More or Less: A Model and Empirical Evidence on Preferences for Under and Over-Payment in Trade-in Transactions

Jungkeun Kim, Raghunath Singh Rao, Kyeongheui Kim, Akshay R. Rao

WEB APPENDIX

Derivation of Proof for the Existence of Preference Reversal for the Case $\alpha \neq \beta$

The value function can be written as below:

$$\upsilon(x) = \begin{cases} x^{\alpha} & if\ x \geq 0 \\ (-\lambda)(-x)^{\beta} & if\ x < 0 \end{cases},$$

where $0 < \alpha \leq \beta < 1$. Loss aversion is captured through $\lambda > 1$, while α and β capture the diminishing sensitivity of the value function in the gain and loss domains respectively. A consumer's most preferred pricing format would be based upon the following optimization problem:

$$\arg\max_{d} (-\lambda)(p_n + d)^{\beta} + (p_u + d)^{\alpha} \qquad \text{(B1)}$$

Since $p_u / p_n \equiv k$, we can re-write this expression as:

$$\arg\max_{d} (-\lambda)(p_n + d)^{\beta} + (kp_n + d)^{\alpha}$$

The first order condition yields:

$$(-\lambda)\beta(p_n + d)^{\beta-1} + \alpha(kp_n + d)^{\alpha-1} = 0$$

This implies that the optimal allowance *d** is the solution of the following non-linear equation:

$$\frac{(p_n + d^*)^{1-\beta}}{(kp_n + d^*)^{1-\alpha}} = \left(\frac{\lambda\beta}{\alpha}\right) \qquad \text{(B2)}$$

Notice that unlike the solution obtained in equation (A5), this equation (B2) cannot be solved analytically to yield an expression for *d**. But using arguments similar to the ones employed in solving the non-parametric form problem above, we can conclude that the preference reversal, if it occurs, would be at a value of *k* that yields *d**=0. Let us denote this critical value of *k* as $\hat{k}$. By substituting *d**=0 in equation (B2) we get:

$$\hat{k} = \left[\left(\frac{\alpha}{\lambda\beta}\right) \frac{1}{(p_n)^{\beta-\alpha}} \right]^{\frac{1}{1-\alpha}} \qquad \text{(B3)}$$

Notice that if $\alpha = \beta$, then $\hat{k} = \lambda^{\frac{1}{\alpha-1}}$, which = *Q*.

A preference reversal would occur if $0<\hat{k}<1$, which in turn requires:

$$\left[(\frac{\alpha}{\lambda\beta}) \frac{1}{(p_n)^{\beta-\alpha}} \right]^{\frac{1}{1-\alpha}} < 1.$$

Simplifying the above expression yields:

$$p_n > \left[\frac{\alpha}{\lambda\beta} \right]^{\frac{1}{\beta-\alpha}} \qquad \text{(B4)}$$

Notice that this is not a very restrictive condition. The term inside the square brackets in equation (B4) is less than 1 and hence the RHS of equation (B4) is less than 1. In other words, except for the cases where p_n is very small, even with $\alpha \neq \beta$, we always get a preference reversal.

Hence we can restate the proposition contained in the paper as below:

Proposition 1: *As long as condition (B4) is satisfied, for k< $\hat{k}$, consumers prefer a pricing format that yields d>0, while for k≥ $\hat{k}$, consumers prefer a pricing format that yields* $d \leq 0$.

Proof: As discussed above, $\hat{k} = \left[(\frac{\alpha}{\lambda\beta}) \frac{1}{(p_n)^{\beta-\alpha}} \right]^{\frac{1}{1-\alpha}}$ is the critical point which separates the preference for H-H pricing from L-L pricing.

We check for the second order condition. If indeed *d** maximizes equation (B1) then it needs to satisfy the second order condition. The second derivative of equation (B1) yields:

$(-\lambda)\beta(\beta-1)(p_n+d)^{\beta-2} + \alpha(\alpha-1)(kp_n+d)^{\alpha-2}$. Therefore at *d** it must be true that:

$(-\lambda)\beta(\beta-1)(p_n+d^*)^{\beta-2} + \alpha(\alpha-1)(kp_n+d^*)^{\alpha-2} < 0$.

This implies:

$(\lambda)\beta(1-\beta)(p_n+d^*)^{\beta-2} < \alpha(1-\alpha)(kp_n+d^*)^{\alpha-2}$,

or,

$$\frac{(\lambda)\beta(1-\beta)}{(p_n+d^*)^{2-\beta}} < \frac{\alpha(1-\alpha)}{(kp_n+d^*)^{2-\alpha}}.$$

Re-writing the above expression yields

$$\frac{(\lambda)\beta(1-\beta)}{(p_n+d^*)^{1-\beta}(p_n+d^*)} < \frac{\alpha(1-\alpha)}{(kp_n+d^*)^{1-\alpha}(kp_n+d^*)},$$

or,

$$\frac{(\lambda)\beta(1-\beta)}{(p_n+d^*)} < \frac{\alpha(1-\alpha)(p_n+d^*)^{1-\beta}}{(kp_n+d^*)(kp_n+d^*)^{1-\alpha}}.$$

This implies:

$$\frac{(\lambda)\beta(1-\beta)}{(p_n+d^*)} < \frac{\alpha(1-\alpha)}{(kp_n+d^*)} \left\{ \frac{(p_n+d^*)^{1-\beta}}{(kp_n+d^*)^{1-\alpha}} \right\}.$$

Substitute equation (B2) in the equation above on the RHS:

$$\frac{(\lambda)\beta(1-\beta)}{(p_n+d^*)} < \frac{\alpha(1-\alpha)}{(kp_n+d^*)}\left\{(\frac{\lambda\beta}{\alpha})\right\},$$

and the simplification yields:

$$\frac{(1-\beta)}{(1-\alpha)} < \frac{(p_n+d^*)}{(kp_n+d^*)}. \tag{B5}$$

Note that since $0<\alpha\le\beta<1$, so:

$\frac{(1-\beta)}{(1-\alpha)} \le 1$ and by definition $\frac{(p_n+d^*)}{(kp_n+d^*)}>1$.

So, equation (B5) is true and hence the second order condition is satisfied as well.