp. 387–90] and the Sadistic Conclusion [Arrhenius, 2000].[17] But the preceding argument shows us that both these views are in tension with Negative Dominance and Personal Good, taken together:

Proposition 4.2 (Variable-Population Incomparability)**.** *Given Richness, the principles of Negative Dominance, Personal Good, and Small Improvements rule out Variable-Population Incompleteness.*

To see this, let b in Table 2 represent nonexistence and a be a life of the sort described by Variable-Population Incompleteness, such that adding someone

[17] Gustafsson [2020] proposes a version of critical-range utilitarianism that is motivated by a fourth category of value called “undistinguishedness” and shows that it escapes versions of the Repugnant and Sadistic Conclusions.

to the population with life a yields incomparability. Then Variable-Population Incompleteness implies that (a, b) and (b, a) are incomparable with (a, a) and (b, b).[18] The rest of the proof then follows the proof of Proposition 4.1. The upshot is that Negative Dominance and Personal Good together tell strongly against views that see changes in population size as a source of incomparability.

5 Incomparable outcomes

In the last section, we focused on incompleteness in the personal ranking of lives, and those forms of incompleteness in the impersonal ranking of outcomes that arise from it. But incomparability between outcomes can have other sources as well. For example, those who resist interpersonal tradeoffs or aggregation might hold that, in some or all cases where o is better than o' for some individuals and worse for others, o and o' are impersonally incomparable. Or one might hold that there are imprecise "exchange rates" between different desirable features of a population, like total vs. average welfare or average welfare vs. equality, so that pairs of populations that trade off these dimensions are sometimes incomparable. (Critical-range utilitarianism, discussed above, can be seen as a view of this kind, in which "small population size" is a desideratum that trades off imprecisely with total welfare.)

This section will show that, as long as population-level incomparability has a plausible "separability" property analogous to Incomparability Transmission, these views are vulnerable to essentially the same difficulty that we explored in the last section. In other words, Negative Dominance and Personal Good support

[18] Note that, while the argument as stated appeals to the "empty population" (b, b), we can avoid this by simply adding another person living life c to all outcomes under consideration.

a general argument against incompleteness in impersonal value, whether or not it arises from incompleteness in personal value.

Our argument will roughly parallel the argument in the last section, with *subpopulations* taking the place of single individuals. A subpopulation is a set of possible individuals that have been arbitrarily indexed (i.e., assigned positive integers from 1 up to the number of individuals in the subpopulation). We also introduce the concept of a *distribution*, which stands in approximately the relation to subpopulations that lives stand in to individuals. A distribution is a function from the first n positive integers to the set of lives (including nonexistence). The *size* of a subpopulation is the number of possible individuals it contains, and the size of a distribution is the number of lives it assigns. For any subpopulation S consisting of possible individuals $S_1, S_2, \ldots, S_n$, and any distribution D of size n or less, we say that S *realizes* D when S_1 experiences the life $D(1)$, S_2 experiences the life $D(2)$, and so on, with any leftover individuals in S being assigned nonexistence.

We then give the following definition:

Strong Incomparability Distributions A and B are *strongly incomparable* if, whenever outcomes o and o' are identical except that some subpopulation realizes distribution A in outcome o and distribution B in outcome o', o is impersonally incomparable with o'.

To say that A and B are strongly incomparable, in other words, is analogous to saying that lives a and b are personally incomparable in a way that satisfies Incomparability Transmission: exchanging one for the other, while holding everything else fixed, always generates incomparability.[19]

[19]Note, however, that we have not formally introduced a betterness ordering on distributions.

It is plausible that, whenever two outcomes are incomparable, the two distributions they realize (relative to a given indexing of the set of individuals who exist in either outcome) are strongly incomparable. This would amount, however, to a fairly strong "separability" assumption, to the effect that in comparing two outcomes, we can focus only on those individuals whose lives differ between the outcomes and ignore everyone else. We choose, therefore, to focus on the weaker claim that, if impersonal betterness is incomplete, then at least *some* distributions are strongly incomparable. For example, perhaps a sufficiently sharp tradeoff between total welfare and inequality always generates incomparability (e.g., a choice between giving everyone in a particular subpopulation a modestly good life or giving half of them excellent lives and the other half mediocre lives), even if for less sharp tradeoffs, the impersonal betterness ranking can depend on features of the unaffected population. As we will see, just as in the last section, our puzzle does not require the full force of even this modest existential claim.

We now come to our result:

Proposition 5.1 (Population-Level Incomparability)**.** *Given Richness, the principles of Negative Dominance, Personal Good, and Small Improvements rule out strong incomparability.*

Proof. Suppose that A and B are strongly incomparable. Letting n be the size of the larger of A and B, choose two disjoint subpopulations $S = \{S_1, S_2, \ldots, S_n\}$ and $T = \{T_1, T_2, \ldots, T_n\}$. Now, consider the two lotteries in Table 3.

These are exact analogues of the two lotteries used to prove Proposition 4.1, with individuals P_1 and P_2 replaced by subpopulations S and T, and lives a and

To say that A and B are strongly incomparable does not mean that they are impersonally incomparable in our formal sense, because the latter relation is only defined on outcomes.

(a) Lottery 1

	Heads	Tails
S	A	B
T	B	A

(b) Lottery 2

	Heads	Tails
S	A^+	B
T	A	B

Table 3

b replaced by distributions A and B. Each outcome assigns a distribution to each subpopulation: in the Heads outcome of Lottery 1, for instance, S_1 receives $A(1)$, S_2 receives $A(2)$, etc, and likewise T_1 receives $B(1)$, T_2 receives $B(2)$, etc, with any leftover individuals being assigned nonexistence.

Now, it follows from the definition of strong incomparability that (A, B) and (B, A) are both incomparable with (A, A) and (B, B) (where the first and second terms of each ordered pair represent the distributions realized by S and T respectively). By Small Improvements, there exists either an outcome (A^+, A) or (A, A^+) that slightly improves one life in (A, A), or else an (A^-, A) or (A, A^-) that slightly worsens one life in (A, A), that is still incomparable with both (A, B) and (B, A). As before, we focus on the first case. Richness guarantees that the domain of the impersonal betterness relation includes the two lotteries in Table 3. And in comparing these two lotteries, Negative Dominance implies that Lottery 2 is not better than Lottery 1, while Personal Good implies that it is.

□

As before, we note that all our argument really requires is that we can find distributions A, A^+, and B such that (A^+, A) and (B, B) are both incomparable with (A, B) and (B, A). We think it will be hard for proponents of incomparability between outcomes to deny this claim (we ourselves cannot think of any plausible incomplete outcome axiology that does so), even if they reject our Small Improvements assumption or do not believe that any distributions are strongly incomparable. It seems hard, therefore, to maintain that the impersonal betterness ranking of outcomes is incomplete without denying either Negative Dominance or Personal Good.[20]

6 From outcomes to lotteries

In the last section, we presented an argument for completeness in the impersonal betterness ranking of outcomes. But the impersonal betterness relation, we have assumed, is defined not just on outcomes but on lotteries. And a complete ordering of outcomes need not entail a complete ordering of lotteries. For instance, even if outcomes are completely ordered by total welfare, the ordering of lotteries might be incomplete because it supervaluates over a range of reasonable risk attitudes. At an extreme, for instance, lotteries might be ordered only by Stochastic Dominance [Tarsney, 2020].

In this section, however, we take our argument a step further by showing that Personal Good goes a long way toward bridging the gap between completeness for outcomes and completeness for lotteries. Thus, if we accept completeness for outcomes on grounds that include Personal Good, there is at least significant reason to accept completeness for lotteries as well. This argument has somewhat limited scope (limited to what we will call "finite rational lotteries"), and it depends more heavily than the arguments of the last two sections on assumptions

[20] Note also that the preceding argument applies to infinite distributions just as much as to finite distributions. Thus it rules out, for instance, strong incomparability between distributions like $(1, 0, 1, 0, \ldots)$ and $(0, 1, 0, 1, \ldots)$.

that go beyond Negative Dominance and Personal Good. So we do not think that this pair of principles support completeness for lotteries as strongly as they support completeness for lives and outcomes. But as we will see, the auxiliary assumptions we require are still quite plausible.

Those assumptions consist of Stochastic Dominance and two new principles:

Substitution For any outcome o in and any sufficiently large subpopulation S (where what counts as "sufficiently large" can depend on o), there is an outcome exactly as impersonally good as o in which only individuals in S exist.

Transitivity If o_1 is at least as impersonally good as o_2, and o_2 is at least as impersonally good as o_3, then o_1 is at least as impersonally good as o_3.

Transitivity is a familiar principle that is very widely (though not universally) accepted, and we won't have anything new to say for or against it. Substitution says, intuitively, that any degree of impersonal value can be realized by any sufficiently large group of possible individuals. This principle might be denied, for instance, by those who think that populations in which different individuals exist are always incomparable. But against the background assumption of completeness for outcomes, it is hard to see how one might deny it. Given completeness, in particular, Substitution follows from

Weak Anonymity For any outcome o and permutation π of the set of possible individuals, o is neither strictly better nor strictly worse than $\pi(o)$ (where $\pi(o)$ is the outcome in which $\pi(o)(x) = o(\pi(x))$.

Weak Anonymity seems like a minimal expression of impartiality. Given completeness for outcomes, Substitution can be understood as a still weaker sort of anonymity principle, which permits significant (though not unlimited) partiality between possible individuals.

Our argument will be restricted to *finite rational lotteries*: lotteries that have only finitely many possible outcomes, with each outcome having a rational probability. The "finitude" restriction is substantive, but we regard the "rationality" restriction as merely technical—it seems obvious that if lotteries with rational probabilities are completely ordered, then lotteries with real probabilities are as well.

Here is our next result:

Proposition 6.1 (Completeness for Lotteries)**.** *If outcomes are completely ordered by impersonal betterness then, given Richness, the principles of Personal Good, Stochastic Dominance, Substitution, and Transitivity imply that the set of finite rational lotteries is also completely ordered by impersonal betterness.*

Proof. Our strategy is to show that, given Personal Good, Stochastic Dominance, Substitution, and Transitivity, the ranking of finite rational lotteries "supervenes on" the ranking of outcomes: that is, for any finite rational lotteries L_1 and L_2, we can find a pair of outcomes o_1 and o_2 such that L_1 is at least as good as L_2 if and only if o_1 is at least as good as o_2.[21] This implies that, if the ranking of outcomes is complete, then the ranking of finite rational lotteries is as well.

Consider, then, an arbitrary pair of finite rational lotteries L_1 and L_2. To

[21] We take inspiration here from Thomas [2023], who proves a similar supervenience result from a different but overlapping set of premises. (Roughly, Thomas does not assume Transitivity or Stochastic Dominance, but makes an anonymity assumption stronger than Substitution.)

reduce the comparison between L_1 and L_2 to a comparison between outcomes, we will subject them to a series of transformations. First, we find the least common denominator of all the outcome probabilities in L_1 and L_2, which we will call n. We can then imagine L_1 and L_2 as gambles in a situation with n equiprobable states. (As in our previous arguments, imagining lotteries as gambles is simply a useful presentational device—states don't play any formal role in our argument.) Take the set of individuals who exist in any outcome of L_1 or L_2, index them arbitrarily, and call the resulting subpopulation S_1. L_1 and L_2 can then be represented as assigning distributions to this subpopulation in each state, as in Table 4.

(a) Lottery 1

	$s_1\left(\frac{1}{n}\right)$	$s_2\left(\frac{1}{n}\right)$	$\ldots$	$s_n\left(\frac{1}{n}\right)$
S_1	$D_{1.1}$	$D_{1.2}$	$\ldots$	$D_{1.n}$

(b) Lottery 2

	$s_1\left(\frac{1}{n}\right)$	$s_2\left(\frac{1}{n}\right)$	$\ldots$	s_n
S_1	$D_{2.1}$	$D_{2.2}$	$\ldots$	$D_{2.n}$

Table 4

The next step is to replace each outcome of each lottery with an equally good outcome, using "fresh" sets of individuals, so that no possible individual exists in more than one state in a given lottery. (It doesn't matter whether the set of individuals who might exist in one lottery overlaps with the set of individuals who might exist in the other.) Call the resulting lotteries $L_{1'}$ and $L_{2'}$. These transformed lotteries are illustrated in Table 5. Substitution (together with Richness) guarantees that we can find lotteries with the required properties, since for each outcome of each lottery, we can find an equally good outcome involving a fresh set of individuals. Stochastic Dominance guarantees that $L_{1'}$ is exactly as good as L_1, and $L_{2'}$ is exactly as good as L_2.

The final step is to "stack" all the individual outcomes other than nonexistence,

(a) Lottery 1′

	s_1 $\left(\frac{1}{n}\right)$	s_2 $\left(\frac{1}{n}\right)$	$\dots$	s_n $\left(\frac{1}{n}\right)$
$S_{1'.1}$	$D_{1.1}$	—	$\dots$	—
$S_{1'.2}$	—	$D_{1'.2}$	$\dots$	—
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$S_{1'.n}$	—	—	$\dots$	$D_{1'.n}$

(b) Lottery 2′

	s_1 $\left(\frac{1}{n}\right)$	s_2 $\left(\frac{1}{n}\right)$	$\dots$	s_n $\left(\frac{1}{n}\right)$
$S_{2'.1}$	$D_{2.1}$	—	$\dots$	—
$S_{2'.2}$	—	$D_{2'.2}$	$\dots$	—
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$S_{2'.n}$	—	—	$\dots$	$D_{2'.n}$

Table 5

in each lottery, into state s_1. Since all the possible outcomes of $L_{1'}$ involve disjoint populations, and likewise for $L_{2'}$, this stacking is possible. The result is a pair of lotteries $L_{1''}$ and $L_{2''}$ which guarantee an "empty" outcome in every state except s_1, while yielding a potentially massive population in that single state. These lotteries are illustrated in Table 6. Since all the states are equiprobable, this second transformation does not affect any individual's personal lottery, so Personal Good implies that $L_{1''}$ is exactly as good as $L_{1'}$, and likewise $L_{2''}$ is exactly as good as $L_{2'}$.

(a) Lottery 1″

	s_1 $\left(\frac{1}{n}\right)$	s_2 $\left(\frac{1}{n}\right)$	$\dots$	s_n $\left(\frac{1}{n}\right)$
$S_{1'.1}$	$D_{1.1}$	—	$\dots$	—
$S_{1'.2}$	$D_{1'.2}$	—	$\dots$	—
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$S_{1'.n}$	$D_{1'.n}$	—	$\dots$	—

(b) Lottery 2″

	s_1 $\left(\frac{1}{n}\right)$	s_2 $\left(\frac{1}{n}\right)$	$\dots$	s_n $\left(\frac{1}{n}\right)$
$S_{2'.1}$	$D_{2.1}$	—	$\dots$	—
$S_{2'.2}$	$D_{2'.2}$	—	$\dots$	—
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$S_{2'.n}$	$D_{2'.n}$	—	$\dots$	—

Table 6

Let o_1 designate the outcome of $L_{1''}$ in state s_1, and o_2 designate the outcome of $L_{2''}$ in state s_1. Since $L_{1''}$ and $L_{2''}$ yield the same outcome in every state except s_1, Stochastic Dominance implies that, if o_1 is at least as good as o_2, then $L_{1''}$ is at least as good as $L_{2''}$; and likewise, if o_2 is at least as good as o_1, then $L_{2''}$ is at least as good as $L_{1''}$. Since we have assumed that outcomes are completely ordered, we know that at least one of these conditions obtains. Therefore, we know that either $L_{1''}$ is at least as good as $L_{2''}$ or vice versa (or both). Finally, Transitivity implies that L_1 is exactly as good as $L_{1''}$, L_2 is exactly as good as $L_{2''}$, and therefore that L_1 is at least as good as L_2 if and only if $L_{1''}$ is at least as good as $L_{2''}$. Thus either L_1 is at least as good as L_2, or L_2 is at least as good as L_1, or both. Since L_1 and L_2 were chosen arbitrarily, this shows that finite rational lotteries are completely ordered.

□

If you were convinced by the arguments for outcome completeness in the last two sections, should you be convinced by this argument as well? We think so. The additional premises are Stochastic Dominance, Substitution, and Transitivity. Stochastic Dominance is, as we have said, a very plausible and widely accepted principle. And, although it neither implies nor is implied by Negative Dominance, both principles reflect the basic idea that we should evaluate lotteries based on the overall value and probability of their possible outcomes, so the principles have some overlapping appeal.[22] Transitivity is likewise widely accepted. And although Substitution could certainly be questioned, it is hard to reject if we have already

[22]On the other hand, those who accept Negative Dominance because they accept the stronger principle of Statewise Negative Dominance will, as we have seen, reject Stochastic Dominance. But we don't think they'll reject the particular uses we make of it in our proof. In fact, in the framework of states and gambles, we could replace Stochastic Dominance in the preceding argument with

Strong Statewise Dominance If in every state, the outcome of gamble G_1 is at least as good as the outcome of gamble G_2, then G_1 is at least as good as G_2. If in addition there is some state in which the outcome of G_1 is strictly better than the outcome of G_2, then G_1 is strictly better than G_2.

Anyone who accepts Statewise Negative Dominance should, we think, find this principle compelling. (Our argument would then also require the innocuous principle that subdividing a state, without changing the outcome of a given gamble in that state, does not affect the value of the gamble.)

accepted completeness for outcomes, since it is then entailed by Weak Anonymity.

7 Responses

The last three sections have shown that, against very weak background assumptions, Negative Dominance and Personal Good, are incompatible with various forms of value incompleteness. So far, we have presented our results as arguments against these forms of incompleteness, and we expect that many readers will wish to take them this way. But, of course, one could also choose to give up Negative Dominance or Personal Good.[23] In this section we will consider both possibilities, focusing especially on the former.

There are at least three reasons, beyond intuitions about cases, why one might be motivated to hang onto incomparability despite the preceding arguments. First, it seems strange that principles about the betterness ordering of lotteries (like Negative Dominance and Personal Good) should constrain the betterness ordering of outcomes. This seems backwards, given the plausible thought that the betterness of outcomes is more fundamental (with betterness of lotteries being grounded in betterness of outcomes plus decision-theoretic principles of rational preference under risk). It would seem particularly odd if principles about the *impersonal* betterness of lotteries forced us to accept particular conclusions about the *personal* betterness of lives—the comparative personal value of, say, the lives of an artist and a banker seems like a prior question to principles like Negative Dominance and Personal Good. If those lives are incomparable, one might think, that is a fact that ethical and decision-theoretic principles must bend

[23] Indeed, of the three of us, one (DS) takes our results as an argument against incompleteness, one (HL) rejects Negative Dominance, and one (CT) rejects Personal Good.

to accommodate.

Second, there are powerful theoretical arguments for incompleteness in certain infinitary contexts: "constructive incompleteness" results in infinite ethics [Zame, 2007, Lauwers, 2010] and the decision theory of lotteries without finite expected value [Lauwers, 2016] show that given some relatively weak assumptions, any ranking of infinite outcomes or lotteries respectively must be either incomplete or non-constructive (having no finite description). Many find constructiveness a very compelling constraint on a theory of value: a non-constructive betterness relation seems objectionably arbitrary (for instance, in the context of infinite ethics, it would require infinitely many apparently-arbitrary choices about which of two countably infinite sets of possible individuals should be prioritized over the other) and would be impossible for any finite agent to represent, let alone know.

Third and finally, even without incompleteness, Personal Good and Negative Dominance conflict with one another in infinitary contexts—-when comparing lotteries with infinitely many possible outcomes and that involve infinite or unboundedly large populations. For example, Table 7 describes a pair of lotteries in which, for every possible individual, the personal lottery given by Lottery 1 stochastically dominates the personal lottery given by Lottery 2; and yet Lottery 2 is certain to result in a positive total of personal value, while Lottery 1 is certain to result in a negative total of personal value. Thus Personal Good implies that Lottery 1 is better than Lottery 2 while, at least given a total utilitarian theory of impersonal value, Negative Dominance implies that it cannot be.[24] While it

[24]This example is inspired by one in Kowalczyk (ms). In Kowalczyk's example, Personal Good conflicts with Statewise Equivalence (the principle that, if two gambles have equally good outcomes in every state, then they're equally good). We have modified the example to create a conflict with Negative Dominance. (Indeed, in our case, Personal Good conflicts with every dominance principle we can think of. In particular, it conflicts with Superdominance—the

is not at all obvious *which* principle we should give up in these contexts, there is strong reason to think that Personal Good and Negative Dominance cannot *both* be completely universal truths. Whichever principle we choose to give up in infinite contexts, this should arguably undercut its intuitive appeal in finite contexts as well, and make us more willing to sacrifice that principle in order to preserve space for incomparability.

So there is reason to consider giving up either Personal Good or Negative Dominance. We won't have much new to say about the former possibility, but will mention a few familiar considerations that might push one in that direction.[25] First, as noted earlier, egalitarians may reject Personal Good for reasons unrelated to incompleteness, since it requires insensitivity to distributive features of outcomes that egalitarians think are important. For example, in the following table, Personal Good prefers the lottery on the right, while *ex post* egalitarians have reason to

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principle that, if each possible outcome of L_i is strictly better than every possible outcome of L_j, then L_i is better than L_j. *A fortiori*, it conflicts with Stochastic Dominance and Statewise Dominance as well.) Related examples of conflicts between dominance principles and *ex ante* Pareto-like principles appear in Goodsell [2021], Wilkinson [2023], and Nebel [2023].

While we have used totalism to illustrate the argument in the main text, the axiological assumptions required to generate this sort of conflict between Personal Good and Negative Dominance are very weak. It would suffice, for instance, to assume a principle we might call "Inverse Lives": There exists a pair of lives l^+ and l^- such that, among outcomes in which everyone who exists has one of those two lives, any outcome where most people have l^+ is better than any outcome where most people have l^-. This assumption is endorsed, for instance, by standard versions of total utilitarianism, critical-level utilitarianism, prioritarianism, average utilitarianism, variable-value utilitarianism, and many forms of egalitarianism.

Finally, we have presented the example as involving only finite (though unboundedly large) populations. We could switch to an infinite-population example by replacing every instance of nonexistence with Table 7 with a neutral life, with welfare 0. This would allow us to weaken Personal Good by restricting it to fixed-population contexts, where every individual who might exist in either lottery is certain to exist in both lotteries. Then the principle could simply assert that if one lottery gives every individual a stochastically dominant personal lottery, then it's impersonally better.

[25] In conversation, Ralf Bader (who independently noticed the central puzzle of this paper) says that he would give up Personal Good.

(a) Lottery 1: Better for All, but Surely Bad

	s_1 $\left(\frac{1}{3}\right)$	s_2 $\left(\frac{2}{9}\right)$	s_3 $\left(\frac{4}{27}\right)$	s_4 $\left(\frac{8}{81}\right)$	s_5 $\left(\frac{16}{243}\right)$	$\cdots$	$\Pr(+\mid\exists)$
P_1	-1	1	1	1	1	$\cdots$	2/3
P_{2-3}	—	-1	1	1	1	$\cdots$	2/3
P_{4-7}	—	—	-1	1	1	$\cdots$	2/3
P_{8-15}	—	—	—	-1	1	$\cdots$	2/3
P_{16-31}	—	—	—	—	-1	$\cdots$	2/3
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Total	-1	-1	-1	-1	-1	$\cdots$	

(b) Lottery 2: Worse for All, but Surely Good

	s_1 $\left(\frac{1}{3}\right)$	s_2 $\left(\frac{2}{9}\right)$	s_3 $\left(\frac{4}{27}\right)$	s_4 $\left(\frac{8}{81}\right)$	s_5 $\left(\frac{16}{243}\right)$	$\cdots$	$\Pr(+\mid\exists)$
P_1	1	-1	-1	-1	-1	$\cdots$	1/3
P_{2-3}	—	1	-1	-1	-1	$\cdots$	1/3
P_{4-7}	—	—	1	-1	-1	$\cdots$	1/3
P_{8-15}	—	—	—	1	-1	$\cdots$	1/3
P_{16-31}	—	—	—	—	1	$\cdots$	1/3
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Total	1	1	1	1	1	$\cdots$	

Table 7 An illustration of the conflict between Personal Good and Negative Dominance (and other dominance principles) in contexts involving infinitely many possible outcomes and unbounded/infinite populations. Each individual has the same probability of existing in both lotteries (with "—" representing nonexistence), and two possible welfare levels conditional on existence. The rightmost column gives each individual's total probability of positive welfare, conditional on existence. The bottom row gives the total welfare realized in each state.

prefer the lottery on the left, since it guarantees perfect equality.[26]

	Heads	Tails
P_1	1	0
P_2	1	0

	Heads	Tails
P_1	$1+\epsilon$	0
P_2	0	$1+\epsilon$

Second, as already noted, you might be willing to reject Personal Good because of its incompatibility with dominance principles in infinite contexts. Finally, while the idea that "what is at least as good for everyone, and better for some, is better overall" is extremely compelling, it is not obvious whether a lottery can be "good for" an individual—we derive no benefit from how our lives *might have* turned out, over and above how they *in fact* turn out. The "*ex post* individualism" expressed by Pareto is, in this respect, more compelling than the "*ex ante* individualism" expressed by Personal Good.

None of these considerations, though, have any obvious connection with incompleteness. So the desire to preserve incompleteness in the face of the results from the last two sections does not seem to provide any distinctive motivation for jettisoning Personal Good. It would be interesting if it turns out to be possible to find such motivations. Perhaps, for instance, there is a view which endorses population-level incompleteness without endorsing incompleteness for individual lives, and which sees incompleteness as arising from global distributive features of outcomes whose significance counts against Personal Good in something like the way that egalitarian concerns do. But we won't try to develop such a view here.

[26]It's not obvious how much this point undercuts the preceding arguments, though, since we can plausibly adapt those arguments to guarantee that the outcome preferred by Personal Good involves equal or greater *ex post* equality. In Table 2, for instance, this will be the case if we sour an outcome in Lottery 1 rather than sweetening an outcome in Lottery 2. Guaranteeing that we can do this, though, requires a modest strengthening of Small Improvements.

A route that is more directly motivated by incompleteness, we think, is to reject Negative Dominance.[27] Someone who accepts Personal Good might motivate the rejection of Negative Dominance on "individualist" grounds as follows: "As individualists, our fundamental concern is with the personal good of each individual considered separately. 'Impersonal value' is just a sort of summary statistic. For instance, the claim that $(2, 0)$ is impersonally better than $(0, 1)$ is a useful but incomplete way of summarizing more fundamental facts, namely, that $(2, 0)$ is better for P_1, worse for P_2, and the first difference in personal value is greater than the second. The concept of impersonal value plays a useful role—for instance, it is an appropriate guide to action under certainty. But because it doesn't capture everything we fundamentally care about, comparisons of lotteries based only on the impersonal value of their possible outcomes—like the comparisons expressed by dominance principles—will sometimes overlook important considerations. These oversights show up in infinite contexts, where dominance reasoning can recommend options that are worse for all concerned. What the arguments of §§4–5 show is that similar oversights can happen in finite contexts: Negative Dominance, because it is sensitive only to the 'overall' value of outcomes, can miss a decisive reason for ranking one lottery above another." This strikes us as at least a plausible reason why sufficiently committed individualists might not feel attached to Negative Dominance.

A second point is that we can soften the blow of rejecting Negative Dominance, by replacing it with a weaker principle that captures some of its intuitive appeal:

[27] One view that takes this route is Gustafsson's (2020) Undistinguished Critical-Range Utilitarianism (see note 17). Although he does not discuss this point explicitly, Gustafsson's preferred rule for evaluating risky prospects (p. 94) satisfies Personal Good but not Negative Dominance.

Supervaluational Negative Dominance Lottery L_1 is better than lottery L_2 only if, on every admissible completion of personal and impersonal betterness, some possible outcome o_1 of L_1 is better than some possible outcome o_2 of L_2.

An "admissible completion" is one that satisfies some set of fundamental constraints on the ranking of outcomes. Suppose that these constraints include transitivity and (*ex post*) Pareto. Then we conjecture that Supervaluational Negative Dominance will be compatible, in finite contexts, with the various sets of assumptions that were shown to conflict with Negative Dominance in §§4–5. For instance, in the case in Table 2, every admissible completion of betterness rankings of lives and outcomes must rank either (a^+, a) or (b, b) above (a, b) and (b, a).[28]

Arguably, the intuitive appeal of Negative Dominance is explained by its similarity to the slightly subtler principle of Supervaluational Negative Dominance. This conclusion will be especially appealing to those who understand incompleteness in terms of indeterminacy between complete orderings (e.g. Broome [1997], Dorr et al. [2021]). On this understanding, Negative Dominance says that for L_1 to be better than L_2, there must be some outcome of L_1 that is determinately better than some outcome of L_2; but all that is really required is that *it's determinately the case that* some outcome of L_1 is better than some outcome of L_2—it can be indeterminate *which* pair of outcomes those are.

It is worth noting, however, that Supervaluational Negative Dominance still

[28] Suppose that, on a given completion, a is at least as good as b in terms of personal value. Then, by transitivity, a^+ is better than b, and by Pareto, (a^+, a) is strictly better than (a, b) and (b, a). The other possibility is that b is strictly better than a, in which case (b, b) is strictly better than (a, b) and (b, a) by Pareto.

conflicts with Personal Good in infinite contexts (e.g., in Table 7 above). So the retreat to this weaker principle holds less appeal for those whose rejection of Negative Dominance is motivated by an absolute commitment to individualistic principles like Personal Good.

Whatever its motivation, the rejection of Negative Dominance would be a surprising and significant conclusion. It's an intuitively compelling principle in its own right. But also, as we have seen, many fans of incompleteness (like Schoenfield [2014], Bales et al. [2014], and Doody [2023]) have adopted the stronger *Statewise* Negative Dominance as a central principle for evaluating risky options with incomparable outcomes. If the preceding arguments force us to give up Negative Dominance then, *a fortiori*, they force us to reject this family of views as well.

8 Conclusion

Against weak background assumptions, Personal Good and Negative Dominance rule out many intuitive forms of value incomparability. Determining which of these should be rejected will require making progress on deep and pressing questions. To mention a few salient ones: is the betterness of lives and outcomes conceptually prior to the betterness of lotteries, in such a way that we cannot reason from principles about lotteries to conclusions about the comparative betterness of lives or outcomes? Is there a distinctive "impersonal" notion of betterness – betterness from the perspective of the world – that can't be reduced to facts about how things go for particular individuals? How should we understand phenomena which putatively support the existence of value incomparability?

Our incompatibility results display surprising constraints on the answers to these questions, and connections between them. But far from being resisted, we think these constraints should be welcomed. They illuminate the difficult choices that must be made in developing a general theory of the good, and thus may point the way toward more satisfactory theories.

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