1 e-Companion "*Conspicuous Consumption and Dynamic Pricing*" by Raghunath Singh Rao and Richard Schaefer

This appendix sets up a discrete version of the model where both WTP for quality and WTP for status utility are independently distributed. This model is briefly discussed towards the end of section 6 (footnote 18) in body of the paper.

Let ν^i reflect WTP for quality, distributed by pmf $f_\nu(0) = f_\nu(1) = \frac{1}{2}$. Individuals, unlike the baseline scenario, also vary in their valuation of social status, where this preference is formally denoted as $w^i \in \{\underline{w} < \frac{1}{2}, \overline{w} = 1 - \underline{w}\}$. w^i is distributed by pmf $f_w(\underline{w}) = f_w(\overline{w}) = \frac{1}{2}$ and is independent of ν^i, implying that $f_{\nu,w}(\nu^i, w^i) = \frac{1}{4}\ \forall\ \{\nu^i, w^i\} \in \{\{0,1\} \times \{\underline{w}, \overline{w}\}\}$.

To fully characterize each consumer's purchase timing problem, we first denote each $\{\nu^i, w^i\}$ pair type: $\{\nu^1, w^1\} = \{0, \underline{w}\}$, $\{\nu^2, w^2\} = \{0, \overline{w}\}$, $\{\nu^3, w^3\} = \{1, \underline{w}\}$, and $\{\nu^4, w^4\} = \{1, \overline{w}\}$. Here, each consumer $i = 1, ..., 4$ optimizes the following:

(1a) $$\max_{x_1^i, x_2^i} x_1^i \left[U(\nu^i, w^i, 1)\right] + x_2^i \left[U(\nu^i, w^i, 2)\right] + \left(1 - x_1^i - x_2^i\right)\left[U(\nu^i, w^i, N)\right]$$

(1b) $$s.t.\ x_1^i + x_2^i \leq 1$$

where

(2a) $$U(\nu^i, w^i, 1) = \nu^i + (1-\delta)\lambda w^i \left(\frac{\nu^i + \sum_{j \in C_1^{-i}} \nu^j}{1 + \sum_{j \in C_1^{-i}} 1}\right) + \delta\lambda w^i \left(\frac{\nu^i + \sum_{j \in C_2^{-i}} \nu^j}{1 + \sum_{j \in C_2^{-i}} 1}\right) - P_1$$

(2b) $$U(\nu^i, w^i, 2) = \delta\nu^i + (1-\delta)\lambda w^i \left(\frac{\nu^i + \sum_{j \in C_{-1}^{-i}} \nu^j}{1 + \sum_{j \in C_{-1}^{-i}} 1}\right) + \delta\lambda w^i \left(\frac{\nu^i + \sum_{j \in C_2^{-i}} \nu^j}{1 + \sum_{j \in C_2^{-i}} 1}\right) - \delta P_2$$

(2c) $$U(\nu^i, w^i, N) = (1-\delta)\lambda w^i \left(\frac{\nu^i + \sum_{j \in C_{-1}^{-i}} \nu^j}{1 + \sum_{j \in C_{-1}^{-i}} 1}\right) + \delta\lambda w^i \left(\frac{\nu^i + \sum_{j \in C_{-2}^{-i}} \nu^j}{1 + \sum_{j \in C_{-2}^{-i}} 1}\right)$$

and

$$C_k^{-i} = \{j \neq i : x_t^j = 1 \text{ for some } t \leq k\} \tag{3a}$$

$$C_{-k}^{-i} = \{j \neq i : x_t^j = 0 \text{ for all } t \leq k\} \tag{3b}$$

The producer optimally responds to the above preferences with price-skimming sequence $\{P_1^\star, P_2^\star(P_1^\star)\}$:

$$\max_{P_1} \quad \left(P_1 - \frac{k}{2}Q^2\right)\sum_{i=1}^{4} x_1^i + \delta\left[\left(P_2^\star(P_1) - \frac{k}{2}Q^2\right)\sum_{i=1}^{4} x_2^i\right] \tag{4a}$$

$$s.t. \quad P_2^\star(P_1) = \arg\max_{P_2} \left(P_2(P_1) - \frac{k}{2}Q^2\right)\sum_{i=1}^{4} x_2^i \tag{4b}$$

A full characterization of the producer's pricing solution proved intractable. We instead chose four values of cost parameter k which allow, but do not necessarily guarantee, sales to occur in both periods. Pricing sequences are listed below for each sampled value of k.

Proposition 1 does not hold over the entire domain, as $\frac{\partial}{\partial\lambda}\frac{P_2^*(P_1^*)}{P_1^*} > 0$ for the pricing sequences $\{P_1^\star = \frac{3Q(1-\delta)+2\lambda(1-\overline{w})(1-\delta)+2\lambda\delta\overline{w}}{3},\ P_2^\star(P_1^\star) = \frac{2\lambda\overline{w}}{3}\}$ and $\{P_1^\star = \frac{2Q(1-\delta)+\lambda\overline{w}(1-\delta)+\lambda\delta(1-\overline{w})}{2},\ P_2^\star(P_1^\star) = \frac{\lambda(1-\overline{w})}{2}\}$. Notably, the producer sells to $\{\nu^3, w^3\} = \{1, \underline{w}\}$ and $\{\nu^4, w^4\} = \{1, \overline{w}\}$ in $t = 1$ under both price schemes. In the final period, the firm sells to $\{\nu^2, w^2\} = \{0, \overline{w}\}$ when $\{P_1^\star = \frac{3Q(1-\delta)+2\lambda(1-\overline{w})(1-\delta)+2\lambda\delta\overline{w}}{3},\ P_2^\star(P_1^\star) = \frac{2\lambda\overline{w}}{3}\}$ and both $\{\nu^1, w^1\} = \{0, \underline{w}\}$ and $\{\nu^2, w^2\} = \{0, \overline{w}\}$ for $\{P_1^\star = \frac{2Q(1-\delta)+\lambda\overline{w}(1-\delta)+\lambda\delta(1-\overline{w})}{2},\ P_2^\star(P_1^\star) = \frac{\lambda(1-\overline{w})}{2}\}$. While neither $i = 1, 2$ place a premium on quality, $\{\nu^2, w^2\}$ greatly cares about social status; here, the second period cohort values status at least as much, if not more than, the early buyer cohort. This drives the result that $\frac{\partial}{\partial\lambda}\frac{P_2^*(P_1^*)}{P_1^*} > 0$, although it is not clear whether this occurs in equilibrium when both ν^i and w^i are continuously distributed.