Rex Y. Du & Wagner A. Kamakura

Where Did All That Money Go? Understanding How Consumers Allocate Their Consumption Budget

All types of consumer expenditures ultimately vie for the same pool of limited resources—the consumer's discretionary income. Consequently, consumers' spending in a particular industry can be better understood in relation to their expenditures in others. Although marketers may believe that they are operating in distinct and unrelated industries, it is important to understand how consumers, with a given budget, make trade-offs between meeting different consumption needs. For example, how much would escalating gas prices affect consumer spending on food and apparel? Which industries would gain most in terms of extra consumer spending as a result of a tax rebate? Answers to these questions are also important from a public policy standpoint because they provide insights into how consumer welfare would be affected as consumers reallocate their consumption budget in response to environmental changes. This study proposes a structural demand model to approximate the household budget allocation decision, in which consumers are assumed to allocate a given budget across a full spectrum of consumption categories to maximize an underlying utility function. The authors illustrate the model using Consumer Expenditure Survey data from the United States, covering 31 consumption categories over 22 years. The calibrated model makes it possible to draw direct inferences about the trade-offs individual households make when they face budget constraints and how their relative preferences for different consumption categories vary across life stages and income levels. The study also demonstrates how the proposed model can be used in policy simulations to quantify the potential impacts on consumption patterns due to shifts in prices or discretionary income.

Keywords: consumer expenditures, demand system, consumption, household budget allocation

The majority of models of consumer demand in the marketing literature focus on within-category purchase decisions—for example, incidence, brand choice, and quantity within a single product category (e.g., Chiang 1991; Chintagunta 1993; Gupta 1988). More recently, several models have been developed to analyze choice behavior across multiple categories using shopping basket data (Seetharaman et al. 2005). For example, Manchanda, Ansari, and Gupta (1999), Russell and Petersen (2000), and Chib, Seetharaman, and Strijnev (2002) examine multicategory purchase incidence decisions. Russell and Kamakura (1997), Ainslie and Rossi (1998), Iyengar, Ansari, and Gupta (2003), and Singh, Hansen, and Gupta (2005) investigate multicategory brand choice decisions. Song and Chintagunta (2006) model multicategory incidence and brand choice decisions jointly, and Song and Chintagunta (2007) allow for incidence, brand choice, and quantity decisions across multiple categories.

In contrast to research that focuses on multicategory choice behavior, in which the budget for a particular shopping trip is allocated across a few selected product cate-

gories, few empirical studies in the marketing literature have focused on modeling how consumers allocate their limited discretionary income to meet different consumption needs, in which trade-offs must be made across a wide range of expenditure categories (e.g., food, apparel, recreation, transportation, medical and personal care). The empirical studies that examine consumer expenditures either are descriptive in nature (e.g., Ferber 1956; Ostheimer 1958), focusing on a particular demographic group (e.g., elderly [Goldstein 1968], working wives [Bellante and Foster 1984]) or a particular consumption category (e.g., food [Rogers and Green 1978], energy [Fritzsche 1981], services [Soberon-Ferrer and Dardis 1991]), or use univariate models that ignore the interdependencies across consumption categories (e.g., Du and Kamakura 2006; Rubin, Riney, and Molina 1990; Wagner and Hanna 1983; Wilkes 1995).

Empirical studies on consumption have been far more common in economics than in marketing. In economics, the main issue has been the intertemporal trade-offs consumers make when choosing between current and future consumption (e.g., Deaton 1992; Gourinchas and Parker 2002); here, all the consumption expenditures are typically lumped into one aggregate account, and the real focus is on the accumulation of assets/debts. In the instances when economists consider the allocation of current consumption budget into different products and services, the focus has been either on a small number of broad commodity groups, such as food, clothing, and housing (e.g., Deaton and Muellbauer 1980;

Rex Y. Du is Assistant Professor of Marketing, Terry College of Business, University of Georgia (e-mail: rexdu@terry.uga.edu). Wagner A. Kamakura is Ford Motor Co. Professor of Global Marketing, Fuqua School of Business, Duke University (e-mail: kamakura@duke.edu).

Pollak and Wales 1978), or on a limited set of more narrowly defined goods, such as different types of recreation (Phaneuf, Kling, and Herriges 2000), urban transportation (Kockelman 2001), food consumed at home (Kao, Lee, and Pitt 2001; Kiefer 1984), energy (Bousquet, Chakir, and Ladoux 2004), and print/online newspapers (Gentzkow 2007).

This relative disregard for consumption budget allocation in marketing research might be due to the primary interest of manufacturers and retailers in influencing consumer choices toward their brands. However, consumer expenditures across seemingly unrelated industries are ultimately interrelated through the budget constraint and therefore should be viewed from a systemic perspective. Marketing researchers should devote more attention to understanding how consumers allocate their consumption budget to meet all kinds of competing needs and how the resultant expenditure patterns are influenced by factors such as income levels, inflation rates, and family life stages. For example, recent economic trends in the United States have focused consumer attention on concerns such as spiraling price increases in energy, health care, and education. Firms such as supermarkets that expand into mass-merchandising and fuel centers or manufacturers that diversify into conglomerates with multiple lines of business need to understand and anticipate how industry demands change as consumers respond to these dramatic shifts in prices by adjusting their consumption patterns while satisfying their budget constraints. Which industries are most affected? How do the responses differ from low- to high-income households and from younger to older families? Similar questions have also been raised in the public policy arena because policy makers need to understand how spikes in the costs of health care, education, and energy affect consumers' everyday lives or to make projections about the potential impact on consumption of a new tax levy or rebate. To answer such questions, demand models that can approximate individual households' budget allocation decisions for any given prices across a full spectrum of consumption categories are needed.

Furthermore, as is often noted, tomorrow's consumers will have more choices, and as a result, tomorrow's firms will face increasingly more intense cross-industry competition as consumers make trade-offs not only within product categories but also across categories. For example, a recent study showed a decline in confectionery sales among teens because teens were using their pocket money for text messaging, not candy bars; similarly, after analyzing consumer food expenditure data, Campbell Soup Company realized that it was competing across a variety of industries that included McDonald's and Burger King (Allen and Rigby 2005). Thus, marketers need to understand better how consumer spending in one industry may be substituted by expenditures in others and how such cross-industry substitution patterns may vary across the population. To do this, again, it is necessary to model consumer budget allocations.

Finally, as households evolve through the family life cycle, their consumption priorities and, therefore, expenditure pattern will change. In a society such as the United States, in which demographic composition has been going through major shifts, the proportion of consumers in different life stages will vary substantially over time. Thus, to project primary demand (i.e., the demand for a general product category), it is necessary to consider simultaneously population trends and differences in consumption priorities of different demographic groups.

In summary, we believe that modeling consumer budget allocation should be of particular importance to marketers and public policy makers, especially in light of the dynamics of economic conditions (e.g., different inflation rates in different industries), cross-industry competition, and demographic trends (e.g., aging population, more nontraditional households). However, to develop a proper budget allocation model, the following three major challenges must be addressed.

The first challenge is that of high dimensionality. A compressive study of consumer budget allocation should include as complete a set of consumption categories as possible. This makes high dimensionality a challenge from the modeling standpoint. For example, the consumer products and services hierarchy used by the Bureau of Labor Statistics (www.bls.gov/data/) to calculate the Consumer Price Index (CPI) comprises 11 major categories (food, alcohol, housing, apparel, transportation, medical care, recreation, education, communication, tobacco, and personal care), and 34 subcategories (e.g., food at home and away from home; alcoholic beverages at home and away from home; shelter, fuels, and utilities; household furnishings and operations). At the lowest level, the CPI hierarchy consists of approximately 200 mutually exclusive items. It would be intractable to build a budget allocation model that includes 200-plus categories. This raises the issue of the appropriate level of aggregation. At one extreme, for example, a model that considers only the 11 major CPI categories could be built. The upside would be the relative ease of implementation, but the downside would be a potential loss of important behavioral and managerial insights. Currently available consumer demand models, such as the linear expenditure system (LES) (Kao, Lee, and Pitt 2001; Pollak and Wales 1969; Stone 1954), the almost ideal demand system (AIDS) (Barnett and Seck 2006; Deaton and Muellbauer 1980; Dreze, Nisol, and Vilcassim 2004), the Rotterdam model (Barten 1964; Clements and Selvanathan 1988; Theil 1965; Vilcassim 1989), or the translog model (Christensen, Jorgenson, and Lau 1975; Srinivasan and Winer 1994), would be impractical to account for the interdependencies across more than a few consumption categories because of the large number of covariance terms and/or cross-category interaction effects to be estimated. This is also the case for the existing multicategory choice models in the marketing literature. Thus, to strike a balance between practicality and richness, we propose a factor-analytic extension of the demand system that Kao, Lee, and Pitt (2001) propose, making it feasible for inclusion of a set of 31 expenditure categories, which are aligned (to the degree data are available) with the CPI subcategory structure mentioned previously.

The second challenge is that of binding nonnegative constraints. A common feature of consumer expenditure data is that most households spend money in only a subset

of categories. The pattern of zero consumptions varies from household to household and contains important information about individual preferences, making it crucial to model them explicitly. Unfortunately, such a requirement rules out any demand systems in which all categories are assumed to be consumed by all households, an assumption we demonstrate subsequently to be vastly violated in household expenditure data. In other words, heavily censored expenditure data render many popular demand systems, such as the AIDS, Rotterdam, or translog, inapplicable because they are all derived from first-order conditions of constrained utility maximization problems, assuming the existence of interior solutions for all goods. As a result, they would predict positive expenditures in all categories for all households, which would be biased and inconsistent with the actual data. In contrast, we accommodate the binding nonnegative constraints observed in household budget allocations by proposing a budget allocation model in which both the "whether-to-spend" and the "how-much-to-spend" decisions result from a common utility maximization problem, allowing for inferences about a unified preference structure from both zero and nonzero observations. An important advantage of using a common utility function for both incidence and quantity decisions is parsimony, which is desirable given the high dimensionality of the demand system.

The third challenge is that of unobservable heterogeneity. Consumer expenditure data consistently depict large variations in the pattern of budget allocation across households. Manifested in these patterns are the different consumption priorities of different households. Capturing these individual differences is important for models of budget allocation because doing so can provide valuable insights into how consumers would respond differently to changes in external (e.g., price shocks, tax rebates) and internal (e.g., life stage changes) conditions. To account for heterogeneity in preferences, existing studies of household budget allocation have relied solely on demographic variables, such as age, income, ethnicity, and family composition. However, as the choice modeling literature has shown, demographics can often explain only a small portion of heterogeneity in consumer preferences. In the context of budget allocation, unobservable heterogeneity may be revealed through the interdependencies of consumption across categories. This is not a trivial task given the two challenges (i.e., high dimensionality and binding nonnegative constraints) we discussed previously. Conversely, as we show in our empirical application, accounting for unobservable heterogeneity is beneficial in that it leads to a more flexible demand system at the aggregate level.

In summary, we believe that it is important for marketers to be able to infer the underlying cross-category trade-offs households make in allocating their consumption budget, so that we can then predict how these allocations will change in response to shifts in prices or discretionary income and how differences in preferences across households lead to different consumption patterns. To understand how households allocate their consumption budget across a full spectrum of expenditure categories, we use a budget allocation model built on an approach first proposed by Wales and Woodland (1983) and then extended by Kao,

Lee, and Pitt (2001) (hereinafter, the KLP model), assuming that households allocate their consumption budget to maximize a utility function that is linear in the logarithms of quantity consumed less a constant for each category (this is also referred to as the "Stone–Geary utility"). This budget allocation model tackles the aforementioned three challenges, leading to a demand system that (1) is feasible for a large number of consumption categories (which is not the case with the KLP model), (2) allows for corner solutions observed in censored expenditure data, and (3) results in a globally flexible aggregate demand system for each consumption category by obtaining household-level estimates of the direct utility function. Next, we briefly review the LES model and describe our more flexible extension to the KLP model. Then, we apply our proposed factor-analytic random coefficients budget allocation model to the Consumer Expenditure Survey (CEX) data from the United States, covering 31 consumption categories over a period of 22 years. We conclude with a discussion and directions for further research.

Modeling Consumption Budget Allocation

Our main purpose in this study is to develop a reasonable "as-if" model that approximates how individual households allocate their consumption budget across a comprehensive set of expenditure categories. Because each household consumes only a subset of all consumption categories, our observed expenditure data are censored. Thus, we need a demand system that can accommodate many consumption categories and allow for corner solutions. As we mentioned previously, these requirements rule out the most popular demand systems, such as the AIDS, Rotterdam, and translog models, all of which (1) quickly become impractical for more than a couple dozen consumption categories and (2) require nonzero consumption in all categories. For these reasons, we extend the KLP budget allocation model to consider simultaneously whether-to-spend and how-much-to-spend decisions. Our model is distinguished from the KLP model in two important aspects. First, the KLP model is feasible only when the focus is on a small number of consumption categories, accounting for only part of the household budget (e.g., seven types of food items). However, for a comprehensive analysis of household budget allocation, the high-dimensionality challenge is inevitable. Rather than limiting the analysis to a high level of aggregation or lumping a large number of distinct expenditure items into an "other" category, our proposed approach affords the flexibility to cover the full spectrum of household consumption budget allocation at a low level of aggregation. Second, to account for a large number of consumption categories while allowing for individual differences in category preferences and a rich pattern of correlation in these preferences across categories, we extend the KLP model by imposing a flexible factor structure to the covariance matrix of the stochastic taste parameters that govern individual households' consumption priorities.

We assume that household h maximizes a continuously differentiable, quasi-concave direct utility function $G(x_h)$

over a set of J nonnegative quantities $x_h = (x_{1h}, x_{2h}, ..., x_{Jh})$, subject to a budget constraint $p'x_h \leq m_h$, where $p = (p_1, p_2, ..., p_J)' > 0$, p_i is the price of good i, and m_h is household h's total consumption budget/discretionary income. Following the work of Wales and Woodland (1983) and Kao, Lee, and Pitt (2001), we use the Stone–Geary utility function, which has the following form:

$$(1) \qquad G(x_h) = \sum_{i=1}^{J} \alpha_{ih} \ln(x_{ih} - \beta_i),$$

where $\alpha_{ih} > 0$, $(x_{ih} - \beta_i) > 0$, and J is the number of all available consumption categories. The h subscript in α_{ih} implies that the utility function is household specific. The Kuhn–Tucker conditions for the household's optimization problem are as follows: $\partial G(x_h)/\partial x_{ih} - \xi p_i \leq 0$ for $x_{ih} = 0$, and $\partial G(x_h)/\partial x_{ih} - \xi p_i = 0$ for $x_{ih} > 0$, such that $p'x_h - m_h \leq 0 \leq \xi$, where ξ denotes the Lagrange multiplier or marginal utility per dollar.

The budget allocation problem we described implies that the household incrementally allocates its discretionary income to the consumption category that produces the highest marginal utility per dollar,

$$\frac{\partial G(x_h)}{\partial x_{ih}} \frac{1}{p_i} = \frac{\alpha_{ih}}{(p_i x_{ih} - p_i \beta_i)},$$

given the current consumption levels x_h, until the budget is reached, $\Sigma_{i=1}^{J} p_i x_{ih} = m_h$. The solution of this optimization problem leads to the following expenditure system, which is linear in discretionary income and prices (thus the label LES in the literature):

$$(2) \qquad p_i x_{ih} = p_i \beta_i + \theta_{ih}^* \left(m_h - \sum_{j=1}^{J^*} p_j \beta_j \right), \quad i = 1, 2, ..., J^*,$$

where $\theta_{ih}^* = \alpha_{ih}/\Sigma_{j=1}^{J^*} \alpha_{jh}$ and J* is the set of consumed goods (i.e., with positive expenditures).

Note that the demand system given by Equation 2 is defined only for a particular consumption regime J*. If the pattern of nonzero expenditures changes, both the intercepts and the slopes of the demand system will also change. In other words, the model implies that there is an optimal consumption regime for each household at each combination of budget and prices, and only within a particular consumption regime are category expenditures linear functions of budget and prices; across consumption regimes, the demand system is piecewise linear. In addition, rather than imposing arbitrary censoring mechanisms, such as the Tobit regression model (Amemiya 1974), the approach based on the Kuhn–Tucker condition allows for zero consumption as a corner solution to a constrained utility maximization problem. It also ensures that predicted expenditures will always be nonnegative and sum to the budget.

Unlike existing budget allocation analyses, which either ignore heterogeneity or allow for heterogeneity only through demographics, we take advantage of the multivariate nature of the estimation problem (i.e., 31 points of expenditure data per household) and obtain household-specific estimates of the preference parameters (α_{ih}), which

means that the slope parameters (θ_{ih}^*) of the demand system will be unique to each household as well.[1] Because each individual household has a unique consumption regime (and, therefore, a different regime switching point) and a different set of demand function parameters, the implied aggregate demand can be highly nonlinear, overcoming a common criticism of the inflexibility of the original LES model.

Given the model we described, the researcher's problem is to infer consumers' utility function parameters (i.e., α_{ih} and β_i) given the observed budget allocations and prices. The KLP model deals with variation in preferences across households by treating α_{ih} as stochastic; that is, $\alpha_{ih} = \exp(\gamma_i + \varepsilon_h)$, where $\varepsilon_h \sim N(0, \Sigma)$. Kao, Lee, and Pitt (2001) demonstrate that estimation through maximum likelihood is feasible only for simple problems with few consumption categories because it requires the evaluation of a multivariate normal cumulative density function. To circumvent this serious limitation, Kao, Lee, and Pitt propose to estimate their model with simulated maximum likelihood.

Although the KLP approach simplifies estimation considerably, the model formulation still limits the number of consumption categories that can be handled (e.g., only seven categories are considered). Specifically, the KLP model requires $[(J - 1) \times J]/2$ parameters for the covariance matrix Σ. An analysis of the CEX data with 31 consumption categories would require the estimation of $(30 \times 31)/2 = 465$ covariance terms. In addition, the stochastic formulation of α_{ih} in the KLP model does not allow for household-level estimates of the taste parameter, which can be achieved through our formulation. This distinguishing characteristic of our model is particularly important because the ability to estimate the household-specific taste parameter α_{ih} enables us to perform more realistic policy simulations that account for individual differences in consumption priorities and for a rich pattern of correlation in preferences across categories.

To account for unobserved heterogeneity in the taste parameter (α_{ih}) for each category i and still have a model of feasible size, we propose a factor-analytic extension of the KLP random coefficients model by extracting the principal components of the covariance matrix of the stochastic terms:

$$(3) \qquad \alpha_{ih} = \exp(\gamma_i + \lambda_i Z_h + \varepsilon_{ih}),$$

and

$$(4) \quad \beta_i = \min(x_i) - \exp(\eta_i), \text{ to ensure that } x_{ih} - \beta_i > 0 \text{ for } \forall h,$$

where

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[1] For identification purposes, in our empirical analysis, α_h is set to 1 for food at home. In other words, all the preferences are relative to the consumption of food at home. For identification purposes, we assume that β_{ih} is the same across households (i.e., $\beta_{ih} = \beta_i$) as a result of an indeterminacy that would produce the same marginal utility, $\partial G(x_h)/\partial x_{ih} = \alpha_{ih}/(x_{ih} - \beta_{ih})$, at any given consumption point for an infinite pairs of α_{ih} and β_{ih}. In other words, this parametric assumption is imposed without loss of generality and has no impact on any of our substantive findings.

e^{γ_i} = the geometric mean of the taste parameter α_{ih} for category i across the sample,

Z_h = a p-dimensional vector of i.i.d. standard normal factor scores for household h,

λ_i = a p-dimensional vector of factor loadings for category i, and

ε_{ih} = a random disturbance normally distributed with mean zero and standard deviation σ_i.

Although γ_i and η_i provide insights into the average preference for category i, the product of the factor loadings (λ_i) and factor scores (Z_h) will show how much higher or lower the (log) taste of household h is relative to the average. Moreover, the factor loadings $\Lambda = \{\lambda_i\}$ capture the essential information about how (log) tastes covary across categories and households because the covariance matrix for their distribution can be directly obtained as $\Lambda'\Lambda$. Thus, if two categories i and j have high loadings (λ) of the same sign on the same dimensions of the latent factors (Z), households assigning a high (low) utility to one category will also assign a high (low) utility to the other. In other words, the results from our factor-analytic model can provide valuable insights into how interrelated the consumption categories are across consumers.

In summary, a key benefit of our proposed factor-analytic random coefficients LES model (compared with the KLP model) is that it enables estimation of the direct utility function for each household in more realistic applications with a large number of consumption categories (high dimensionality), many of which may not be consumed by all households (censored data). Moreover, our proposed factor-analytic extension to the KLP model allows not only for unobserved heterogeneity in the household's taste (α_{ih}) for each consumption category but also for a rich pattern of correlation in these tastes across categories. Details about the estimation of our model with simulated maximum likelihood appear in the Appendix.

Implications of the Budget Allocation Model

A common criticism of the Stone–Geary utility function assumed in Equation 1 is that it does not allow for potential complementarity between categories. Although this assumption could be limiting when studying consumption at a more micro level (e.g., pasta and pasta sauce should be complementary because the utility derived from consuming them together is greater than the sum of utilities derived from consuming them separately), the additive separable utility assumed by the Stone–Geary function is not as restrictive in a broader analysis of how consumers allocate their discretionary income, because at that point, all consumption categories are ultimately substitutes as they compete for the same budget.

Furthermore, as Gentzkow (2007, pp. 714–15) discusses, separating complementarity between consumption categories from correlation of consumer preferences across categories would require additional variables that discriminate among consumption categories and/or longitudinal data for each household. Unfortunately, because consumer expenditure surveys (e.g., the CEX) are usually done on an annual basis, so that the analyst has only one observation on how the household allocated its consumption budget across various expenditure categories, it is not possible to discern true complementarity between categories from correlation in consumer preferences across categories. Finally, given the high dimensionality involved in modeling households' budget allocations across a comprehensive list of expenditure categories, it is empirically intractable to consider potential interaction effects among all the categories (e.g., $[30 \times 31]/2 = 465$ additional parameters would be needed to allow for all the potential interaction effects in the utility function in our analysis). In summary, for both theoretical and practical purposes, in a household budget allocation analysis such as ours, a main-effect-only utility function, such as the Stone–Geary utility, should be viewed as a reasonable as-if model, which precludes complementarity between consumption categories.

By accounting for unobservable heterogeneity in preferences across households, our factor-analytic random coefficients LES model produces a globally flexible demand system when aggregated cross-sectionally. At the individual level, the additive separable Stone–Geary utility function implies that consumption of one category does not interact with consumption of another category. This means that preferences for different consumption categories are locally independent (i.e., within a particular household, consumption of one category does not affect the marginal utility of consuming another category). However, in our proposed model, preferences for different consumption categories do not need to be independent across households, because the factor structure embedded in α_{ih} allows tastes to be globally correlated, making it possible, for example, that consumers who have a high (relative to other households) preference for tobacco products also have a high preference for alcohol consumption.

The own- and cross-price elasticities implied in our model for consumption categories i and j are defined at the household level, respectively, as follows:

$$(5) \qquad \frac{\partial \ln x_{ih}}{\partial \ln p_i} = -1 + \frac{(1 - \theta_{ih}^*)\beta_i}{x_{ih}},$$

and

$$(6) \qquad \frac{\partial \ln x_{ih}}{\partial \ln p_j} = -\theta_{ih}^* \frac{p_j \beta_j}{p_i x_{ih}},$$

where

$$\theta_{ih}^* = \frac{\alpha_{ih}}{\sum\limits_{j=1}^{J^*} \alpha_{jh}} = \frac{\exp(\gamma_i + \lambda_i Z_h + \varepsilon_{ih})}{\sum\limits_{j=1}^{J^*} \exp(\gamma_j + \lambda_j Z_h + \varepsilon_{jh})}.$$

Note that these elasticities depend on the household's factor scores (Z_h) and the factor loadings for the particular categories involved (λ_i).

Note also that the elasticities are defined at the household level. Given that the taste parameters α_{ih} are correlated across consumption categories and households (according to the pattern reflected in the latent factors), our model

allows for covariation in the consumption pattern across households, and the resultant aggregate demand system is much more flexible than the traditional (no unobservable heterogeneity) LES demand system. Despite this flexibility, the proposed model still assumes an additive separable utility function, which precludes complementarity between categories. To demonstrate this, using Equation 6, we can readily derive the aggregate (population-wide) cross-elasticities (Equation 7) and decompose them into income and substitution effects (Equations 8 and 9, respectively) (Bohm and Haller 1987). By definition, Equation 7 equals Equation 8 plus Equation 9.

$$(7) \quad \text{Aggregate cross-elasticities:} \quad \frac{\partial \ln \sum\limits_{h} x_{ih}}{\partial \ln p_j} = -\frac{\sum\limits_{h} \theta^*_{ih} \beta_j}{\sum\limits_{h} x_{ih}} \frac{p_j}{p_i};$$

$$(8) \quad \text{Income effects:} \quad \frac{\partial \ln \sum\limits_{h} x_{ih}^{\text{Income}}}{\partial \ln p_j} = -\frac{\sum\limits_{h} \theta^*_{ih} x_{jh}}{\sum\limits_{h} x_{ih}} \frac{p_j}{p_i}; \text{ and}$$

$$(9) \quad \text{Substitution effects:} \quad \frac{\partial \ln \sum\limits_{h} x_{ih}^{\text{Substitution}}}{\partial \ln p_j}$$

$$= \frac{\sum\limits_{h} \theta^*_{ih} \left(x_{jh} - \beta_j \right)}{\sum\limits_{h} x_{ih}} \frac{p_j}{p_i}.$$

Note that after the income effects (Equation 8), which are all negative, are accounted for, the substitution effects (Equation 9) are all positive. This implies that after the budget constraint is accounted for, all categories are substitutes, ruling out the possibility for complementarity. In other words, consumption of one category does not increase the marginal utility of any other category, and therefore the resultant demand system cannot predict that consumption of pasta will positively affect consumption of pasta sauce. Although this limitation might be critical in detailed analyses among a few product categories, such as within food consumption, it is less important in studies of household expenditures involving a larger number of broadly defined commodity groups, particularly when data are available only at one period for each household.

In addition to price elasticities, another property of demand systems commonly considered in consumption analysis is the Engel curve, which relates consumption in each category to the total consumption budget. If defined on quantity, the Engel curve implied by our model is linear in the budget m_h, with the slope proportional to θ^*_{ih} and inversely proportional to price p_i:

$$(10) \quad x_{ih} = \beta_i + \frac{\theta^*_{ih}}{p_i} \left(m_h - \sum\limits_{j=1}^{J^*} p_j \beta_j \right),$$

which implies that the income (or budget) elasticities are as follows:

$$\frac{\partial \ln x_{ih}}{\partial \ln m_h} = \theta^*_{ih} \frac{m_h}{p_i x_{ih}}.$$

Note that the Engel curve implied by our model applies to a specific consumption regime—namely, the set of positive consumption goods, J*. When the consumption budget (m_h) increases, the demand regime (J*) may change, which means that the intercepts and slopes of Equation 10 will also change. More specifically, as the budget is distributed across a larger consumption set, intercepts will continue to increase, and slopes will continue to decrease; that is, the Engel curve becomes flatter. Consequently, when significant shifts are observed in the demand regime over a range of incomes, we can note only that the Engel curve is piecewise linear. Moreover, the Engel curve is typically defined for the same household across different income levels. Again, because we allow for heterogeneities in the taste parameter α_{ih}, we account for the possibility that changes in discretionary income also result in changes in the utility function, so that shifts in income may result in cross-sectional changes in the Engel curve as well as movements along an individual Engel curve, thus producing highly nonlinear Engel curves. In other words, although the traditional LES demand system implies linear Engel curves for an individual household at its current consumption regime, our extension produces globally flexible aggregate demand structures with nonlinear Engel curves, again leading to a more flexible and realistic demand system at the population level than the traditional LES system. Our empirical results (which we discuss in detail subsequently) illustrate how well our factor-analytic extension of the LES demand system produces more flexible and realistic Engel curves than the traditional system.

If defined on expenditure share, the Engel curve is inversely proportional to m_h,

$$(11) \quad s_{ih} = \frac{p_i x_{ih}}{m_h} = \frac{p_i}{m_h} \beta_i + \theta^*_{ih} \left(1 - \sum\limits_{j=1}^{J^*} \frac{p_j}{m_h} \beta_j \right),$$

and the slope of the Engel curve is inversely proportional to the square of m_h,

$$(12) \quad \frac{\partial s_{ih}}{\partial m_h} = \frac{p_i \beta_i}{m_h^2} \left(\theta^*_{ih} \frac{\sum\limits_{j=1}^{J^*} p_j \beta_j}{p_i \beta_i} - 1 \right).$$

Given the consumption regime J*, Equation 12 suggests that expenditure share can be an increasing or decreasing function of consumption budget, depending on the relative sizes of θ^*_{ih} and $p_i \beta_i / \Sigma_{j=1}^{J^*} p_j \beta_j$. However, regardless of the direction of change, the Engel curve of expenditure share becomes less sensitive to consumption budget as the latter increases.

Consumption Patterns in the United States: 1982–2003

As an illustration of our proposed consumption budget allocation model, we apply it in an analysis of household

expenditures in the United States for a period of 22 years. The goal here is to obtain household-level estimates of their direct utility functions across a comprehensive set of consumption categories and use these estimates to gain insights into the consumption priorities across different types of households and income levels over the 22 years covered by our data. Rather than directly relating the utility functions to demographics, we first use our factor-analytic random coefficients approach to obtain estimates of the utility function for each household in the sample, leveraging the pattern of covariation across consumption categories, and then we investigate how these estimates differ across household types and income levels.

Data Description

To estimate our model, we use the CEX family extracts made available by the National Bureau of Economic Research (NBER) for the 1982–2003 period (http://www. nber.org/data/ces_cbo.html). The CEX is collected from different samples each year, so that each of the 66,368 households in this sample reports its consumption expenditures for only one year, and therefore the sample cannot be treated as a longitudinal panel. These NBER extracts from the CEX database contain the dollar amounts allocated by each sample household during a one-year window across 31 consumption categories.[2] In defining these broad consumption categories, we followed the typology NBER uses.

For each of the 31 consumption categories and 22 years in the CEX–NBER data, we collected the relevant price index from the Bureau of Labor Statistics, which we normalized with 1982 as the base year. Figure 1 summarizes these price indexes. Each vertical line within a bar represents one year of data; note the large variation in inflation rates across categories, with the largest increases in tobacco, education, and health care.

Figures 2 and 3 provide a summary description of the consumption data from 1982 to 2003. Figure 2 shows the percentage of households in the CEX sample reporting expenditures in each category, with low incidence rates (less than 40% of the sample) for several categories, such as hospital and related services, airline fare, public transportation, nonprescription drugs, and medical supplies. The figure shows some clear trends in consumption in the two decades under study. For example, the reporting of tobacco and alcohol consumption declined during this period. There is a similar decline in incidence rate for doctors, dentists, and hospitals, which might be due to the increase in incidence rate for health insurance. Another category worth noting is jewelry and watches, for which there was a substantial drop in incidence rate. Most important, Figure 2 shows that other demand systems, such as the popular AIDS and Rotterdam models, would be inappropriate for this type of analysis because they assume that all households spend

on all categories, even when this assumption clearly does not hold, except for food at home.

Figure 3 shows how households have allocated budget across categories. On average, food and clothing took a smaller portion of the household budget over the years, whereas health insurance took a larger share. Although the incidence rate of tobacco usage decreased during the 22-year period, tobacco consumption took an increasing portion of the budget among those who still smoke. The portion of the budget allocated to motor and home fuel decreased substantially in the 1980s but seems to have increased in the last few years.

Aside from the expenditure data, we also used demographic data from the CEX–NBER extracts to classify each sample household into a specific life stage, using the typology that Du and Kamakura (2006) propose. Table 1 describes the typical profile of each life stage. The last column of Table 1 reports the results from our classification of the CEX–NBER sample into this typology. This typology proves useful when we compare the patterns of preferences across households and consumption categories.

Estimation Results

To find the best compromise between parsimony and goodness of fit, we applied our proposed budget allocation model to a random sample of 9526 households from the CEX–NBER extracts, varying the number of factors (Z_h) from p = 3 to p = 8. From Bayesian information criterion and the interpretability of the factor solution, we chose the six-factor solution and applied it to the entire sample of 66,683 households in the CEX–NBER data. We report the parameter estimates and the goodness-of-fit measures in Table 2. The R-squares reflect our model's overall performance in predicting budget shares for each category, including observations with zero expenditures. It is calculated as follows: 1 – (Sum of squared prediction error/Variance of observed shares). At first glance, some of the R-squares in Table 2 may seem low. However, we computed these R-squares across a large sample of households according to estimates obtained from a smaller sample of households. Note also that for low incidence categories, the dependent variable is highly censored (i.e., with a lot of zero shares and relatively few positive values). For these categories, the R-squares tend to be the lowest because the model must correctly predict both incidence and the budget share conditional on incidence. The more pertinent goodness-of-fit measure for these categories is the hit ratio, which represents the percentage of correct predictions with respect to whether a household has positive consumption in a category.

Engel Curves by Consumption Category

We can attain a better sense of the flexibility afforded by our proposed model by comparing the cross-sectional Engel curves implied by our model with their observed counterparts. To obtain the cross-sectional Engel curves, we first computed the expected expenditure shares for each household ($\hat{s}_{ih}$), as shown in Equation 11, according to the individual estimates of $\hat{\alpha}_{ih} = \exp(\hat{\gamma}_i + \hat{\lambda}_i\hat{Z}_h + \frac{1}{2}\hat{\sigma}_i^2)$ and the other

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FIGURE 1
Price Indexes 1982–2003 (1982 = 100)
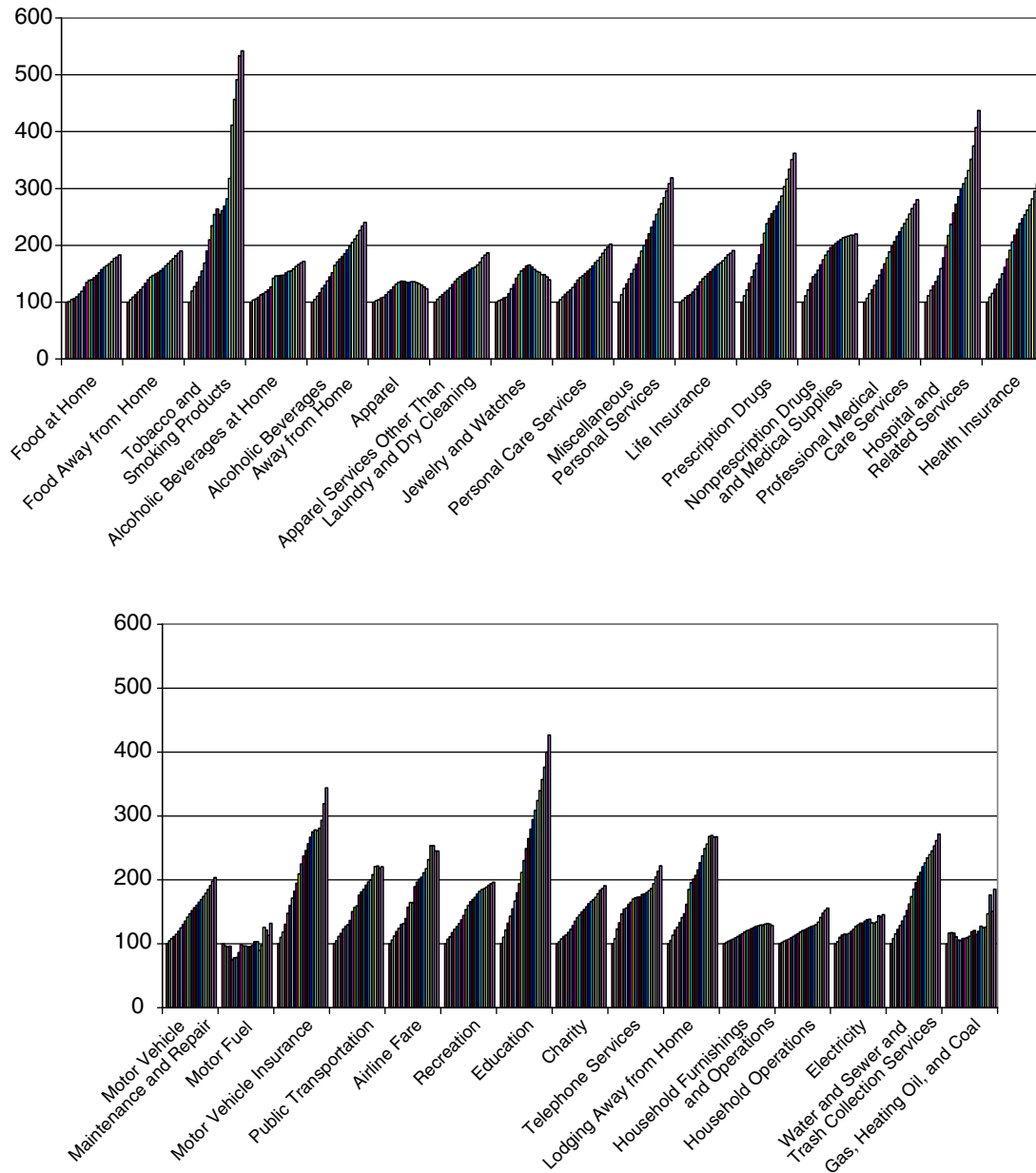
model parameters ($\hat{\beta}_i$), and then we averaged these expected shares within each income decile. We report the actual and model-implied cross-sectional Engel curves for some of the product categories in Figure 4, which shows that the curves implied by our proposed model conform well to their observed counterparts. Most important, Figure 4 shows that our factor-analytic random coefficients formulation can accommodate monotonically increasing shares (as income decreases) for essential categories (e.g., electricity, tele-phone, food at home) and decreasing shares (as income decreases) for nonessential categories (e.g., food outside the home, recreation, education, lodging away from home). Moreover, the model is flexible enough to capture certain inflections in the Engel curve, as is evident for the food-at-home category (from concave to convex as income decreases). It also captures nonmonotonic shapes, such as an inverted U shape for motor fuel and a U shape for public transportation, reflecting that at the lowest income deciles, more private transportation is substituted with public transportation.

The Principal Components of Consumption

From Equations 1 and 3, we can write the log-marginal util-ity given x_{ih} as $\gamma_i + \lambda_i Z_h + \varepsilon_{ih} - \ln(x_{ih} - \beta_i)$. Accordingly, $\gamma_i - \ln(-\beta_i)$ represents the average initial (when the quantity consumed is still zero) log-marginal utility for consumption

FIGURE 2
Percentage of Households Reporting Expenditures in Each Consumption Category
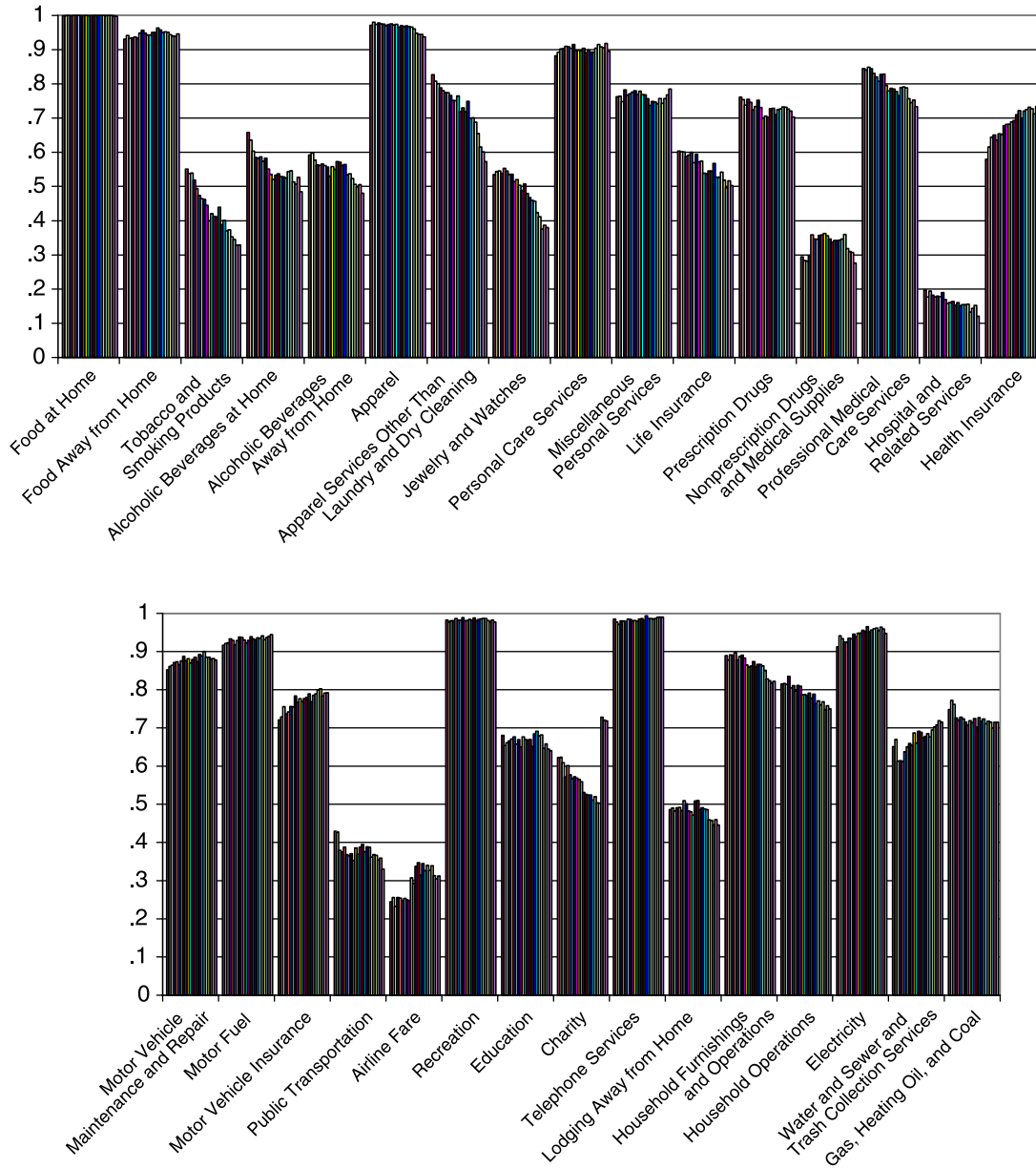
category i. The factor scores (Z_h) for a household, weighted by the factor loadings (λ_i) for a consumption category, show whether the household's log-marginal utility is higher or lower than average for that category at a given consumption level. Therefore, consumption categories that have large loadings of the same sign for the same factor have positively correlated log-marginal utilities across households. For example, because jewelry and watches as well as education have large loadings of the same sign for Factor 1, households with a higher-than-average score on this factor will have higher-than-average log-marginal utilities for both consumption categories. By the same token, through the sign and magnitude of these loadings, it is possible to identify sets of consumption categories that tend to have higher

(or lower) marginal utilities for the same households. For easier interpretation, Table 2 shows in bold the largest loadings (in absolute value) for each consumption category. From these loadings, households with a higher-than-average score on Factor 1 would be expected to have higher log-marginal utilities for categories, such as jewelry and watches, education, alcohol, and recreation, suggesting that this factor is associated with nonessential consumption. Factor 2 is associated with smoking and drinking and could be labeled a "sin" factor. Factor 3 is associated with large family-oriented consumption needs, such as insurance, household operations, electricity, and utilities. Factor 4 is associated with health care. Factor 5 has the highest loadings for public transportation (which includes trains and
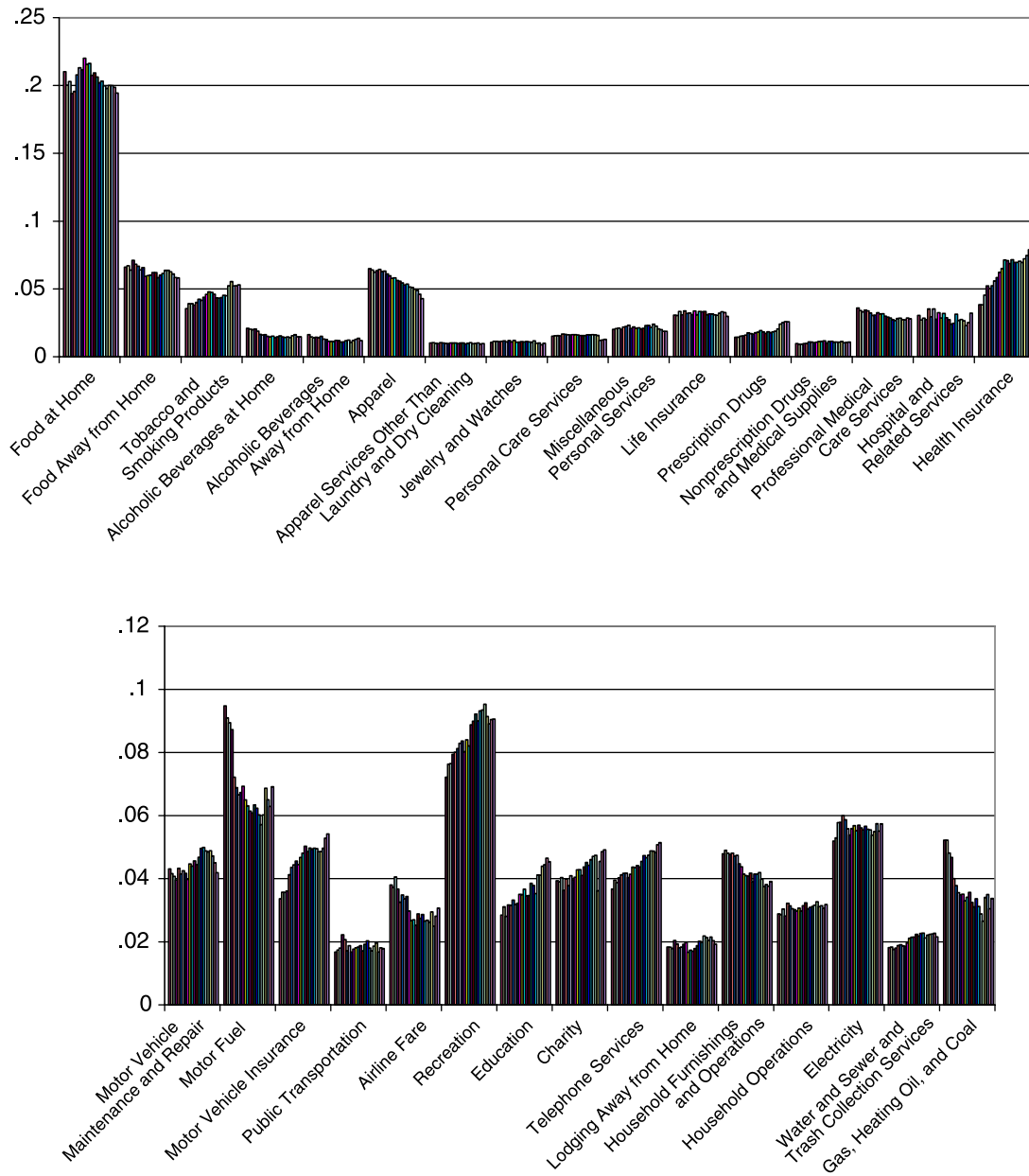
taxis), airfare, and lodging away from home and therefore could be labeled a "travel" factor. Finally, Factor 6 has relatively weaker loadings than the other factors, but it has higher loadings for categories related to the operation of motor vehicles.

Most important, these factors account for both heterogeneity in the log-marginal utilities across households and the correlation among these log-marginal utilities across consumption categories. We further investigated how consumption preferences differ across households by performing an analysis of variance (ANOVA) on the factor scores using three variables that describe the households: (1) life stage, as we defined it previously; (2) income quintile, relative to other households reporting expenditures in the same

year; and (3) year of data collection, which is classified into five categories (early or late 1980s, early or late 1990s, and early 2000s). This ANOVA, which we performed across all 66,683 households for Factor 1 (nonessential consumption), showed that only the life stage × income quintiles and life stages × year interactions (along with the main effects) were statistically significant at the .01 level. Therefore, we report averages only for these two interactions in Figure 5, Panel A. Moreover, to simplify the exposition, we focus only on the six life stages with the largest number of households, which account for more than 70% of our sample. Figure 5, Panel A, shows that average log-marginal utilities for "nonessential" consumption have increased over time and, as expected, are higher for the high-income quintiles. How-

TABLE 1
Summary Description of Life Stages

| Life Stage	Head of Household Marital Status	Head of Household Age	Head of Household Employment	Spouse Employment	Other Adults	Kids in College	Family Size	Kids Age 6 or Younger	Kids Ages 7–14	Kids Ages 15–18	% of Sample Households
Co/So	Couple/single	22–30	Working	N.A./working	None	No	One or two	No	No	No	16.6
C1	Couple	25–35	Working	Working/home	None	No	Three or Four	Yes	No	No	9.6
C2	Couple	33–41	Working	Working/home	None	No	Five or more	Yes	Yes	No	3.7
C3	Couple	40–50	Working	Working/home	None/one	Yes	Five or more	No	Yes	Yes	4.7
C4	Couple	34–44	Working	Working/home	None	No	Four or three	No	Yes	Yes	5.6
S1	Single	26–42	Working	N.A.	None	No	One or two	No	No	No	14.7
C5	Couple	45–57	Working	Working/home	One/none	Yes	Three or four	No	No	Yes	6.3
C6	Couple	51–73	Retired/working	Working/home	None	No	Two	No	No	No	11.2
S2	Divorced/single	27–41	Working	N.A.	None	No	Three or two	Yes	Yes	Yes	8.8
S3	Divorced/widowed	44–62	Working	N.A.	One/two	Yes	Two or three	No	No	Yes	7.0
S5	Widowed	66–84	Retired/home	N.A.	None	No	One	No	No	No	8.6
S4	Divorced/single	49–71	Working/retired	N.A.	None	No	One	No	No	No	2.1
C7	Couple	63–77	Retired	Home/retired	One/none	No	Three or two	No	No	No	1.0

Notes: N.A. = not applicable.

TABLE 2
Parameter Estimates for the Six-Factor Model

Consumption Category/Price Index	Gamma	Sigma	Beta	Factor Loadings						R² (%)	Hit Ratio (%)
				Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6		
Food at home	.000	.000	-.701	.00	.00	.00	.00	.00	.00	90	100
Food away from home	-1.252	.501	-.212	**.51**	.12	.24	.06	.21	.10	18	95
Tobacco and smoking products	-2.193	.886	-.229	-.10	**.40**	.10	-.05	-.07	.03	8	63
Alcoholic beverages at home	-3.059	.664	-.118	.32	**.60**	.09	-.06	.21	.02	40	79
Alcoholic beverages away from home	-4.007	.796	-.035	**.76**	**.80**	.12	-.06	.48	.03	42	82
Apparel	-1.402	.487	-.190	**.55**	.01	.09	.04	.09	-.02	29	97
Apparel services other than laundry and dry cleaning	-3.905	1.047	-.027	**.52**	.12	.06	-.04	**.34**	-.06	11	74
Jewelry and watches	-4.493	1.267	-.032	**.97**	.05	.10	.10	.14	-.05	9	65
Personal care services	-2.667	.604	-.063	**.27**	.01	**.23**	.11	.16	-.02	6	91
Miscellaneous personal services	-3.234	1.184	-.026	**.55**	.10	**.29**	.22	.10	.18	0	78
Life insurance[a]	-1.982	.740	-.310	.24	.00	**.32**	.14	.02	.01	4	65
Prescription drugs	-3.558	1.098	-.020	-.04	-.11	**.41**	**.83**	.04	-.01	35	77
Nonprescription drugs and medical supplies	-3.603	.905	-.091	.24	.01	**.28**	**.37**	.13	.06	1	63
Professional medical care services	-2.429	.890	-.067	**.33**	-.03	.23	**.52**	.06	.04	16	83
Hospital and related services	-5.287	1.863	-.064	.15	-.05	.14	**.80**	-.13	.02	6	69
Health insurance	-1.486	.738	-.271	-.12	-.03	**.42**	**.40**	.16	.08	33	75
Motor vehicle maintenance and repair	-1.847	.705	-.130	**.52**	.09	**.29**	.11	.02	**.33**	19	91
Motor fuel	-1.128	.403	-.413	**.28**	.10	**.25**	.00	-.08	**.20**	35	94
Motor vehicle insurance	-1.410	.496	-.250	**.21**	.08	**.33**	.11	.01	**.24**	18	85
Public transportation	-4.756	1.452	-.035	.33	.02	-.10	-.02	**1.14**	-.14	31	69
Airline fare	-2.463	.711	-.329	.44	.09	.23	.10	**.72**	.18	39	85
Recreation	-.950	.503	-.203	**.54**	.08	.18	.11	.17	.09	19	98
Education	-3.802	1.712	-.014	**1.18**	-.16	-.05	.12	.22	.29	5	69
Charity[a]	-2.741	1.268	-.111	**.49**	-.27	**.48**	**.37**	.26	.04	14	66
Telephone services	-1.769	.501	-.074	.14	.05	**.25**	.04	.08	.10	19	99
Lodging away from home	-2.699	.674	-.145	**.61**	.07	.27	.13	**.46**	.22	30	78
Household furnishings and operations	-2.099	.930	-.106	**.68**	.01	.18	.17	.04	-.04	11	87
Household operations	-2.161	.770	-.155	**.32**	.00	**.42**	**.27**	.13	-.07	12	83
Electricity	-1.366	.421	-.243	.05	.04	**.30**	.06	-.05	.04	35	96
Water and sewer and trash collection services	-2.131	.473	-.177	.05	.03	**.38**	.13	.03	.03	13	79
Gas, heating oil, and coal	-1.830	.667	-.313	.04	.08	**.30**	.08	.02	-.04	6	76

[a]General price index was used for these two consumption categories.
Notes: Larger loadings are marked in bold for easier interpretation of the factors.

FIGURE 4
Actual and Estimated Engel Curves for Some of the Consumption Categories
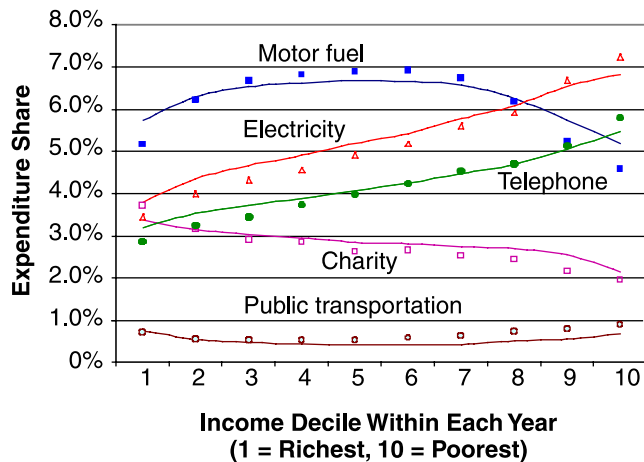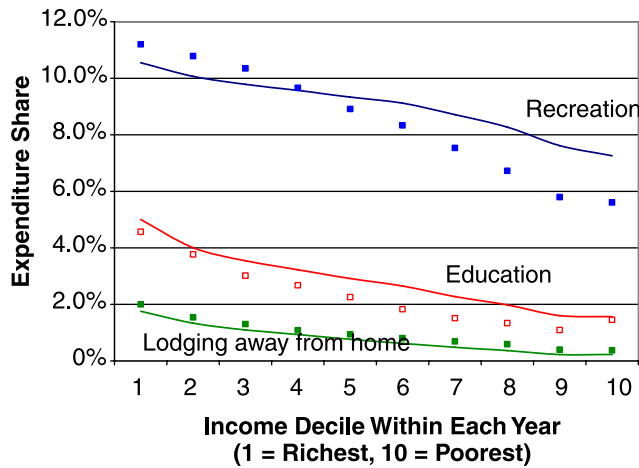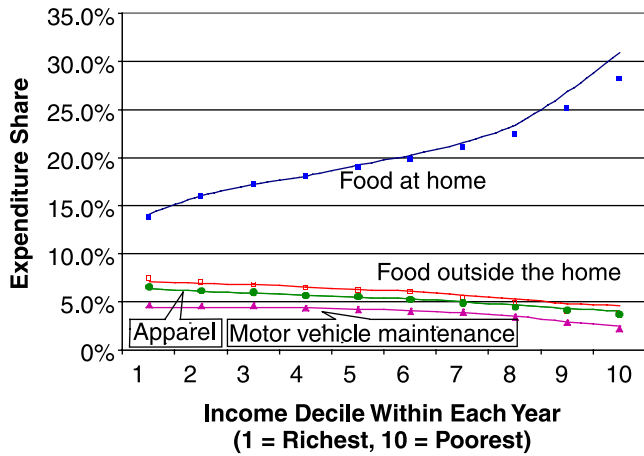
Notes: Engel curves implied by the proposed model are represented by solid lines; Engel curves based on observed data are represented by dots.

ever, these patterns of change are distinct for each life stage. As would be expected, the nonessential factor scores are higher in the richest quintiles. They also tend to be lower in the later life stages (S5) than in the earlier stages (Co/So, C1, and S1). To our surprise, these scores declined at different rates for different life stages in the 22 years covered by our data, suggesting that the marginal utility for nonessential consumption (compared with food at home) decreased over time.

Similar results for the health-related Factor 4 (see Figure 5, Panel B) suggest that average log-marginal utilities for the older life stages (C6 and S5) do not vary substantially with income, but they increase with income for the younger life stages (C1 and S1). Figure 5, Panel B, also shows a trend upward in the average log-marginal utilities for health care.

Consumption Priorities, Household Life Stage, and Income

In addition to investigating each factor separately, we can capture the variation of preferences for a particular consumption category across households by the "preference shares,"

$$\hat{\theta}_{ih} = \frac{e^{\hat{\alpha}_{ih}}}{\sum_{j=1}^{J} e^{\hat{\alpha}_{jh}}} = \frac{e^{\hat{\gamma}_i + \hat{\lambda}_i \hat{Z}_h + \frac{1}{2}\hat{\sigma}_i^2}}{\sum_{j=1}^{J} e^{\hat{\gamma}_j + \hat{\lambda}_j \hat{Z}_h + \frac{1}{2}\hat{\sigma}_j^2}},$$

which represent the expected expenditure shares for household h when there is no budget constraint. We report the average estimated preference shares in Table 3. For clarity, we report these averages only for the six most populated life stages and for the richest/poorest income quintiles. These results show that though there are substantial differences in preferences between the two extreme income quintiles, the differences across the six main life stages are relatively minor, particularly for the richest quintile. In general, the poorest 20% of our sample have higher preference shares than the richest 20% for food at home; tobacco and smoking products; health insurance; telephone services; electricity; water and sewer and trash collection services; and gas, heating oil, and coal, suggesting that these are the more essential consumption categories. For categories such as motor fuel, being considered essential depends on the life stage; the poorest 20% have higher preference shares than the richest 20% among some life stages (Co/So, C1, S1, and C6) but lower shares in other stages (S2 and S5), again demonstrating the importance of accounting for unobserved heterogeneity in tastes in modeling consumption budget allocation.

Policy Simulations

A major advantage of demand systems, such as the one we propose here, is that they are consistent with budget-constrained utility-maximizing behavior, leading to predicted expenditures that are always logically consistent (i.e., nonnegative and sum up to the budget). Such a multicategory structural approach makes our proposed budget allocation model valuable in anticipating consumers' reactions to environmental shocks, such as price hikes or shifts in dis-

FIGURE 5
Average Scores by Income Quintile and Year

A: Factor 1: Nonessential Consumption
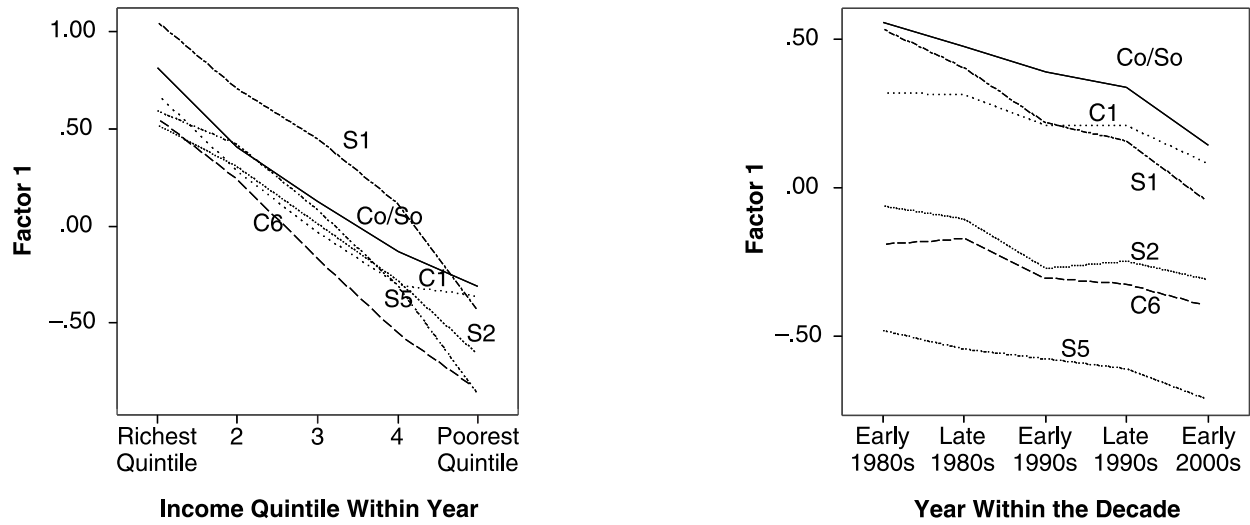
B: Factor 4: Health Care
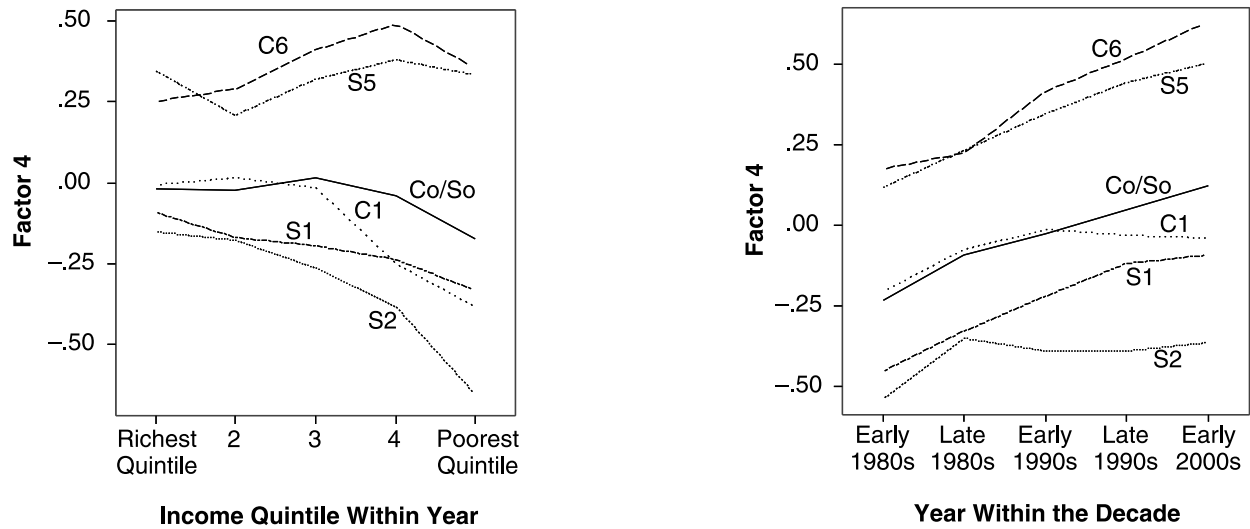
cretionary income, under the premise that the consumers' underlying preference structures are more stable and will remain unchanged, at least in the short run. Our factor-analytic approach has the added advantages of allowing for diversity in consumption priorities or tastes across households and capturing the correlation among these priorities, leading to a more flexible demand system. In contrast, any model that treats each category independently (rather than jointly) would be inapplicable because, by definition, category-specific models are not bound by the budget constraint, and as a result, the predicted expenditures would not sum up to the household consumption budget, which is a prerequisite for any meaningful policy simulations.

To illustrate, we conducted three policy simulations of current relevancy. First, we consider the not-so-hypothetical scenario in which prices for nonrenewable energy sources (i.e., motor fuel and gas, heating oil, and coal) increase by 50%. This policy simulation exemplifies how the model can be used to project shifts in consumer spending in response to projected price increases in some consumption categories. In the second policy simulation, we consider a scenario in which the federal government gives each household a $500 tax rebate specifically earmarked for stimulating current consumption. This second scenario illustrates how a policy maker can anticipate the effects on consumer spending of policies that increase or decrease discretionary income, such as programs to provide federal health insur-

TABLE 3
Average Estimated Preference Shares for Major Life Stages and Richest/Poorest Income Quintiles

Consumption Categories	Co/So Richest Quintile (%)	Co/So Poorest Quintile (%)	C1 Richest Quintile (%)	C1 Poorest Quintile (%)	S1 Richest Quintile (%)	S1 Poorest Quintile (%)	C6 Richest Quintile (%)	C6 Poorest Quintile (%)	S2 Richest Quintile (%)	S2 Poorest Quintile (%)	S5 Richest Quintile (%)	S5 Poorest Quintile (%)
Food at home	13.4	19.5	14.7	22.0	11.7	20.4	14.1	20.0	15.9	25.0	12.5	19.5
Food away from home	6.4	5.3	6.6	5.7	6.2	5.2	5.9	4.2	6.5	5.7	5.3	4.0
Tobacco and smoking products	2.3	3.8	2.3	4.0	2.3	4.2	2.3	3.6	2.7	4.4	2.2	3.4
Alcoholic beverages at home	1.2	1.2	1.0	1.2	1.4	1.3	1.0	.9	1.2	1.1	1.1	.8
Alcoholic beverages away from home	.8	.6	.6	.5	1.2	.6	.6	.3	.8	.4	.7	.3
Apparel	5.7	4.4	5.7	4.5	5.8	4.3	5.4	3.5	5.6	4.4	5.0	3.5
Apparel services other than laundry and dry cleaning	.8	.6	.7	.6	.9	.6	.7	.4	.7	.6	.7	.5
Jewelry and watches	.8	.4	.7	.4	.9	.4	.7	.3	.7	.3	.7	.3
Personal care services	1.4	1.4	1.4	1.3	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4
Miscellaneous personal services	1.7	1.2	1.6	1.2	1.8	1.2	1.6	1.1	1.5	1.0	1.7	1.1
Life insurance	2.9	3.0	3.0	2.9	2.7	2.9	3.0	3.2	2.8	2.9	3.1	3.3
Prescription drugs	.8	1.0	.8	.8	.6	.9	1.1	1.8	.7	.7	1.3	1.9
Nonprescription drugs and medical supplies	.7	.7	.7	.6	.6	.6	.7	.8	.6	.6	.8	.8
Professional medical care services	2.3	2.1	2.3	1.9	2.1	1.9	2.6	2.5	2.2	1.6	2.7	2.5
Hospital and related services	.5	.5	.5	.5	.4	.4	.6	.8	.4	.4	.6	.7
Health insurance	3.9	5.6	4.0	4.8	3.5	5.6	4.9	8.4	4.0	4.9	5.5	8.8
Motor vehicle maintenance and repair	4.2	3.3	4.0	3.2	4.3	3.1	3.7	2.8	3.9	2.6	3.8	2.6
Motor fuel	5.8	6.2	5.8	6.4	5.5	6.1	5.4	5.6	5.9	5.8	5.4	5.3
Motor vehicle insurance	4.4	4.8	4.4	4.7	4.2	4.7	4.3	4.9	4.4	4.2	4.4	4.7
Public transportation	.7	.6	.6	.5	1.0	.7	.8	.4	.7	.8	.7	.6
Airline fare	2.6	1.8	2.3	1.6	3.3	2.0	2.6	1.6	2.4	1.7	2.8	1.9
Recreation	9.3	6.9	8.9	6.6	10.0	6.8	8.7	5.7	8.7	6.2	8.7	5.8
Education	3.9	1.7	3.6	1.5	4.4	1.4	3.3	1.0	3.4	1.2	3.1	.8
Charity	3.1	2.2	3.0	1.9	3.1	2.0	3.4	2.5	2.6	1.7	3.8	2.9
Telephone services	2.9	3.4	3.0	3.4	2.8	3.4	2.9	3.5	3.0	3.5	2.9	3.5
Lodging away from home	2.1	1.3	1.9	1.2	2.5	1.3	2.0	1.1	1.8	1.1	2.1	1.2
Household furnishings and operations	4.4	2.9	4.3	2.8	4.6	2.7	4.2	2.4	4.0	2.5	4.2	2.4
Household operations	2.7	2.5	2.7	2.2	2.7	2.4	3.0	2.8	2.5	2.1	3.3	3.0
Electricity	3.9	5.1	4.1	5.2	3.5	5.1	4.1	5.7	4.1	5.3	4.2	5.7
Water and sewer and trash collection services	1.9	2.4	1.9	2.3	1.7	2.4	2.0	2.8	1.9	2.3	2.2	2.9
Gas, heating oil, and coal	2.8	3.7	2.9	3.6	2.6	3.8	3.0	4.1	2.9	3.8	3.1	4.2

ance and funding or rebates for child care, to certain segments of the population.

The third simulation is an attempt to quantify consumer welfare losses due to the dramatic increases in prices for prescription drugs in the past 22 years, which grew by 262% from 1982 to 2003 (an annual rate of 6.3%), compared with an inflation rate of 91% (or 3.1% per year) during the same period. Here, we consider a hypothetical scenario in which the costs of prescription drugs followed the general inflation rate observed in the past 22 years. In this counterfactual simulation, we consider the income effects of the dramatic price increases, attempting to estimate how households shifted their discretionary income away from other consumption categories to pay for the increasing costs of prescription drugs. In this scenario, we consider all households that spent discretionary income on prescription drugs in the past 22 years and compare their actual expenditures with those predicted by our budget allocation model, assuming that the extra discretionary income resulting from the lower prescription drug prices would be allocated to the other categories according to the estimated utilities for each household and consumption category. This comparison shows how these households reduced their expenditures in all the other categories to compensate for the dramatic increases in prescription drug prices in the past 22 years, thus providing some insights into welfare losses potentially caused by these price increases.

For each household in our sample, we simulate their budget reallocation decisions by solving the constrained utility maximization problem, using the estimated parameters ($\hat{\alpha}_{ih}$ and $\hat{\beta}_i$ for household h and category i) and observed versus simulated prices (p_i versus $p_i + \Delta p_i$) and budget (m_h versus $m_h + \Delta m_h$). The solution can be derived through a five-step procedure, which we detail in the Appendix.

Policy Simulation 1: reactions to shifts in energy costs. Table 4 reports the simulated effects of a dramatic increase in oil prices, showing the percentage changes in quantity consumed in response to a 50% increase in prices for motor fuel and gas, heating oil, and coal. As would be expected, the price increases affect the poorest quintile more dramatically than the richest quintile. The difference between the two income quintiles is the largest in the demand for motor fuel among households in the S5 stage, in which the poorest (richest) quintile reduces the quantity consumed by 43% (20%). In other words, the demand among the poorest households in the S5 stage is elastic, whereas those in the richest quintile have an inelastic demand. Table 4 also shows how households in different life stages would adjust their expenditures in other categories to compensate for the shift of discretionary income toward motor and home fuels. As would be expected, the more essential categories, such as food at home, telephone services, electricity, and water and sewer and trash collection, are the least affected, along with "addictions," such as tobacco and alcoholic beverages at home. The consumption categories most affected are the less essential ones, such as education (which includes books and other educational expenses), miscellaneous personal services, and charity. The category showing substantial

differences in response across life stages is jewelry and watches, for which the highest percentage drop in demand occurs in the richest quintile of S5 (31%), compared with a drop of only 3%–13% in the lowest quintiles, probably because they already spend the least in this category. A surprising and worrisome effect is the substantial drop in prescription drugs and other health care expenses, particularly among the older (C6 and S5) and poorer households.

The price effects we report in Table 4 show negative cross-elasticities, so that increases in fuel prices produce decreases in demand for all other categories, which happens because income effects dominate substitution effects, as discussed previously. In other words, because demand for motor and home fuel is inelastic, increases in fuel prices leave less discretionary income to be spent elsewhere, leading to a decrease in expenditures in all other categories. After we partial out these income effects (Equation 8), the substitution effects (Equation 9) are all indeed positive, which implies that there is no complementarity between categories. It might be argued that price increases in motor fuel would lead consumers to reduce their car use, which would lead to spending less on motor vehicle maintenance and repair and, thus, complementarity between these two categories (though it could also be argued that the impact on motor vehicle maintenance and repair might take longer to observe). However, after we account for income effects, these two categories become substitutes because the utility function is assumed to be additive separable, and accounting for complementarity with the CEX data is infeasible (because it is not a truly longitudinal panel).

Policy Simulation 2: reaction to a tax rebate. Table 5 reports the simulated effects of a hypothetical $500 tax rebate for each household, distributed by the federal government to boost demand and thus earmarked for consumption. Simulations such as this could help policy makers anticipate and quantify the differential impacts of such a shift in discretionary income on different population sectors across different consumption categories.[3] Across the six main life stages, food at home receives the highest share of the extra $500 in discretionary income, more so among the poorest quintile. In general, for essential categories, such as health insurance; telephone services; electricity; and gas, heating oil, and coal, there is a larger increase in spending among poorer households. After food retailing, recreation would be one of the industries that would benefit the most from the $500 tax rebate, particularly among the wealthier households. Similarly, we observe a larger increase in spending among richer households for other nonessential categories, such as airline fare, education, charity, and household furnishings.

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[3]Our analysis of preferences (Table 3) across life stages and income quintiles showed that preferences vary with income (cross-sectionally). In this policy simulation, we assumed that a relatively small increment of $500 in discretionary income would not cause substantial shifts in preferences.

TABLE 4

Expected Percentage Change in Demand in Response to a 50% Price Increase in Motor Fuel and Gas, Heating Oil, and Coal

Consumption Categories	Co/So Richest Quintile (%)	Co/So Poorest Quintile (%)	C1 Richest Quintile (%)	C1 Poorest Quintile (%)	S1 Richest Quintile (%)	S1 Poorest Quintile (%)	C6 Richest Quintile (%)	C6 Poorest Quintile (%)	S2 Richest Quintile (%)	S2 Poorest Quintile (%)	S5 Richest Quintile (%)	S5 Poorest Quintile (%)
Food at home	-2	0	-2	0	-4	1	-4	0	-1	-2	-6	1
Food away from home	-2	0	-1	2	-2	-1	-2	-2	-1	0	-2	-1
Tobacco and smoking products	-2	-2	-3	1	1	2	-2	3	0	2	1	2
Alcoholic beverages at home	-2	-1	-4	0	-4	-5	-4	-2	-2	0	-1	1
Alcoholic beverages away from home	-6	-9	-1	-4	-12	-15	-2	-6	-11	-10	1	-7
Apparel	-1	0	0	-1	-2	1	-2	-2	-2	-3	-1	0
Apparel services other than laundry and dry cleaning	-4	-6	-2	-8	-7	-8	-3	-3	-3	-14	-2	-5
Jewelry and watches	-17	-5	-12	-3	-19	-10	-16	-8	-13	-11	-31	-13
Personal care services	-2	-1	-1	-1	-1	-1	-2	0	-2	-1	-1	-1
Miscellaneous personal services	-17	-26	-14	-16	-21	-16	-18	-24	-20	-23	-13	-22
Life insurance	-3	3	-3	-5	0	2	-8	-1	-1	2	-18	2
Prescription drugs	-9	-18	-5	-12	-3	-22	-13	-26	-8	-12	-11	-27
Nonprescription drugs and medical supplies	-1	-1	-1	5	-2	-1	-1	-2	-4	1	0	-6
Professional medical care services	-6	-10	-4	-11	-6	-8	-10	-11	-5	-8	-15	-14
Hospital and related services	-14	-19	-5	-9	-11	-13	-9	-15	-1	-17	-12	-24
Health insurance	-2	1	-2	0	-4	1	-2	0	-4	4	-1	0
Motor vehicle maintenance and repair	-7	-5	-4	-5	-10	-5	-3	-6	-4	-4	-9	-5
Motor fuel	-18	-26	-18	-23	-25	-32	-17	-27	-17	-32	-20	-43
Motor vehicle insurance	-2	-1	-4	1	-4	4	-1	1	-4	2	2	2
Public transportation	-10	-12	-6	-8	-9	-17	-14	-15	-9	-15	-4	-12
Airline fare	-3	-3	-4	-2	-3	-2	-3	0	-6	3	1	-4
Recreation	-6	-3	-3	-4	-7	-4	-7	-3	-2	-1	-3	0
Education	-28	-30	-31	-29	-25	-30	-27	-25	-30	-29	-31	-23
Charity	-14	-13	-11	-6	-16	-10	-16	-17	-10	-3	-21	-14
Telephone services	-2	-5	-1	-7	-4	-10	-2	-4	-3	-12	-2	-6
Lodging away from home	-1	-3	-4	-3	-2	-1	-2	0	-3	1	-2	-1
Household furnishings and operations	-15	-10	-10	-15	-15	-12	-15	-12	-7	-11	-19	-12
Household operations	-4	-5	-7	-4	-2	-3	-4	-1	-10	-2	-3	-3
Electricity	-3	0	-3	0	-1	0	-3	0	-2	-1	-4	0
Water and sewer and trash collection services	-1	5	0	1	0	3	-2	1	-3	2	-8	3
Gas, heating oil, and coal	-25	-36	-24	-39	-32	-42	-25	-34	-22	-33	-20	-34

TABLE 5
Expected Allocation of an Incremental $500 in the Consumption Budget

Consumption Categories	Co/So Richest Quintile ($)	Co/So Poorest Quintile ($)	C1 Richest Quintile ($)	C1 Poorest Quintile ($)	S1 Richest Quintile ($)	S1 Poorest Quintile ($)	C6 Richest Quintile ($)	C6 Poorest Quintile ($)	S2 Richest Quintile ($)	S2 Poorest Quintile ($)	S5 Richest Quintile ($)	S5 Poorest Quintile ($)
Food at home	67	101	72	116	56	110	69	102	79	135	53	103
Food away from home	36	26	28	21	45	28	32	20	32	23	34	21
Tobacco and smoking products	9	19	8	21	9	24	9	16	11	22	6	10
Alcoholic beverages at home	6	6	5	6	7	7	6	4	6	5	5	2
Alcoholic beverages away from home	5	3	3	2	8	4	3	1	5	2	3	1
Apparel	29	22	29	26	29	21	27	16	31	30	22	18
Apparel services other than laundry and dry cleaning	4	4	3	4	6	5	3	2	3	5	3	3
Jewelry and watches	5	2	4	2	5	2	4	1	4	1	4	1
Personal care services	7	7	6	6	6	7	7	7	7	7	7	8
Miscellaneous personal services	8	10	7	6	11	5	8	7	12	6	19	5
Life insurance	17	12	16	12	13	10	18	16	13	10	16	15
Prescription drugs	3	6	3	3	2	5	6	14	3	3	5	13
Nonprescription drugs and medical supplies	3	2	3	3	3	2	4	3	3	1	3	4
Professional medical care services	3	2	3	2	3	2	4	5	3	1	3	3
Hospital and related services	11	11	10	11	11	9	15	14	11	8	27	13
Health insurance	18	24	18	20	16	25	24	46	19	18	23	51
Motor vehicle maintenance and repair	22	17	21	17	24	16	17	15	19	13	13	12
Motor fuel	28	34	26	36	22	31	24	29	30	28	20	23
Motor vehicle insurance	22	22	20	22	20	20	19	23	22	16	20	19
Public transportation	4	3	2	3	5	5	4	2	4	5	2	3
Airline fare	13	5	10	4	17	4	13	4	11	3	12	5
Recreation	49	33	49	32	54	33	45	26	44	30	38	26
Education	12	12	28	9	8	10	9	2	18	9	15	2
Charity	16	9	14	8	17	8	24	13	12	6	31	15
Telephone services	14	19	14	19	15	22	12	16	16	23	11	19
Lodging away from home	11	5	9	4	13	3	11	4	8	2	10	3
Household furnishings and operations	24	14	24	16	22	12	23	13	18	12	20	12
Household operations	15	12	22	12	16	11	18	13	16	10	25	18
Electricity	19	29	18	28	17	29	20	29	19	33	21	32
Water and sewer and trash collection services	9	10	10	12	9	10	10	14	10	11	10	15
Gas, heating oil, and coal	13	19	13	19	12	20	13	22	14	23	14	27
Total	500	500	500	500	500	500	500	500	500	500	500	500

Policy Simulation 3: welfare losses due to spiraling costs of prescription drugs. Table 6 reports the simulated percentage changes in household expenditure if the prices of prescription drugs had increased at the same rate as the CPI. If that had been the case, consumers could have reduced their prescription drug expenditure by an average of 37%, while maintaining the same level of treatment. The savings could then have been spent in other categories. Older, retired, and poor households (bottom income quintiles of the C6 and S5 life stages) would have benefited more, percentage-wise, than younger, working, and wealthier households. For example, the oldest and poorest households could have increased their spending on additional life insurance (3.5%), water and sewer and trash collection services (3.2%), motor vehicle insurance (3.1%), tobacco and smoking products (3%), and motor fuel (2.6%).

Conclusions and Directions for Further Research

The main purpose of this study was to develop a feasible demand system that would enable us to investigate budget allocation decisions by individual households across a comprehensive set of consumption categories. This development was motivated by the belief that marketers, research analysts, and policy makers need a better understanding of how consumers allocate their discretionary income to meet different consumption needs, and they must be able to anticipate how the resultant consumption patterns will change in response to changes in prices and budgets.

When studying consumption at this basic level, it is important to emulate the consumer's resource allocation problem. Our basic premise is that every household allocates its discretionary income among competing needs and wants so that when the consumption budget is exhausted, all expenditure categories offer the same marginal utility per dollar. Because we attempt to develop a reasonable as-if model to approximate the household's basic resource allocation problem, when we obtain estimates of each household's direct utility function, we can simulate the household's reaction to changes in prices or income and understand how these changes will affect different consumption categories representing different industries across different consumer segments.

By definition, a model is a simplified representation of observed phenomena, and therefore we made some simplifying assumptions in developing our proposed factor-analytic random coefficients budget allocation model. An important assumption to make the model parsimonious and feasible is that of a direct utility function that is additive separable across consumption categories. In other words, we assume that for an individual household, expenditure in one category does not increase or decrease the marginal utility derived from consuming another category. This does not imply that consumption is independent across categories, because all categories compete for the same budget. It also does not imply that preferences are independent across categories and households, because our factor structure accounts for possible correlations of preferences among the categories across households. However, the addi-

tive separable utility assumption implies that the resultant demand system will not be able to capture potential complementarities among consumption categories, which could not be discerned from correlations in preferences because of the one-shot nature of the CEX (Gentzkow 2007). We believe that this is a critical limitation only in detailed analyses that consider a limited number of potentially complementary product categories, such as the type of cross-category choice modeling commonly performed for consumer packaged goods in the marketing literature using longitudinal scanner panels, but this is not as critical in expenditure analyses performed across a comprehensive set of broad consumption categories. We leave for further research the methodological challenge of extending our model beyond additive separable utilities, while addressing the issues of high dimensionality (more than a couple dozen categories), binding nonnegative constraints (households have no spending in many categories), unobservable heterogeneity (unique household preferences that cannot be captured by demographics), and correlation in individual households' category preferences.

Substantively, there are several directions for extending our work. First, the proposed framework for modeling household consumption budget allocation could be adapted for forecasting industry sales and assessing market potential. As a model of primary demand, our structural approach has the advantage of simultaneously considering household spending across a full spectrum of expenditure categories. Because all types of consumer expenditures ultimately vie for the same household budget, primary demand in one industry can be better predicted in relation to consumer expenditures in other industries. For example, fast-food restaurant chains may be able to forecast sales trends better if they can understand how consumers' spending on food away from home is influenced by their spending on food at home, apparel, motor fuel, and so on. Moreover, our approach explicitly links household discretionary income, category price indexes, and demographics (through the household-specific taste parameters) to household category expenditures. This enables analysts in forecasting industry sales or assessing market potential to factor in projections about household income, inflation rates, and demographic trends through an integrated framework.

Second, although developed under a different context, we believe that there is a potential to adapt our consumption budget allocation model for shopping-basket data analysis. As such, researchers can study many more categories jointly without needing to focus on a few categories selected a priori or lump a large number of distinct items into an "other" category.

Finally, in a more realistic setting, consumers make not only cross-category allocations but also intertemporal allocations (e.g., consume more today versus save more for tomorrow). In our work, we ignored the intertemporal aspect by treating the consumption budget as exogenously determined. Further research could relax this assumption and model both cross-category and intertemporal allocations explicitly. However, this would require a true panel with longitudinal information about individual households' expenditure patterns.

TABLE 6
Expected Percentage Change in Expenditure if the Prices of Prescription Drugs Had Increased at the Same Rate as CPI

Consumption Categories	Co/So Richest Quintile (%)	Co/So Poorest Quintile (%)	C1 Richest Quintile (%)	C1 Poorest Quintile (%)	S1 Richest Quintile (%)	S1 Poorest Quintile (%)	C6 Richest Quintile (%)	C6 Poorest Quintile (%)	S2 Richest Quintile (%)	S2 Poorest Quintile (%)	S5 Richest Quintile (%)	S5 Poorest Quintile (%)
Food at home	.3	.8	.2	.3	.3	1.1	.6	1.6	.3	.4	.6	1.9
Food away from home	.3	.7	.2	.4	.2	.9	.5	1.5	.2	.4	.4	2.1
Tobacco and smoking products	.6	1.3	.6	.5	.4	1.6	1.2	3.1	.5	.7	2.0	3.0
Alcoholic beverages at home	.3	.7	.3	.5	.2	.8	.6	1.4	.3	.4	.5	2.2
Alcoholic beverages away from home	.3	.6	.2	.3	.2	.4	.4	1.1	.2	.3	.3	1.5
Apparel	.3	.6	.2	.3	.2	.9	.4	1.5	.3	.3	.4	1.8
Apparel services other than laundry and dry cleaning	.3	.8	.2	.3	.2	.8	.5	1.7	.2	.3	.4	2.2
Jewelry and watches	.3	.6	.2	.4	.2	.8	.4	1.1	.2	.4	.2	1.5
Personal care services	.3	.8	.2	.4	.3	1.3	.6	1.9	.3	.5	.6	2.0
Miscellaneous personal services	.2	1.0	.2	.4	.3	.9	.6	1.2	.2	.3	.4	1.6
Life insurance	.4	1.1	.3	.6	.4	2.4	.6	2.8	.4	1.0	.5	3.5
Prescription drugs	−36.9	−36.5	−38.7	−35.5	−37.6	−38.9	−37.3	−37.2	−35.1	−35.0	−37.7	−37.9
Nonprescription drugs and medical supplies	.6	1.6	.6	1.0	.5	1.8	1.1	2.8	.6	.9	1.0	2.4
Professional medical care services	.4	1.3	.3	.4	.3	1.5	.6	1.6	.3	.5	.4	1.6
Hospital and related services	.6	1.4	.5	.4	.4	2.0	1.1	2.1	.5	.4	.6	1.7
Health insurance	.5	1.5	.3	.6	.5	2.0	1.0	2.2	.4	1.0	.8	2.3
Motor vehicle maintenance and repair	.3	.7	.2	.3	.2	.8	.5	1.5	.2	.4	.4	1.9
Motor fuel	.3	.8	.2	.4	.3	1.0	.6	1.7	.2	.4	.5	2.6
Motor vehicle insurance	.4	1.3	.3	.5	.3	1.6	.8	2.7	.3	.8	.7	3.1
Public transportation	.3	.7	.2	.3	.2	.7	.3	.9	.3	.3	.6	1.3
Airline fare	.4	.8	.3	.5	.3	1.1	.5	1.2	.4	.4	.7	1.8
Recreation	.3	.6	.2	.3	.2	.8	.4	1.3	.2	.3	.4	1.9
Education	.2	.3	.2	.2	.2	.3	.3	1.1	.2	.2	.2	1.5
Charity	.3	.8	.2	.3	.2	1.3	.4	1.6	.3	.6	.4	1.5
Telephone services	.3	.7	.2	.3	.2	.8	.5	1.6	.2	.3	.5	1.6
Lodging away from home	.3	.8	.3	.3	.2	1.0	.5	1.2	.4	.5	.6	1.8
Household furnishings and operations	.2	.7	.2	.2	.2	.8	.4	1.2	.3	.3	.3	1.6
Household operations	.3	.8	.2	.4	.2	1.5	.5	1.8	.2	.6	.4	1.8
Electricity	.3	.9	.2	.4	.3	1.1	.6	1.8	.3	.4	.5	1.9
Water and sewer and trash collection services	.5	1.5	.4	.7	.5	2.0	.9	3.0	.5	.8	.9	3.2
Gas, heating oil, and coal	.4	1.2	.3	.5	.4	1.6	.8	2.3	.4	.5	.6	2.3

Appendix

Estimating the Proposed Budget Allocation Model

The problem faced by household h is to choose a consumption plan, $x_h(x_{1h}, \ldots, x_{Jh} \geq 0)$, that maximizes the utility function, $G(x_h) = \Sigma_{i=1}^{J} \alpha_{ih} \ln(x_{ih} - \beta_i)$, where $\alpha_{ih} > 0$, $(x_{ih} - \beta_i) > 0$, and J is the number of all available expenditure categories. Given the prices $(p_1, \ldots, p_J)$ of unit consumption in each category, the household's allocation plan must satisfy the budget constraint, $\Sigma_{i=1}^{J} p_i x_{ih} = \Sigma_{i=1}^{J} m_{ih} \leq m_h$, where m_{ih} represents household h's expenditures (in dollars) in category i. The household's optimization problem implies the following Kuhn–Tucker conditions:

$$(A1) \qquad \frac{\partial G(x_h)}{\partial x_{ih}} = \frac{\alpha_{ih}}{(x_{ih} - \beta_i)} \leq \xi p_i, \;\; \text{for } x_{ih} = 0,$$

and

$$(A2) \qquad \frac{\partial G(x_h)}{\partial x_{ih}} = \frac{\alpha_{ih}}{(x_{ih} - \beta_i)} = \xi p_i, \;\; \text{for } x_{ih} > 0,$$

where $\xi \geq 0$ is the Lagrange multiplier. Given our parameterization of $\alpha_{ih} = \exp(\gamma_i + \lambda_i Z_h + \varepsilon_{ih})$ in Equation 3, Equations A1 and A2 lead to, respectively,

$$(A3) \qquad \gamma_i + \lambda_i Z_h + \varepsilon_{ih} - \ln(x_{ih} - \beta_i) - \ln(p_i) \leq \ln(\xi), \;\text{for } x_{ih} = 0,$$

and

$$(A4) \qquad \gamma_i + \lambda_i Z_h + \varepsilon_{ih} - \ln(x_{ih} - \beta_i) - \ln(p_i) = \ln(\xi), \;\text{for } x_{ih} > 0,$$

where Z_h is a p-dimensional vector of i.i.d. standard normal factor scores for household h and ε_{ih} is a random disturbance normally distributed with mean zero and standard deviation σ_i. The parameters to be estimated, given the observed consumptions $x_{ih} (= m_{ih}/p_i)$ and prices p_i, are γ_i, λ_i, $\beta_i [= \min(x_i) - \exp(\eta_i)]$, and σ_i, which we collect in the set θ. Every household in our sample has positive consumption for the first category, food at home (i.e., $x_{1h} > 0$ for $\forall$h), and for identification purposes, we set γ_1, λ_1, and σ_1 to zero, which means that Equations A3 and A4 can be simplified as, respectively,

$$(A5) \qquad \varepsilon_{ih} \leq (\tau_{ih} - \tau_{1h}) - (\gamma_i + \lambda_i Z_h), \;\text{for } x_{ih} = 0,$$

and

$$(A6) \qquad \varepsilon_{ih} = (\tau_{ih} - \tau_{1h}) - (\gamma_i + \lambda_i Z_h), \;\text{for } x_{ih} > 0,$$

where $\tau_{ih} = \ln(x_{ih} - \beta_i) + \ln(p_i) = \ln(p_i x_{ih} - p_i \beta_i)$.

Let $\varepsilon_{ih}^*(x_h) \equiv \ln(p_i x_{ih} - p_i \beta_i) - \ln(p_1 x_{1h} - p_1 \beta_1) - (\gamma_i + \lambda_i Z_h)$. Based on Equations A5 and A6, for a given set of model parameters θ and factor scores Z_h, the likelihood contribution of household h, $L_h(Z_h)$, or $L(\theta | Z_h, x_h, p) = L(\gamma_{2\ldots l_h}, l_{h+1\ldots J}, \lambda_{2\ldots l_h}, l_{h+1\ldots J}, \sigma_{2\ldots l_h}, l_{h+1\ldots J}, \beta_{1\ldots J} | Z_h, X_{1h\ldots Jh}, p_{1\ldots J})$, is

$$(A7) \qquad L_h(Z_h) = L(\theta | Z_h, x_h, p) = f_{x_h}(x_h | \theta, Z_h, p)$$

$$= \prod_{i=2}^{l_h} f_{\varepsilon_{ih}}\left[\varepsilon_{ih}^*(x_h)\right] \left|\frac{\partial \varepsilon_{2\ldots l_h, h}^*}{\partial x_{2\ldots l_h, h}}\right| \times \prod_{i=l_h+1}^{J} \int_{-\infty}^{\varepsilon_{ih}^*(x_h)} f_{\varepsilon_{ih}}(\varepsilon_{ih}),$$

where $x_{ih} > 0$ for $\forall i \in (2, \ldots, l_h)$, $x_{ih} = 0$ for $\forall i \in (l_h + 1, \ldots, J)$, $f_{\varepsilon_{ih}}(\cdot)$ is the density function of ε_{ih}, and

$$\left|\frac{\partial \varepsilon_{2\ldots l_h, h}^*}{\partial x_{2\ldots l_h, h}}\right|$$

is the determinant of the $(l_h - 1) \times (l_h - 1)$ Jacobian of the transformation from $x_{2\ldots l_h, h}$ to $\varepsilon_{2\ldots l_h, h}^*$, which is a continuous one-to-one mapping, because $p_1 x_{1h} = (m_h - \Sigma_{i=2}^{l_h} p_i x_{ih})$.

Given that $\varepsilon_{ih} \sim N(0, \sigma_i)$, we can write Equation A7 as follows:

$$(A8) \quad L_h(Z_h) = \prod_{i=2}^{l_h} \frac{1}{\sigma_i} \phi\left(\frac{\varepsilon_{ih}^*}{\sigma_i}\right) \times \left|\frac{\partial \varepsilon_{2\ldots l_h, h}^*}{\partial x_{2\ldots l_h, h}}\right| \times \prod_{i=l_h+1}^{J} \Phi\left(\frac{\varepsilon_{ih}^*}{\sigma_i}\right),$$

where φ and Φ are, respectively, the probability and cumulative density function of the standard normal distribution and

$$\left|\frac{\partial \varepsilon_{2\ldots l_h, h}^*}{\partial x_{2\ldots l_h, h}}\right| \propto \frac{\sum_{i=1}^{l_h} (p_i x_{ih} - p_i \beta_i)}{\prod_{i=1}^{l_h} (p_i x_{ih} - p_i \beta_i)}.$$

(For a detailed derivation, see Kao, Lee, and Pitt 2001, pp. 210–11.)

Because Z_h is unobservable, the likelihood contribution of household h, L_h, needs to be integrated over $Z_h \sim N(0, I_p)$:

$$(A9) \qquad L_h \propto \int_{-\infty}^{\infty} \left[\prod_{i=2}^{l_h} \frac{1}{\sigma_i} \phi\left(\frac{\varepsilon_{ih}^*}{\sigma_i}\right) \times \frac{\sum_{i=1}^{l_h} (p_i x_{ih} - p_i \beta_i)}{\prod_{i=1}^{l_h} (p_i x_{ih} - p_i \beta_i)} \right.$$

$$\left. \times \prod_{i=l_h+1}^{J} \Phi\left(\frac{\varepsilon_{ih}^*}{\sigma_i}\right)\right] \times \phi(Z_h) dZ_h.$$

We evaluate Equation A9 with simulation (Gourieroux and Monfort 2002) by replacing the multidimensional integration with a summation over K Halton-sequence draws (Train 2003) of Z_h from the standardized normal distribution, which leads to the following:

$$(A10) \; L_h = \frac{1}{K} \sum_{k=1}^{K} \left[\prod_{i=2}^{l_h} \frac{1}{\sigma_i} \phi\left(\frac{\varepsilon_{ihk}^*}{\sigma_i}\right) \times \frac{\sum_{i=1}^{l_h} (p_i x_{ih} - p_i \beta_i)}{\prod_{i=1}^{l_h} (p_i x_{ih} - p_i \beta_i)}\right.$$

$$\left. \times \prod_{i=l_h+1}^{J} \Phi\left(\frac{\varepsilon_{ihk}^*}{\sigma_i}\right)\right].$$

A gradient search on the log-likelihood function $L = \Sigma_{h=1}^{N} \ln(L_h)$, leads to the parameter estimates of our demand model (i.e., γ_i, λ_i, β_i, and σ_i). After the parameters of the demand model have been estimated, factor scores can be estimated for each household using the same likelihood function in Equation A10, except that the parameters of the model are known and the estimation is done on the factor scores Z_h. Rather than using a gradient search to estimate the factor scores for each consumer, we use a sampling-importance-resampling procedure (Smith and Gelfand 1992).

Procedure for Policy Simulation

For the households in our sample, we simulate their budget reallocation decisions by solving the constrained utility maximization problem, using the estimated parameters ($\hat{\alpha}_{ih}$ and $\hat{\beta}_i$ for household h and category i), observed versus simulated prices (p_i versus $p_i + \Delta p_i$), and budget (m_h versus $m_h + \Delta m_h$). We derive the solution in five steps.

Step 1. Calculate all categories' marginal utilities at zero expenditure (i.e., $m_{ih} = 0$):

$$\partial \hat{U}_{ih}(0) = \frac{\partial \hat{U}_{ih}(m_{ih} = 0)}{\partial m_{ih}}$$

$$= \exp(\hat{\alpha}_{ih}) \frac{1}{\left(\dfrac{m_{ih}}{p_i} - \hat{\beta}_i\right)} \frac{1}{p_i} = \frac{\exp(\hat{\alpha}_{ih})}{m_{ih} - p_i\hat{\beta}_i} = \frac{\exp(\hat{\alpha}_{ih})}{-p_i\hat{\beta}_i}.$$

Step 2. Rank the categories from the lowest to the highest by $\partial \hat{U}_{ih}(0)$, and denote the lowest with 1′ and the highest with J′ (i.e., 1′, 2′, …, k′, …, n′, …, J′). Note that the marginal utility for any category is always decreasing as category expenditure increases, and if category k′ is consumed, category n′ must be consumed as well, as long as n′ > k′. As such, we know that only J category consumption regimes are possible: (1′, 2′, …, k′, …, n′, …, J′), (2′, …, k′, …, n′, …, J′), …, (k′, …, n′, …, J′), …, (n′, …, J′), or (J′).

Step 3. Assume that Category 1′ has \$.01 consumption. It is fairly straightforward to calculate the expenditures for other categories (i):

$$\hat{m}_{i'h} = \exp(\hat{\alpha}_{i'h} - \hat{\alpha}_{1'h})(.01 - p_1\hat{\beta}_{1'}) + p_i\hat{\beta}_{i'},$$

which means that

$$\hat{m}_h = \sum_{i'=1'}^{J'} \exp(\hat{\alpha}_{i'h} - \hat{\alpha}_{1'h})(.01 - p_1\hat{\beta}_{1'}) + \sum_{i'=1'}^{J'} p_i\hat{\beta}_{i'}.$$

Step 4. Compare $\hat{m}_h$ with m_h. If $\hat{m}_h$ is smaller than m_h, the household has a large enough budget to spend money on the least essential category. If $\hat{m}_h$ is greater than m_h, Category 1′ will not receive any spending. Then, we go back to Step 1 and repeat Steps 2 and 3 until the first category—for example, k′—such that the sum of $\hat{m}_{i'h}$ is smaller than m_h. We then know the categories consumed will be (k′, k + 1′,…, J′ – 1, J′).

Step 5. After we know the categories consumed, solving for category expenditures is straightforward. Because $m_h = \Sigma_{i'=k'}^{J'} \exp(\hat{\alpha}_{i'h} - \hat{\alpha}_{k'h})(\hat{m}_{k'h} - p_k\hat{\beta}_{k'}) + \Sigma_{i'=k'}^{J'} p_i\hat{\beta}_{i'}$, we have the following:

$$\hat{m}_{k'h} = \frac{\left(m_h - \sum_{i'=k'}^{J'} p_i\hat{\beta}_{i'}\right)}{\sum_{i'=k'}^{J'} \exp(\hat{\alpha}_{i'h} - \hat{\alpha}_{k'h})} + p_{k'}\hat{\beta}_{k'}.$$

In short, if i′ < k′, then $\hat{m}_{i'h} = 0$; otherwise, $\hat{m}_{i'h} = \exp(\hat{\alpha}_{i'h} - \hat{\alpha}_{k'h})(\hat{m}_{k'h} - p_k\hat{\beta}_{k'}) + p_i\hat{\beta}_{i'}$.

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