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Bridge, Focus, Attack, or Stimulate: Retail Category Management Strategies with a Store Brand

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Abstract. We investigate a monopolist retailer's category management strategy where the main strategic decisions are how to horizontally position a store brand relative to the incumbent national brands and how to price the store and national brands for retail category profit maximization. We analyze a market composed of two consumer segments with differing tastes and heterogeneity with respect to willingness to pay and a product category consisting of two competing national brands and one store brand. We find that contrary to the existing literature, it is not always optimal for a retailer to position its store brand against the leading national brand; instead there are many situations where it is best to position the store brand close to the weaker national brand or to position it in the "middle" so it appeals to both national brands' target segments. In the process we identify four distinct category management strategies that a retailer can use with a store brand. In three of these the optimal store brand price is the brand's monopoly price, while in the remaining one strategy the price is lower. We also suggest an easy to implement means for a retailer to determine which strategy is best to use, depending on the particular competitive environment present before the introduction of the store brand and the relative quality of the store brand. We find that the store brand entry is most beneficial to the retailer when the national brands are moderately differentiated. Finally we show that introducing a store brand not only allows the retailer to garner a higher share of the channel profits through higher retail margins, but also often provides the retailer the benefit of increases in national brand unit sales as well as incremental sales from the store brand.

Key words. store brands, retailing, category management, positioning, channel strategy, game theory, strategic pricing

JEL Classification: M310

1. Introduction

Store brands now account for one of every five items sold every day in U.S. supermarkets, drug chains and mass merchandisers. They represent more than $50 billion of current business at retail, are achieving new levels of growth every year, (Private Label Manufacturers Association, www.plma.com) and have been expanding into non-grocery categories, such as personal computers (Wall Street Journal, May 3, 2002). Yet, optimal pricing and positioning of store brands still pose a challenging problem as witnessed by the following two quotes: "A retailer's pricing strategy should be based upon the role of private label. A number of people now realize they don't have the answer to that question." (Bill Bishop, President of Willard Bishop Consulting, Progressive Grocer, 2000) "A lot of things still get put together in the old way. For example, 'Tide is doing well, so we ought to emulate Tide. We will be 10% to 15% below Tide and make a better profit.' That is an answer, but in a world where retail is rapidly changing it may not be the optimal answer." (Ron Lunde, a consultant and former retailer, Progressive Grocer, 2000).

Given the growing importance of store brands and how to manage them, it is not surprising that marketing scholars have paid more attention to this issue in the recent years.[1] For example Scott Morton and Zettelmeyer (2004, SZ hereafter) and Sayman et al. (2002, SHR hereafter) analyze the issue of store brand positioning[2] and suggest that, in general, a store brand should be positioned as close to the leading national brand as possible, a la "Emulate Tide." SHR further show that store brand introduction under this positioning strategy has an asymmetric impact on the incumbent national brands, causing a greater decrease in wholesale price and a greater increase in retail margin for the leading national brand than for the secondary national brand.

Interestingly, the few published empirical studies on store brand positioning do not suggest the "Emulate Tide" strategy is the universally used strategy. For instance, SHR analyzed 75 product categories in two grocery chains and found this strategy was followed in less than 1/3 of the categories. Similarly, SZ surveyed two stores and found only 15–20% of the store brands matched a major national brand in size, shape, color, lettering and art although 63–65% of the store brands were placed next to a major national brand on the store shelves. Pauwels and Srinivasan's (2004) investigation of four product categories in Dominick's Finer Foods suggests that store brands typically compete more closely with second-tier national brands than against premium national brands, contrary to the results of the two theoretical studies. These findings, as well as SHR's analysis of secondary data, all suggest that retailers might be pursuing a variety of different positioning strategies for store brands.

Our own observation of three additional product categories in the Dominick's database further supports the possible presence of multiple store brand strategies across product categories. For instance, the changes in manufacturer revenue in Table 1 show that the major "victim" of the store brand entry in one instance is the premium national brand manufacturer (Welch's cranberry juice), and in other instances is the non-premium national brand manufacturer (Kellogg's raisin bran cereal and Geisha canned tuna). The table also shows store brand introduction can affect national brands in a variety of ways—increasing the wholesale and retail prices with little impact on the quantity sold (Total Raisin Bran cereal), decreasing the wholesale and retail prices as well as the quantity (Chicken of the Sea tuna), decreasing the wholesale and retail prices while increasing the quantity (Bumble Bee tuna), or causing little changes in the wholesale and retail prices while significantly decreasing the quantity (Welch's cranberry juice).

1 We follow the lead of most of the literature and use the term "store brand" to refer to all merchandise sold under a retail store's private label. That label can be the store's own name or a brand name created exclusively by the retailer for that store.

2 As in these studies, we define positioning strictly as horizontal positioning. As for vertical positioning of store brands, Raju et al. (1995) as well as our analysis show a retailer is always better off with higher levels of store brand quality as long as the incremental cost is not too high.

Table 1. The impact of store brand entry on incumbent national brands*.

Product category	Brand	Change in wholesale price (%)	Change in retail price (%)	Change in quantity sold (%)	Change in manufacture revenue (%)
Canned Tuna (Solid White)	Chicken of the Sea	−11.4	−10.43	−11.53	−21.67
	Geisha	−6.91	−7.28	−27.33	−32.35
	3 Diamond	−2.04	−12.01	+6.73	+4.55
	StarKist**	−10.27	−9.3	+15.32	+3.47
	Bumble Bee	−14.21	−10.37	+42.19	+21.98
Raisin Bran Cereal	Total**	+2.62	+2.69	−0.75	+1.85
	Kellogg	−5.2	−3.22	−4.78	−9.73
	Post	−3.52	−2.99	+17.77	+13.62
Frozen Cranberry Juice	Welch's**	−0.7	−1.34	−19.99	−20.56
	Tropicana	−17.01	−17.96	+23.03	+2.1

*The numbers in this table represent % changes from the average for the period until 24 weeks prior to the time of the store brand introduction to the average for the period since 24 weeks after the store brand introduction. For raisin bran cereal, however, the "before store brand introduction" period was defined as the period until 12 weeks prior to the time of store brand introduction, due to limited number of observations.
**Premium national brand (characterized by the highest retail price following Pauwels and Srinivasan's 2004 definition).

SHR attribute the discrepancies between their theoretical analysis conclusion and empirical evidence to factors such as a high cost involved in imitating the leading national brands, the presence of a price-sensitive segment, and the lack of sufficient convexity of the function that links horizontal differentiation into cross price sensitivity of demand. While accepting these as plausible explanations, our study demonstrates that there exist more fundamental strategic forces in retail category management that lead to multiple store brand strategies. We do this by analyzing a parsimonious two-manufacturer, one-retailer game theoretic model to explore the following research questions:

1. Excluding the situations noted by SHR which preclude a retailer from positioning the store brand against the leading national brand, is it otherwise always optimal to follow SHR's and SZ's suggestion to position the store brand as close as possible to the leading national brand?
2. If positioning against the leading national brand is not always optimal, what are the other alternative strategies?
3. Under what conditions should a retailer use each of the alternative strategies?

Our model builds on the relative strengths of SZ's and SHR's analytic models. In order to ensure transparent connections between the underlying market characteristics and the demand structure, we derive demand functions from an explicit buyer behavior model that is identical to SZ's. Then, we follow the lead of SHR and relax three of SZ's restrictions on strategic alternatives by allowing the retailer to (a) position the store brand anywhere between the two national brands (instead of selecting one of two pre-specified positions), (b) have stronger bargaining power against a store brand manufacturer (instead of assuming the same bargaining power as against national brand manufacturers), and (c) carry two (instead of one) national brands in addition to a store brand.[3] In this way, our model *does not* impose any new assumptions that have not been used in the existing studies, yet *does* enable us to expand their findings and provide richer insights into the retailer's category management strategies with a store brand.

Our analysis produces three key results. First, in contrast to the extant analytic literature that suggests a retailer position the store brand as close as possible to the leading national brand, we show that the retailer often earns higher category profits by positioning the store brand either close to the weaker national brand or roughly halfway between the two national brands. Interestingly, the existence of diverse optimal store brand positions does not require asymmetric sizes between consumer segments nor a store brand manufacturer's inability to fully imitate a national brand. This leads to our second major finding, i.e., these variations in optimal store brand position reflect four distinct types of retail category management strategies with a store brand. We label these strategies "Bridge", "Focus", "Attack", and "Stimulate" and describe each with specific positioning and pricing actions. Third, we identify the conditions under which each of the four strategies is optimal for a retailer. These conditions are described in terms of measurable real world factors, thereby enabling academics to empirically test our conclusions and practitioners to directly apply our suggestions to their specific circumstances.

We note that these new insights are due in large part to two factors. First our derived demand function has multiple regions with different slopes, a characteristic empirically observed in brand competition within categories. Faced by such a demand structure, an important strategic issue for all the channel members is to ensure their strategic actions let them "play in" the demand region most favorable to themselves. In this sense, one can consider a store brand as a strategic tool that a retailer can use to reshape the competitive environment within the category for its own advantage. Second our approach is less restrictive than previous work and thus we are able to identify new strategic alternatives.

Consistent with previous analytical studies we find *a* major benefit of introducing a store brand comes from the retailer increasing its share of the channel profits associated with the national brand sales. However, a retailer can also benefit from increases in national brand unit sales caused by a lower retail price and profits generated from the store brand sales. The relative magnitudes of these different benefits systematically depend upon the underlying market conditions. Therefore, it is imperative that a retailer understands the linkages between the underlying market conditions and the optimality of each of the category management strategies and their impact on the market outcomes.

2. Model

In this section, we develop a model of buyer behavior and market structure and use this model to derive demand functions. This demand derivation approach is similar to SZ's and allows us to ensure that the demand models before and after the store brand introduction represent the same underlying market environment. In Section 3, we analyze this model by applying a set of rules of the game that are comparable to SHR's to obtain equilibrium solutions.

3 By allowing the retailer to add a third brand, we implicitly assume no binding constraint on shelf space.

2.1. Consumer utility model

Our model of consumer behavior is based on Desai's (2001) utility function for horizontally and vertically differentiated products. Specifically:

$$U_{ij} = \beta_i v_j - k_i t_{ij} - p_j \tag{1}$$

where β_i = consumer i's willingness to pay for quality; v_j = quality level of brand j; k_i = unit cost of mismatch (or transportation) for consumer i; t_{ij} = mismatch (or distance) between consumer i's ideal point and brand j's position; p_j = retail price of brand j.

Equation (1) indicates consumers are heterogeneous in three ways—willingness to pay for quality (β_i), cost of mismatch (k_i) and degree of product mismatch (t_{ij}). In addition, "to model the possibility that higher valuation consumers also have stronger taste preferences," Desai assumes if $\beta_H > \beta_L$ then $k_H \geq k_L$. Thus, β_i and k_i are either positively correlated ($k_H > k_L$) or independent ($k_H = k_L$). As we discuss below, these two situations lead to two alternative utility models for our study.

Assume first that β_i and k_i are perfectly positively correlated. Next, define a rescaled mismatch variable, $m_{ij} = t_{ij}/b$ where b is the constant that perfectly maps k_i into β_i, i.e. $\beta_i = bk_i$. Substituting this into equation (1), the utility function simplifies to:

$$U_{ij} = \beta_i(v_j - m_{ij}) - p_j \tag{2}$$

Alternatively, let β_i and k_i be independent ($k_H = k_L = k$). Then rescale t_{ij} into $m_{ij} = kt_{ij}$ so that $t_{ij} = m_{ij}/k$. Substituting this into equation (1), the utility function simplifies to:

$$U_{ij} = \beta_i v_j - m_{ij} - p_j \tag{3}$$

without loss of generality.[4]

Equations (2) and (3) represent two alternative models of consumer behavior. In words, equation (2) implies that a consumer who highly values quality will also highly disvalue a horizontal mismatch (i.e., highly value a horizontal fit) between her ideal point and the horizontal brand position. This is identical to the consumer utility model used by SZ. In contrast, equation (3) states that regardless of how much (little) a person values quality, this person always disvalues horizontal mismatch (i.e., values horizontal fit) to the same degree. For parsimony, we provide a full discussion of our analyses only for the first model in this paper. However, we later discuss the results obtained from the latter model in order

4 We thank one of the anonymous reviewers for suggesting this utility formulation.

to examine the robustness of our main conclusions to alternative assumptions of consumer behavior.

2.2. *Underlying market structure*

We assume a market composed of two consumer segments, each with its own ideal point. Within each segment consumers are heterogeneous with respect to willingness to pay, β_i, which is uniformly distributed between 0 and 1 with a density of 1. We assume each segment is targeted by one national brand and the characteristics of the brand fit its target segment's tastes perfectly. More technically, national brand j (hereafter referred to as NB_j) is positioned at segment j's ideal point ($j = 1, 2$). Let parameter d ($0 \leq d \leq 1$) denote the "perceived distance" between the positions of NB_1 and NB_2 in the product space. A higher d simultaneously implies greater horizontal differentiation between the NB's in the minds of the consumers and a greater mismatch between NB_j and segment 3-j's tastes. If a store brand (SB) also exists in the category, its position relative to the NB's is captured by x, $0 \leq x \leq 1$. xd and $(1-x)d$ capture the perceived distances from the SB's position to NB_1 and NB_2, respectively. $x < .5$ implies the SB is positioned closer to NB_1 than to NB_2.[5]

We allow for vertical differentiation among the competing brands by setting the quality levels of NB_1, NB_2 and SB to be 1, α and α_S, respectively ($0 < \alpha, \alpha_S \leq 1$). In this way, NB_1 is the leading brand in quality and SB's quality can be higher or lower than that of NB_2. See Figure 1 for a graphic illustration of the model.

The above formulation of market structure relaxes SZ's assumption of equal quality of the two NB's and thus reflects the reality that SB's are sometimes perceived to have higher quality than some NB's (Chintagunta, 2002; Harrison, 2000; USDA, 2000). It also allows for three brands to be sold at one time and allows the SB to be positioned anywhere on the line between the two NB's. On the other hand, our assumption of an equal mass of consumers in each segment is a simplification of SZ's model of two segments with unequal masses.[6]

For a given set of horizontal product positions (d and x) and quality levels (1, α and α_S), one can determine each consumer's utility by using equation (2). This leads to the following utility functions for NB_1, NB_2 and SB for consumer i in segment 1:

$$U_{i1} = \beta_i - p_1 \quad (4)$$

$$U_{i2} = \beta_i(\alpha - d) - p_2 \quad (5)$$

$$U_{iS} = \beta_i(\alpha_S - xd) - p_S, \quad (6)$$

where the second subscript refers to the brand, and p_1, p_2 and p_S are the retail prices of NB_1, NB_2 and the SB, respectively. Likewise, the utility functions for consumer i in segment 2

5 We do not assume a third, unique target segment for SB. This rules out the (realistic but less interesting) possibility that the store brand position is determined by the unique consumer needs not met by the NB's.

6 We later relax this assumption and discuss the impact on our results.

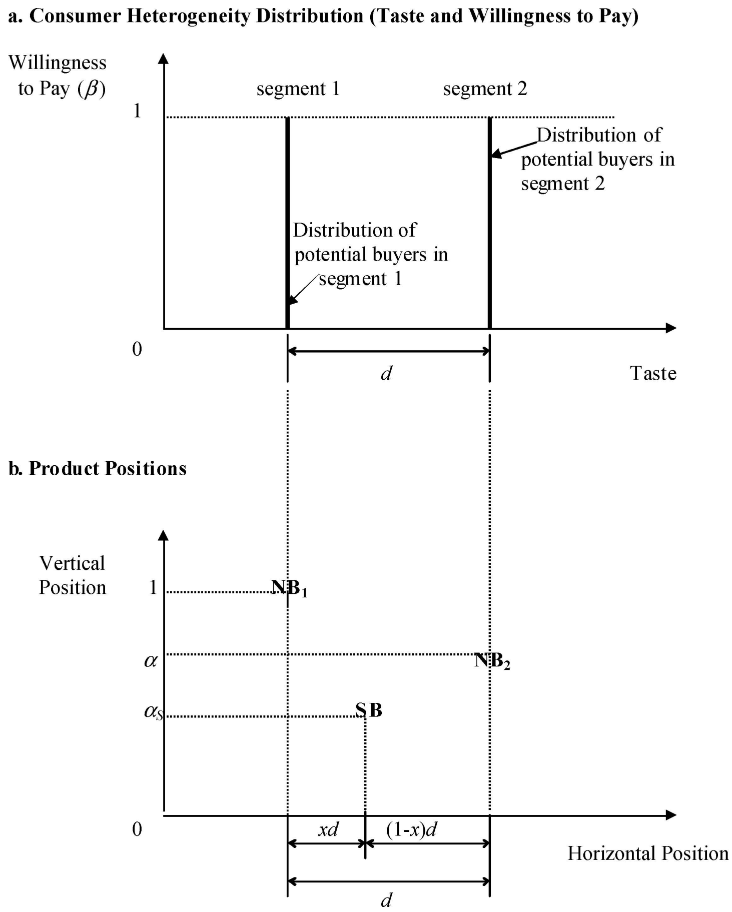


Figure 1. Model of the underlying market.

are as follows:[7]

$$U_{i1} = \beta_i(1 - d) - p_1 \tag{7}$$

$$U_{i2} = \beta_i(\alpha) - p_2 \tag{8}$$

$$U_{iS} = \beta_i(\alpha_S - d(1 - x)) - p_S, \tag{9}$$

Note that when β_i is close to 0, consumer i's net utility depends almost entirely upon prices and little on the products' quality levels and positions, indicating high price sensitivity and little brand loyalty.[8] As is standard, we assume the utility for the outside option is zero for all customers.

7 Note that consumer i in segment 1 is not the same consumer i found in segment 2.

8 Note that, in our second buyer behavior model (equation (3)), every consumer values a good fit equally regardless of their willingness to pay for quality. Thus, even low willingness to pay consumers exhibit "loyalty" and thus are less price sensitive under this second buyer behavior model.

2.3. *Derivation of demand*

We make the typical assumption that each consumer purchases either one unit of the brand that yields the highest positive net utility or nothing if none of the brands yields positive net utility. Then, the market demand for each brand can be derived from equations (4) to (9) in a straightforward four-step process. As a first step, we determine the rank order of the three brands in terms of gross utility (i.e., the non-price component of utility) for each segment.[9] For each segment, let A denote the brand with the highest gross utility, B the next highest, C the lowest, and D the no purchase option. Note that the identity of A, B and C is a function of the category characteristics, i.e., α, α_S, d and x.

Second, for each segment we determine the six values of β_i that represent the marginal consumers who are indifferent between a pair of alternatives (including the no purchase option), using equations (4)–(9). For instance, the marginal consumer in segment 1 who is indifferent between purchasing NB_1 and SB can be identified by equating equations (4) and (6) and solving for $\beta^*_{11S}(= (p_1 - p_S)/(1 - \alpha_S + xd))$, where the first subscript denotes the segment and the next two the identity of the two brands. Similarly, the marginal consumer in segment 2, who is indifferent between NB_2 and no purchase, is identified by equating equation 8 to zero. In this way, we determine the six critical β's for segment h $(= 1, 2)$, labeled as β^*_{hAB}, β^*_{hAC}, $\beta^*_{hAD}, \ldots,$ and β^*_{hCD}. Note that these six values are functions of prices as well as α, α_S, d and x.

Third, we determine the within-segment demand for each brand by partitioning the unit line representing each segment as shown in Figure 1(a). To partition the market segment into discrete groups of consumers making different brand choices, we use the following Lemma (proof in Appendix 1, which is available at http://www.fuqua.duke.edu/faculty/alpha/staelin.htm):

Lemma 1. *For a given set of values for α, α_S, x and d, there exists a rank order of the competing brands in terms of their gross utility in each segment. If for a given set of prices a consumer with a certain level of willingness to pay purchases a particular brand, another consumer with lower willingness to pay will never buy a higher ordered brand at that set of prices.*

In the context of our illustrative example, Lemma 1 implies that if a consumer with β_H purchases brand B (second highest gross utility) instead of brand A (highest gross utility) because of B's relatively low price, another consumer with β_L $(<\beta_H)$ will not purchase A, either.

Applying this principle to all pairs of choice alternatives and the fact that β_i is distributed uniformly between zero and one, the demand for each brand from segment h is derived as follows:

$$q_{hA} = 1 - \text{Min}[1, \text{Max}(\beta^*_{hAB}, \beta^*_{hAC}, \beta^*_{hAD}, 0)] \tag{10}$$

$$q_{hB} = 1 - q_{hA} - \text{Min}[1 - q_{hA}, \text{Max}(\beta^*_{hBC}, \beta^*_{hBD}, 0)] \tag{11}$$

$$q_{hC} = 1 - q_{hA} - q_{hB} - \text{Min}[1 - q_{hA} - q_{hB}, \text{Max}(\beta^*_{hCD}, 0)]. \tag{12}$$

9 In analyzing our second model of buyer behavior, the ranking is done with respect to just quality. Thus for this model of buyer behavior NB_1 is ranked first in both segments as long as α and α_S are less than 1.

Table 2. Partitioning possibilities in a given segment for two and three brand cases*.

Two brands	Three brands
A B	A B C
A	A B
B	A C
	A
	B C
	B
	C

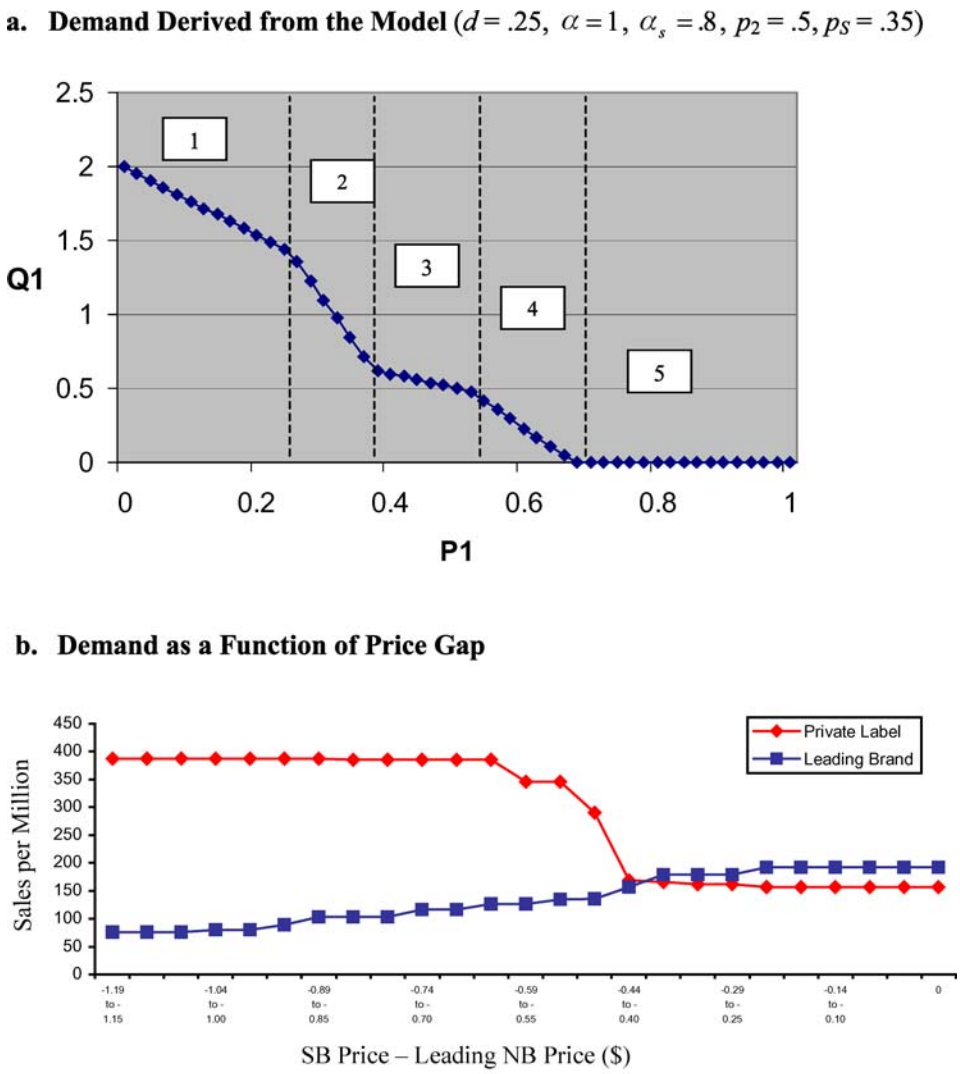
*The brand rank ordering in gross utility is descending alphabetically.

Since the β^*'s are simple linear functions of prices, equations 10–12 represent linear demand functions. However, note that as prices change, the rank order of $\beta^*_{hAB}, \ldots,$ and β^*_{hCD} might also change, leading to multiple linear regions in the above demand functions, i.e., a kinked demand system. Specifically there are seven possible ways of partitioning a segment as shown in Table 2. (The table also displays the three possible ways of partitioning the segment if only two competing brands, A and B exist.) The fourth and final step of demand derivation is to simply sum up the brand demands in the two segments. This creates a maximum of $7 \times 7 = 49$ ($3 \times 3 = 9$) possible regions for the three (two) brand situation.

2.4. Demand characteristics

Figure 2(a) displays an example of NB_1's demand function derived through this four step process for $\alpha = 1$, $\alpha_S = .8$, $d = .25$, $x = .5$, $p_2 = .5$, and $p_S = .35$. Note that the demand function is continuous and downward sloping in own price but also kinked with five regions of linear demand with different slopes. These varying slopes reflect the effect of p_1 on the patterns of brand competition in each segment. In region 1, p_1 is sufficiently lower than p_2 and p_S so that NB_1 is the only brand purchased by consumers in both segments. In region 2, p_1 is still low enough to attract price sensitive (low β) consumers of segment 2 to buy NB_1, but segment 2 consumers with higher β's are now willing to pay the higher price ($p_2 = .5$) to purchase NB_2 since it fits their tastes better. In region 3, p_1 is too high for NB_1 to attract any buyer from segment 2 but still low enough to keep the segment 1 buyers from switching to the SB. In region 4, the high p_1 causes some price sensitive consumers in segment 1 to switch from NB_1 to the SB. Finally, in region 5, p_1 is so high that NB_1 loses all its segment 1 customers to the SB.

This type of kinked demand has been used in other studies analyzing spatial models (e.g., Salop, 1979; Vandenbosch and Weinberg, 1995; Chiang et al., 2003). Empirical studies also show the presence of kinked demand in an oligopoly (Bhaskar et al., 1991; Awh and Primeaux 1992; Dickson and Urbany, 1994). More importantly, a study by ACNielsen



(Source: "Private Label Grows Up," Consumer Insight Magazine, ACNielsen, September, 1999).

Figure 2. Demand characteristics.

(1999) finds kinked demand a key characteristic for a national brand and a store brand competing in the same product category. Specifically, as Figure 2(b) shows, there exists a price range over which price changes cause active switching from one brand to the other. Outside this range, demand is much less sensitive to price changes. Our model captures this demand characteristic very well. In contrast SHR's demand function (which is linear over the entire price range) is not able to reflect this varying price sensitivity. Our formulation also implies that the competitive environment depends in part on the specific price levels since the slopes differ over the range of prices. It is this changing competitive environment that allows us to provide new insights into inter-brand strategic interactions within a category and broaden our understanding of optimal category management with a store brand.

2.5. Rules of the game

To date two different approaches have been used to model the game between channel members while studying the introduction of a store brand. SZ use a bargaining model first proposed by Shaffer and Zettelmeyer (2002) that implicitly assumes the channel members are able to perfectly coordinate the channel (i.e., charge the channel profit maximizing retail prices) both before and after the introduction of the SB. SHR assume no channel coordination and independent profit maximization by each channel member. Given our interest in category management (which includes not only the positioning decision, but also the pricing strategy) and our belief that most channels are not fully coordinated, we use SHR's approach.

We treat the horizontal and vertical positions of the NB's (d and α) and the quality level of the SB (α_S) as exogenous factors characterizing the market environment. Then, within this environment, we apply the following sequence of moves:

(1) The retailer, if it offers a SB, selects the SB's position, x, relative to the NB's.
(2) The two NB manufacturers set their respective wholesale prices, w_1 and w_2, to maximize their own respective profits taking into consideration the retailer's reaction to the wholesale price changes.
(3) The retailer sets retail prices for NB_1, NB_2 and the SB conditional on wholesale prices, w_1 and w_2, to maximize the retailer category profits.

As in previous studies (Raju et al., 1995; Vandenbosch and Weinberg, 1995; SHR, 2002; SZ, 2004), we assume the marginal cost of production is zero regardless of the quality level for all three brands. While this assumption may be unrealistic, the first three studies mentioned above show their main results remain qualitatively unchanged when the production cost is assumed to be an increasing function of quality. In addition, since the SB is still a commodity until it has the store label, SB manufacturers have little market power (Connor and Peterson, 1992; Mills, 1995; Ailawadi and Harlam, 2004). Consequently, we assume the SB is not subject to double marginalization and, thus, is obtained by the retailer at the manufacturer's cost ($= 0$) (Raju et al., 1995; SHR, 2002). This makes the SB almost always cheaper for the retailer to acquire than a NB although the SB could be of higher quality than a NB, which is consistent with previous empirical observations (USDA, 2000; Chintagunta, 2002).

These assumptions lead to the following objective functions for the manufacturers and the retailer:

$$\Pi_{M1} = w_1 q_1, \tag{13}$$

$$\Pi_{M2} = w_2 q_2, \quad \text{and} \tag{14}$$

$$\Pi_R = (p_1 - w_1)q_1 + (p_2 - w_2)q_2 + p_S q_S, \tag{15}$$

where p_S and q_S are held to zero before SB introduction.[10]

3. Model analysis

The kinked demand functions derived from our model are not continuously differentiable over the entire range of prices. Consequently, using the standard mathematical method (e.g., McGuire and Staelin, 1983) for obtaining the equilibrium conditions requires solving the problem for each region of the kinked demand curve and then comparing all local optimums and corner solutions to find the global optimum. This approach can be very complicated even with a simpler demand structure as shown by Chiang et al. (2003). Given this complexity we start our model analysis with two polar cases for d (i.e., $d = 0$ and d large enough to insure the two NB's don't compete), which can be easily solved using the standard mathematical approach. Later, we generalize our analysis to the entire range of d by taking a more efficient approach of quadratic programming and numerical optimization.

3.1. When d = 0

$d = 0$ represents an environment where there are no taste differences among consumers and no horizontal differentiation between the NB's. Since the SB position is assumed to be between the two NB's, it, too, is positioned at the ideal point of the one consumer segment which is twice the density of the single segment in our general case. Since there is no horizontal differentiation between the available brands, consumers only consider quality and price when deciding which brand to buy. Thus, the retailer only needs to decide what to charge for each brand. This simplification allows us to analyze the model mathematically (Details are shown in Technical Appendix 1, which, along with Technical Appendix 2 and 3, is available at http://www.fuqua.duke.edu/faculty/alpha/staelin.htm). The closed form solutions are presented in Table 3.[11]

We note three main results from Table 3. First, before the introduction of the SB, the premium national brand (NB_1) has a higher wholesale price, retail price, and quantity sold than the weaker national brand (NB_2), allowing manufacturer 1 to earn more profits than manufacturer 2. This indicates the weaker national brand with no horizontal differentiation must offer a significantly lower price in order to motivate some low willingness to pay consumers to buy the brand. As α approaches 1, the wholesale prices and, thus, profits for both manufacturers go to zero as expected for perfectly substitutable products. In this

10 This profit function does not consider any fixed costs associated with introducing a store brand.

11 Note that since $d = 0$, our two models of buyer behavior are identical. Thus, the Table 3 results hold for both models of buyer behavior.

Table 3. Equilibrium results for vertically differentiated product category ($d = 0$).

	Before SB	After SB ($1 > \alpha > \alpha_S$)	After SB ($1 > \alpha_S \geq \alpha$)
w_1	$\frac{2(1-\alpha)}{4-\alpha}$	$\frac{2(1-\alpha)(1-\alpha_S)}{4-\alpha-3\alpha_S}$	$\frac{1-\alpha_S}{2}$
w_2	$\frac{\alpha(1-\alpha)}{4-\alpha}$	$\frac{(1-\alpha)(\alpha-\alpha_S)}{4-\alpha-3\alpha_S}$	
p_1	$\frac{3(2-\alpha)}{2(4-\alpha)}$	$\frac{6-3\alpha-5\alpha_S+2\alpha\alpha_S}{2(4-\alpha-3\alpha_S)}$	$\frac{3-\alpha_S}{4}$
p_2	$\frac{\alpha(5-2\alpha)}{2(4-\alpha)}$	$\frac{5\alpha-2\alpha^2-\alpha_S-2\alpha\alpha_S}{2(4-\alpha-3\alpha_S)}$	
p_s		$\frac{\alpha_S}{2}$	$\frac{\alpha_S}{2}$
q_1	$\frac{2}{4-\alpha}$	$\frac{2(1-\alpha_S)}{4-\alpha-3\alpha_S}$	$\frac{1}{2}$
q_2	$\frac{1}{4-\alpha}$	$\frac{1-\alpha_S}{4-\alpha-3\alpha_S}$	0
q_s		$\frac{1-\alpha}{4-\alpha-3\alpha_S}$	$\frac{1}{2}$
Π_{M1}	$\frac{4(1-\alpha)}{(4-\alpha)^2}$	$\frac{4(1-\alpha_S)(1-\alpha_S)^2}{(4-\alpha-3\alpha_S)^2}$	$\frac{1-\alpha_S}{4}$
Π_{M2}	$\frac{\alpha(1-\alpha)}{(4-\alpha)^2}$	$\frac{(\alpha-\alpha_S)(1-\alpha_S)^2}{(4-\alpha-3\alpha_S)^2}$	0
Π_R	$\frac{4+5\alpha}{2(4-\alpha)^2}$	$\frac{4+5\alpha-\alpha_S-2\alpha_S^2-18\alpha\alpha_S+\alpha^2\alpha_S+11\alpha\alpha_S^2}{2(4-\alpha-3\alpha_S)^2}$	$\frac{1+3\alpha_S}{8}$

Demand structure before SB

$$q_1 = 2\Big(1 - \frac{p_1 - p_2}{1-\alpha}\Big), \quad q_2 = 2\Big(\frac{p_1 - p_2}{1-\alpha} - \frac{p_2}{\alpha}\Big)$$

Demand structure after SB ($1 > \alpha > \alpha_S$)

$$q_1 = 2\Big(1 - \frac{p_1 - p_2}{1-\alpha}\Big), \quad q_2 = 2\Big(\frac{p_1 - p_2}{1-\alpha} - \frac{p_2 - p_S}{\alpha - \alpha_S}\Big), \quad q_S = 2\Big(\frac{p_2 - p_S}{\alpha - \alpha_S} - \frac{p_S}{\alpha_S}\Big)$$

Demand structure after SB ($1 > \alpha_S \geq \alpha$)

$$q_1 = 2\Big(1 - \frac{p_1 - p_S}{1-\alpha_S}\Big), \quad q_2 = 0, \quad q_S = 2\Big(\frac{p_1 - p_S}{1-\alpha_S} - \frac{p_S}{\alpha_S}\Big).$$

situation, the retailer extracts all the profits in the channel. This shows how the competitive environment between the two NB's affects the retailer's ability to increase its share of channel profits.

Second, comparing the "Before SB" column and the next column, we note that when the SB has the lowest level of quality ($1 > \alpha > \alpha_S$), the SB entry leads to a larger percent decrease in the wholesale price for NB_2 than for NB_1 while the quantities for the NB's decrease proportionally. Consequently, the profit for manufacturer 2 is more severely affected by the SB entry. As expected, the negative impact of the SB entry on manufacturer profits increases as α_S becomes larger. In particular, as α_S approaches α, w_2 and Π_{M2} are driven toward zero. Interestingly we note that in these situations the retailer always prices the SB at its monopoly price, $\alpha_S/2$, which is a function of its own quality level, but is not influenced by the quality levels of the NB's.

Third, as seen in the last column of Table 3, when the quality level of the SB exceeds that of NB_2 ($1 > \alpha_S \geq \alpha$), the retailer completely displaces the lower quality NB_2, again pricing the SB at its monopoly price. As before, the SB entry results in decreases in w_1, p_1 and q_1, making manufacturer 1 worse off. Nevertheless, it is clear that the main "victim" of the SB entry is still NB_2 as is the case when $\alpha > \alpha_S$. This result is in agreement with Pauwels and Srinivasan's (2004) empirical finding that a SB introduction affects a second tier NB much more than a premium NB. This suggests that perhaps the product categories analyzed by Pauwels and Srinivasan consist mainly of vertically differentiated NB's with little horizontal differentiation.

The above results reflect how the SB interacts with the two NB's when $d = 0$. Note that when $1 > \alpha > \alpha_S$, the SB only directly affects the demand for NB_2 (i.e., the demand function for NB_1 does not contain p_S). However, NB_1 is still impacted by the SB entry since the SB's "attack" on NB_2 causes the NB_2 manufacturer to react (by lowering w_2) and this reaction directly affects NB_1. In this way, the SB is able to "stimulate" the NB_1 manufacturer to react. As a result, wholesale prices decrease and retail margins increase for both NB's, enhancing the retailer's category profits. We label this the "*Stimulate*" strategy since the retailer uses the SB to stimulate the price competition between the two NBs.

In contrast, when $1 > \alpha_s \geq \alpha$, the SB enters the category as the medium quality brand, competing directly against both NB's. In this case, the retailer directly attacks both NB's with the SB. We label this the "*Attack Both*" strategy. Within this parameter range, the retailer finds it best to displace NB_2, since the SB is of higher quality and thus can garner a higher retail price. Consequently, the retailer only carries two brands.

3.2. When d is large.

We next consider the situation where the two NB's are highly differentiated horizontally. With a sufficiently large d, we can safely assume that before the introduction of the SB, NB_j is the most preferred brand (before price) in segment j and neither its manufacturer nor the retailer finds it profitable to try to sell it to segment 3-j. This allows each NB to operate at the bilateral monopoly solution in its target segment resulting in the prices and profits as shown in the column labeled " Before SB" in Table 4.[12]

A sufficiently large value of d also simplifies the after-SB situation since Lemma 1 implies only four of the 49 possible demand situations can occur in this case. They are:

(1) NB_1 and SB sold in segment 1; NB_2 sold in segment 2
(2) NB_1 sold in segment 1; NB_2 and SB sold in segment 2
(3) NB_1 and SB sold in segment 1; NB_2 and SB sold in segment 2
(4) NB_1 sold in segment 1; SB sold in segment 2 (possible only if $1 > \alpha_S \geq \alpha$)

Given this smaller number of demand regions, it is feasible to solve the game mathematically for each region and check for boundary conditions. (Details are available in Technical Appendix 2.) The closed form solutions are provided in Table 4. These solutions are functions of the SB position, x, and, thus, provide initial answers to our research questions regarding whether or not the retailer should always position the SB as close as possible to NB_1 and if not, where the retailer should position the SB and why.

From Table 4(a) we see that there exist five distinct possibilities for the equilibrium after the SB introduction when d is large. These five possible sets of equilibrium solutions

12 We again note that when d is large enough to preclude inter-segment competition, our two models of buyer behavior are identical. Thus Table 4 results apply to both cases.

Table 4. Equilibrium results for high level of horizontal differentiation (large d).

a. **When** $\alpha > \alpha_S$

	Before SB	After SB				
		FO1	BG1	BGb	BG2	FO2
w_1	$\frac{1}{2}$	$\frac{1-\alpha_S+xd}{2}$	$\frac{1-\alpha_S+xd}{2}$	$\frac{1-\alpha_S+xd}{2}$	$\overline{w}_1$	$\frac{1}{2}$
w_2	$\frac{\alpha}{2}$	$\frac{\alpha}{2}$	$\overline{w}_2$	$\frac{\alpha-\alpha_S+(1-x)d}{2}$	$\frac{\alpha-\alpha_S+(1-x)d}{2}$	$\frac{\alpha-\alpha_S+(1-x)d}{2}$
p_1	$\frac{3}{4}$	$\frac{3-\alpha_S+xd}{4}$	$\frac{3-\alpha_S+xd}{4}$	$\frac{6\alpha_S-2\alpha_S^2-3d-\alpha_S d+6\alpha_S xd+xd^2-4x^2d^2}{4(2\alpha_S-d)}$	$\frac{1+\overline{w}_1}{2}$	$\frac{3}{4}$
p_2	$\frac{3\alpha}{4}$	$\frac{3\alpha}{4}$	$\frac{\alpha+\overline{w}_2}{2}$	$\frac{6\alpha\alpha_S-2\alpha_S^2+5\alpha_S d-3\alpha d-3d^2-6\alpha_S xd+7xd^2-4x^2d^2}{4(2\alpha_S-d)}$	$\frac{3\alpha-\alpha_S+(1-x)d}{4}$	$\frac{3\alpha-\alpha_S+(1-x)d}{4}$
p_S		$\frac{\alpha_S-xd}{2}$	$\frac{\alpha_S-xd}{2}$	$\frac{\alpha_S(\alpha_S-d)+xd^2(1-x)}{2\alpha_S-d}$	$\frac{\alpha_S-(1-x)d}{2}$	$\frac{\alpha_S-(1-x)d}{2}$
q_1	$1-p_1$	$1-\frac{p_1-p_s}{1-\alpha_S+xd}$	$1-\frac{p_1-p_s}{1-\alpha_S+xd}$	$1-\frac{p_1-p_S}{1-\alpha_S+xd}$	$1-p_1$	$1-p_1$
q_2	$1-\frac{p_2}{\alpha}$	$1-\frac{p_2}{\alpha}$	$1-\frac{p_2}{\alpha}$	$1-\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}$	$1-\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}$	$1-\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}$
q_S		$\frac{p_1-p_S}{1-\alpha_S+xd}-\frac{p_S}{\alpha_S-xd}$	$\frac{p_1-p_S}{1-\alpha_S+xd}-\frac{p_S}{\alpha_S-xd}$	$\frac{p_1-p_S}{1-\alpha_S+xd}-\frac{p_S}{\alpha_S-xd}+\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}-\frac{p_S}{\alpha_S-(1-x)d}$	$\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}-\frac{p_S}{\alpha_S-(1-x)d}$	$\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}-\frac{p_S}{\alpha_S-(1-x)d}$

b. **When** $\alpha_S > \alpha$

	Before SB	After SB					
		FO1	BG1	BGb	BG2/FO2 ($\alpha \geq \alpha_S-(1-x)d$)	BG2 ($\alpha < \alpha_S-(1-x)d$)	FO2
w_1						$\overline{w}_1$	$\frac{1}{2}$
w_2						0	0
p_1						$\frac{1+\overline{w}_1}{2}$	$\frac{3}{4}$
p_2				Same as when $\alpha > \alpha_s$		$\frac{\alpha}{2}$	$\frac{\alpha}{2}$
p_S						$\frac{\alpha_S-(1-x)d}{2}$	$\frac{\alpha_S-(1-x)d}{2}$
q_1						$1-p_1$	$1-p_1$
q_2						$\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}-\frac{p_2}{\alpha}=0$	$\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}-\frac{p_2}{\alpha}=0$
q_S						$1-\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}$	$1-\frac{p_2-p_S}{\alpha-\alpha_S+(1-x)d}$

$\bar{w}_1 = \frac{(2x-1)d\sqrt{(\alpha_S-xd)(2\alpha_S-d)(1-\alpha_S+xd)}}{(\alpha_S-xd)(2\alpha_S-d)}$, $\bar{w}_2 = \frac{(1-2x)d\sqrt{\alpha(\alpha_S-(1-x)d)(2\alpha_S-d)(\alpha-\alpha_S+(1-x)d)}}{(\alpha_S-(1-x)d)(2\alpha_S-d)}$;

BGj: "Bridge" by attacking NB_j and threatening NB_{3-j}; BGb: "Bridge" by attacking both NB's; FOj: "Focus" by attacking NB_j and leaving NB_{3-j} unthreatened.

represent five conceptually different category management strategies available to the retailer when introducing a SB to a category characterized by highly differentiated NB's. We illustrate these five different strategies in Figure 3, which plots the retailer's equilibrium profits as a function of the SB's position.

The first example shown in Figure 3(a) is for the parameter setting $\alpha = .95$, $\alpha_S = .85$, and $d = .65$. Starting from $x = 0$, the first region ($0 \leq x \leq .2$) maps to the equilibrium solution in the "FO1" column of Table 4(a). In this region, the SB is positioned close to NB_1 and far from NB_2. Consequently the SB only affects the demand for NB_1 and not NB_2. (Table 4(a) shows w_2, p_2 and q_2 remain at the bilateral monopoly solution as found for the "Before SB" condition). We label this strategy of positioning the SB to attack one NB while leaving the other NB unaffected "*Focus*". Since the SB attacks NB_1 and does not impact NB_2, we call the strategy "*Focus 1*" (FO1). The downward slope of the retail profit function in the region of $0 \leq x \leq .2$ indicates that the retailer using the FO1 strategy finds it optimal to position the SB as close as possible to NB_1 (i.e., $x = 0$) and charge the monopoly price for the SB.

In the second region of Figure 3(a) ($.2 < x \leq .3$), the SB still only attacks NB_1. However, since the SB now is positioned closer to NB_2, the SB, by lowering its price below the monopoly price, is able to pose a competitive threat to NB_2. Perceiving this competitive threat, the NB_2 manufacturer lowers w_2 below the bilateral monopoly wholesale price of $\alpha/2$ in order to ensure that none of its customers in segment 2 buy the SB. Consequently, although NB_2 remains the only brand selling to segment 2, the retailer successfully introduces competition to NB_2 and benefits from the increased retail margin and sales volume due to the lower wholesale and retail prices of NB_2.[13] We label this strategy of positioning the SB between the two NB's to lower the previously charged monopoly prices of both NB's as "*Bridge*" (as in bridge the gap between the two highly differentiated NB's). The specific strategy used in this region is "*Bridge-Attack* 1 *and Threaten* 2" (BG1), which has NB_1 as the target of the SB attack while the SB threatens NB_2. The closed form solution for this strategy is presented in the (BG1) column of Table 4.

The third region of Figure 3(a) ($.3 < x < .7$) shows another variant of the Bridge strategy labeled "*Bridge-Attack Both*" (BGb). In this situation, the retailer positions the SB roughly half way between the two NB's, again setting the SB price below the monopoly price so that the SB attracts consumers from both segments. The retailer's strategy in the fourth region ($.7 \leq x < .8$) is "*Bridge-Attack* 2 and *Threaten* 1" (BG2) which is conceptually analogous to BG1. However, the identities of the NB being attacked and the NB being threatened are reversed. The fifth region of Figure 3(a) ($.8 \leq x \leq 1$) represent the "*Focus* 2" (FO2) strategy, which is the exact mirror image of the FO1 strategy in the first region. The positive slope of the retail category profit function in this region indicates that the retailer should position the SB as close to NB_2 as possible (i.e., $x = 1$), when using the FO2 strategy.

Faced by such a profit function as Figure 3(a), the retailer must then choose among these five strategies and within the chosen strategy, the optimal positioning. For the particular parameter setting assumed in Figure 3(a), the retailer's category profits are maximized when

13 Chiang, Chhajed, and Hess (2003) capture a similar effect by showing that a manufacturer can profitably introduce a direct channel as a threat to an indirect channel without taking away any sales form the latter.

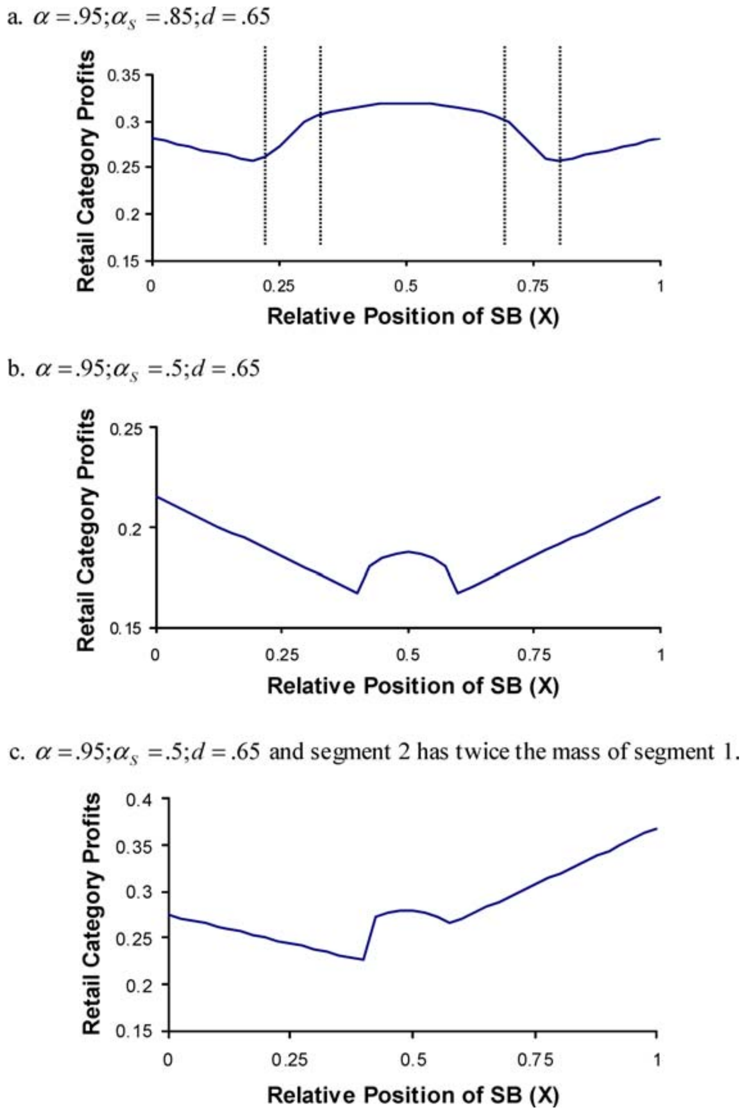


Figure 3. Impact of the Retailer's SB Positioning on the Retail Category Profits.

$x = .5$. Therefore, the optimal category management strategy in this particular market environment is BGb.

In the situation shown in Figure 3(a), the SB has a reasonably high quality SB ($\alpha_S = .85$), making the *Bridge* strategy more profitable than *Focus*. However, if α_S becomes smaller and/or d becomes larger, the competitive pressure on the NB's from the SB positioned in the middle will get weaker, making the *Bridge* strategy less profitable or possibly even infeasible. Figure 3(b) shows such an example. Here α and d remain the same as in Figure 3(a), but the SB quality parameter, α_S, is reduced to .5. Now the only way the retailer can bridge the gap is to lower the SB price to such a degree that this strategy becomes less profitable than focusing on one of the NB's and charging the SB's monopoly price while leaving the other NB unaffected. Thus the optimal SB position is $x = 0$ or 1.

The optimality of the *Focus* strategy shown in Figure 3(b) is somewhat consistent with the conclusions of SHR and SZ, who suggest that the retailer should generally position the SB as close to the leading NB as possible. Note, however, that our closed form solutions in Table 4(a) indicate FO1 and FO2 lead to the exactly same retail category profits, making the retailer indifferent between targeting the stronger or weaker national brand. This difference is partially due to how the three different research teams model the asymmetry between the NB's. In our model, the asymmetry comes from a quality difference between the two NB's while holding the size of the two segments the same. In contrast, SZ assume identical product quality but more consumers in segment 1, while SHR's demand specification implicitly assumes both types of asymmetries.

To see the implications of these assumptions, look at Figure 3(c), which is based upon the same parameter values as in Figure 3(b) but with double the mass of consumers in segment 2. This represents a situation where NB_1 is the leading NB in terms of quality but targets a niche market. Consequently NB_2 is the leading brand in terms of market share before the SB entry. In such a situation, Figure 3(c) clearly shows that the retailer is better off by selecting the FO2 strategy and positioning the SB at $x = 1$, contrary to SHR's conclusion. On the other hand, if we assume a larger mass of consumers in segment 1 relative to segment 2, the optimal strategy will be FO1 with $x = 0$ as found by SHR and SZ. This implies when considering where to position the SB, the retailer must first understand how the "leading NB" is defined.

Note that the examples shown in Figure 3 are based on the assumption that the two NB's have higher quality than the SB ($\alpha > \alpha_S$). Table 4(b) shows that if the SB's quality is higher than the weaker NB ($\alpha_S > \alpha$), the retailer can completely displace NB_2 with the SB by positioning the SB close to NB_2 (i.e., x is close to 1), resulting in $q_2 = 0$. In this way, we find situations similar to SZ where only two brands are sold even after the SB introduction, although we do not require shelf space scarcity to obtain this result. More importantly, when $\alpha_S > \alpha$, FO2 is more profitable than FO1 and BG2 can be more profitable than BG1. Consequently, if $\alpha_S > \alpha$ and d is too large for the retailer to use the BG strategy profitably, the optimal strategy is FO2 with $x = 1$.

3.3. *General case*

3.3.1. Analysis method. Our analyses of the two polar cases for d provide initial answers to the three research questions we listed in the beginning of the paper. First, we have shown that it is not always optimal for the retailer to position the SB as close to the leading NB as possible. Second, we find there are three generic SB positioning strategies (i.e., close to the leading NB, close to the weaker NB, or in the middle) and two generic SB pricing strategies (same as or below the monopoly price) that, in combination, yield a number of distinct category management strategies. Third, the choice of the best category strategy depends on the degree of horizontal differentiation of the two NB's as well as the relative quality levels of the three competing brands.

We next generalize these findings by analyzing situations between the two polar cases. Due to the complexity associated with our kinked demand structure, we take a numerical approach for obtaining equilibrium solutions for the general case. Specifically, we first specify proper boundary conditions for each demand region and then use a quadratic programming algorithm to solve for the profit maximizing set of quantities (and thus prices) for the region conditional upon given levels of w_1, w_2, x, d, α and α_S.[14] We then select the solution from the region that yields the global maximum category profits. This efficiently yields the retailer's optimal category pricing response to a particular vector of wholesale prices and brand positions. Once the optimal retailer pricing response is identified, we are able to obtain the manufacturer level equilibrium solutions as well as the optimal store brand positions using a numerical optimization procedure described in Technical Appendix 3.[15]

Note that the equilibrium solution obtained from this numerical procedure is equivalent to an analytically derived solution although it only pertains to the specific set of parameter values used. Consequently, in order to produce sufficiently generalizable results, we perform our analysis following an "experimental design" over a wide range of feasible values for the three key model parameters, d, α and α_s. A preliminary analysis revealed that the closed form solutions found in Table 4 hold for $d \geq .65$. Therefore, we vary d between 0 and .65, letting it take four levels, .1, .175, .25 and .5.[16] For the other two parameters, we found it sufficient to vary α across the four values .7, .8, .9, and 1 and α_S across the six values, .5, .6, .7, .8, .9 and 1. This allows us to span the relevant "Before SB" situations by analyzing 16 (4 levels of d times 4 levels of α) different parameter settings and the relevant "After SB" situations by analyzing, 96 ($4 \times 4 \times 6$) different cases.

3.3.2. Results Table 5 shows the conditions under which the retailer finds it optimal to utilize each of the four strategies over this wider range of settings analyzed numerically as well as the polar extremes analyzed mathematically. The table shows that the numerically obtained solutions blend in nicely with the mathematically obtained solutions. For instance, the mapping of optimal strategies for $d = .5$ is similar to that for $d = .65$, the only difference being that the retailer can use the *Bridge* strategy with a lower quality SB since the "gap" is smaller. In the same way, the mapping of optimal strategies for $d = .1$ is very similar to that for $d = 0$. Thus, the implications of the mathematical analysis for $d = 0$ generalize to low values of d. For the remaining two levels of d, the mapping reflects a gradual transition between more extreme values of d.

Interestingly, when we use our second model of buyer behavior (i.e., where consumers' willingness to pay for quality is independent of their willingness to pay for product match), we find the same general pattern of strategies. The only difference is the correspondence

14 The retailer's profit can be expressed as a quadratic function of prices (or quantities).

15 We solved the manufacturer level pricing game in two ways, once with the NB1 manufacturer being the Stackelberg leader to ensure the existence of a unique equilibrium and once by assuming a Bertrand Nash game. Except for the fact that in a few instances we did not find a unique equilibrium in the Bertrand game, the two game rules produced very similar results. Thus we only report the Stackelberg game results.

16 Since our second model of buyer behavior is less competitive we found that values of $d > .40$ yielded monopoly solutions. Consequently we let d take on the values .40, .175, .1 and .05 when analyzing this model.

Table 5. Effects of d, α and α_S on optimal category management strategy.

d	α	$\alpha_S = .5$	$\alpha_S = .6$	$\alpha_S = .7$	$\alpha_S = .8$	$\alpha_S = .9$	$\alpha_S = 1$	Competitive intensity
0**	.7	ST2	ST2	Ab*	Ab*	Ab*	Ab*	.90
	.8	ST2	ST2	ST2	Ab*	Ab*	Ab*	.93
	.9	ST2	ST2	ST2	ST2	Ab*	Ab*	.97
.1	.7	ST2	ST2	ST2	Ab*	Ab*	Ab*	.82
	.8	ST2	ST2	ST2	Ab	Ab*	Ab*	.78
	.9	ST2	ST2	ST2	ST2	Ab	Ab*	.79
	1	ST2	ST2	ST2	ST2	ST2	Ab	.83
.175	.7	ST1	ST1	ST1	ST1*	A2*	A2*	.22
	.8	A2	A2	A2	A2*	A2*	A2*	.34
	.9	A2	A2	A2	A2	A2*	A2*	.43
	1	A2	A2	A2	A2	A1/2	A1/2	.53
.25	.7	BG2	BG2	BG2	BG2*	BG2*	BG2*	0
	.8	A2	A2	A2	A2	A2*	A2*	.01
	.9	A2	A2	A2	A2	A2	A2*	.08
	1	A2	A1/2	A1/2	A1/2	A1/2	A1/2	.14
.5	.7	BG1	BG2	BG2	BG2*	BG2*	BG2*	0
	.8	BG1	BG2	BG2	BG2	BG2	BG2*	0
	.9	BG1	BG2	BG2	BG2	BG2	BG2	0
	1	FO	BG1/2	BG1/2	BG1/2	BG1/2	BG1/2	0
.65**	.7	FO	FO	BGb	BGb	BGb	BGb	0
	.8	FO	FO	BGb	BGb	BGb	BGb	0
	.9	FO	FO	BGb	BGb	BGb	BGb	0
	1	FO	FO	BGb	BGb	BGb	BGb	0

The case of $d = 0$ and $\alpha = 1$ is not included since, in this perfect competition situation, there is no need for a SB.
*NB_2 is completely displaced.
**Based on closed form solutions.
BGj: "Bridge" by attacking NB_j and threatening NB_{3-j}; BGb: "Bridge" by attacking both NB's; FO: "Focus" by attacking one NB and leaving the other NB unthreatened; STj: "Stimulate" NB_j to attack NB_{3-j}; Aj: "Attack" NB_j while threatening NB_{3-j}; Ab: "Attack" both NB's.

between the specific parameter values and the optimality of a given strategy. This difference occurs because the same parameter values imply different buyer behavior and thus different demand characteristics which in turn lead to different levels of competition between the two NB's. Since the optimality of a given strategy depends on this level of competition, we next introduce a measure of competitive intensity which not only can be used to relate our findings from the two different buyer behavior models, but also provide testable hypotheses on when each of the identified strategies is the best.

3.4. Impact of competitive intensity on optimal SB strategies

Conceptually, competitive intensity prior to the introduction of the SB depends on the degrees of vertical and horizontal differentiation between the two NB's, the consumers' taste distribution, and the consumers' choice rules. One measure that captures all these factors, is easy to obtain empirically and analytically, and has theoretical underpinnings is the percent of current buyers who find both NB's to be viable options (i.e., who receive positive net utilities from both NB's) at the equilibrium retail prices. Such a measure is conceptually similar to the proportion of consumers whose consideration sets contain multiple brands. When this measure equals zero, each brand enjoys a "monopoly" position since there are no viable alternatives for the current customers other than the one they purchased. Likewise when this measure equals 100%, all current buyers are potential switchers, finding both brands acceptable.

This measure of competitive intensity is given in the last column of Table 5 for various values of d and α based on equation (2) buyer behavior. As shown in the table, this measure varies with both the horizontal and vertical differentiation between the NB's. This same general pattern was found for the alternative buyer behavior model (equation (3)), although the mapping from competitive intensity to specific values of d and α differed in a systematic fashion.

More importantly, as seen in Table 6, we found a very strong correspondence for both sets of buyer behavior results between the competitive intensity measure and the optimal category management strategy. Thus, under both buyer behavior models the retailer uses *Focus* or *Bridge* when our competitive intensity measure is zero. For moderate levels of competitive intensity (i.e., for values greater than zero but less than .6), the results from both

Table 6. Category conditions and retailer actions associated with each category management strategy.

	Strategy	Category conditions: Competitive intensity	Category conditions: Product quality	SB positioning (x)	SB pricing* (p_S/p_S^{MON})
Focus	FO	None	Low α_S	0 or 1	Always 1
Bridge	BG	None	α_S is sufficiently large to bridge the gap.	near middle ($.3 \le x \le .85$)	.73 on average (.54∼.96)
Attack	A1/A2**	Moderate	$\alpha = 1$ and α_S is not too low	close to NB1 ($x \le .2$)	.97 on average (.88∼1.05)
	A2	Moderate	α is not too low or $\alpha_S > \alpha$.	close to NB2 ($x \ge .75$)	
	Ab	High	$\alpha_S \ge \alpha$	.69 on average ($.45 \le x \le 1$)***	
Stimulate	ST1	Moderate	α is much lower than 1 and $\alpha_S \le \alpha$.	.35 on average ($x \le .5$)	.94 on average (.89∼1.05)
	ST2	High	$\alpha_S < \alpha$	.72 on average ($x = 1$ with exceptions)***	

*Ratio of the equilibrium SB price to what the retailer would charge if the SB is in monopoly ($p_S^{MON} = \alpha_S/2$).
**When A1 is the optimal strategy, A2 is also optimal.
***Ab or ST2 is the optimal strategy when there exists little or no horizontal differentiation between the NBs. Consequently, the retailer's choice of x has little or no impact on both SB position and retail category profits.

buyer behavior models indicate that the retailer finds it best to attack the weaker NB and threaten the stronger NB (A2) unless both the weaker NB and the SB are very low quality (ST1) or there is no quality difference in the two NB's (A1 or A2). When the competitive intensity is high prior to the SB introduction (i.e., greater than .6), both models have the retailer using the *Stimulate* strategy against the weaker NB (ST2) if the quality level of the SB is the lowest of the three brands, and the *Attack Both* (Ab) strategy if the SB is the middle quality brand.

Table 6 also summarizes the actual positioning and pricing actions associated with each strategy. As in the polar cases, we find many instances where the optimal SB position is closer to the weaker NB. In addition, as seen before, the SB is almost always priced close to its monopoly price except when the retailer is using the *Bridge* strategy. Here we note that the SB is priced low enough to entice customers from both market segments to consider the SB. The degree to which the price must be lowered depends on the degree of differentiation between the two NB's and the quality level of the SB.

3.5. *Impact of store brand introduction*

We further analyzed our results to better understand the sources of the retailer's benefit of introducing a SB by decomposing the retailer's profit enhancement in the 138 different cells found in Table 5 in two different ways. The results are summarized in Table 7(a).

The first three columns of Table 7(a) show the relative contribution of each of three sources to the retailer's profit improvement due to the introduction of a SB. The relative contribution of the first source, improved retail margins from the NB's, is remarkably stable

Table 7. Impact of store brand introduction.

a. *Sources of Retailer's Category Profit Increase*

Competitive intensity	Higher NB retail margin	Increase (or decrease) in NB sales	SB sales	Increased share of channel profit	Increase (or decrease) in channel profit
Low (= 0)	36%	10%	54%	79%	21%
Moderate	38%	16%	46%	78%	22%
High (>.6)	37%	−161%	224%	101%	−1%

b. *% Changes in National Brand Prices and Retailer's Category Profits*

Competitive intensity	Strategy	Change in w_1	Change in w_2	Change in p_1	Change in p_2	Change in Π_R
Low (= 0)	FO1	−54%	0%	−18%	0%	+88%
	FO2	0%	−64%	0%	−21%	+88%
	BG	−58%	−70%	−19%	−23%	+185%
Moderate	A1	−68%	−75%	−19%	−25%	+149%
	A2	−69%	−81%	−19%	−26%	+157%
	ST1	−83%	−94%	−33%	−25%	+227%
High (>.6)	Ab	−69%	−99%	−11%	−10%	+42%
	ST2	−29%	−51%	−5%	−7%	+24%

across different levels of competitive intensity. In contrast, the relative contributions of the other two sources, changes in the NB quantity and the increases due to the SB sales, are highly sensitive to the level of competitive intensity. When this measure is greater than .6, the SB's entry cannabalizes sales from the two NB's, resulting in a net decrease in retail profits from these products. However, this loss is more than compensated for by the profits from the SB sales. In contrast, when the NB's are at least moderately differentiated (competitive intensity is less than .6), the SB's entry leads to an increased NB quantity, allowing the retailer to benefit positively from all three sources. In any case, for our setting the largest contribution, on average, comes from the SB sales, which is in contrast to SHR and SZ. Their models suggest the major benefit comes from a reduction of the NB manufacturers' margins.

Another way of understanding the impact of the SB on retailer category profits is to decompose this impact into changes in total category profits and changes in the retailer's share of channel profits. As seen in the last two columns of Table 7(a), we find the majority of the retail profit improvement comes from the increased share of the total channel profits for the retailer. This is due to the fact that the SB's entry not only increases the retailer's margins on the NB's but also allows the retailer to keep 100% of channel profit associated with selling SB. In addition, we find the SB entry normally enhances channel coordination thereby increasing the total channel profits especially when the competitive intensity is moderate to low. (In a few cases when the competitive intensity is very high, a small decrease in total channel profits is observed after SB's entry.[17]) This indicates that the SB's entry generally improves channel coordination but this effect is small or even negative when there already exists intense competition between the NB's.

Table 7(b) provides more details on how these increases in profits occur. We note that in almost every case NB_2 is affected more than NB_1, both in terms of wholesale price and retail price. This is counter to the findings of SHR but consistent with the empirical finding of Pauwels and Srinivasan (2004). The only exceptions are found when the retailer uses FO1 or ST1. In the latter case the retailer not only positions its SB near NB_1, but also lowers the retail price of NB_1 more than for NB_2 in order to have NB_1 attack NB2's target market. In short, the diverse set of retail category management strategies provides intuitive explanations for why the empirically observed impact of a SB entry on existing NB's varies across categories. Finally we see that the SB entry is most beneficial to the retailer when the NB's are moderately differentiated since such a condition provides the opportunity for the SB to significantly increase competitive pressure on both NB's.

4. Discussion

Our main message is that it is not always optimal for a retailer to position its SB against the leading NB; instead it is often optimal to position the SB close to the weaker NB or to introduce a SB that appeals to both market segments. In deciding where to position the

17 In contrast, Narasimhan and Wilcox (1998) find the SB introduction always decreases total channel profits. Their result is due to their assumption of fixed total category demand, which rules out the presence of the double marginalization problem.

SB and how to price each of the brands, the retailer needs to consider how to "play in" the most favorable demand region thereby reshaping the pattern of intra-category brand competition. We identify four distinct types of category management strategies. We also demonstrate that the optimal strategy depends upon the degree of competitive intensity between the two NB's prior to the introduction of the SB and the relative quality levels of the three brands. Moreover, these relationships are robust to two different assumptions of buyer behavior. Since both our measure of competitive intensity and product quality are easy to obtain empirically, we believe our results as summarized in Table 6 can be readily tested empirically and implemented in practice.

Consistent with previous analytical studies, we find *a* major benefit of introducing a store brand is increased retail margins on the NB's. However, we show the retailer can also benefit from increases in NB unit sales caused by a lower retail price and profits generated from the SB sales. In fact we find this latter source of profits to be the largest of the three sources in many situations. We also find that the retailer normally finds it best to price its SB near its monopoly price. The only exception to this is when the NB's are highly differentiated and the retailer finds it best to use the *Bridge* strategy in order to impact both NB's. In addition we find a SB is most beneficial when there was only moderate competitive intensity between the two NBs prior to the introduction of the SB.

One might ask why our results are different from those of SZ's and SHR's despite our model's similarities to theirs. The answer to this question rests in the few new features incorporated in our model. Although SZ's market model is similar to ours, they only allow the retailer to carry two brands in the category and to position the store brand at one of the two segment ideal points (i.e., $x = 0$ or $x = 1$). In addition, they assume retail prices are set with complete channel coordination and the resulting channel profits are split between the retailer and the manufacturers via a bargaining game.[18] In contrast, we follow SHR's lead and assume double marginalization within the channel.

The difference between our results and SHR's is a result of the two studies using different demand models, since all other aspects of the analysis remain almost identical. These two demand models differ in two important ways. First, we derive a demand structure from an explicit market model. This yields a kinked demand function. In contrast SHR assume a demand model that is continuously linear over the total range of prices. As a result they were not able to uncover the four category management strategies reported in this paper. Second, SHR's demand specification restricts their cross price effects to be no higher than 1/3 of the own price effects. In contrast we allow the cross price effects to vary from zero to one in relation to the own price effects. Consequently their analyzed markets do not capture the competitive intensity found in situations with highly substitutable brands, thereby limiting the generalizability of their results. In contrast, our results span the total competitive environment.[19]

18 Interestingly, when we assume complete channel coordination, we also find the optimal strategy is always to sell just two brands and that, if a NB is displaced, it is always the weaker NB. However, the optimal SB position, if it is introduced, is $x = 1$, not $x = 0$ as concluded by SZ.

19 By assuming a larger size for segment 1 and limiting the competitive intensity to low levels in our model, our analysis replicated SHR's result of optimal SB position at $x = 0$.

With this said, any generalization of our findings also must be done after considering the simplifying assumptions in our model. For one, our model consists of two discrete segments of consumers in terms of tastes. This is an abstraction of a more realistic situation where consumer tastes follow a continuous bimodal distribution. As long as the two modes are sufficiently pronounced (i.e., well-defined clusters of consumer tastes exists), we believe our findings should hold. However, it is not clear how well our results will apply to the case of a more uniform distribution of tastes. Analyzing such a case will require not only changing the consumer taste distribution assumption but also determining the equilibrium number and positions of national brands before the SB entry.

We also acknowledge that our model results do not capture the size asymmetry between the two consumer segments, which is considered in SZ's and SHR's models. When we reanalyzed our model by allowing for this target market size asymmetry (while holding $\alpha = 1$) we found a few differences in the optimal product positioning (x^*) but little difference in our basic findings on optimal category management strategy. Thus, we do not believe this set of assumptions is driving our main results.

Our model is also silent on specifically how a retailer goes about positioning the SB in the real world. Clearly the retailer can use such tools as local advertising, shelf location, and package design to 'locate' the store brand in the minds of the consumer. In addition, the retailer might be able to specify certain product characteristics the SB manufacturer must deliver. For example Food Lion, a major regional grocery chain, carries two major national brands of yogurt, Dannon and Yoplait, as well as its own SB. Dannon has fruit at the bottom and comes in 8 oz. containers. Yoplait blends the fruit with the yogurt and comes in a distinctive 6 oz. container. The SB comes in a container very similar to Dannon's 8 oz. container, but the yogurt is blended with the fruit. Casual empiricism would classify this SB position as being in the middle of two differentiated NB's. One might also argue that a retailer cannot always position the SB at any desired position due to the retailer's need for establishing a consistent image for all of its SB's. However, horizontal features are usually category-specific as seen in the yogurt example while a storewide image is often vertically defined (quality level). Therefore, the category specific SB positioning strategy and the storewide SB positioning strategy do not have to be linked.

Finally, our analysis is from a monopolist retailer's point of view, ignoring the possible role of SB's as a competitive strategic tool against other retailers. Considering competing retailers, each with the ability to introduce a store brand, raises a number of interesting questions regarding a retailer's store brand strategy. If one retailer introduces a store brand, what would be the best reaction by the competing retailer? What would be the resulting equilibrium? Could it be that the rapid proliferation of store brands is partially due to retail competition following the pattern of the "prisoner's dilemma"? These questions, as well as the empirical testing of the implications of our findings using our proposed measure of competitive intensity, represent interesting future research directions.

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Appendix 1
Proof of Lemma 1

Define $\gamma_j = v_j - m_j$, where v_j and m_j measure the quality and the degree of mismatch for brand *j* perceived by the consumers of a certain segment.

Suppose brand *a* is evaluated higher than brand *b* by the buyers of this segment before prices are considered. Thus:

$$\gamma_a > \gamma_b . \tag{A1}$$

Consider two buyers, *h* and *i*, belonging to this segment. Let buyer *h* have higher willingness to pay than buyer *i*, i.e.,:

$$\beta_h > \beta_i . \tag{A2}$$

If buyer *h* purchases brand *b*, it must be because $U_{hb} = \beta_h \gamma_b - p_b > U_{ha} = \beta_h \gamma_a - p_a$.

Thus, $$\beta_h < \frac{p_a - p_b}{\gamma_a - \gamma_b} . \tag{A3}$$

From Equations A2 and A3, we get

$$\beta_i < \frac{p_a - p_b}{\gamma_a - \gamma_b} . \tag{A4}$$

From (A4), one can see that

$p_a - p_b > \beta_i (\gamma_a - \gamma_b)$, which can be rearranged as

$$\beta_i \gamma_b - p_b > \beta_i \gamma_a - p_a . \tag{A5}$$

Thus,

$$U_{ib} = \beta_i \gamma_b - p_b > U_{ia} = \beta_i \gamma_a - p_a . \tag{A6}$$

Therefore, buyer *i*, who has lower willingness to pay than buyer *h*, will not buy brand *a*, the higher ordered brand of the two.

Technical Appendix 1
Derivation of Closed Form Solutions for *d* = 0

Before the SB entry, the demand functions for the two NBs' are:

$$q_1 = 2(1 - \frac{p_1 - p_2}{1-\alpha}) \text{ and} \tag{1-1}$$

$$q_2 = 2(\frac{p_1 - p_2}{1-\alpha} - \frac{p_2}{\alpha}) . \tag{1-2}$$

By substituting these demand functions into the retailer's profit function, $\Pi_R = (p_1 - w_1)q_1 + (p_2 - w_2)q_2$, and solving the resultant first order conditions, we obtain the following reaction functions for the retailer:

$$p_1^* = \frac{1+w_1}{2} \tag{1-3}$$

$$p_2^* = \frac{\alpha + w_2}{2} \tag{1-4}$$

By substituting these reactions functions into the manufacturers' profit functions, $\Pi_{M1} = w_1 q_1$ and $\Pi_{M2} = w_2 q_2$, and solving the resultant first order conditions, we obtain the equilibrium solutions shown in Table 3.

Next, we consider the case after the SB entry, when $1 > \alpha > \alpha_S$. With three brands available for the retailer to include in its assortment, there exist 7 possible demand structures, as indicated in Table 2. However, at equilibrium, all three brands receive positive quantities because it can be shown that 1) the retailer profits are higher with SB than without SB in the assortment and that 2) a manufacturer has an obvious incentive to set its wholesale price low enough to avoid selling zero quantity. Therefore, the relevant demand structure is as follows:

$$q_1 = 2(1 - \frac{p_1 - p_2}{1-\alpha}) \tag{1-5}$$

$$q_2 = 2(\frac{p_1 - p_2}{1-\alpha} - \frac{p_2 - p_S}{\alpha - \alpha_S}) \tag{1-6}$$

$$q_S = 2(\frac{p_2 - p_S}{\alpha - \alpha_S} - \frac{p_S}{\alpha_S}) \tag{1-7}$$

which leads to the equilibrium solutions presented in Table 3 via the standard mathematical process outlined above.

For the case of $1 > \alpha_S \ge \alpha$, once again it is true that 1) the retailer profits are higher with SB than without SB in the assortment and that 2) a manufacturer has an obvious incentive to set its wholesale price low enough to avoid selling zero quantity. This leads to the following demand structure:

$$q_1 = 2(1 - \frac{p_1 - p_S}{1 - \alpha_S}) \tag{1-8}$$

$$q_2 = 2(\frac{p_S - p_2}{\alpha_S - \alpha} - \frac{p_2}{\alpha}) \tag{1-9}$$

$$q_S = 2(\frac{p_1 - p_S}{1 - \alpha_S} - \frac{p_S - p_2}{\alpha_S - \alpha}) \tag{1-10}$$

From this demand structure, we derive (via the same mathematical approach used above) the following equilibrium solution:

$w_1 = \frac{1 - \alpha_S}{2}$, $w_2 = 0$, $p_1 = \frac{3 - \alpha_S}{4}$, $p_2 = \frac{\alpha}{2}$, $p_S = \frac{\alpha_S}{2}$, $q_1 = q_S = \frac{1}{4}$, and $q_2 = 0$.

This indicates that, even though manufacturer 2 tries its best ($w_2 = 0$) to sell a positive quantity, the retailer finds it optimal to sell only NB_1 and SB and, thus, eliminates NB_2 from its assortment. Given this, it is not surprising that the exactly same equilibrium solution can be derived from the demand structure composed of NB_1 and SB only as follows:

$q_1 = 2(1 - \frac{p_1 - p_S}{1 - \alpha_S})$ and $q_S = 2(\frac{p_1 - p_S}{1 - \alpha_S} - \frac{p_S}{\alpha_S})$.

Technical Appendix 2
Derivation of Closed Form Solutions for a Sufficiently Large *d*

In this technical appendix, we use the following assumptions:

1. $\alpha, \alpha_S, d \le 1$
2. d is sufficiently large to ensure that NB1 is the most preferred in segment 1 and NB2 is the most preferred in segment 2 (i.e., $1-d<\alpha$ and $\alpha_S<\alpha$).
3. d is sufficiently large to ensure that each manufacturer focuses on its own target segment (ruling out NB1 selling in seg. 2 and vice versa). Therefore, before the SB entry, the equilibrium is localized monopoly.
4. $\alpha_S > d/2$ so that SB can sell to both segments, if the retailer chooses to do so.

The equilibrium condition before the SB entry is two localized bilateral monopolies. The solutions are easily obtained from $q_1 = 1 - p_1$ for NB1 and $q_2 = 1 - \dfrac{p_2}{\alpha}$ for NB2.

When SB is introduced, if R chooses to sell SB <u>only to segment 1</u>, the demand structure is:

$$q_1 = 1 - \frac{p_1 - p_S}{1-\alpha_S + xd}, \tag{2-1}$$

$$q_2 = 1 - \frac{p_2}{\alpha}, \tag{2-2}$$

$$q_S = \frac{p_1 - p_S}{1-\alpha_S + xd} - \frac{p_S}{\alpha_S - xd} \tag{2-3}$$

Then,

$$\Pi_R = q_1(p_1 - w_1) + q_2(p_2 - w_2) + q_S p_S \,. \tag{2-4}$$

Solving the FOC's yields the following reactions functions:

$$p_1^* = \frac{1+w_1}{2} \tag{2-5}$$

$$p_2^* = \frac{\alpha + w_2}{2} \tag{2-6}$$

$$p_S^* = \frac{\alpha_S - xd}{2} \tag{2-7}$$

Let's call this (the set of Eq. 2-5, 2-6 and 2-7) "A1" reaction.

If R chooses to sell SB <u>only to segment 2</u>, the demand structure is:

$$q_1 = 1 - p_1, \tag{2-8}$$

$$q_2 = 1 - \frac{p_2 - p_S}{\alpha - \alpha_S + (1-x)d}, \tag{2-9}$$

$$q_S = \frac{p_2 - p_S}{\alpha - \alpha_S + (1-x)d} - \frac{p_S}{\alpha_S - (1-x)d} \tag{2-10}$$

Solving the FOC's under this demand structure yields the following reactions functions:

$$p_1^{**} = \frac{1 + w_1}{2} \qquad \text{(same as 2-5)}$$

$$p_2^{**} = \frac{\alpha + w_2}{2} \qquad \text{(same as 2-6)}$$

$$p_S^{**} = \frac{\alpha_S - (1-x)d}{2} \tag{2-11}$$

Let's call this (the set of Eq. 2-5, 2-6 and 2-11) "A2" reaction.

If R chooses to sell SB to both segments, the demand structure is:

$$q_1 = 1 - \frac{p_1 - p_S}{1 - \alpha_S + xd}, \tag{2-12}$$

$$q_2 = 1 - \frac{p_2 - p_S}{\alpha - \alpha_S + (1-x)d}, \tag{2-13}$$

$$q_S = \frac{p_1 - p_S}{1 - \alpha_S + xd} - \frac{p_S}{\alpha_S - xd} + \frac{p_2 - p_S}{\alpha - \alpha_S + (1-x)d} - \frac{p_S}{\alpha_S - (1-x)d} \tag{2-14}$$

Solving the FOC's under this demand structure, we get:

$$p_1^{***} = \frac{2\alpha_S - \alpha_S d - d + 2\alpha_S xd + xd^2 - 2x^2d^2}{2(2\alpha_S - d)} + \frac{w_1}{2} \tag{2-15}$$

$$p_2^{***} = \frac{2\alpha\alpha_S - \alpha d + \alpha_S d - d^2 - 2\alpha_S xd + 3xd^2 - 2x^2d^2}{2(2\alpha_S - d)} + \frac{w_2}{2} \tag{2-16}$$

$$p_S^{***} = \frac{\alpha_S(\alpha_S - d) + xd^2(1-x)}{2\alpha_S - d} \tag{2-17}$$

Let's call this "AB" reaction.

Now, R's choice among A1, A2, and AB depends upon 1) whether a particular reaction is feasible (i.e., the resulting demand structure is as intended by R) and 2) whether a particular reaction is more profitable than the other two.

In order for A1 to be feasible, the following conditions must hold:

$$\frac{p_1^* - p_S^*}{1 - \alpha_S + xd} > \frac{p_S^*}{\alpha_S - xd} \tag{2-18}$$

(i.e., marginal consumer between NB1 and SB has higher β than marg. consumer between SB and no purchase. This ensures SB sells to segment 1.)

$$\frac{p_2^* - p_S^*}{\alpha - \alpha_S + (1-x)d} < \frac{p_S^*}{\alpha_S - (1-x)d} \qquad (2\text{-}19)$$

(i.e., marginal consumer between NB2 and SB has lower β than marg. consumer between SB and no purchase. This ensures SB doesn't sell to segment 2)

Plugging (2-5) and (2-7) into (2-18), one can easily show that (2-18) is true for any positive w_1.
Plugging (2-6) and (2-7) into (2-19) yields:

$$w_2 < \frac{\alpha d(1-2x)}{\alpha_S - (1-x)d} = A \qquad (2\text{-}20)$$

Note that the above condition can exist only if $x<1/2$ (since $w_2>0$ and $0<x<1$).

In order for A2 to be feasible, the following conditions must hold:

$$\frac{p_2^{**} - p_S^{**}}{\alpha - \alpha_S + (1-x)d} > \frac{p_S^{**}}{\alpha_S - (1-x)d} \qquad (2\text{-}21)$$

(i.e., marginal consumer between NB2 and SB has higher β than marg. consumer between SB and no purchase. This ensures SB sells to segment 2.)

$$\frac{p_1^{**} - p_S^{**}}{1 - \alpha_S + xd} < \frac{p_S^{**}}{\alpha_S - xd} \qquad (2\text{-}22)$$

(i.e., marginal consumer between NB1 and SB has lower β than marg. consumer between SB and no purchase. This ensures SB doesn't sell to segment 1)

Plugging (2-6) and (2-11) into (2-21), one can easily show that (2-21) is true for any positive w_2.
Plugging (2-5) and (2-11) into (2-22) yields:

$$w_1 < \frac{(2x-1)d}{\alpha_S - xd} = B \qquad (2\text{-}23)$$

Note that the above condition can exist only if $x>1/2$ (since $w_1>0$ and $0<x<1$).

Eq. 2-20 and 2-23 jointly imply that when SB is positioned at x = 1/2 (& d is constrained to be smaller than as), NB manufacturers can stop SB attacking both only by setting w = 0. Thus, AB is the only relevant reaction function at x=1/2.

In general, for AB to be feasible, the following conditions must hold:

$$\frac{p_1^{***} - p_S^{***}}{1 - \alpha_S + xd} > \frac{p_S^{***}}{\alpha_S - xd} \qquad (2\text{-}24)$$

(i.e., marginal consumer between NB1 and SB has higher β than marg. consumer between SB and no purchase. This ensures SB sells to segment 1.)

$$\frac{p_2^{***} - p_S^{***}}{\alpha - \alpha_S + (1-x)d} > \frac{p_S^{***}}{\alpha_S - (1-x)d} \qquad (2\text{-}25)$$

(i.e., marginal consumer between NB2 and SB has higher β than marg. consumer between SB and no purchase. This ensures SB sells to segment 2.)

By plugging (2-15), (2-16) and (2-17) into (2-24) and (2-25), we obtain:

$$w_1 > \frac{d(2x-1)(1-\alpha_S + xd)}{2\alpha_S - d} = C \tag{2-26}$$

$$w_2 > \frac{d(1-2x)(\alpha - \alpha_S + (1-x)d)}{2\alpha_S - d} = D \tag{2-27}$$

Note that these two conditions are guaranteed to hold for $x = ½$. They also imply that R should be able to employ AB as long as x is reasonably in the middle.

Comparing eq. 2-20 and 2-27 reveals A>D. Therefore, **if *x*<1/2,** R's reaction is:

1. A1 if $w_2 < D$
2. A1 or AB depending on profits if $D \le w_2 \le A$, and
3. AB if $w_2 > A$.

Further investigating condition 2 above, we compare $\Pi_R = q_1(p_1 - w_1) + q_2(p_2 - w_2) + q_S p_S$ between A1 reaction and AB reaction. This reveals:

R prefers A1 if $w_2 < \overline{w}_2$ and AB if $w_2 > \overline{w}_2$ where

$$\overline{w}_2 = \frac{(1-2x)d\sqrt{\alpha(\alpha_S - (1-x)d)(2\alpha_S - d\ )(\alpha - \alpha_S + (1-x)d)}}{(\alpha_S - (1-x)d)(2\alpha_S - d)} \tag{2-28}$$

It can be shown that $D < \overline{w}_2 < A$.

Therefore, when x<1/2, R's best reaction is

A1 for $w_2 \le \overline{w}_2$ and AB for $w_2 > \overline{w}_2$.

Knowing the retailer's reaction function, we now solve the Bertrand Nash game between the two M's.

For M1, if $w_2 \le \overline{w}_2$, M1 knows the retailer's reaction is A1. Thus, its profit function is $\Pi_{M1} = w_1(1 - \frac{p_1^* - p_S^*}{1-\alpha_S + xd})$. From the resulting FOC, we derive, $w_1^* = \frac{1-\alpha_S + xd}{2}$.

Similarly, by solving the FOC from $\Pi_{M2} = w_2(1 - \frac{p_2^*}{\alpha})$, we get, $w_2^* = \frac{\alpha}{2}$.

Thus, if $w_2^* = \frac{\alpha}{2} \le \overline{w}_2$,

$$w_1^* = \frac{1-\alpha_S + xd}{2}, \; w_2^* = \frac{\alpha}{2},$$

$$p_1^* = \frac{1+w_1^*}{2} = \frac{3-\alpha_S + xd}{4}, \; p_2^* = \frac{\alpha + w_2^*}{2} = \frac{3\alpha}{4} \text{ and } p_S^* = \frac{\alpha_S - xd}{2}$$

is the Nash equilibrium. This is the "**FOCUS 1**" solution. Note that the resulting quantities are $q_1 = q_2 = q_S = .25$. Also note that R's profit under Focus 1 is maximized at $x = 0$ since retail margin on NB1 and SB are decreasing in x. Thus, if R is to employ Focus 1, it should position SB at $x = 0$.

If $w_2 > \overline{w}_2$, M1 knows the retailer's reaction is AB. Thus, its profit function is $\Pi_{M1} = w_1(1 - \frac{p_1^{***} - p_S^{***}}{1-\alpha_S + xd})$, the FOC of which leads to $w_1^{***} = \frac{1-\alpha_S + xd}{2}$.

For M2, $\Pi_{M2} = w_2(1 - \frac{p_2^{***} - p_S^{***}}{\alpha - \alpha_S + (1-x)d})$, the FOC of which yields $w_2^{***} = \frac{\alpha - \alpha_S + (1-x)d}{2}$.

Thus, $$p_1^{***} = \frac{6\alpha_S - 2\alpha_S^2 - 3d - \alpha_S d + 6\alpha_S xd + xd^2 - 4x^2 d^2}{4(2\alpha_S - d)},$$

$$p_2^{***} = \frac{6\alpha\alpha_S - 2\alpha_S^2 + 5\alpha_S d - 3\alpha d - 3d^2 - 6\alpha_S xd + 7xd^2 - 4x^2 d^2}{4(2\alpha_S - d)}, \text{ and}$$

$$p_S^{***} = \frac{\alpha_S(\alpha_S - d) + xd(1-x)}{2\alpha_S - d}.$$

This is the "**ATTACK BOTH**" solution, which is the Nash equilibrium if $w_2^{***} > \overline{w}_2$.

Note that the resulting quantities are $q_1 = q_2 = .25$ and $q_S = .5$ (Total quantity of 1, as often seen in our numerical analysis).

Note that the conditions for FOCUS 1 ($w_2^* \le \overline{w}_2$) and for ATTACK BOTH ($w_2^{***} \ge \overline{w}_2$) are not collectively exhaustive, since there exists a range of x that leads to $w_2^{***} = \frac{\alpha - \alpha_S + (1-x)d}{2} < w_2^* = \frac{\alpha}{2}$. Consequently, there exist the following three possibilities:

1) $w_2^*, w_2^{***} \le \overline{w}_2$: FOCUS 1 is the solution.
2) $\overline{w}_2 \le w_2^*, w_2^{***}$: ATTACK BOTH is the solution.
3) $w_2^{***} \le \overline{w}_2 \le w_2^*$: The interior solutions are not feasible.

Under condition 3), in the region where $w_2 \le \overline{w}_2$ (SB is not selling to seg. 2), M2 will keep raising w_2 toward w_2^*. In the region where $w_2 \ge \overline{w}_2$ (SB sells to both segments), M2 will keep lowering w_2 toward w_2^{**}. The end result is that $w_2 = \overline{w}_2$, where w_2 is set just low enough to keep SB from attacking segment 2.

Thus, this is the "**ATTACK 1 and THREATEN 2**" solution.

<u>For *x*>1/2</u>, comparing eq. 2-23 and 2-26 reveals B>C. Therefore, R's reaction is:

1. A2 if $w_1 < C$
2. A2 or AB depending on profits if $C \le w_1 \le B$, and
3. AB if $w_1 > B$.

Further investigating condition 2 above, we compare $\Pi_R = q_1(p_1 - w_1) + q_2(p_2 - w_2) + q_S p_S$ between A2 reaction and AB reaction. This reveals:

R prefers A2 if $w_1 < \overline{w}_1$ and AB if $w_1 > \overline{w}_1$ where

$$\overline{w}_1 = \frac{(2x-1)d\sqrt{(\alpha_S - xd)(2\alpha_S - d\ )(1-\alpha_S + xd)}}{(\alpha_S - xd)(2\alpha_S - d)} \qquad (2\text{-}29)$$

It can be shown that $C < \overline{w}_1 < B$.

Therefore, when x>1/2, R's best reaction is

A2 for $w_1 \le \overline{w}_1$ and AB for $w_1 > \overline{w}_1$.

Knowing the retailer's reaction function, we now solve the Bertrand Nash game between the two M's.

For M2, if $w_1 \le \overline{w}_1$, M2 knows the retailer's reaction is A2. Thus, its profit function is $\Pi_{M2} = w_2(1 - \frac{p_2^{**} - p_S^{**}}{\alpha - \alpha_S + (1-x)d})$. From the resulting FOC, we derive,

$$w_2^{**} = \frac{\alpha - \alpha_S + (1-x)d}{2}.$$

Similarly, by solving the FOC from $\Pi_{M1} = w_1(1 - p_1^{**})$, we get, $w_1^{**} = \frac{1}{2}$.

Thus, if $w_1^{**} = \frac{1}{2} \le \overline{w}_1$,

$$w_1^{**} = \frac{1}{2},\ w_2^{**} = \frac{\alpha - \alpha_S + (1-x)d}{2},$$

$$p_1^{**} = \frac{3}{4},\ p_2^{**} = \frac{3\alpha - \alpha_S + (1-x)d}{4} \text{ and } p_S^{**} = \frac{\alpha_S - (1-x)d}{2}$$

is the Nash equilibrium. This is the "**FOCUS 2**" solution. Note that the resulting quantities are $q_1 = q_2 = q_S = .25$. Also note that R's profit under Focus 2 is maximized at $x = 1$ since retail margin on NB2 and SB are increasing in x. Thus, if R is to employ Focus 2, it should position SB at $x = 1$.

The "**ATTACK BOTH**" solution, derived earlier, holds for x>1/2 if $w_1^{***} > \overline{w}_1$.

$$w_1^{***} = \frac{1-\alpha_S + xd}{2},\ w_2^{***} = \frac{\alpha - \alpha_S + (1-x)d}{2},$$

$$p_1^{***} = \frac{6\alpha_S - 2\alpha_S^2 - 3d - \alpha_S d + 6\alpha_S xd + xd^2 - 4x^2d^2}{4(2\alpha_S - d)},$$

$$p_2^{***} = \frac{6\alpha\alpha_S - 2\alpha_S^2 + 5\alpha_S d - 3\alpha d - 3d^2 - 6\alpha_S xd + 7xd^2 - 4x^2d^2}{4(2\alpha_S - d)}, \text{ and}$$

$$p_s^{***} = \frac{\alpha_S(\alpha_S - d) + xd(1-x)}{2\alpha_S - d},$$

with $q_1 = q_2 = .25$ and $q_S = .5$.

Since it is possible that $w_1^{***} = \frac{1-\alpha_S + xd}{2} < w_1^{**} = \frac{1}{2}$, there exist the following three possibilities:

1) $w_1^{**}, w_1^{***} \le \overline{w}_1$: FOCUS 2 is the solution.
2) $\overline{w}_1 \le w_1^{**}, w_1^{***}$: ATTACK BOTH is the solution.
3) $w_1^{***} \le \overline{w}_1 \le w_1^{**}$: The interior solutions are not feasible.

Under condition 3), in the region where $w_1 \le \overline{w}_1$ (SB is not selling to seg. 1), M1 will keep raising w_1 toward w_1^{**}. In the region where $w_1 \ge \overline{w}_1$ (SB sells to both segments), M1 will keep lowering w_1 toward w_1^{***}. The end result is that $w_1 = \overline{w}_1$, where w_1 is set just low enough to keep SB from attacking segment 1.

Thus, this is the "**ATTACK 2 and THREATEN 1**" solution.

The solutions for the case of $1 > \alpha_S \ge \alpha$ can be obtained through a similar process.

Technical Appendix 3
Numerical Algorithm for Equilibrium Analysis

The algorithm consists of four major routines solving, respectively, (1) the retailer's pricing problem, (2) NB_2 manufacturer's pricing problem, (3) NB_1 manufacturer's pricing problem, and (4) the retailer's store brand positioning problem – all for a given set of environmental parameters, α, α_S and d.

Routine I -- optimize the retailer's pricing decision

This routine produces numerical answers to the following question: for any given pair of wholesale prices, how should the retailer set prices for NB_1, NB_2 and SB? The most straightforward way of solving this problem is to maximize retail profit over retail prices, using Equation 10-12 as the demand functions. However, since Equation 10-12 are not "well behaved", no standard optimization procedure can guarantee global optimal. To get around this problem, we reframe the problem using reverse demand functions as a mathematically equivalent decision of how many units of each brand to sell to each segment, i.e., making six quantity decisions [Q_{11}, Q_{21}, Q_{31}, Q_{12}, Q_{22}, Q_{32}]. There exist 49 scenarios regarding which of the six quantities are non-negative (as implied by Table 2), each associated with a set of constraints on retail prices following Equation 10-12. For each of the 49 scenarios, the retailer's optimization problem is characterized by linear constraints and a quadratic objective function as follows:

$$Maximize[\, f'(i) \cdot q - q' \cdot H(i) \cdot q \,],$$

subject to: $A_1(i) \cdot q <= b_1(i)$, $A_2(i) \cdot q = b_2(i)$, and $q >= 0$

where q' = [Q_{11}, Q_{21}, Q_{31}, Q_{12}, Q_{22}, Q_{32}]. H(i), f(i), A_j(i) and b_j(i) are unique to scenario i. The objective function can be rearranged as $(f'(i) - q' \cdot H(i)) \cdot q$ in which the term $(f'(i) - q' \cdot H(i))$ represents the retail margin vector. The three constraints are the segment size constraint, the equal price constraint between the two segments, and the non-negative quantity constraint, respectively.

For illustration, take a scenario where the brand before price valuation ranking is $NB_1 > NB_2 > SB$ in both segments, and all three brands have positive sales in both segments. The corresponding $H(i)$, $f(i)$, $A_1(i)$, $A_2(i)$, $b_1(i)$ and $b_2(i)$ are as follows:

$$H = \begin{bmatrix} \gamma_{11} & \gamma_{21} & \gamma_{31} & 0 & 0 & 0 \\ \gamma_{21} & \gamma_{21} & \gamma_{31} & 0 & 0 & 0 \\ \gamma_{31} & \gamma_{31} & \gamma_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_{12} & \gamma_{22} & \gamma_{32} \\ 0 & 0 & 0 & \gamma_{22} & \gamma_{22} & \gamma_{32} \\ 0 & 0 & 0 & \gamma_{32} & \gamma_{32} & \gamma_{32} \end{bmatrix},$$

$$f' = [\gamma_{11} - w_1, \gamma_{21} - w_2, \gamma_{31} - w_3, \gamma_{12} - w_1, \gamma_{22} - w_2, \gamma_{32} - w_3],$$

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} \gamma_{11} & \gamma_{21} & \gamma_{31} & -\gamma_{12} & -\gamma_{22} & -\gamma_{32} \\ \gamma_{21} & \gamma_{21} & \gamma_{31} & -\gamma_{22} & -\gamma_{22} & -\gamma_{32} \\ \gamma_{31} & \gamma_{31} & \gamma_{31} & -\gamma_{32} & -\gamma_{32} & -\gamma_{32} \end{bmatrix},$$

$$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{ and } b_2 = \begin{bmatrix} \gamma_{11} - \gamma_{12} \\ \gamma_{21} - \gamma_{22} \\ \gamma_{31} - \gamma_{32} \end{bmatrix},$$

where $\gamma_{ij} = v_i - m_{ij}$ (i.e., quality minus mismatch) for brand i in segment j. We solve this optimization problem for each scenario using the standard quadratic programming routine with guaranteed global optimality of the solution. (We use the QUADPROG routine from MATLAB 6.0, and codes are available on request.) Finally, we compare the optimization outputs from all the 49 scenarios, and pick the one that leads to the highest retail profit as the solution to the retailer's pricing problem.

Routine II -- optimize NB_2 manufacturer's pricing decision

For any given wholesale price of NB_1, the manufacturer of NB_2 searches over its strategy space and picks the wholesale price that leads to the highest profit level, taking into account that retail prices will be determined through Routine I, for any given pair of wholesale prices. We mimic this search behavior by conducting a numerical grid search. In order to guarantee that this routine produces accurate results, the search is conducted over the entire feasible range of w_2, from 0 (marginal cost) to 1 (the highest reservation price for any consumer), in steps of 1/2000, resulting in a near-continuous search over the range.

Routine III -- optimize NB_1 manufacturer's pricing decision

Routine III is similar to II. We conduct an exhaustive search over a bounded and discrete strategy space for the manufacturer of NB_1, taking into account that w_2 is determined through Routine II, in which Routine I is called to determine the retail prices for each pair of wholesale prices.

Routine IV – optimize store brand positioning

We search over the entire feasible range of SB positioning ($0 \le x \le 1$) taking 21 steps with an increment of 0.05. Our investigation of the retailer profit function with respect to x confirms that this discrete strategy space is fine enough to ensure practically the same results as its continuous counterpart would produce.

Routine IV is the outmost loop in the sense that, when evaluating the profitability of each feasible store brand position, Routine IV calls Routine III as a subroutine to determine the NB_1 manufacturer's response, w_1. In a similar way, during each run of Routine III, Routine II is called to determine the NB_2 manufacturer's response to w_1 and each run of Routine II calls Routine I as a subroutine to determine optimal retail prices for each pair of wholesale prices.