ARTICLE IN PRESS

International Journal of Research in Marketing xxx (xxxx) xxx

Contents lists available at ScienceDirect

International Journal of Research in Marketing

journal homepage: www.elsevier.com/locate/ijresmar

Full length article

Stay positive or go negative? Memory imperfections and messaging strategy

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ARTICLE INFO

Article history:
Received 10 March 2020
Available online xxxx

Keywords:
Political advertising
Memory models
Persuasion
Game theory

ABSTRACT

This paper studies the optimal mix of message content in elections while explicitly accounting for voters' memory imperfections. We build an analytical model of a political contest between two candidates facing an election with an electorate consisting of supporters, opponents, and undecided voters. The candidates take decisions on advertising sequence and content (positive vs. negative). Our model explicitly considers the role of memory processes, in particular *decay* (the idea that memories fade with time) and *rehearsal* (the idea that accessing a memory eases its recall,) that crucially affect how effective ads are in influencing choice. The model yields several interesting insights: (a) when both candidates have low initial support, they invest only in positive messages; (b) when both candidates are endowed with high initial support, their messaging strategies take a "pulsing" shape involving negative advertising accompanied by positive advertising; (c) when one candidate has low initial support while the other has high initial support, the former adopts a "pulsing" strategy while the latter adopts only positive advertising. Furthermore, we show that a candidate with low initial support facing a candidate with high initial support responds with a messaging strategy bunched with negative content towards the end of the election cycle. Our model's predictions are shown to find empirical support in a dataset assembled from 2016 U.S. Senate races.

1. Introduction

During the summer of 2015, former Democratic senator Russ Feingold announced his candidacy for the Wisconsin Senate to face off Republican Ron Johnson. From June to October 2016, Feingold maintained a healthy lead over Johnson of 5–11% (see Fig. 1) while lagging in the money raised during these months (see Fig. 2). The tone of campaign messaging varied significantly across the two candidates – from August to October, Johnson released a large number of negative TV ads attacking Feingold while Feingold released a mix of positive and negative ads, with the overall tone being mainly positive (see Fig. 3).

The Feingold-Ross battle provides motivation for the kinds of questions that this paper attempts to answer. What determines the mix of positive and negative advertising for a candidate, given budget constraints? What determines the intertemporal pattern of this advertising, i.e., should a candidate mix positive and negative ads throughout, or show a preponder-

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https://doi.org/10.1016/j.ijresmar.2022.01.001
0167-8116/

Please cite this article as: X. Li, Raghunath Singh Rao, O. Narasimhan et al., Stay positive or go negative? Memory imperfections and messaging strategy, International Journal of Research in Marketing, https://doi.org/10.1016/j.ijresmar.2022.01.001

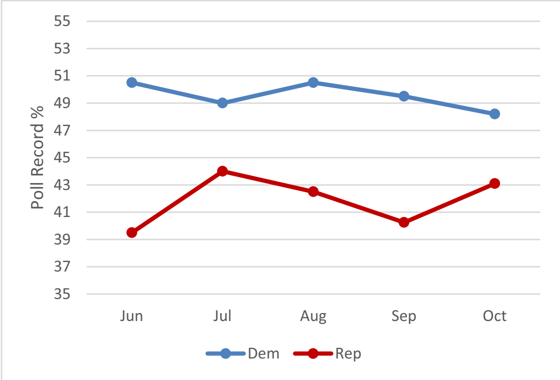

Fig. 1. Monthly average polling numbers for Feingold (D) and Johnson (R) during the 2016 Senate Elections.

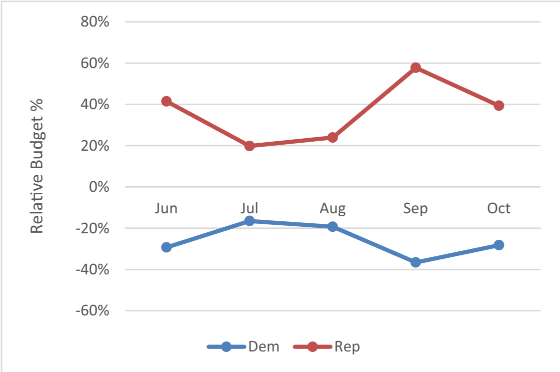

Fig. 2. Relative Budget Ratio for Feingold and Johnson in the 2016 Senate Elections.

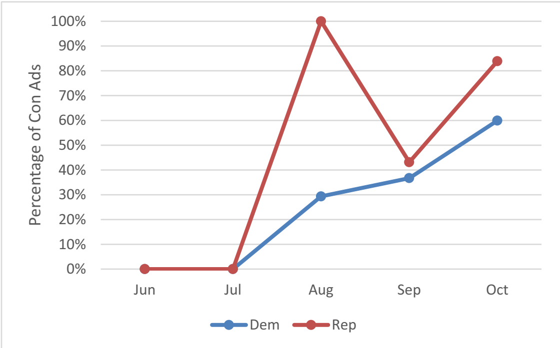

Fig. 3. Message Tone for Feingold and Johnson in the 2016 Senate Elections.

ance of positive ads early on and negative ads later, or lead with negative ads and move to a positive message later? We show that the pattern of messaging seen in the Feingold-Johnson example is far from random and is broadly consistent with a model of message communication that accounts for candidates' poll positions and their budgets, in conjunction with how such messages affect voters' memories.

Understanding political advertising is important for both economic and policy reasons. Political advertising has been an activity of significant and growing importance in the U.S. for many decades now. In the Presidential election alone, advertising spending increased from about $700 million in 2008 to exceed $1 billion in 2016 and 2020. It is estimated that total advertising expenditure in all U.S. political campaigns in 2012 was close to $6 billion (Issenberg 2012), and increased to about $8.5 billion in 2020 (Homonoff, 2020). Political advertising is expected to continue to play a critical role in determining the outcome of elections in the U.S., given the 2010 *Citizens United vs. Federal Election Commission* Supreme Court ruling that removed most limits on campaign contributions. Further, the increase in spending on political advertising has been accompanied by a rise in unease around the role played by 'negative' advertising in influencing the outcomes of these races. For example, one study (Page, 2006) found that 90% of ads run in the final 60 days of all House and Senate campaigns in the 2006 midterm elections nationwide were negative. Of the $522 million spent in the 2004 Bush vs. Kerry presidential campaign, it is estimated that about 70% was spent on negative advertising (Gandhi, Iorio, & Urban, 2015).

While there is a long history of theoretical and empirical research in marketing and economics studying various aspects of advertising, most of this literature is in the context of advertising for goods and services offered by for-profit firms (Stigler, 1961; Nelson, 1974; Ackerberg, 2001; Lodish et al., 1995; Hu, Lodish, & Krieger, 2007). Historically, issues related to political marketing have primarily been pursued by scholars in areas like political science and communication (notable exceptions are Ahluwalia, 2000; Gordon & Hartmann, 2013; Shachar, 2009). This is unfortunate, since the vast theoretical and empirical literature in marketing devoted to the study of various aspects of consumer choice provides a unique and rich foundational basis to study political marketing in general, and political advertising in particular. To quote Kim, Rao, and Lee (2009), "*...studying consumers' choice of political candidate is arguably at least as important as studying which brand of carbonated soft drink they prefer or purchase*." At the same time, it is worth emphasizing that while choices made by voters share many similarities with consumer decisions in other contexts, the political context is somewhat unique and hence requires different models and methods to study. For example, unlike the majority of purchase scenarios, the "sale" in an election cycle is typically concentrated on a single day (the election date).[1]

This paper attempts to answer several open questions related to political advertising, with a particular focus on the tone and the inter-temporal pattern of advertising. In doing so, it seeks to partly rectify the lack of formal studies in the context of political marketing within the marketing discipline. Answers to these questions carry both positive and normative implications. A proper analytical framework has the potential to enhance our understanding of the rationale behind candidates' choice of message content and possibly allow us a more nuanced understanding of the rise of negative advertising in political campaigns. Normatively, findings from such a model could help advertisers design their message content strategies in political marketing. Beyond political marketing, the results of this paper provide a deeper understanding of many marketing contexts where competitive advertising involves a mix of positive and negative messaging.

We depart from most prior work in the area of political advertising (see DellaVigna & Gentzkow, 2009 for a review) by explicitly bringing in the psychological literature on the importance of memory imperfections, especially in the context of evaluating and responding to advertising. In particular, we emphasize two processes – *decay* (the idea that memories fade with time) and *rehearsal* (the idea that accessing a memory eases its recall) that crucially affect how effective ads are in influencing choice. Voter memory is likely to be an essential determinant of election outcomes, given that in most important elections, voters are often bombarded with a myriad of messages, both positive and negative, over a long duration. For example, a randomized study conducted during Rick Perry's 2006 gubernatorial campaign showed that a given week's advertising (in the absence of opponent advertising) significantly raised Perry's vote share (4.73 percentage points per 1,000 GRPs). Still, within a week, this number had fallen close to zero (Issenberg, 2012).

Nevertheless, past research has demonstrated (e.g., Lodge, Steenbergen, & Brau, 1995) that while many voters often do not remember the specifics of the messages, they do make use of summary evaluations of candidates on the day of the election to cast their vote. Thus, it is not surprising that political campaigns and pollsters eagerly track the "likability" ratings of candidates (a summary of the affective evaluation of a candidate, independent of his or her policy positions). The dominant narrative of the outcome of the 2012 U.S. Presidential elections was that while Mitt Romney was perceived as a candidate with rich experience on important issues like the economy, he could never overcome Barack Obama's significant likability edge (Jones, 2012). In sum, both voters' memory processes and the valence of the messages received (which are likely to impact likability significantly) during the election cycle are likely to be crucial drivers of the election outcomes.

Our approach is game-theoretic: We build a model of a political contest wherein two candidates endowed with initial budgets face an election at a future date with an electorate consisting of supporters, opponents, and undecided voters. We model the candidates making decisions on advertising quantity and content (positive vs. negative) over two periods. Our analysis yields many interesting insights. First, when both candidates have low initial support, they invest only in positive messages. In contrast, when both candidates are endowed with high initial support, their messaging strategies take a "pulsing" shape involving an initial negative (positive) advertising combined with positive (negative) advertising. Second, when one candidate is endowed with low initial support, whereas the other is endowed with high initial support, the former follows a "pulsing" messaging strategy while the latter employs only positive advertising. For these equilibrium-advertising strategies, we reveal that a candidate with low initial support who is confronted with a candidate with high initial support responds with a messaging strategy weighted towards negative content at the end of the election cycle. Third, we show that memory decay decreases the likelihood of overall positivity in campaigns while the rehearsal effect increases this likelihood. Our findings suggest asymmetric candidate budgets as a significant impetus for negative advertising and help rationalize the

[1] This, of course, changed somewhat dramatically in the US Presidential elections in 2020, where early voting accounted for more than 100 million ballots cast (Garrison, 2020).

[2] Indeed, an average Senate incumbent raised close to 7.42 MM USD in the 1998 election cycle while an average challenger raised about 1.52 MM – a ratio of about 5 to 1. This ratio increased to about 8 to 1 in the 2016 Senate election cycle, wherein an average incumbent raised about 12.71 MM compared to an average challenger raising only about 1.59 MM. The intervening years have seen a steady rise in this lopsided fundraising ratio. (Source: www.opensecrets.org).

rise in negative campaigning observed in recent years.[2] Finally, we collect data on support, budget, and the tone of advertising for competitive U.S. Senate races during 2016 and find face validity for our key results.

The remainder of the paper starts with a review of the literature in political advertising, with a specific focus on negative ads, followed by the literature on memory imperfections. Next, we provide the details of our model, followed by analysis and results. We then present a number of extensions of the main model followed by an empirical analysis of the key results using data from U.S. Senate elections. We conclude with limitations and suggestions for future research.

2. Literature

We organize our literature review around three themes: (1) the impact of political advertising on election outcomes, (2) negative political advertising, and (3) the role of memory processes in advertising. The academic literature in these areas is vast, and we only highlight critical papers that have a bearing on our research questions and our model development.

2.1. Impact of political advertising on election outcomes

At its core, political advertising by candidates is an attempt at persuading voters to go to the polls and vote for them. Early studies examining the persuasion effects directed at voters found "minimal effects" (Berelson, Lazarsfeld and McPhee 1954; Lazarsfeld, Berelson, & Gaudet, 1944), i.e., political advertising does very little to change voters' allegiance (DellaVigna and Gentzkow 2009). In a similar vein, Gordon and Hartmann (2013), using advertising expenditures by county, find that advertising elasticities are 0.03 - much below estimated elasticities for packaged consumer goods (the bulk of which range from 2 to 5). These results showing minuscule effects are at odds with work on advertising in other contexts, which finds that exposure almost always helps in building brand awareness and frequently results in higher sales. Indeed, more recent studies using randomized experiments have found that voting may be habit-forming (Gerber, Green, & Shachar, 2003) and that there is a significant impact of advertising (Gerber, Karlan, & Bergan, 2009) on voter attitudes towards candidates and election outcomes. Shachar (2009), using state-by-state data on presidential elections, "solves" the empirical puzzle of higher participation under close races through the inclusion of marketing variables. Gordon and Hartmann (2013) build a structural model of U.S. Presidential elections to construct a counterfactual of the direct vote in place of the existing Electoral College and conjecture that this would result in higher advertising expenditures and consequently a higher turnout.

2.2. Negative political advertising

Several studies have specifically examined the impact of negative advertising on election outcomes. Using voter survey data from the American National Election Studies (ANES) project, combined with advertising information from the 2000 elections, Che, Iyer, and Shanmugam (2007) find that negative advertising positively affects both turnout and the likelihood of voting for the featured candidate, and that voters have a higher preference for negative ads closer to the election date. Goldstein and Freedman (2002), also through the use of 2000 U.S. election data, empirically illustrate that negative campaigning could increase voter involvement. Lovett and Shachar (2011), using election data from U.S. house elections, find that the tendency to go negative under close races is mediated by voters' knowledge and candidates' budgets. Gandhi et al. (2015) posit that the duopolistic nature of electoral contests could explain the prominence of negativity in U.S. elections. The rationale is that in contests with more than two candidates, engaging in negativity creates positive externalities for candidates who are not objects of the attack, thus dissuading candidates from the practice. They empirically test and find support for this argument using U.S. election data from non-presidential primary races.

2.3. Empirical evidence on memory and advertising

The decay of memory over time and the strategies used by consumers to preserve memories have received much attention from researchers (e.g., Ebbinghaus, 1885 for an early discussion of a decay function). Of these strategies, one of the best-studied is the use of *rehearsal* (Schacter, 1996), which refers to the idea that recall or repetition of past events increases their recall at a future date. Consumer choice is obviously impacted by memory, and one of the critical roles of advertising is to improve accessibility via external cues (see Bettman, 1979 for an early survey). A large body of literature has looked at how features of advertising, such as message content, media format, and message dispersion, impact memory and consumer choice (see Janiszewski, Noel, & Sawyer, 2003 for a meta-analysis). Other studies have sought to link the impact of subsequent competitor advertising to recall of past advertising (e.g., Burke & Srull, 1988; Pechmann & Stewart, 1990).

2.4. Formal models of memory and advertising dynamics

While equilibrium models of outcomes with individual memory imperfections are somewhat rare, there are aggregate models on advertising dynamics that have connections to our work. Freimer and Horsky (2012) use a discrete-time period setup to analyze when two firms might engage in an ad pulsing strategy, but they do not explicitly model the role of memory. Prasad and Sethi (2004) consider a stochastic response to advertising in a duopoly model where the advertising dynamics are driven by market level exogenous parameters for ad effectiveness, discounting, churn and cost. However, they do not explicitly consider memory parameters, although one could think of ad effectiveness being driven by limitations on consumers' memories. In a discrete setting, Bronnenberg (JMR 1998) analyzes a single firm's decision on the level and frequency of advertising, rationalizes a "pulsing" strategy, and connects it to memory effects via Markov carryover. In two important contributions, Aravindakshan and Naik (2011, 2015) explicitly incorporate memory awareness while including delayed forgetting to study issues such as pulsing strategies for a single brand.

In terms of specifics of memory formulation informed by the psychological literature, Mullainathan (2002) provides a foundation for formally incorporating rehearsal and the dependence on cues to study the impact of limited memory on the economic decision-making of agents. Dow (1991) and Chen, Iyer, and Pazgal (2010) study the effects of limited memory on price competition, with memory modeled as a coarse partition of prices.[3] Our modeling approach is closest to Sarafidis (2007), who extends Mullainathan's approach to model memory effects in the presence of a strategic actor (firm or political candidate) who needs to decide upon his messaging strategy. We advance Sarafidis's approach to the context of political competition and explicitly model a candidate's endogenous mixing of negative and positive messaging for a given budget at the start of a political cycle. The problem of investing in positive versus negative advertising has previously been studied by Harrington and Hess (1996) and Skaperdas and Grofman (1995) using a Hoteling-like spatial approach. These authors show that a candidate who is perceived as having less attractive personal attributes runs a relatively more negative campaign. However, these models do not incorporate voters' limited memory. Finally, Soberman and Sadoulet (2007), in a paper closest in spirit to ours, study the impact of campaign spending limits on advertising strategies, and explain the recent rise in partisan campaigns. Their main finding is that higher spending levels lead campaigns to focus on their own constituents with high intensity.

3. Model

The phenomenon we seek to understand, namely advertising content and its pattern over time in the context of a political election, requires us to pay careful attention to several features unique to the problem. The first set of such considerations relates to the uniqueness of elections as a context. As Gordon et al. (2012) point out, elections differ from regular goods and services in that: i) they are typically "winner-takes-all" contests; ii) while voters may enjoy the process of voting, their preferred candidate might not win (unlike regular consumers who typically get to enjoy the goods and services they purchase); iii) the probability of any single voter impacting the outcome is minuscule, and iv) elections have a clear endpoint, so all the action is focused on making voters behave in a certain way on a given date.

The second set of considerations relates to the specific features of how voter memory and information processing are crucial to explaining the impact of political advertising on choices. Important features here are the following. First, memory imperfections have important implications in political contests. As an example, Neuman (1986), using National Election Study (NES) series survey data for the period 1956–1980, found that about 56% of the voters could not name either senatorial candidate even at the height of Senate campaigns. Second, as pointed out earlier, while the decay of details of the messages is very swift, campaigns have an impact independent of recall because they help voters form an overall impression of the candidate based on a "summary affective evaluation of the candidates" (Lodge et al., 1995). In other words, what matters is not so much the content of the message but the valence (positivity or negativity), which persists and influences choice. Third, elections are often driven by undecided voters (Hayes & McAllister, 1996), and these voters are likely to be "less involved" and more influenced by advertising (Fiske, Kinder, & Larter, 1983).

The third set of considerations relates to the features of negative advertising per se, and how it is processed, in contradistinction to a positive message. Damore (2002) points out that positive advertising attracts undecided voters and reduces uncertainty. By contrast, negative advertising undermines support for the opponent – negative messages generally "do not offer any direct reason to support a candidate, only reasons not to support the opposition" (Damore, 2002, page 672, Skaperdas & Grofman, 1995). This difference is crucial and will play a significant role in how we model the two types of advertising. It is also essential in that it suggests a role for both positive and negative advertising in any political campaign.

The final set of considerations is more by way of stylized facts that provide useful directions for building a model of political advertising. Thus, related to the intertemporal pattern of political advertising, Damore (2002) empirical analysis of the campaigns of presidential candidates from 1976 to 1996 suggests that: i) candidates trailing are more likely to engage in negative advertising; ii) negative advertising becomes more common as the election date approaches; iii) as the difference in the candidate's support base increases, so does the likelihood of negative advertising; and iv) there is a great deal of reciprocity - negative advertising from one candidate frequently triggers a rebuttal negative ad. These findings suggest the importance of incorporating the distance from the election date and the gap between the contestants' support in any analytical framework that seeks to explain the dynamics of a messaging strategy. To preview the predictive power of our analytical model, findings (i) and (iii) are consistent with Proposition 3 in the next section.

We now turn to a formal exposition of our model, taking care to point out how we include most of the points mentioned above.

[3] There is little marketing academic work using field data for memory-based applications (exceptions are Mehta, Rajiv, & Srinivasan, 2004 and Sahni, 2015).

3.1. Preliminaries

We model a game of electoral contest between two candidates $i \in \{A, B\}$ over two periods (stages) consisting of $t = 1, 2$. Each candidate is assumed to be endowed at the beginning of the campaign with a total budget $b^i \leq 2$ and an initial supporter base $0 \leq r_0^i \leq 1$. Also, a fraction of the voter population S_0 is undecided. Hence, $r_0^A + r_0^B + S_0 = 1$. In any period, a candidate can use the endowed budget to engage in either positive (spreading positive messages about oneself) or negative (negative statements about the rival candidate) messaging. Crucially, the two types of advertising impact voter choice differently. Following prior work (Skaperdas and Grofman, 1995), positive advertising from candidate A makes voters favorably inclined to vote for A. By contrast, negative advertising from A succeeds in turning off some loyal supporters of candidate B into undecided voters but does not attract them to A, per se. The only kind of advertising that changes voters from undecided to supporters is positive advertising. To reiterate, candidate A can use negative advertising to 'convert' some of B's loyal supporters to the undecided pool. He can then use positive advertising to convert them into his loyal supporters. (Positive advertising acts on the pool of undecided voters and directly influences them into becoming loyalists). Therefore, the only way to steal the other candidate's vote share is to engage in negative followed by positive advertising.

Formally, let z_t^i be an indicator that takes a value of -1 if candidate i has invested in a negative message during period t and +1 if candidate i has invested in a positive message during period t, with the constraint that in any given period, a candidate can invest in at most one positive or one negative message. z_t^i takes a value of 0 if candidate i has invested in no advertising during the period t. For candidates A and B, the set of strategies for advertising investment can therefore be denoted as $(z_1^A, z_2^A; z_1^B, z_2^B)$. Furthermore, the budget constraint implies $|z_1^i| + |z_2^i| \leq b^i$.[4]

3.2. Memory formulation

We make use of the psychological findings discussed in the last section on memory decay and rehearsal and follow the modeling approach used by Mullainathan (2002) and Sarafidis (2007) to incorporate memory imperfections. First, we need to formulate how messages released by a candidate evolve in a voter's mind. Our base model is a two-period game, implying that to determine the outcome of the election, we need to consider the final memory of each candidate in a voter's mind in period 2, at the end of which the voting takes place. In period 2, the memory of a message released in period 1 is determined by both memory decay and message rehearsal. Specifically, we have the following expression for m_2^{1A+}, a voter's memory in period 2 of candidate A's positive message released in period 1:

$$m_2^{1A+} = \rho m_1^{1A+} + I\{z_1^A = +1\}\kappa I\{z_2^A = +1\} \tag{1a}$$

where ρ is the memory decay parameter, κ is the rehearsal parameter and $I\{x\}$ is an indicator variable that takes a value of 1 if x is true and 0 otherwise. Note that a voter's memory of candidate A's positive message in period 1 (this is the *initial state* of memory) is $m_1^{1A+} = 1$ if $z_1^A = +1$ and $m_1^{1A+} = 0$ otherwise (because a message within a given period has neither memory decay nor message rehearsal). The first term on the right-hand side of (1a) represents the decay of memory from period 1 to period 2, with a higher ρ representing lower decay. The second term captures the rehearsal effect; the release of a new positive message in period 2 improves the memory of the previous positive message released in period 1, through rehearsal. That is, the rehearsal effect of a past message in the current period occurs when a message of similar valence appears in the current period and thus a voter can "rehearse" it.

We obtain a similar expression for m_2^{1B+}, a voter's memory in period 2 of candidate B's positive message released in period 1:

$$m_2^{1B+} = \rho m_1^{1B+} + I\{z_1^B = +1\}\kappa I\{z_2^B = +1\} \tag{1b}$$

where the initial state satisfies $m_1^{1B+} = 1$ if $z_1^B = +1$ and $m_1^{1B+} = 0$ otherwise.

Note that the aim of both candidates' advertising investment is to influence voters in the final period, which depends on the message stock at the end of period 2.The total positive message stock at the end of period 2 for each candidate, including the memory of the message of period 1 (m_2^{1A+} and m_2^{1B+}) and the current message in period 2 (m_2^{2A+} and m_2^{2B+}), can be written as: $M_2^{A+} = m_2^{1A+} + m_2^{2A+}$ and $M_2^{B+} = m_2^{1B+} + m_2^{2B+}$.

On the other hand, we can write an expression for the negative message about candidate A (released by B in period 1) remembered in the period 2 as:

$$m_2^{1A-} = \rho m_1^{1A-} + I\{z_1^B = -1\}\kappa I\{z_2^B = -1\} \tag{2a}$$

[4] We assume one unit of budget brings one piece of positive or negative message; modeling the decreasing marginal impact of advertising does not change our analysis. For example, we can assume one message requires monetary amount b_1 in the first period and b_2 in the second-period, with $b_1 < b_2$ to capture the decreasing marginal impact of advertising. Then, we could assume the total budget to be b_1+b_2, without affecting our results

where a voter's memory in period 1 of negative message about candidate A in the same period (initial memory state) is $m_1^{1A-} = 1$ if $z_1^B = -1$ and $m_1^{1A-} = 0$ otherwise. Likewise, the expression for a negative message about candidate B (released by A in period 1) remembered in period 2 is:

$$m_2^{1B-} = \rho m_1^{1B-} + I\{z_1^A = -1\}\kappa I\{z_2^A = -1\} \tag{2b}$$

where a voter's memory in period 1 of a negative message about candidate B in the same period (initial state) is $m_1^{1B-} = 1$ if $z_1^A = -1$ and $m_1^{1B-} = 0$ otherwise. Similarly, the total negative message stock about each candidate at the end of period 2, including the memory of the message of period 1 and the current message in period 2, becomes: $M_2^{A-} = m_2^{1A-} + m_2^{2A-}$ and $M_2^{B-} = m_2^{1B-} + m_2^{2B-}$.

Using Eqs. (1a,b) and (2a,b), we get the following for candidate A's strategy of advertising investment:

For period 1, $m_1^{1i+} = I\{z_1^i = +1\} = \begin{cases} 1 & z_1^i = +1 \\ 0 & otherwise \end{cases}$, and

$m_1^{1i-} = I\{z_1^{-i} = -1\} = \begin{cases} 1 & z_1^{-i} = -1 \\ 0 & otherwise \end{cases}$, where –$A$ denotes B and –B denotes A.

For period 2,

$$m_2^{1A+}(z_2^A = +1|z_1^A, z_1^B) = \rho m_1^{1A+} + I\{z_1^A = +1\}\kappa = I\{z_1^A = +1\}(\rho + \kappa)$$

$$m_2^{1A+}(z_2^A = -1|z_1^A, z_1^B) = \rho m_1^{1A+} = I\{z_1^A = +1\}\rho$$

$$m_2^{1B-}(z_2^A = +1|z_1^A, z_1^B) = \rho m_1^{1B-} = I\{z_1^A = -1\}\rho$$

$$m_2^{1B-}(z_2^A = -1|z_1^A, z_1^B) = \rho m_1^{1B-} + I\{z_1^A = -1\}\kappa = I\{z_1^A = -1\}(\rho + \kappa).$$

To summarize, the outcome of the election game is given by the final stock of the memories of the two candidates and by the initial conditions of the optimization problem (the initial shares of the candidates). The memories evolve via Eqs. (1) and (2) based on the advertisement investments by the candidates. The total memory stock for each candidate is determined by Period 1 investment (if any), Period 2 investment (if any), and the decay and rehearsal parameters.

3.3. Value functions

We put more structure to the above basic framework to model competition in the environment. Let $L^i\left(M_2^{i-}\right) = LM_2^{i-}$ represent the fraction of candidate i's loss of support at the end of period 2 because of the total memory of candidate j's negative campaign during the election cycle (summarized by M_2^{i-}), where L is assumed to be small enough to ensure $0 < LM_2^{i-} \leq 1$. This equation formulates the loss of share of a candidate to the undecided pool as being proportional to the amount of negative advertising by the opponent. As a result, under the memory mechanisms characterized by Eqs. (1) and (2), candidate i's support share evolves from r_0^i in the initial period to $r_0^i - r_0^i L^i(M_2^{i-}) = r_0^i[1 - L^i(M_2^{i-})]$ in the final period, in which $r_0^i L^i(M_2^{i-})$ denotes the share of voters who convert from candidate i's supporters in the initial period to switchers in the final period. Meanwhile, considering the added switchers converting from candidate i,$r_0^i L^i(M_2^{i-})$, we can write the total share of undecided voters at the end of period 2 as[5]:

$$S_2 = S_0 + r_0^A L^A(M_2^{A-}) + r_0^B L^B(M_2^{B-}) \tag{3}$$

In other words, the share of undecided voters evolves from S_0 in the initial period to S_2 in the final period, following the memory mechanisms described in Eqs. (1) and (2).

As discussed before, this pool of undecided voters would go to each of the candidates in proportion to their respective positive advertising levels. Note that this is based on two simple and intuitive assumptions: (a) Undecided voters do not have a strong loyalty to any candidate, and (b) Their vote is thus more likely to be influenced by advertising (that provides them with the reason to vote for a specific candidate) and a candidate with higher positive advertising is more likely to provide an undecided voter to vote in favor of a specific candidate. Indeed, the detailed empirical evidence in Franz and Ridout (2007) is consistent with our assumption that the undecided ("low-information voters" in their terminology) are most likely to be persuaded via advertising. Furthermore, recent detailed evidence in Spenkuch and Toniatti (2018) suggests, consistent with our assumption, that "a one standard deviation increase in the partisan difference in advertising raises the partisan difference in vote shares by about 0.5 percentage points". We can then express the share of undecided voters supporting candidate i at the end of period 2 as:

[5] Note that within our setting, the entire history of advertising matters, unlike in Markov models where only the current state matters. Furthermore, we can explicitly include error terms in the share equations to incorporate uncertainty in the model. However, inclusion of error terms does not provide any additional insights. We consider our simplified approach to approximate a model that embeds uncertainty explicitly.

$$q^i(M_2^{A+}, M_2^{B+}) = \begin{cases} 0.5 \;\; if \; M_2^{A+} + M_2^{B+} = 0 \\ \frac{M_2^{i+}}{M_2^{A+} + M_2^{B+}} \;\; if \; M_2^{A+} + M_2^{B+} \neq 0 \end{cases}$$

The expression above says that if both candidates refrain from positive advertising, then they split the undecided pool equally, else the split is in the ratio of their positive advertising levels. As a result, $q^i(M_2^{A+}, M_2^{B+})S_2$ is the share of supporters earned by candidate i at the end of period 2. Hence, candidate i's support at the end of the election cycle (period 2) is given by:

$$V_2^i = E[r_2^i + S_2 q^i(M_2^{A+}, M_2^{B+})] = E\left[r_0^i[1 - L^i(M_2^{i-})] + [S_0 + r_0^A L^A(M_2^{A-}) + r_0^B L^B(M_2^{B-})]q^i(M_2^{A+}, M_2^{B+})\right] \tag{4}$$

In this equation, the first term represents the total vote share of candidate i at the end of the final period 2, and the second term represents the share of switchers captured by candidate i during the final period. Note that the total share *lost* by candidate i depends upon the total of the negative memories of her in the terminal period (which are a consequence of negative advertising by the other candidate). In contrast, the total share *captured* by candidate i from the undecided pool depends upon the total of the positive memories for her (which are a consequence of her positive advertising). The objective of candidate i is to maximize[6]:

$$W_2^i = V_2^i - V_2^j \tag{5}$$

subject to the budget constraints of the candidates and Eqs. (2), (3), and (4) specified earlier ($i, j = A, B; i \neq j$). Note that because it is a "winner-takes-all contest", for each candidate the objective function (5) is equivalent to maximizing the probability of a win in the election. Since $V_2^A + V_2^B = 1$, the objective of the candidate i can be re-written as

$$W_2^i = V_2^i - (1 - V_2^i) = 2V_2^i - 1 \tag{6}$$

By our partisan rule, we have:

$$V_2^i = r_0^i(1 - LM_2^{i-}) + (S_0 + r_0^A LM_2^{A-} + r_0^B LM_2^{B-})\frac{M_2^{i+}}{M_2^{A+} + M_2^{B+}} \tag{7}$$

4. Analysis and results

While there are several possible combinations of the primitives of our model, each of which gives us a likely scenario to analyze, the ones we focus upon here are the ones that we feel best answer the real-world research questions we had posed earlier, on the tone of the campaign and on the impact of each candidate's support base.

First, political races are often characterized by a large pool of undecided voters. Is the nature of political advertising different in a race with a large pool of undecided voters than in a race with a sharply polarized electorate and very few undecided voters? For example, there is evidence to suggest that state-level races in the U.S. typically feature a bigger pool of undecided voters with a less sharply polarized electorate than do national level House, Senate, and Presidential races (Shor and McCarty, 2011). Relatedly, what if there is an asymmetry between the candidates in terms of the share of loyal voters? As an example, most incumbents in U.S. Senate races typically have a considerable advantage in terms of name recognition as well as a loyal supporter base (Cover, 1977; Gelman & King, 1990).

Second, election races generally stretch over a specific time period. Is there a tendency for negative campaigning to be concentrated at either end of the race or is it more evenly spread out? The anecdotal evidence seems to suggest that the tone of most campaigns gets more negative as the election date approaches (e.g., Kaplan, 2013), but there exist many prominent counterexamples as well. For example, in the 2012 U.S. Presidential elections, Obama's campaign went into a very early and highly effective negative advertising offensive that sought to define Mitt Romney as a "private equity executive unfriendly to the middle class" (Mason, 2012).

The cases we consider address each of the situations above. As discussed previously, we consider a two-period game in which each candidate chooses advertising decisions sequentially (period 1followed by period 2). We start by assuming both candidates are endowed with equal budgets of two, $b^A = b^B = 2$. In other words, both face binding budget constraints.[7] Before proceeding further, it is useful to point out one intuitive result: no candidate will invest solely in negative advertising. This is because while negative advertising pushes some of the opponent's voters into the undecided pool, it does not get votes for the candidate and is a strictly dominated strategy for each candidate.

[6] Our objective function is consistent with a simple two-person zero sum election game (e.g., most Senate and House races in the US, or most elections in the UK which have historically been a fight between the Conservatives and Labor). Elections that involve additional features, such as an Electoral College system (e.g., US presidential elections) or proportional voting (e.g., France, Israel), will likely need to be represented by considerably more complex objective functions.

[7] It is obvious that within our setting, the candidates will exhaust their budgets to maximize the probability of win. However, in the real world, other considerations (e.g., very low probability of winning) might lead to budgets not being exhausted.

4.1. Baseline

While memory imperfections are unavoidable in political contests, we first consider an extreme scenario to study the baseline scenario in which there is perfect memory, that is, $\rho = \kappa = 1$. In this situation, the candidates' sequences of investment do not matter, so the advertising strategy (z_1^i, z_2^i) is equivalent to (z_2^i, z_1^i). Hence, each candidate just needs to choose between $(+1,+1)$ and $(+1,-1)$. One can observe

$$V_2^A(+1,+1;+1,+1) = r_0^A + \frac{S_0}{2}, V_2^A(+1,-1;+1,+1) = r_0^A + \frac{(S_0 + r_0^B L)}{4},$$

$$V_2^A(+1,+1;+1,-1) = r_0^A(1-L) + \frac{3(S_0 + r_0^A L)}{4} V_2^A(+1,-1;+1,-1) = r_0^A(1-L) + \frac{(S_0 + r_0^A L + r_0^B L)}{2}$$

Simple calculations show that:

(a) $(+1,+1;+1,+1)$ is an equilibrium if and only if $r_0^A + (1+L)r_0^B < 1$ and $(1+L)r_0^A + r_0^B < 1$;
(b) $(+1,+1;+1,-1)$ is an equilibrium if and only if $(1-L)r_0^A + (1+2L)r_0^B < 1$ and $(1+L)r_0^A + r_0^B > 1$, by which $(1+L)r_0^A + r_0^B > (1-L)r_0^A + (1+2L)r_0^B$ or $r_0^A > r_0^B$;
1. (c)$(+1,-1;+1,+1)$ is an equilibrium if and only if $r_0^A + (1+L)r_0^B > 1$ and $(1+2L)r_0^A + (1-L)r_0^B < 1$, by which $r_0^A < r_0^B$; and finally:
(c) $(+1,-1;+1,-1)$ is an equilibrium if and only if $(1-L)r_0^A + (1+2L)r_0^B > 1$ and $(1+2L)r_0^A + (1-L)r_0^B > 1$.

The parameter regions for all equilibria are given by Fig. 4. It is easily seen that both candidates invest only in positive messages when they have low initial support; they invest in a mix of positive and negative messages when they have high initial support. Moreover, when facing very different levels of initial support, the candidate with low initial support invests in varying messages, whereas the other candidate always invests in positive messages. One can observe that the four equilibrium advertising strategies cover the entire available region $\{(r_0^A, r_0^B) | r_0^A + r_0^B < 1\}$. There are no regions with multiple equilibria. Furthermore, one can straightforwardly validate that there are no mixed strategy equilibria for any parameter region.

4.2. The model with memory imperfections

We now formally include memory imperfections into our model and investigate how our results change. We can derive the sub-game perfect equilibrium (SPE) of the game through backward induction. All possible SPE equilibria are listed in Table 1. It suffices to examine the advertising strategies in bold and italics because the other three advertising strategies can be analyzed just by exchanging both candidates' initial support. Although there are six possible equilibrium advertising strategies, we can qualitatively divide them into three categories that correspond with three regions of initial support. We assume throughout that $0 < \kappa < \rho < 1$. Moreover, we assume $\rho + \kappa < 1$ to ensure that in two successive election cycles, for two identical advertising strategies (e.g. (+1, +1)), the effect of the past election cycle is lower than that of the current election cycle.

We start by examining the SPE $(+1,+1;+1,+1)$ in which both candidates always choose positive advertising. Proposition 1 shows this equilibrium arises when both candidates' initial supports are relatively low (details of the equilibrium calculations are in the Technical Appendix).

Proposition 1. $(+1,+1;+1,+1)$*is an SPE when*

$$(r_0^A, r_0^B) \in \left\{ (r_0^A, r_0^B) | r_0^A + \frac{2L\rho + \rho + \kappa}{\rho + \kappa} r_0^B < 1, \frac{2L\rho + \rho + \kappa}{\rho + \kappa} r_0^A + r_0^B < 1 \right\}.$$

Verbally, when both candidates have low initial support, meaning there exists a large pool of undecided voters, then both candidates are *more likely* to invest in positive messaging throughout the election cycle, as seen in Fig. 5.

This result seems counter-intuitive at first glance; one would imagine a large pool of undecided voters would cause candidates to fight bitterly, implying the heavy use of negative advertising. However, a simple examination of the roles played by positive and negative advertising shows us why this result makes sense. To recap, negative advertising by a candidate offers a way to convert a voter who is loyal to the other candidate into an undecided voter, i.e., "turn off" the voter. It does not convince a voter to vote for the candidate who advertises. For that, the candidate has to send a positive message. The result can now be interpreted easily. With a large pool of undecided voters, there is a significant return to converting these to loyal voters through positive advertising. There is little gain to switching the relatively small number of loyal voters of the other candidate to the undecided pool. In a symmetric world, this is true for both candidates, hence the equilibrium in Propo-

[8] In the presence of memory imperfections, one could assume candidates A and B choose advertising strategy $+1$ with probabilities x_1 and $y_1 \in (0,1)$ in the first period and choose $+1$ with probabilities x_2 and $y_2 \in (0,1)$ in the second period, to solve for mixed equilibria. However, it is mathematically intractable to derive and solve for these mixed equilibria, so in what follows we focus on only pure strategy equilibria.

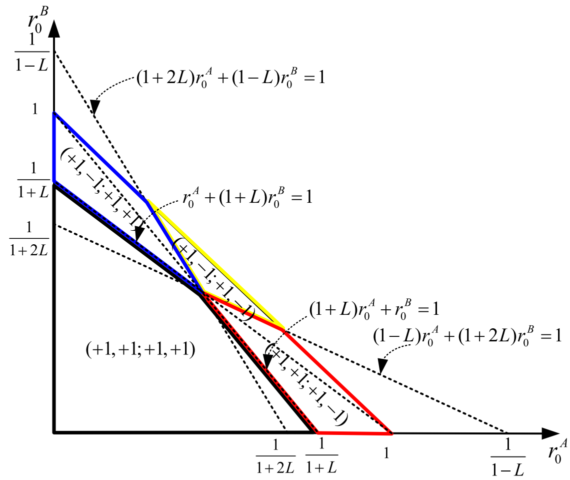

Fig. 4. The parameter regions for four equilibria in the baseline model.

Table 1
Both candidates' strategy matrix with elements denoting $(z_1^A, z_2^A; z_1^B, z_2^B)$.

$(z_1^B, z_2^B)(z_1^A, z_2^A)$	(+1,+1)	(+1,−1)	(−1,+1)
(+1,+1)	**(+1,+1;+1,+1)**	(+1,+1;+1,−1)	(+1,+1;-1,+1)
(+1,−1)	**(+1,−1;+1,+1)**	**(+1,−1;+1,−1)**	(+1,−1;-1,+1)
(−1,+1)	(−**1,+1;+1,+1)**	(−**1,+1;+1,−1)**	(−**1,+1;-1,+1)**

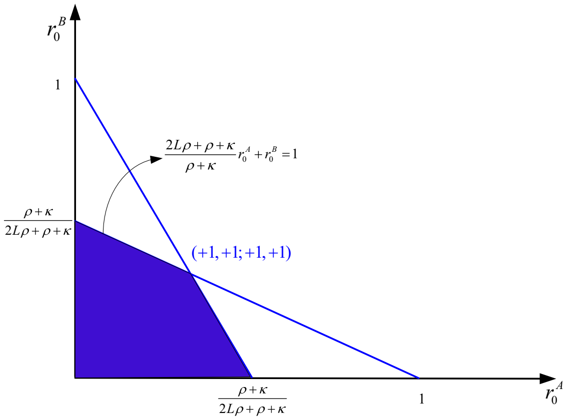

Fig. 5. The equilibrium region in Proposition 1.

sition 1. According to Propositions 2 and 3, one can find that the equilibrium $(+1, +1; +1, +1)$ is unique (and thus multiple equilibria do not exist) in the related parameter region.[8]

Our next proposition analyzes equilibria $(+1, -1; +1, -1)$, $(-1, +1; +1, -1)$, and $(-1, +1; -1, +1)$, in which each candidate adopts the type of advertising strategy that takes a "pulsing" shape ($(+1, -1)$ or $(-1, +1)$). Our analysis reveals that these equilibria arise when both candidates' initial supports remain relatively high.

Proposition 2. $(+1, -1; +1, -1)$*is an SPE when*

$$(r_0^A, r_0^B) \in \left\{ (r_0^A, r_0^B) | (1-L)r_0^A + \left[\frac{(2\rho+\kappa+1)L}{\kappa+1} + 1\right] r_0^B > 1, \left[\frac{(2\rho+\kappa+1)L}{\kappa+1} + 1\right] r_0^A + (1-L)r_0^B > 1 \right\}$$

$(-1, +1; +1, -1)$ *is an SPE when*
$(r_0^A, r_0^B) \in \left\{ (r_0^A, r_0^B) | \left[\frac{(\rho+\kappa+2)L\rho}{\kappa+1} + 1\right] r_0^A + (1-L\rho)r_0^B > 1, (1-L)r_0^A + \left[\frac{(2\rho+\kappa+1)L}{\kappa+\rho} + 1\right] r_0^B > 1 \right\}$, *and*
$(-1, +1; -1, +1)$ *is an SPE when*

$$(r_0^A, r_0^B) \in \left\{ (r_0^A, r_0^B) | (1 - L\rho) r_0^A + \left[\frac{(\rho + \kappa + 2) L\rho}{\rho + \kappa} + 1 \right] r_0^B > 1, \left[\frac{(\rho + \kappa + 2) L\rho}{\rho + \kappa} + 1 \right] r_0^A + (1 - L\rho) r_0^B > 1 \right\}.$$

The equilibrium regions in Proposition 2 are illustrated in Fig. 6.[9]

Intuitively, in a situation of high initial vote shares, both candidates devote a part of their budget to negative messaging. The intuition is similar to the earlier proposition. With high vote shares for both candidates, there is a tiny pool of undecided voters. As such, the only way to expand vote share is to "steal" the other candidate's loyal supporters. The way to do that is to engage in negative advertising, which is precisely what Fig. 6 says.

Fig. 6 provides two other critical insights. First, consider the two symmetric equilibria. Note that the equilibrium region $(-1, +1; -1, +1)$ subsumes the equilibrium region $(+1, -1; +1, -1)$; in other words, the latter region is a subset of the former. The equilibrium $(-1, +1; -1, +1)$ arises when both candidates' initial supports are moderate to extremely high. In contrast, the equilibrium $(+1, -1; +1, -1)$ appears only when both candidates' initial supports are extremely high. Intuitively, when both candidates face extremely high initial supports, each candidate is confronted with high pressure to "steal" the other candidate's loyal supporters. This happens via the adoption of negative advertising in period 2, because negative advertising in that period maximizes the stealing effect. On the other hand, the equilibrium $(-1, +1; -1, +1)$ might exist even when each candidate is confronted with less pressure (moderate support), and thus exists for a broader range of initial supports.

Second, note that the asymmetric equilibrium $(-1, +1; +1, -1)$ is more likely to exist when candidate *A*'s initial support increases and candidate *B*'s initial support decreases. The reason for this result is that candidate *A* has less pressure to "steal" candidate *B*'s loyal supporters and thus does not need to release a negative message in period 2. However, candidate *B* faces more significant pressure and hence invests in negative advertising in the final period. It should be noted that the intuition for the equilibrium $(+1, -1; -1, +1)$ can be arrived at similarly by simply exchanging both candidates' initial support.

Note that one can find the existence of multiple equilibria for some regions. However, since both candidates play a zero-sum game, there exist no Pareto-improving equilibria, and thus these fail in refinement tests. This is also applicable to Proposition 3 below.

We now turn to an analysis of the situation when there is 'sufficient' asymmetry in the initial vote share between the candidates, i.e., one candidate has a substantial lead over the other at the beginning of the election cycle. Proposition 3 states the equilibrium when candidate *B* has a considerable lead over candidate *A*.

Proposition 3. $(+1, -1; +1, +1)$*is an SPE when*

$$(r_0^A, r_0^B) \in \left\{ (r_0^A, r_0^B) | r_0^A + \frac{2L\rho + \kappa + 1}{\kappa + 1} r_0^B > 1, \left[\frac{(2\rho + \kappa + 1)L}{\kappa + \rho} + 1 \right] r_0^A + (1 - L) r_0^B < 1 \right\}$$

and $(-1, +1; +1, +1)$ *is an SPE when*

$$(r_0^A, r_0^B) \in \left\{ (r_0^A, r_0^B) | r_0^A + \frac{2L\rho + \rho + \kappa}{\rho + \kappa} r_0^B > 1, \left[\frac{(\rho + \kappa + 2)L\rho}{\rho + \kappa} + 1 \right] r_0^A + (1 - L\rho) r_0^B < 1 \right\}.$$

As can be seen in the Technical Appendix, the regions for both equilibria in Proposition 3 imply $r_0^A < r_0^B$. For both the equilibria in Proposition 3, candidate *B* always chooses positive advertising in both periods, whereas candidate *A* chooses the advertising strategy that takes a "pulsing" shape, as illustrated by Fig. 7. Hence, when two candidates with initial asymmetric support face each other, the leading candidate (*B*) is likely to be positive throughout. By contrast, the laggard (*A*) is likely to start with a negative message and finish with a positive message or start with a positive message and finish with a negative message. Intuitively, the only way in which a laggard can close the gap with the leader is by "stealing" some of the opponent's supporters, and he does this by deploying a negative message (that expands the pool of undecided voters) accompanied with a positive message that convinces the undecided voters.

It can be observed from Fig. 7 that in the equilibrium region for $(+1, -1; +1, +1)$, candidate *A*'s initial support is much lower than candidate *B*'s initial support. Similar to the rationale in Proposition 2, candidate *A* is under higher pressure and chooses a negative message in the final period to steal candidate *B*'s supporters. However, depending on the degree of asymmetry in the initial vote share, candidate *A* may choose negative advertising in period 1 or period 2. By contrast, candidate *B*, as the leader, uses only positive messaging, which allows her to capture a higher share of undecided voters on the day of the election. The general message that comes out is that if one player starts with a clear advantage in terms of initial support, she is likely to remain more positive throughout the race. This finding is somewhat different from Proposition 1, which reveals that a player with a disadvantage in the initial support would choose positive advertising. The equilibria $(+1, +1; +1, -1)$ and $(+1, +1; -1, +1)$ can be analyzed similarly by exchanging both candidates' initial support.

Our next proposition shows how the memory parameters that capture rehearsal and decay effects affect the overall tone of the campaign.

Proposition 4a. *For higher ρ (i.e., messages have lower memory decay), both candidates are less likely to release only positive messages; (b) however, for higher κ (i.e., messages have higher rehearsal effect), both candidates are more likely to release only positive messages.*

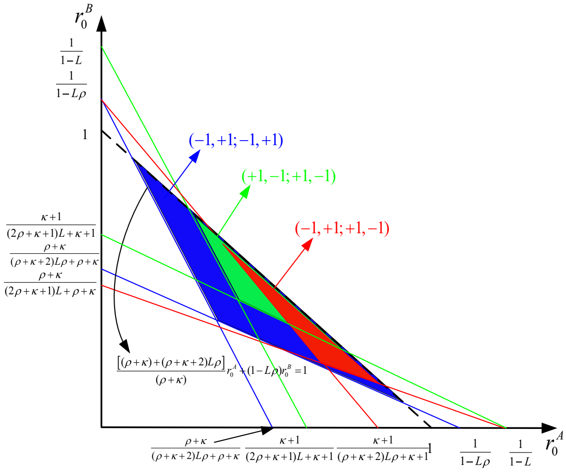

Fig. 6. Equilibria in Proposition 2.

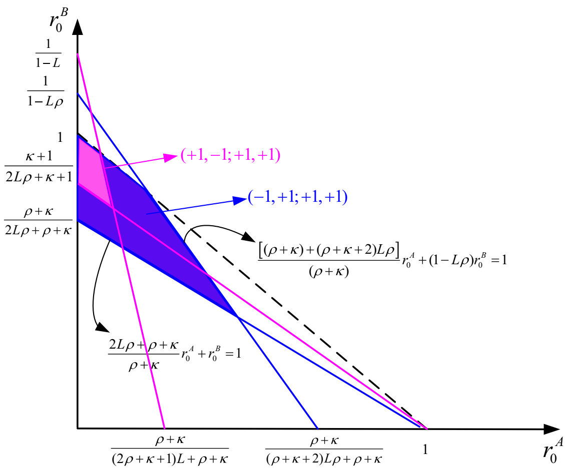

Fig. 7. Equilibria in Proposition 3.

When the decay rate is low, candidates can rely upon fewer positive messages (which remain relatively fresh because decay is slow) and are incentivized to use some of their budgets on negative advertising to expand the pool of undecided voters. While this result makes intuitive sense with our model formulation, it is useful to question what it can usefully tell us about messaging strategy in real-world political contexts, given that memory parameters are individual specific and hence exogenous to the model. One way to answer this is to think of memory parameters (like decay) as being not just psychological traits, but also likely to be influenced by the nature of the electoral contest. In that light, this result provides insights that are useful in informing us about messaging strategy. For example, in an election with low voter interest and a crowded field of candidates (e.g., a Senate primary) the retention of messages is likely to be lower (low ρ), and this would result in a more positive tone of messaging, per Proposition 4. This is consistent with empirical evidence provided by Gandhi et al. (2015).

The intuition for (b) is based on two things. The fact that greater rehearsal implies that a message has greater reinforcement power, in conjunction with our earlier result that at least one positive message will always be used in equilibrium, implies that candidates are more likely to schedule all-positive messaging under higher κ. To connect this to real-world election contests, the greater the consistency of past policy positions taken by a candidate, the higher is κ likely to be; all else being equal, this would result in a more positive tone of messaging.

[9] In Fig. 5, we assume $\rho > \frac{\kappa(1+\kappa)}{(1-\kappa)}$ so that $\frac{(\rho+\kappa)}{[(\rho+\kappa+2)L\rho+\rho+\kappa]} < \frac{(\kappa+1)}{[(2\rho+\kappa+1)L+\kappa+1]}$. Relaxing this assumption does not change the regions for the three equilibria.

5. Extensions

This section takes a closer look at the robustness of our results to relaxing some of the assumptions of our model.

5.1. Extension 1: Positive advertising could turn some of the opponent's supporters into undecided

In the main model, we assume that positive advertising could make undecided voters favorably inclined to vote for a candidate. This is somewhat restrictive – higher positivity from a focal candidate could also possibly make the supporters of the opposing candidate question their own candidate (Harrington & Hess, 1996). In this section, we model this effect of a candidate's positive advertising by assuming that it could convert some of her opponent's supporters into undecided voters. This is modeled by revising the fraction of candidate *i*'s loss of support as $E[L^i(M_2^{i-}, M_2^{j+})] = L(M_2^{i-} + \alpha M_2^{j+})$, where $0 \leq \alpha < 1$ reflects the effect of candidate *j*'s positive advertising on candidate *i*'s loss of support. We assume that this effect is weaker than candidate *j*'s negative advertising and is consistent with the empirical finding that negative advertising is more effective than positive advertising in turning off supporters of the opponent (Damore, 2002). Proposition A1 in the Online Appendix shows that the main results in the base model continue to hold as long as the value of α is not very high.[10] Hence, one can conclude from Proposition A1 that Propositions 1–3 continue to hold even when one candidate's positive advertising could convert its opponent's supporters to 'undecided' (to a limited degree).

5.2. Extension 2: Negative advertising decreases the evaluation of the sponsoring candidate

Some studies show that negative advertising decreases the evaluation of the sponsoring candidate. For example, Merritt (1984) finds that negative advertising harms the perceptions of not just the targeted but also of the sponsoring candidate (termed the 'backlash' effect). In order to examine the robustness of our main results to this effect, we modify our model by revising the fraction of candidate *i*'s loss of support as $E[L^i(M_2^{i-}, M_2^{j-})] = L(M_2^{i-} + \beta M_2^{j-})$, where $0 \leq \beta < 1$ measures the extent of the backlash effect. Proposition A2 in the Online Appendix provides the equilibrium analysis; we find that our main results in Propositions 1–3 continue to hold under the existence of the backlash effect.

5.3. Extension 3: Negative advertising depresses (or facilitates) the turnout

An underlying assumption in the basic model is that the size of the participating electorate (the sum of both candidates' supporters and undecided voters) remains unchanged and thus independent of negative advertising. While this assumption has some empirical support (Clinton & Lapinski, 2004), other studies find that turnout can be lowered (Ansolabehere et al., 1999) or raised (Stevens, Sullivan, Allen, & Alger, 2008) by negative advertising. We modify our main model to accommodate the lowering (enhancing) effect of negative advertising on the turnout by letting r_0^A, r_0^B and S be multiplied by $1 - \gamma(M_2^{A-} + M_2^{B-})$, where $\gamma > 0$ ($\gamma < 0$) describes the strength of the lowering (enhancing) effect. That is, the objectives of both candidates become $[1 - \gamma(M_2^{A-} + M_2^{B-})]W_2^A$ and $[1 - \gamma(M_2^{A-} + M_2^{B-})]W_2^B$.

Such a formulation makes the model quite complicated, making it difficult to obtain closed-form analytical results. As such, we turn to a numerical analysis: Assuming $\rho = 0.5$, $\kappa = 0.4$ and $L = 0.5$, we can get the regions for the equilibria for varying values of γ in Fig. 8 and Fig. 9.

It is evident from Figs. 8 and 9 that both sets of equilibrium regions are similar and consistent with the main results contained in Propositions 1–3. It is important to emphasize that this robustness check is based entirely on numerical simulations.

5.4. Extension 4: Negative advertising increases fundraising

Some prior literature has shown that negative advertising can increase fundraising by energizing the core (Lovett & Shachar, 2011). In this extension, we briefly examine this effect of negative advertising. We consider a two-period game model with both candidates having the same budget constraint, $b^A = b^B = 1$. As pointed out earlier, within the base model, this budget endowment implies that each candidate can only advertise in a single period, and they will do so via positive advertising in the terminal period. However, we show these dynamic changes when negative advertising can increase fundraising. The assumption of symmetric and single budget endowment allows us to simplify the analyses and transparently explicate the fundraising effect of negative advertising. Given this, the first question is: When will either candidate invest in negative advertising? Since negative advertising brings no returns in the second period in the baseline model, no candidate will invest in it then. In contrast, now, an investment in negative advertising in the first period leads to increased fundraising (and an increase in the undecided pool) that can potentially be used in the second period to invest in positive advertising. Given that the discrete advertising budget is 1, if this budget is spent in period 1, then it results

[10] This restriction on α makes sense as the primary role of positive advertising is to move voters from undecided to becoming supporters, while the secondary effect (modeled in this extension) is to affect the supporters of the opposing candidate.

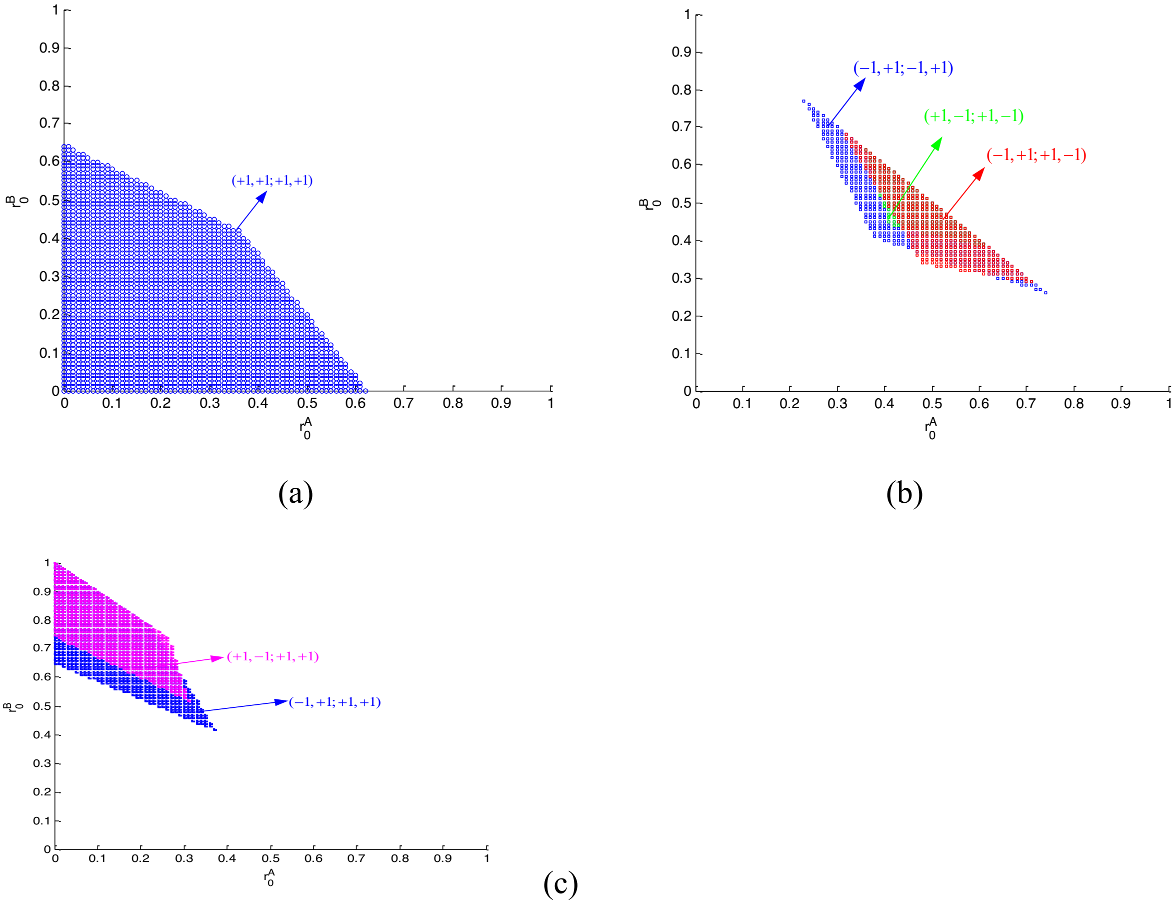

Fig. 8. The equilibrium regions with $\gamma = 0.01$.

in fundraising that equals an advertising budget of f, with $0 \le f \le 1$ (note that the (reasonable) assumption here is that the fundraising effect is not stronger than the investing effect of the negative advertising). Given this, we now have a new possible advertising strategy that a candidate can choose, denoted as $(-1, +1_f)$, where $+1_f$ denotes positive advertising as a result of an investment of f. Because both the candidates have a single unit of budget and negative advertising can increase fundraising, advertising strategy $(0, +1)$ is investing only in positive advertising, whereas $(-1, +1_f)$ can be viewed as a "pulsing" strategy involving both positive and negative advertising. Extension 4 in the Online Appendix provides the equilibrium analysis. Our main conclusions remain largely the same when negative advertising increases fundraising.

5.5. Extension 5: Undecided voters could decide not to vote

In the primary model, we assume all undecided voters finally become voters at the end of the election cycle. In this extension, we consider another scenario wherein undecided voters could decide not to vote as they do not have any clear preferences. In this case, both candidates' supports satisfy $V_2^A + V_2^B = 1 - S_2$, and their objectives become:

$$\begin{aligned} W_2^i &= V_2^i - V_2^j = V_2^i - (1 - S_2 - V_2^i) = 2V_2^i + S_2 - 1 = 2r_0^i[1 - L^i(M_2^{i-})] + S_0 + r_0^A L^A(M_2^{A-}) + r_0^B L^B(M_2^{B-}) - 1 \\ &= r_0^i\left(1 - L^i(M_2^{i-})\right) - r_0^j\left(1 - L^j(M_2^{j-})\right) \end{aligned}$$

which always increase with M_2^{j-} for any M_2^{i-}. It is easily seen that the objective of each candidate is independent of any positive advertising, which implies that no candidate has any incentive to invest in positive advertising. Hence, each candidate would choose the advertising strategy $(-1, -1)$ in equilibrium, and as a result, the candidate with higher initial support likely wins. This somewhat implausible outcome is a consequence of the fact that positive advertising does not affect undecided voters.

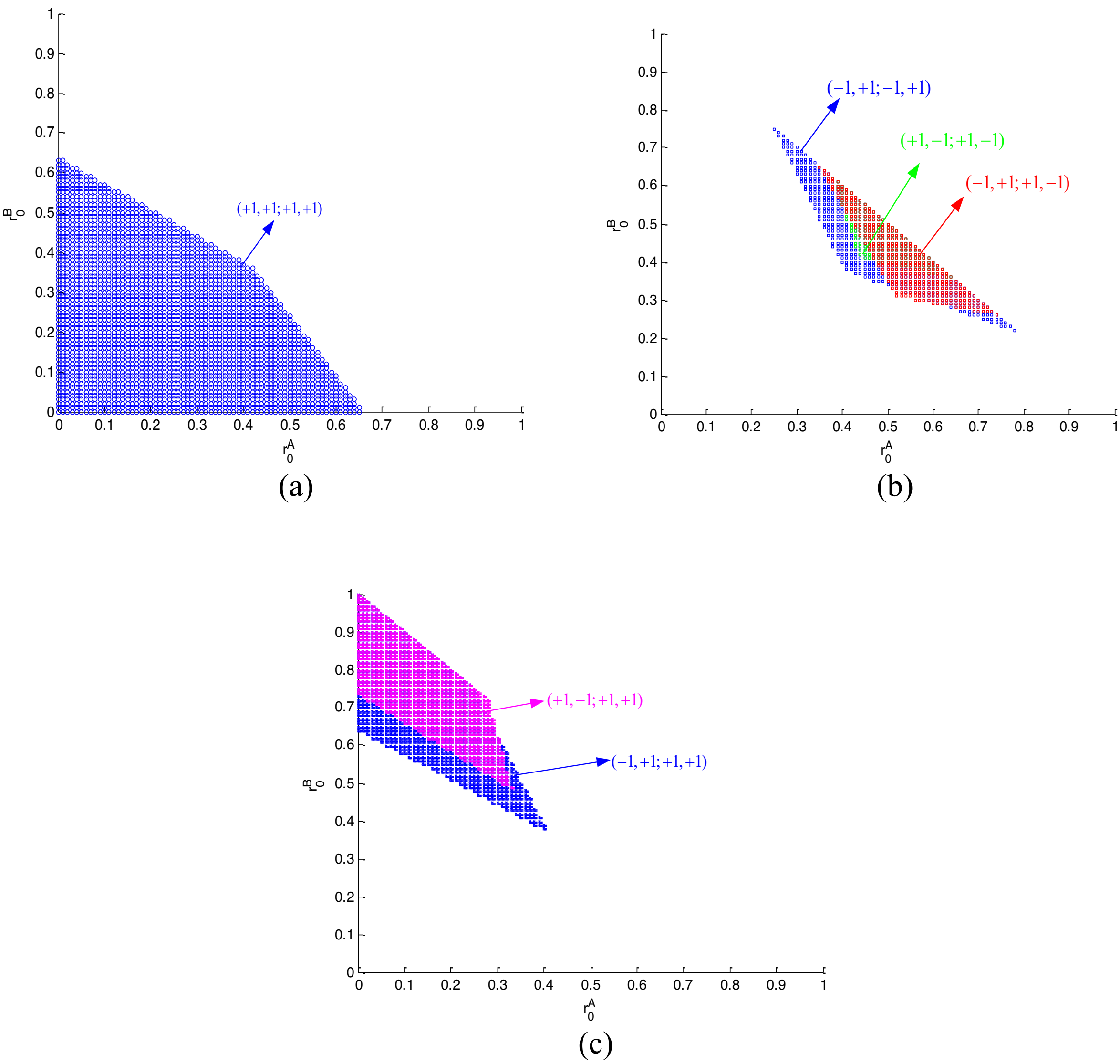

Fig. 9. The equilibrium regions with $\gamma = -0.01$.

5.6. Extension 6: The impact of asymmetric budgets

All our results above have been for the case where both candidates have had equal budgets to spend on advertising. This is clearly not the case in the real world; while highly asymmetric budgets are more likely to be observed in state and local elections (e.g., in the 2006 House elections, an incumbent on average spent close to a million dollars to seek re-election while a challenger spent about a quarter of a million dollars (data from opensecrets.org cited in Meirowitz, 2008, p 689)), it is not uncommon to see such a situation even in a Presidential race. In the 2000 U.S. presidential election, for example, the volume of TV ads released by the Bush campaign in the final two weeks was twice that released by the Gore campaign; similarly, in 2008, the Obama campaign outspent the McCain campaign quite significantly (Sides, 2011). We now discuss the case where candidates differ in their endowed budgets.

Assume that in a three-period world, candidate A has a budget of two whereas candidate B has a budget of three. Because of discounting, a candidate with binding budget constraints (fewer available messages than the number of periods) will wait to release messages until the end of the election cycle. Candidate A will allocate his budget in the last two periods, and therefore we just need to consider the advertising strategy of $(z_2^A, z_3^A; z_1^B, z_2^B, z_3^B)$.

The model and main results in this setup are similar to the case with symmetric budgets (see Online Appendix Proposition A3). While the complete equilibrium analysis is complex, we can look at some representative equilibria to check the robustness of Propositions 1–3. The equilibrium $(+1,+1;+1,+1,+1)$, in which both candidates engage in all positive advertising, arises when their initial supports remain low. The result is a generalization of Proposition 1. In contrast, the equilibrium $(+1, -1; +1, -1, -1)$, in which both candidates engage in negative advertising in at least one period, appears when their initial supports are high. This result is a generalization of Proposition 2. The equilibrium $(+1, +1; +1, -1, -1)$, in which candidate A always chooses positive advertising whereas candidate B chooses negative advertising in the last two periods, appears when candidate A is endowed with a much higher initial support than candidate B. The result is consistent with Proposition 3.

Overall, our results with both symmetric and asymmetric budgets yield a number of common results that can be summarized as follows: (1) In contests where candidates have a small share of loyal supporters, and hence a larger pool of undecided voters, the overall tone of the campaign is likely to be positive. The opposite holds (i.e., the tone is likely to be negative) if the initial shares are higher and the undecided pool is smaller. (2) When the initial support among candidates is asymmetric, the laggard is more likely than the leader to engage in negative advertising.

6. Empirical analysis

In this section, we conduct an exploratory empirical analysis of some of the findings from our model that are amenable to examination using aggregate data. Two hypotheses that flow from the model are readily testable from data:

H1. A candidate with higher support is *less* likely to invest in negative advertising (Proposition 3).

H2. A candidate with a higher budget is *more* likely to invest in negative advertising when she has smaller support (Extension 6; Proposition A3).

To test these hypotheses, we need data on a candidate's support, her budget, and the tone of advertising during a campaign. We focus our data collection on competitive Senate races for the year 2016 because the features of such a race fit the context of our theoretical model: (i) two opposing candidates (similar to the two-player game formulation in our model), and (ii) reasonable variation in the advertising budget and in the support enjoyed by the candidates across the race, two critical parameters of our analytical model. We describe below our sources of data and the construction of the key variables.

6.1. The tone of advertising

Our advertising data comes from *politicaladarchive.org*, which maintains details of the airings of TV political ads. We focus on advertising in major broadcast channels in twenty markets in ten states with competitive Senate races for the year 2016.[11] Internet Archive researchers have coded each advertisement in this database as belonging to one of four categories: *pro* (positive ads), *con* (negative ads), *mix* (a mix of both positive and negative), and *unknown*.

An ad is defined as "pro" when it positively mentions one or more candidates and contains no negative message about any candidate. An advertisement is defined as a "*con*" when it mentions one or more candidates negatively. For example, if an ad that supports a candidate i mentions another candidate j negatively, the ad is coded as "con." "Mix" is defined as an ad that mentions more than one candidate in a particular race, with significant positive content about one or more candidates *and* negative content about one or more candidates. To make a clear distinction between positive and negative ads, we only count the airing of "con" as negative ads.

We count the number of each category of ads for each candidate in each month (from Jan. 2016 to Oct. 2016) and then define the *relative negativity of advertising* for each candidate i in month t:

$$NegI_{it} = \begin{cases} 1 & \%Negative\ \ Ads_{it} > \%Negative\ \ Ads_{-it} \\ 0 & otherwise \end{cases}$$

where $-i$ is the counterparty of candidate i in the same race (rows 3 and 4 in Table 2 provide summary statistics on shares of positive and negative advertising across candidates).

6.2. Candidate budgets

Our data on the budget available to each candidate comes from the website of the Federal Election Commission, which maintains highly updated and detailed files on committees, candidates, and campaign finance data for election cycles from 1980 onwards. We use the total contribution amount at the end of the fiscal year 2015 as our measure of the initial budget level. We then calculate the disposable budget of each candidate at the beginning of a month by subtracting reported expenditures from total accumulated income in the previous month. We assume all unspent income at the end of each month is rolled over to the next month (row 2 in Table 2 provides summary statistics of monthly budgets across candidates in our sample).

To control for income levels across States, we define *Budget indicator* (*BudgetI*) for candidate i in month t as follows:

[11] Major broadcast channels in 10 markets in ten states for competitive U.S. Senate races: Arizona, Colorado, Iowa, Nevada, New Hampshire North Carolina, Florida, Ohio, Pennsylvania and Wisconsin.

Table 2
Summary Statistics of Monthly Budgets and Shares of Negative vs Positive Advertising.

Variables	N of Candidates	Mean	Median	Std. Dev.	Min	Max
Monthly Budgest ($ Million)	20	5.50	4.69	6.51	0	49.40
% Negative Advertising	20	0.54	0.67	0.42	0	1
% Positive Advertising	20	0.34	0.17	0.37	0	1

Note: The negative and positive shares do not add to 1 as there are some ads with mixed messaging (see paper for details).

$$BudgetI_{it} = \begin{cases} 1 & Budget_{it} > Budget_{-it} \\ 0 & otherwise \end{cases}$$

We observe both inter-state and intra-state heterogeneity in budgets. For example, for the senate race in Florida, the average monthly fund for Rubio (R) was less than half of that for Murphy (D) (2.74 M vs. 6.46 M). On the other hand, in the Wisconsin senate race, both Feingold (D) and Johnson (R) had comparable monthly budgets (4.31 M vs. 4.67 M).

6.3. Support

We measure the support for each candidate using monthly average polling data (RealClearPolitics.com). For candidates in each state, we collect all polling data from all reported media, and then calculate the monthly average polling ratio (*Support*) from Mar. 2016 to Nov. 2016. For each candidate *i*, $SupportI_{it}$ is defined as 1 (0) if her spread is higher (lower) than her counterparty in month *t*.

$$SupportI_{it} = \begin{cases} 1 & Support_{it} > Support_{-it} \\ 0 & otherwise \end{cases}$$

6.4. Hypothesis testing

Note that our first hypothesis (H1) says that candidates with a higher level of support are less likely to go negative. As a first cut, we group candidate-month combinations into "leaders" (higher polling support) vs. "laggards" (lower polling support) and find that the mean advertising negativity is 40% for leaders versus 47% for laggards. This provides descriptive evidence that candidates with higher support are relatively less negative in their advertising, consistent with Proposition 3. Fig. 10 plots the distribution of relative negativity across the leaders and laggards – clearly, the laggards use more airings of negative ads.

To formally test H1 and H2, we use the following simple empirical specification:

$$NegI_{it} = \beta_1 * SupportI_{it} + \beta_2 * BudgetI_{it} + \beta_3 * SupportI_{it} * BudgetI_{it} + \varepsilon_{it}$$

The results of this specification (estimated via a logistic regression model) are presented in Table 3. The results provide support for both our hypotheses. Specifically, support has a *negative effect on negative advertising investment* ($\beta_1 = -1.62 < 0, p < 0.05$). These numbers imply that the higher the support, the less negative will be the campaign tone, consistent with H1 and proposition 3. On the other hand, the interaction effect of budget and support is negative

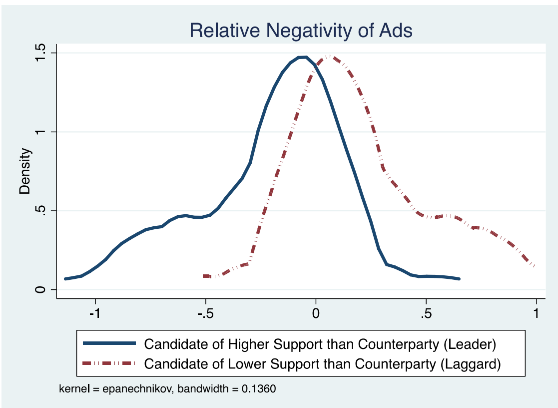

Fig. 10. Distribution of Negative Ads across Leading and Lagging Candidates. Note: A higher number implies more airings of negative ads.

Table 3
Results of Logistic Regression Model.

DV: Relative Negativity of Advertising	Coef. (Std. Err.)
Relative Support	−1.62 (0.78) *
Relative Budget	−0.80 (0.68)
Interaction: Support*Budget	−1.89 (0.61) ***
cons.	0.64 (0.39)
N obs.	80
Log Likelihood	−48.40

* significant at 95% level; ** significant at 99% level.

($\beta_3 = -1.89$, $p < 0.01$), which implies that at lower levels of support, the higher budget has a smaller negative impact on negativity – that is, the tone of messaging will be more negative at higher budgets and lower support. This provides evidence for our theoretical prediction, Proposition A3 in extension 6 and hypothesis H2.

Overall, our simple empirical analysis provides directional evidence of the validity of our model. This obviously comes with some caveats, notably that we are only looking at one election cycle within a single specific context.

7. Concluding remarks

It has become increasingly common to hear concerns in the media and among policymakers about the increasingly negative tone of campaigns in U.S. elections, seemingly coinciding with an ever-increasing amount of money being raised and spent. In this study, we provide a normative model that illustrates how the tone of a campaign might be endogenously affected by candidates' initial support, their budgets, and memory imperfections on the part of the electorate. Our model of electoral competition between two candidates is deliberately kept simple to allow us to focus on how memory imperfections might impact outcomes in conjunction with other key variables. Our main results on the temporal pattern of advertising, as well as on the mix of positive and negative advertising, prove robust to a variety of extensions of the basic model.

While our model seems to suggest that negative advertising might be a rational response by candidates based upon their support and budget levels, there are legitimate concerns that negativity is poisoning the civic discourse and, if accompanied by tools like fake news on social networks, could pose a threat to citizens' faith in democracy. Our model suggests that asymmetric budgets are a significant motivator for negative advertising; this, in turn, suggests that actions that reduce the funding gap among candidates could lessen the intensity of negative advertising. A possible regulatory intervention that would achieve this is public funding for elections.

There are many directions in which our model can be fruitfully extended. For example, the role of messaging in our setup is mainly modeled as improving voters' affective evaluations and hence falls under the rubric of persuasive advertising. It would be useful to consider the informational aspects of messaging in conjunction with memory imperfections. A future model might also consider more robust formulations of memory processes (e.g., differential decay and rehearsal parameters across positive and negative messages) and estimate individual-level memory parameters using more detailed voter-level data. Interestingly, recent trends suggest the need to model the process of US elections differently. For instance, in the 2020 US presidential election, many votes were cast before election day; in such a situation, it would be important to model the dynamics of voting throughout the election cycle. Finally, it is important to extend our setup to a multi-person race and examine the role of memory imperfections therein.

Appendix A. Technical appendix

This appendix presents the proofs of the main results presented in the paper.

Proof of Proposition 1. We can calculate both candidates' objective functions at different strategies of advertising investments as below (as shown in the paper):

$$W_2^A(z_1^A, z_2^A; z_1^B, z_2^B) = 2V_2^A(z_1^A, z_2^A; z_1^B, z_2^B) - 1$$

$$\text{and} W_2^B(z_1^A, z_2^A; z_1^B, z_2^B) = 1 - 2V_2^A(z_1^A, z_2^A; z_1^B, z_2^B),$$

Below we describe the explicit procedure for calculating the value function embedded within objective functions using two example strategies. The other calculations are analogous.

(1) **For strategy (+1, +1; +1, +1):**

In period 1, the state of memory evolution is as below (which we have referred to as *initial memory state* in the paper):

$$m_1^{1A+}(z_1^A=+1,z_1^B=+1)=1,m_1^{1B+}(z_1^A=+1,z_1^B=+1)=1,m_1^{1A-}(z_1^A=+1,z_1^B=+1)=0,m_1^{1B-}(z_1^A=+1,z_1^B=+1)=0.$$

In period 2, the state of positive memory (carried over from period 1) transforms into

$$m_2^{1A+}(z_2^A=+1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=\rho*1+1*\kappa*1$$

$$m_2^{1B+}(z_2^A=+1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=\rho*1+1*\kappa*1$$

The negative memory state is still:

$$m_2^{1A-}(z_2^A=+1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=m_2^{1B-}(z_2^A=+1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=0.$$

During period 2, the *new gained* memory is:

$$m_2^{2A+}(z_2^A=+1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=1,m_2^{2B+}(z_2^A=+1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=1,m_2^{2A-}(z_2^A=+1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=m_2^{2B-}(z_2^A=+1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=0.$$

Therefore, by the end of period 2, the stock of memory is:

$$M_2^{A+}(+1,+1;+1,+1)=m_2^{1A+}+m_2^{2A+}=\rho*1+1*\kappa*1+1=\rho+\kappa+1$$

$$M_2^{B+}(+1,+1;+1,+1)=m_2^{1B+}+m_2^{2B+}=\rho*1+1*\kappa*1+1=\rho+\kappa+1$$

And $M_2^{A-}(+1,+1;+1,+1)=0$ and $M_2^{B-}(+1,+1;+1,+1)=0$
Hence, the value function by the end of period 2 can be written as:

$$V_2^A(+1,+1;+1,+1)=r_0^A+\frac{M_2^{A+}(+1,+1;+1,+1)}{M_2^{A+}(+1,+1;+1,+1)+M_2^{B+}(+1,+1;+1,+1)}S_0=r_0^A+\frac{(\rho+\kappa+1)}{(\rho+\kappa+1)+(\rho+\kappa+1)}S_0$$
$$=r_0^A+\frac{1}{2}S_0$$

(2) **For strategy (+1, −1; +1, +1):**

In period 1, the state of memory evolution is as below:

$$m_1^{1A+}(z_1^A=+1,z_1^B=+1)=1,m_1^{1B+}(z_1^A=+1,z_1^B=+1)=1,m_1^{1A-}(z_1^A=+1,z_1^B=+1)=0,m_1^{1B-}(z_1^A=+1,z_1^B=+1)=0$$

During period 2, the state of memory transforms into:
$m_2^{1A+}(z_2^A=-1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=\rho$, $m_2^{1B+}(z_2^A=-1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=\rho+\kappa$, and

$$m_2^{1A-}(z_2^A=-1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=0,m_2^{1B-}(z_2^A=-1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=0.$$

During period 2, the *new gained* memory is:
$m_2^{2A+}(z_2^A=-1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=0$, $m_2^{2B+}(z_2^A=-1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=1$, and:

$$m_2^{2A-}(z_2^A=-1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=0,m_2^{2B-}(z_2^A=-1,z_2^B=+1|z_1^A=+1,z_1^B=+1)=1.$$

Note that $z_2^A=-1$ affects m_2^{1B-} instead of m_2^{1A-}.
Hence, by the end of period 2, the stock of memory is:

$$M_2^{A+}(+1,-1;+1,+1)=m_2^{1A+}+m_2^{2A+}=\rho$$

$$M_2^{B+}(+1,-1;+1,+1)=m_2^{1B+}+m_2^{2B+}=\rho+\kappa+1$$

$$M_2^{A-}(+1,-1;+1,+1)=m_2^{1A-}+m_2^{2A-}=0$$

$$M_2^{B-}(+1,-1;+1,+1)=m_2^{1B-}+m_2^{2B-}=1$$

The state of undecided voters evolves to:
$S_2=S_0+r_0^AL^A\left(M_2^{A-}(+1,-1;+1,+1)\right)+r_0^BL^B\left(M_2^{B-}(+1,-1;+1,+1)\right)=S_0+0+r_0^BL*1$(notice that we assume L(x) = L*x).
The value function by the end of period 2 can be written as:

$$\begin{aligned}V_2^A(+1,-1;+1,+1) &= r_0^A + \frac{M_2^{A+}(+1,-1;+1,+1)}{M_2^{A+}(+1,-1;+1,+1)+M_2^{B+}(+1,-1;+1,+1)}S_2 \\ &= r_0^A + \frac{M_2^{A+}(+1,-1;+1,+1)}{M_2^{A+}(+1,-1;+1,+1)+M_2^{B+}(+1,-1;+1,+1)}(S_0+r_0^BL) = r_0^A + (S_0+r_0^BL)\frac{\rho}{\rho+(\rho+\kappa+1)} \\ &= r_0^A + (S_0+r_0^BL)\frac{\rho}{2\rho+\kappa+1}\end{aligned}$$

The other values can similarly be derived for different strategies and are produced below:

$$V_2^A(-1,+1;+1,+1) = r_0^A + (S_0+r_0^BL\rho)\frac{1}{\rho+\kappa+2}$$

$$V_2^A(-1,-1;+1,+1) = r_0^A$$

$$V_2^A(+1,+1;+1,-1) = r_0^A(1-L) + (S_0+r_0^AL)\frac{\rho+\kappa+1}{2\rho+\kappa+1}$$

$$V_2^A(+1,-1;+1,-1) = r_0^A(1-L) + \frac{1}{2}(S_0+r_0^AL+r_0^BL)$$

$$V_2^A(-1,+1;+1,-1) = r_0^A(1-L) + (S_0+r_0^AL+r_0^BL\rho)\frac{1}{1+\rho}$$

$$V_2^A(+1,+1;-1,+1) = r_0^A(1-L\rho) + (S_0+r_0^AL\rho)\frac{\rho+\kappa+1}{\rho+\kappa+2}$$

$$V_2^A(+1,-1;-1,+1) = r_0^A(1-L\rho) + (S_0+r_0^AL\rho+r_0^BL)\frac{\rho}{1+\rho}$$

$$V_2^A(-1,+1;-1,+1) = r_0^A(1-L\rho) + \frac{1}{2}(S_0+r_0^AL\rho+r_0^BL\rho).$$

The sub-game perfect equilibrium (SPE) of the game can be derived through backward induction. Specifically, given both candidates' strategies of advertising investments z_1^A and z_1^B in period one, both candidates first choose their advertising strategies in period two to maximize their objective functions $W_2^A(z_1^A,z_2^A;z_1^B,z_2^B)$ and $W_2^B(z_1^A,z_2^A;z_1^B,z_2^B)$,[12] namely,

$$z_2^A = \text{argmax}_{z_2^A} W_2^A(z_1^A,z_2^A;z_1^B,z_2^B) \text{ and } z_2^B = \text{argmax}_{z_2^B} W_2^B(z_1^A,z_2^A;z_1^B,z_2^B)$$

that is equivalent to

$$W_2^A(z_1^A,z_2^A;z_1^B,z_2^B) > W_2^A(z_1^A,-z_2^A;z_1^B,z_2^B) \tag{A1}$$

And

$$W_2^B(z_1^A,z_2^A;z_1^B,z_2^B) > W_2^B(z_1^A,z_2^A;z_1^B,-z_2^B) \tag{A2}$$

forming optimal advertising strategies $z_2^A(z_1^A;z_1^B)$ and $z_2^B(z_1^A;z_1^B)$. Given these optimal advertising strategies in period two, both candidates need to choose the first period strategy z_1^A and z_1^B to maximize their objective functions $W_2^A(z_1^A,z_2^A(z_1^A;z_1^B);z_1^B,z_2^B(z_1^A;z_1^B)$ and $W_2^B(z_1^A,z_2^A(z_1^A;z_1^B);z_1^B,z_2^B(z_1^A;z_1^B))$,equivalently

$$z_1^A = \text{argmax}_{z_1^A} W_2^A(z_1^A,z_2^A(z_1^A;z_1^B);z_1^B,z_2^B(z_1^A;z_1^B))$$

and $z_1^B = \text{argmax}_{z_1^B} W_2^B(z_1^A,z_2^A(z_1^A;z_1^B);z_1^B,z_2^B(z_1^A;z_1^B))$which implies

$$W_2^A(z_1^A,z_2^A(z_1^A;z_1^B);z_1^B,z_2^B(z_1^A;z_1^B)) > W_2^A(-z_1^A,z_2^A(z_1^A;z_1^B);z_1^B,z_2^B(z_1^A;z_1^B)) \tag{A3}$$

and

$$W_2^B(z_1^A,z_2^A(z_1^A;z_1^B);z_1^B,z_2^B(z_1^A;z_1^B)) > W_2^B(z_1^A,z_2^A(z_1^A;z_1^B);-z_1^B,z_2^B(z_1^A;z_1^B)) \tag{A4}$$

[12] Note that to derive the optimal advertising strategies in period 2 for any advertising investment in period 1, z_1^A and z_1^B, we need to compare two candidates' objective functions for their specific advertising strategies in period 2. In this period, objective functions are expressions with z_1^A and z_1^B. For example, one can get $V_2^A(z_1^A,+1;z_1^B,+1)$=$r_0^A - I\{z_1^B=-1\}r_0^AL(1+\rho) + [S_0 + I\{z_1^B=-1\}r_0^AL(1+\rho) + I\{z_1^A=-1\}r_0^BL(1+\rho)] \times \frac{I\{z_1^A=+1\}(1+\rho+\kappa)}{I\{z_1^A=+1\}(1+\rho+\kappa)+I\{z_1^B=+1\}(1+\rho+\kappa)}$.

That is,$(z_1^A, z_2^A; z_1^B, z_2^B)$ is an SPE if and only if Eqs. (A1)–(A4) hold, in which Eqs. (A1) and (A2) characterize the optimization conditions to form an SPE in period two and Eqs. (A3) and (A4) give the optimization conditions in period one.

Hence, $(+1,+1;+1,+1)$ is an SPE if and only if[13]

$$W_2^A(+1,+1;+1,+1) > W_2^A(+1,-1;+1,+1), W_2^B(+1,+1;+1,+1) > W_2^B(+1,+1;+1,-1)$$

$$W_2^A(+1,+1;+1,+1) > W_2^A(-1,+1;+1,+1), W_2^B(+1,+1;+1,+1) > W_2^B(+1,+1;-1,+1)$$

where the first two conditions imply the optimal advertising strategies $z_2^A(+1;+1) = +1$ and $z_2^B(+1;+1) = +1$ in the second period and the optimal advertising strategies $z_1^A = +1$ and $z_1^B = +1$ in the first period.

Through algebraic manipulations, these conditions become
$r_0^A + \frac{2L\rho+\kappa+1}{\kappa+1} r_0^B < 1$ (omitted), $\frac{2L\rho+\kappa+1}{\kappa+1} r_0^A + r_0^B < 1$ (omitted)

$$r_0^A + \frac{2L\rho + \rho + \kappa}{\rho + \kappa} r_0^B < 1, \frac{2L\rho + \rho + \kappa}{\rho + \kappa} r_0^A + r_0^B < 1.$$

Note that the first two conditions are weaker than the following two conditions and thus can be omitted. ■

Proof of Proposition 2. $(+1,-1;+1,-1)$ is an SPE if and only if

$$W_2^A(+1,-1;+1,-1) > W_2^A(+1,+1;+1,-1), W_2^B(+1,-1;+1,-1) > W_2^B(+1,-1;+1,+1)$$

$W_2^A(+1,-1;+1,-1) > W_2^A(-1,-1;+1,-1)$ (always true), $W_2^B(+1,-1;+1,-1) > W_2^B(+1,-1;-1,-1)$ (always true), which become
$(1-L)r_0^A + \left[\frac{(2\rho+\kappa+1)L}{\kappa+1} + 1\right] r_0^B > 1$ and $\left[\frac{(2\rho+\kappa+1)L}{\kappa+1} + 1\right] r_0^A + (1-L) r_0^B > 1$.
Similarly, $(-1,+1;+1,-1)$ is an SPE if and only if
$W_2^A(-1,+1;+1,-1) > W_2^A(-1,-1;+1,-1)$ (always true), $W_2^B(-1,+1;+1,-1) > W_2^B(-1,+1;+1,+1)$,
$W_2^A(-1,+1;+1,-1) > W_2^A(+1,+1;+1,-1)$, $W_2^B(-1,+1;+1,-1) > W_2^B(-1,+1;-1,-1)$ (always true),
which reduces to
$\left[\frac{(\rho+\kappa+2)L\rho}{\kappa+1} + 1\right] r_0^A + (1-L\rho) r_0^B > 1$ and $(1-L)r_0^A + \left[\frac{(2\rho+\kappa+1)L}{\kappa+\rho} + 1\right] r_0^B > 1$.
$(-1,+1;-1,+1)$ is an SPE if and only if
$W_2^A(-1,+1;-1,+1) > W_2^A(-1,-1;-1,+1)$ (always true), $W_2^B(-1,+1;-1,+1) > W_2^B(-1,+1;-1,-1)$ (always true), $W_2^A(-1,+1;-1,+1) > W_2^A(+1,+1;-1,+1)$, $W_2^B(-1,+1;-1,+1) > W_2^B(-1,+1;+1,+1)$,
which becomes:
$(1-L\rho)r_0^A + \left[\frac{(\rho+\kappa+2)L\rho}{\rho+\kappa} + 1\right] r_0^B > 1$ and $\left[\frac{(\rho+\kappa+2)L\rho}{\rho+\kappa} + 1\right] r_0^A + (1-L\rho) r_0^B > 1$. ■

Proof of Proposition 3. Following the procedure employed in Proposition 1, one can show $(+1,-1;+1,+1)$ is an SPE if and only if

$$W_2^A(+1,-1;+1,+1) > W_2^A(+1,+1;+1,+1), W_2^B(+1,-1;+1,+1) > W_2^B(+1,-1;+1,-1)$$

$W_2^A(+1,-1;+1,+1) > W_2^A(-1,-1;+1,+1)$ (always true), $W_2^B(+1,-1;+1,+1) > W_2^B(+1,-1;-1,+1)$.
These conditions become the following three inequalities:
$r_0^A + \frac{2L\rho+\kappa+1}{\kappa+1} r_0^B > 1$, $\left[\frac{(2\rho+\kappa+1)L}{\kappa+1} + 1\right] r_0^A + (1-L) r_0^B < 1$ (omitted),

$$\left[\frac{(2\rho + \kappa + 1)L}{\kappa + \rho} + 1\right] r_0^A + (1-L) r_0^B < 1.$$

Note that the first two inequalities imply $r_0^A < r_0^B$, and that the second inequality, weaker than the third inequality, can be omitted.

$(-1,+1;+1,+1)$ is a SPE if and only if
$W_2^A(-1,+1;+1,+1) > W_2^A(-1,-1;+1,+1)$ (always true), $W_2^B(-1,+1;+1,+1) > W_2^B(-1,+1;+1,-1)$

$$W_2^A(-1,+1;+1,+1) > W_2^A(+1,+1;+1,+1), W_2^B(-1,+1;+1,+1) > W_2^B(-1,+1;-1,+1).$$

These conditions can be simplified as
$\left[\frac{(\rho+\kappa+2)L\rho}{\kappa+1} + 1\right] r_0^A + (1-L\rho) r_0^B < 1$ (omitted),$r_0^A + \frac{2L\rho+\rho+\kappa}{\rho+\kappa} r_0^B > 1$,
$\left[\frac{(\rho+\kappa+2)L\rho}{\rho+\kappa} + 1\right] r_0^A + (1-L\rho) r_0^B < 1$.
Note that the first condition is weaker than the third condition and thus can be omitted. It should be noted that the last two conditions yield $r_0^A < r_0^B$. ■

[13] Note that different from SPE, $(+1,+1;+1,+1)$ is a NE if and only if $W_2^A(+1,+1;+1,+1) > \max\{W_2^A(+1,-1;+1,+1), W_2^A(-1,+1;+1,+1), W_2^A(-1,-1;+1,+1)\}$, $W_2^B(+1,+1;+1,+1) > \max\{W_2^B(+1,+1;+1,-1), W_2^B(+1,+1;-1,+1), W_2^B(+1,+1;-1,-1)\}$.

Proof of Proposition 4. The result immediately follows from the fact that $\frac{2L\rho+\rho+\kappa}{\rho+\kappa}$ increases with ρ and decreases with κ. ∎

Appendix B. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijresmar.2022.01.001.

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