Electrically induced surface instability of a conductive thin film on a dielectric substrate

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The stability of a conductive thin film on a dielectric substrate subjected to a transverse electric field and a residual strain is analyzed. Under a uniform electric field, an equilibrium state exists with a constant thickness reduction of the substrate. The equilibrium state, however, can be unstable, depending on the intensity of the electric field, the stiffness, and Poisson's ratio of the substrate, and on the residual strain in the film. Based on a linear perturbation analysis, the critical condition is determined, beyond which wrinkling of the film is expected.
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A dielectric substrate coated with thin-film electrodes is a common structure in microelectromechanical systems, such as electrostrictive polymer actuators.[1,2] Similar structures are also used in large-area flexible electronics.[3] Understanding the deformation behavior of such structures subjected to electrical and mechanical loading is important for practical applications. This study concerns stability of a conductive thin film on a dielectric substrate (Fig. 1). It is well known that a compressive residual strain in the film can induce wrinkling on a relatively compliant substrate.[4–7] How would this instability be influenced by an electric field? Electrically induced instability of thin liquid films has been studied both theoretically and experimentally,[8–12] based on which a patterning technique has been developed as a promising method for nanoscale fabrication.[13–15] Can an electric field induce similar instability in a solid film bonded to a solid substrate? The answer is "Yes" according to this study. Such instability may be undesirable for some applications, but may also be carefully controlled to generate ordered wrinkle patterns for other applications.

Figure 1 sketches the structure considered in this study, consisting of a conductive thin film of thickness h_f bonded to a dielectric substrate of thickness h_s, which in turn lies on a rigid base. At a reference state, the film is flat and subjected to an equibiaxial residual strain ε_f (e.g., due to differential thermal expansion). Applying an electrical voltage V between the film and the base, the electrostatic force induces a uniform pressure on the surface of the substrate. Let δ be the reduction of the substrate thickness. Assuming linear elastic deformation in the substrate, the strain energy per unit area of the surface is

$$U_s = \frac{(1-\nu_s)E_s h_s}{2(1+\nu_s)(1-2\nu_s)}\left(\frac{\delta}{h_s}\right)^2, \tag{1}$$

where E_s and ν_s are Young's modulus and Poisson's ratio of the substrate, respectively. The electrostatic energy per unit area is

$$\Phi = -\frac{\kappa V^2}{2(h_s-\delta)}, \tag{2}$$

where κ is the dielectric permittivity of the substrate. As the substrate thickness decreases, the electrostatic energy decreases and the strain energy increases. The total free energy per unit area of the system is

$$G = \Phi + U_s + U_f = \frac{E_s h_s}{2}\left[\frac{(1-\nu_s)}{(1+\nu_s)(1-2\nu_s)}\bar{\delta}^2 - \frac{\xi}{1-\bar{\delta}}\right] + U_f, \tag{3}$$

where U_f is the strain energy in the film, which is independent of $\bar{\delta}$, and

$$\bar{\delta} = \frac{\delta}{h_s}, \quad \xi = \frac{\kappa V^2}{E_s h_s^2}. \tag{4}$$

Minimization of the free energy leads to

$$2\bar{\delta}(1-\bar{\delta})^2 = \xi\frac{(1+\nu_s)(1-2\nu_s)}{1-\nu_s}, \tag{5}$$

which solves for the thickness reduction at the equilibrium state.

The dimensionless parameter ξ measures the relative intensity of the electric field. When $\xi=0$, the free energy minimizes at $\bar{\delta}=0$; that is, no thickness reduction at zero voltage. Figure 2 plots the relationship between ξ and $\bar{\delta}$ for different Poisson ratios. It is noted that, when $\nu_s=0.5$, the dielectric is incompressible and thus $\bar{\delta}\equiv 0$. For $\nu_s \neq 0.5$, Fig. 2 shows that the equilibrium thickness reduction decreases as Poisson ration increases. In addition, a pull-in behavior is observed in Fig. 2 at $\bar{\delta}=1/3$, similar to the pull-in instability in elastically suspended parallel-plate electrostatic actuators.[16] However, this is unphysical in the present system because the

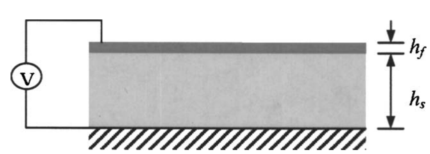

FIG. 1. Schematic of a conductive thin film on a dielectric substrate.

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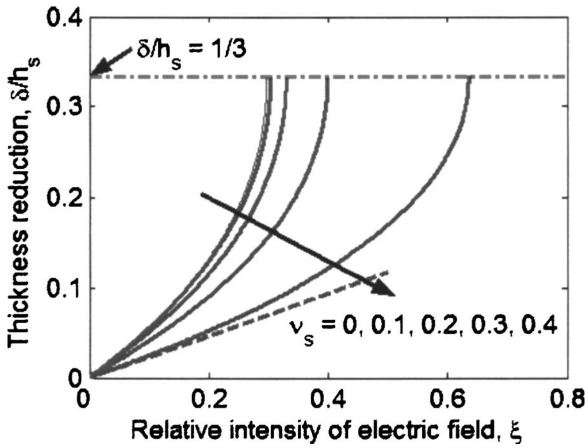

FIG. 2. Thickness reduction of dielectric substrates under electrostatic pressure. The dash-dotted line on top indicates the pull-in limit.

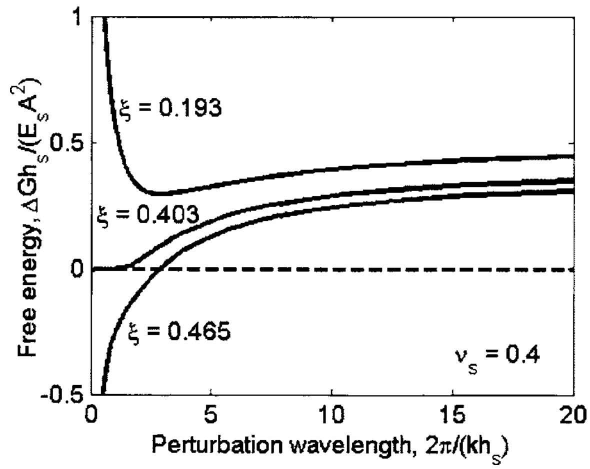

FIG. 3. Change of free energy due to a sinusoidal deflection of the flat film, under an electric field of different intensity.

dielectric substrate would eventually prevent complete pull-in. The erroneous prediction of pull-in in this analysis is due to the assumption of linear elastic deformation in the substrate, which should be restricted to small deformation (say, $\bar{\delta}<0.1$). Nonlinear material behavior must be considered for larger deformation. Focusing on small deformation, a linear approximation of Eq. (5) leads to

$$\bar{\delta}=\frac{(1+\nu_s)(1-2\nu_s)}{2(1-\nu_s)}\xi, \tag{6}$$

as shown by the dashed line in Fig. 2 for $\nu_s=0.4$. It is found that the linear approximation typically agrees closely with the exact solution up to $\bar{\delta}=0.05$. Using typical values of $E_s=1$ MPa, $\nu_s=0.4$, $\kappa=2\times10^{-11}$ F/m, and $h_s=1$ μm, the thickness reduction is about 47 nm under a voltage of 100 V.

The analysis just presented determines one equilibrium state assuming that the film remains flat. However, this equilibrium state may be unstable. Perturb the flat film with a sinusoidal deflection

$$w=A\sin kx, \tag{7}$$

where k and A are the wave number and amplitude, respectively. Assume that the film remains bonded to the substrate. The perturbation induces nonuniform deformation in the film and the substrate. In addition, the electrical field also becomes nonuniform. To the leading order of the perturbation amplitude, the changes to the energy terms are

$$\Delta\Phi=-f(kh_s)\frac{\kappa V^2}{4(h_s-\delta)^3}A^2, \tag{8}$$

$$\Delta U_s=g(kh_s,\nu_s)\frac{E_s}{h_s}A^2, \tag{9}$$

$$\Delta U_f=\frac{E_f h_f}{48(1-\nu_f^2)}[12(1+\nu_f)\varepsilon_f+k^2h_f^2]k^2A^2, \tag{10}$$

where

$$f(kh_s)=\frac{kh_s\cosh(kh_s)}{\sinh(kh_s)}. \tag{11}$$

$$g(kh_s,\nu_s)=\frac{(3-4\nu_s)\cosh^2(kh_s)+(kh_s)^2+(1-2\nu_s)^2}{4(1-\nu_s^2)[(3-4\nu_s)\sinh(2kh_s)-2kh_s]}kh_s. \tag{12}$$

It is noted that the electrostatic energy decreases as the perturbation amplitude increases, thus driving growth of the instability and bifurcation of the equilibrium state. This can be understood as a result of redistributed charge on the surface of the dielectric due to wrinkling. In the limit of a long-wave wrinkle ($kh_s\to0$), Eq. (8) reduces to that by a parallel-plate approximation.[16] The residual strain in the film, if compressive ($\varepsilon_f<0$), provides additional driving force for bifurcation, similar to the classical Euler buckling. On the other hand, the elastic stiffness of the film and the substrate resists growth of the instability, and a tensile residual strain in the film ($\varepsilon_f>0$) further stabilizes the flat film. The competition between the driving forces and the resistances determines the stability of the film. The effect of surface energy is similar to that of a residual strain in the film,[17] thus not included in the present analysis.

First consider a special case when the film is very thin and compliant so that the strain energy in the film is negligible. The change of total free energy is then

$$\Delta G=\Delta U_s+\Delta\Phi =E_sh_s\left[g(kh_s,\nu_s)-f(kh_s)\frac{\xi}{4(1-\bar{\delta})^3}\right]\frac{A^2}{h_s^2}. \tag{13}$$

Figure 3 plots the free energy change as a function of the perturbation wavelength, for various values of ξ. When ξ is less than a critical value, the energy increases for all wavelengths; thus the flat surface is stable. When ξ is greater, the energy decreases for short-wave perturbations; the flat surface becomes unstable and wrinkles. It is noted that the stability is controlled by the short-wave limit of the perturbation (i.e., $kh_s\to\infty$), from which the critical thickness reduction is determined, namely,

$$\bar{\delta}_c=\frac{1-2\nu_s}{5-10\nu_s+4\nu_s^2}. \tag{14}$$

The critical value ξ_c can then be found by substituting Eq. (14) into Eq. (5), as plotted in Fig. 4 versus Poisson's ratio. For incompressible substrates, the thickness reduction is al-

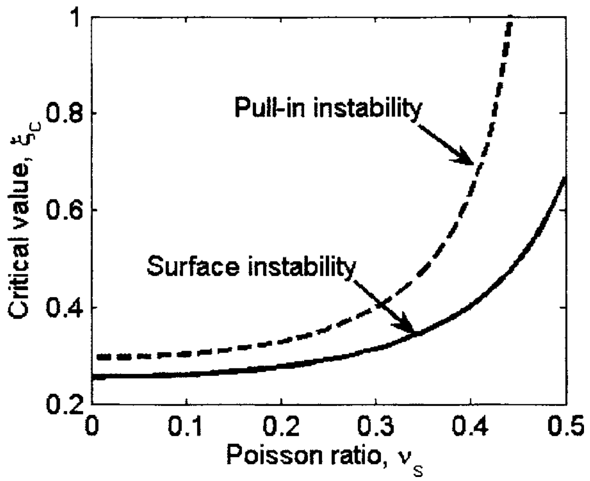

FIG. 4. Critical conditions for surface instability and pull-in instability, loaded with a transverse electric field.

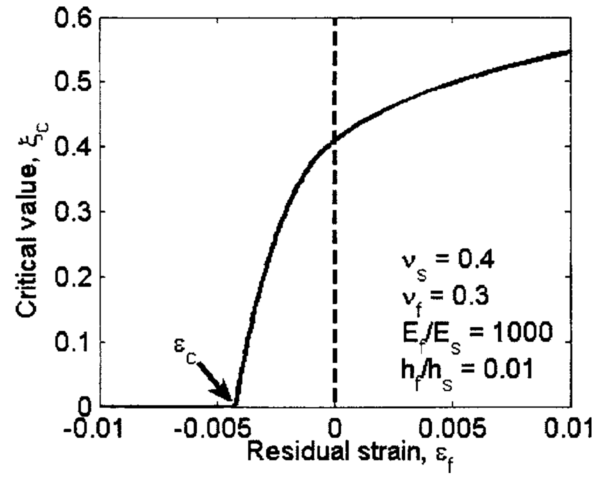

FIG. 5. Critical condition for surface instability loaded with an electric field and a residual strain.

ways zero and $\xi_c = 2/3$. The dashed line in Fig. 4 plots the pull-in condition predicted by the linear analysis. Clearly, the surface instability always occurs before pull-in. Using the same typical values as before, the critical voltage for the onset of surface instability is 142 V.

Next consider the effect of the film. The strain energy in the film [Eq. (10)] consists of two parts: one associated with bending and the other with stretching; the former scales with k^4 and the latter scales with k^2. Consequently, both strongly affect the free energy at the short-wave limit. The bending stiffness of the film always stabilizes short-wave perturbations. The stretching energy can be stabilizing or destabilizing, depending on the sign of the residual strain. Figure 5 plots the critical value ξ_c as a function of the residual strain ε_f. A critical compressive strain exists, below which the flat film is unstable without applying any voltage.[5–7] Assuming that the substrate is much thicker than the film, the critical strain at zero voltage is

$$\varepsilon_c = -\frac{1}{4(1+\nu_f)}\left(\frac{3\bar{E}_s}{\bar{E}_f}\right)^{2/3}, \tag{15}$$

where $\bar{E} = E/(1-\nu^2)$ is the plane-strain modulus. For $\varepsilon_f > \varepsilon_c$, the flat film is stable if $\xi < \xi_c$ but becomes unstable when $\xi > \xi_c$. Therefore, an electric field can destabilize an otherwise stable film. At zero strain, the critical value ξ_c is slightly higher than that predicted in Fig. 4, where the film stiffness is completely ignored. The critical value increases if the strain is tensile and decreases if the strain is compressive.

In summary, it is shown that a transverse electric field can induce surface instability of a conductive thin film on a dielectric substrate. The critical stability condition is determined based on a linear perturbation analysis. It is expected that a nonlinear analysis will determine the equilibrium wavelength and amplitude beyond the onset of the instability.

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