Nonlinear Dynamics of Wrinkle Growth and Pattern Formation in Stressed Elastic Thin Films

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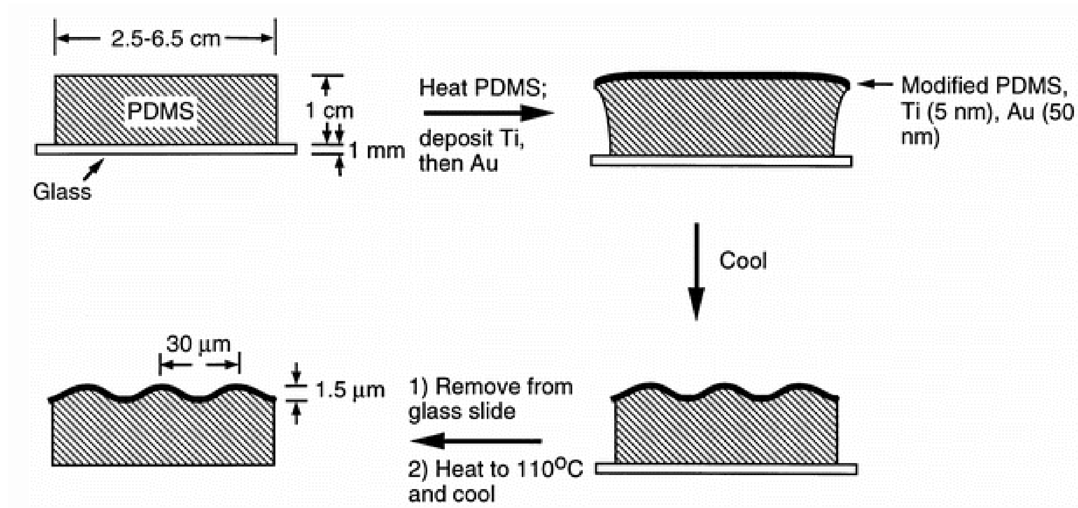
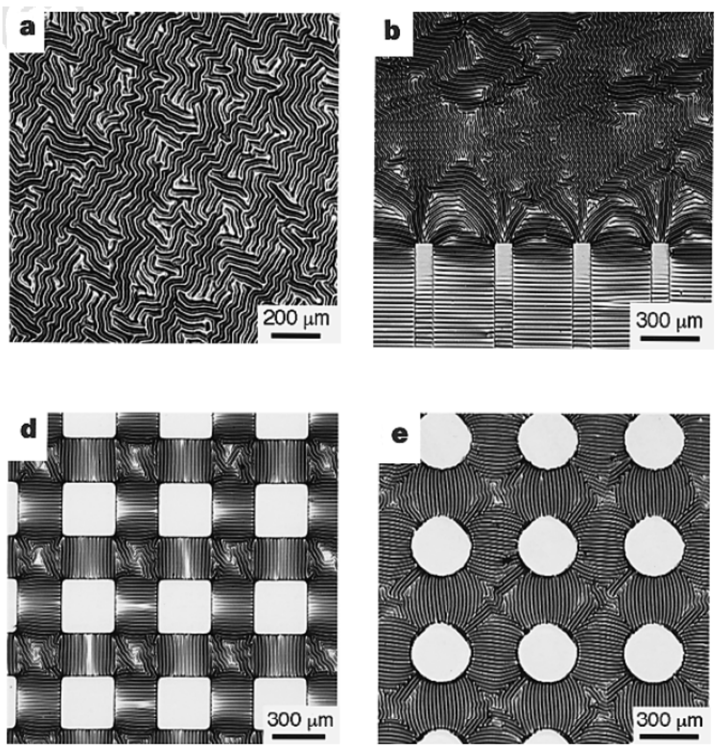
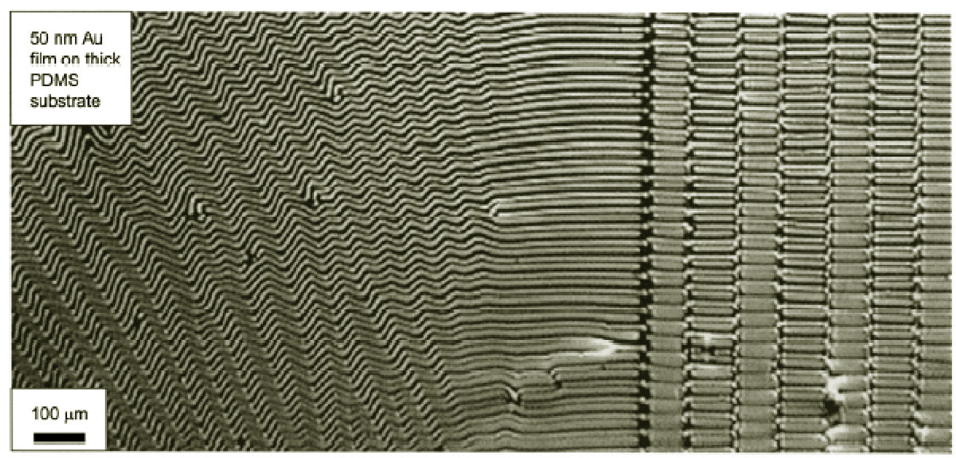
Au film on PDMS: *thermoelastic* wrinkling

- Bowden et al., 1998 & 1999
- Huck et al., 1999
- Efimenko et al., 2005

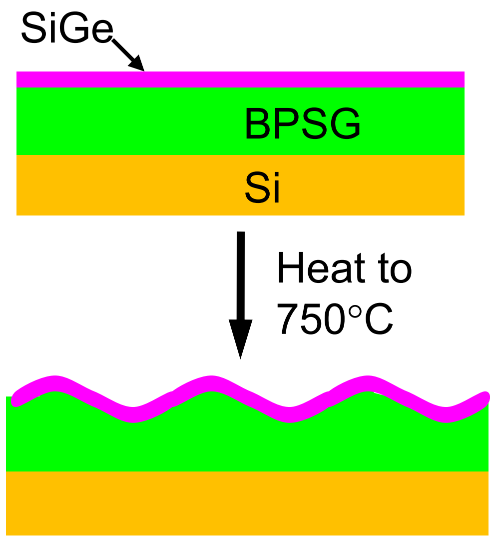


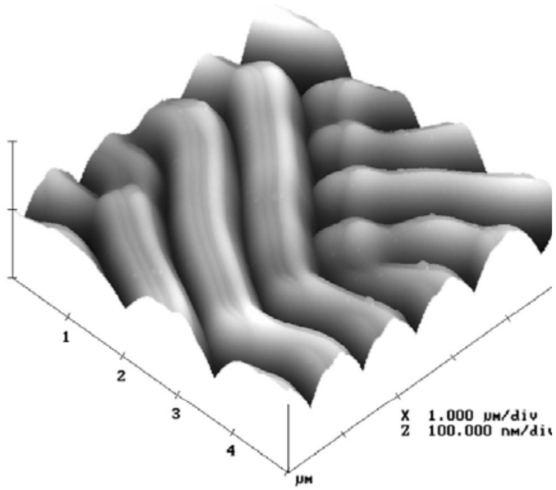


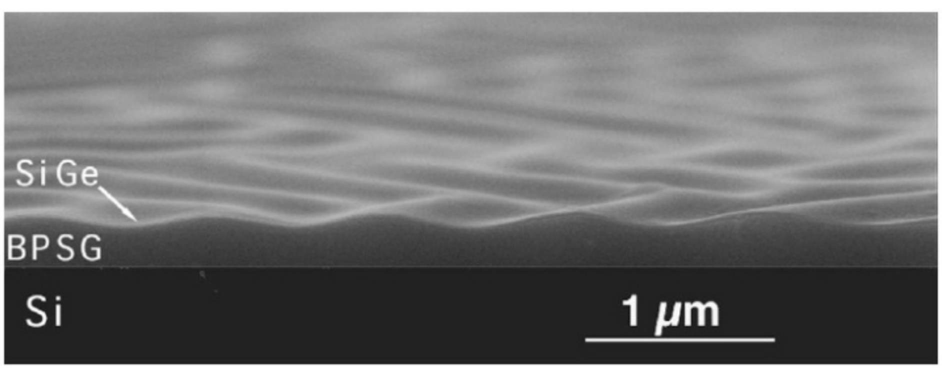


SiGe film on BPSG: *viscous* wrinkling

- Hobart et al., 2000
- Yin et al., 2002 & 2003
- Peterson et al., 2006
- Yu et al., 2005 & 2006







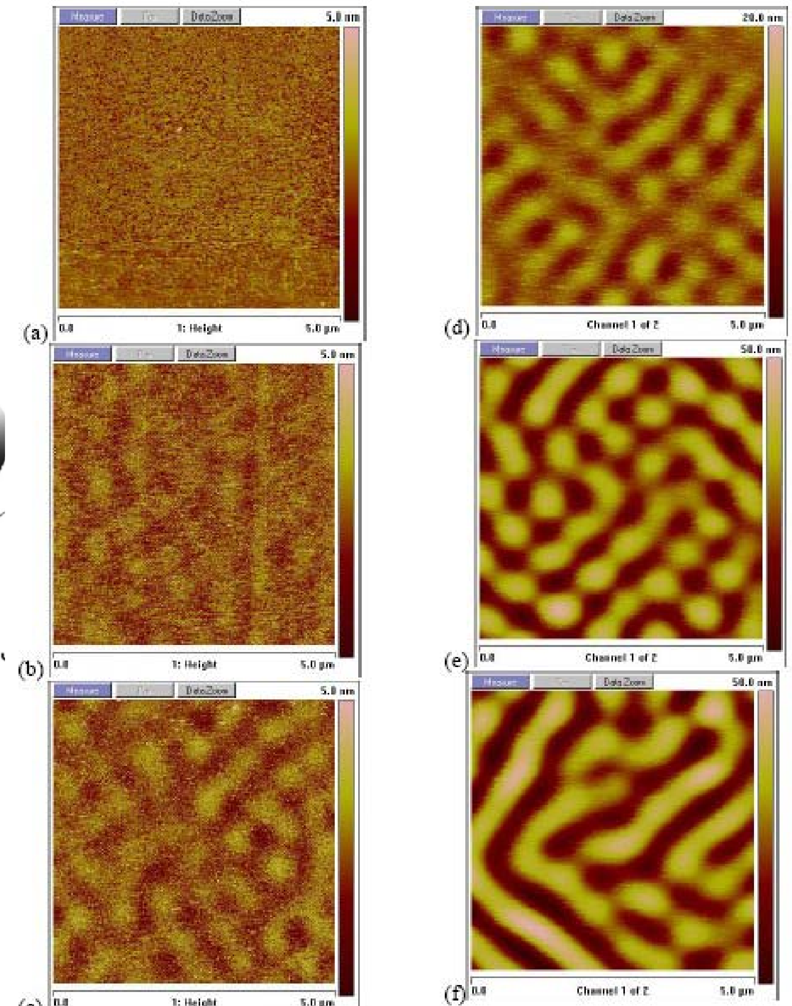


Figure 6.13: AFM images of buckled SiGe on 235-nm BPSG at the center of 150-μm x 150-μm islands after 750°C anneals of various lengths: (a) no anneal, (b) 25 min, (c) 50 min, (d) 105 min, (e) 253 min, and (f) 600 min. The AFM scan size is 5 μm x 5 μm, and the z-axis scale is indicated for each image. The scan edge is along a <110> crystal direction; buckles are along <100>, as before.

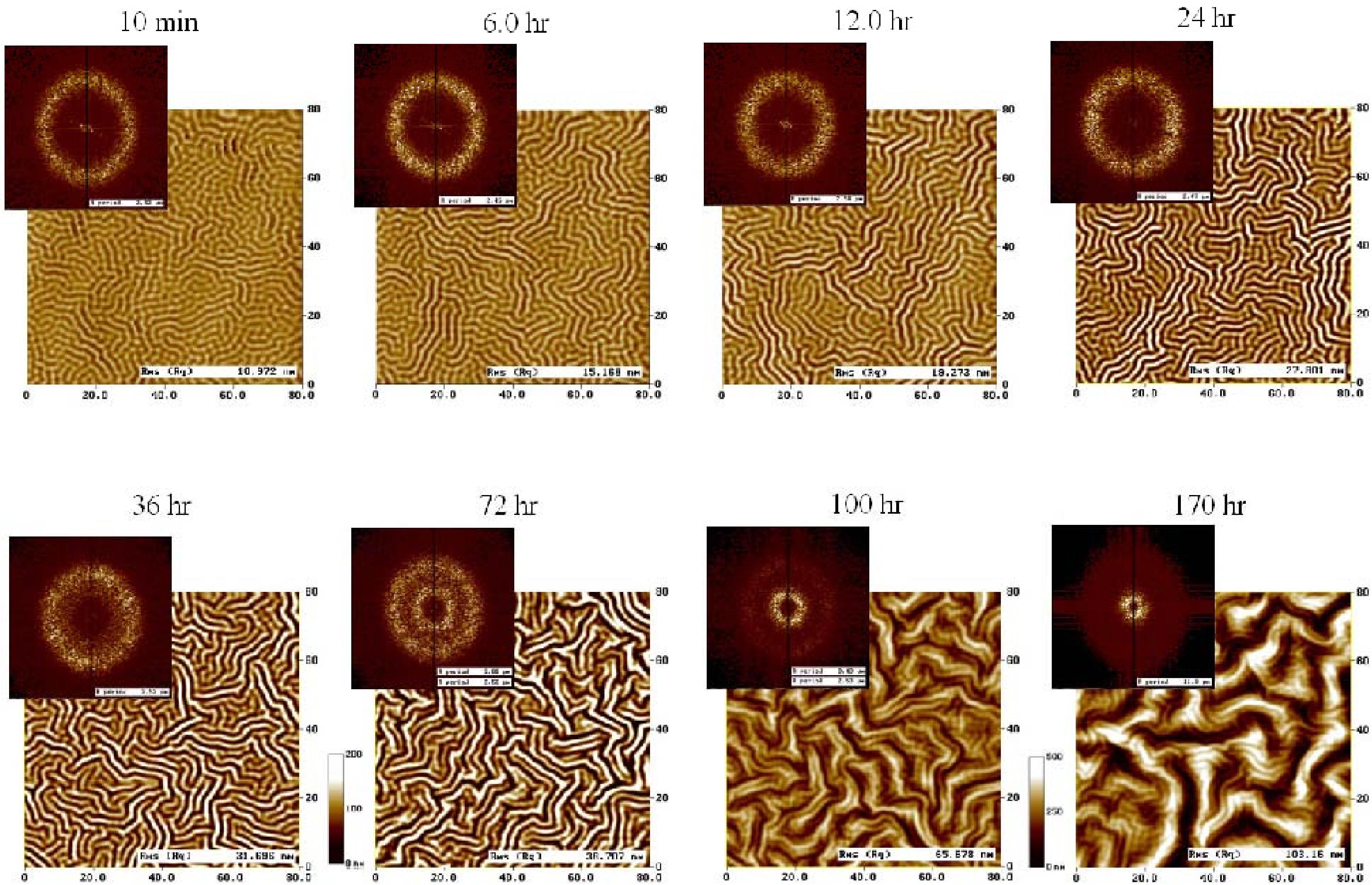
Elastic wrinkling **vs** ***Viscous wrinkling***

film/substrate materials	**elastic/elastic**	**elastic/viscous**
Onset of wrinkling	Critical stress or strain depends on the elastic modulus and thickness of substrate.	Unstable with any compressive stress in a thin film ($L/h >> 1$).
Wrinkle amplitude	Equilibrium amplitude increases with the stress/strain.	Amplitude grows over time.
Wrinkle wavelength	Equilibrium wavelength depends on the elastic modulus and thickness of substrate.	The fastest growing wavelength dominates the initial growth, then coarsening over time.
Energetics	Spontaneous energy minimization.	Energy dissipation by viscous flow.

Al film on PS: *viscoelastic wrinkling*

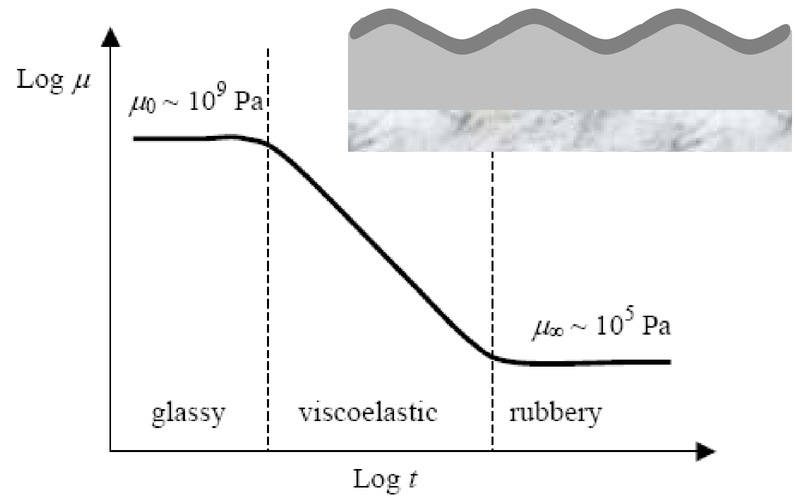
(Yoo and *Lee*, *2003-2005*)

PS 400 nm **(MW 1,340,000)** *Al 40nm, 145 ℃*



A model for viscoelastic wrinkling

Viscoelastic relaxation modulus

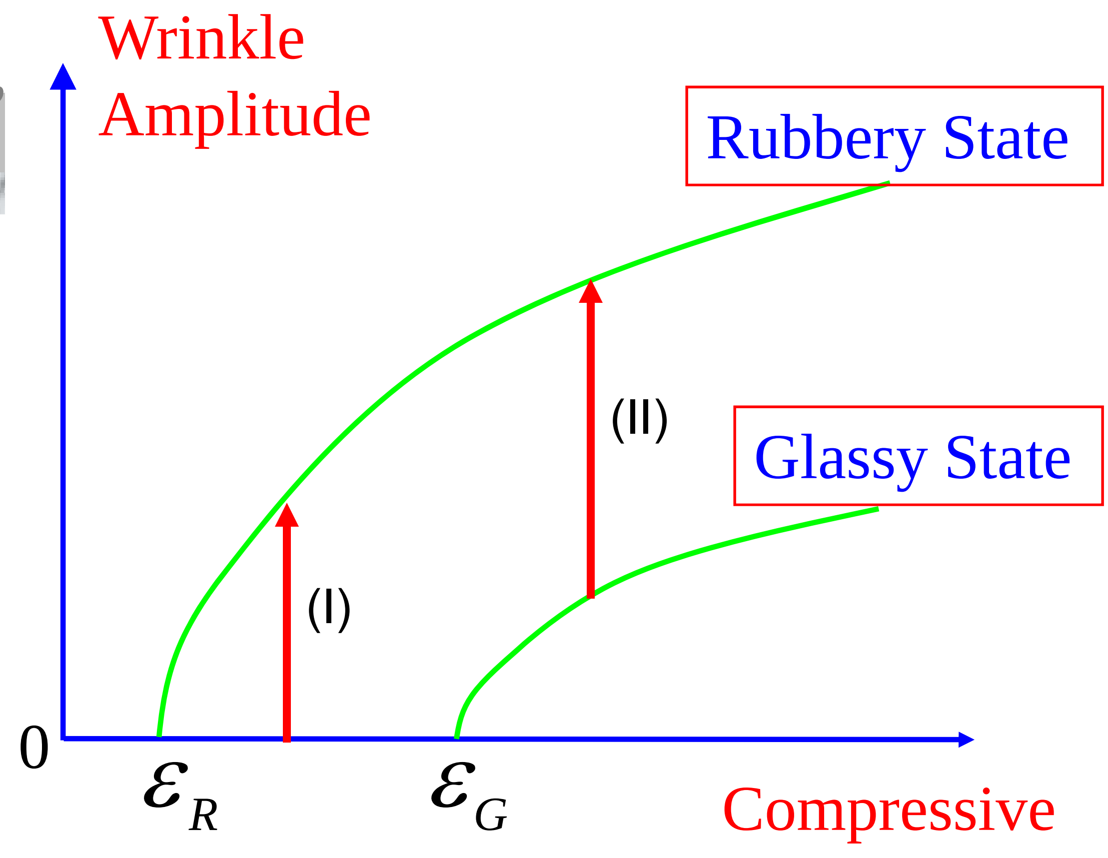


Equilibrium states at the elastic limits (thick substrate):

$$\varepsilon_c = -\frac{1}{4}\left(\frac{3\bar{E}_s}{\bar{E}_f}\right)^{2/3}$$

$$A_{eq} = h_f\left(\frac{\varepsilon}{\varepsilon_c} - 1\right)^{1/2}$$

$$\lambda_{eq} = 2\pi h_f\left(\frac{\bar{E}_f}{3\bar{E}_s}\right)^{1/3}$$



Evolution of wrinkles:

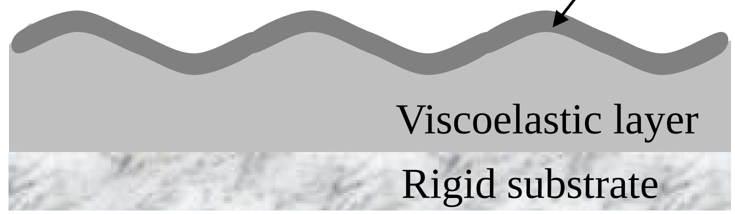
(I) Viscous to Rubbery

(II) Glassy to Rubbery

Huang, JMPS 2005.

Scaling of evolution equation

$$\frac{\partial w}{\partial t} = -K\nabla^2\nabla^2 w + F\nabla\cdot(\boldsymbol{\sigma}\cdot\nabla w) - Rw$$



$$K = \frac{(1-2\nu_s)\mu_f h_f^3 h_s}{12(1-\nu_s)(1-\nu_f)\eta_s} \qquad F = \frac{1-2\nu_s}{2(1-\nu_s)}\frac{h_s h_f}{\eta_s} \qquad R = \frac{\mu_R}{\eta_s}$$

Fastest growing mode:

$$\lambda_m = 2\sqrt{2}\pi L_1 \qquad A = A_0 \exp\left(\frac{t}{4\tau_1}\right)$$

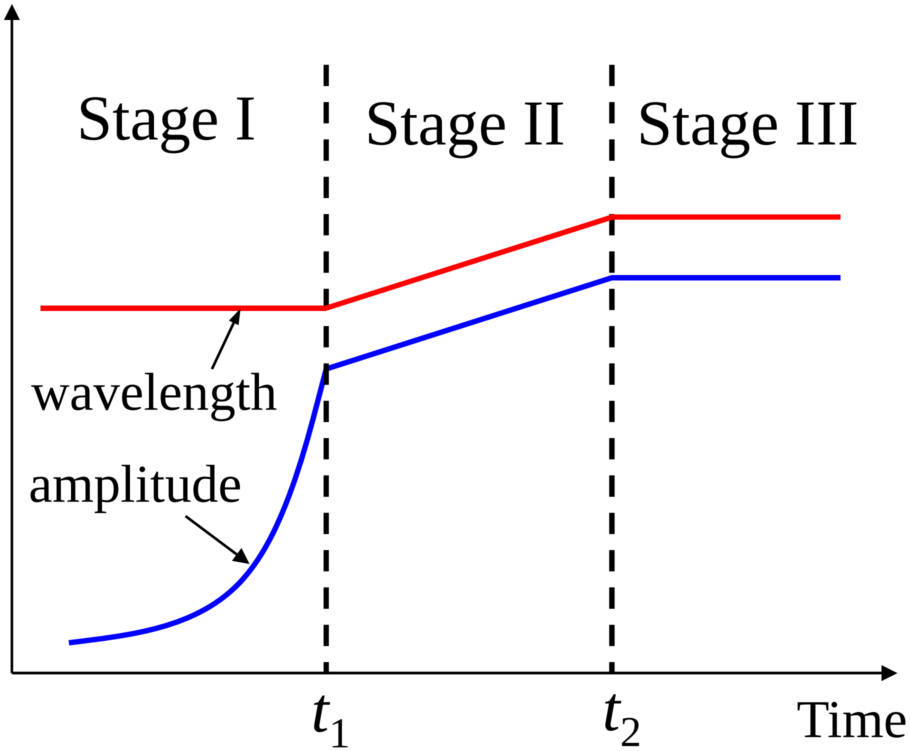
Length and time scales: $L_1 = \sqrt{-\dfrac{K}{F\sigma_0}}$ $\qquad \tau_1 = \dfrac{K}{(F\sigma_0)^2}$

critical stress: $\sigma_c = -\sqrt{\dfrac{4KR}{F^2}}$

Equilibrium state:
$(\sigma_0 < \sigma_c)$

$$\lambda_{eq} = 2\pi\left(\frac{K}{R}\right)^{1/4} > \lambda_m \qquad A_{eq} = h_f\sqrt{\frac{2}{3}\left(\frac{\sigma_0}{\sigma_c} - 1\right)}$$

Three-Stage Viscoleastic Wrinkle Evolution



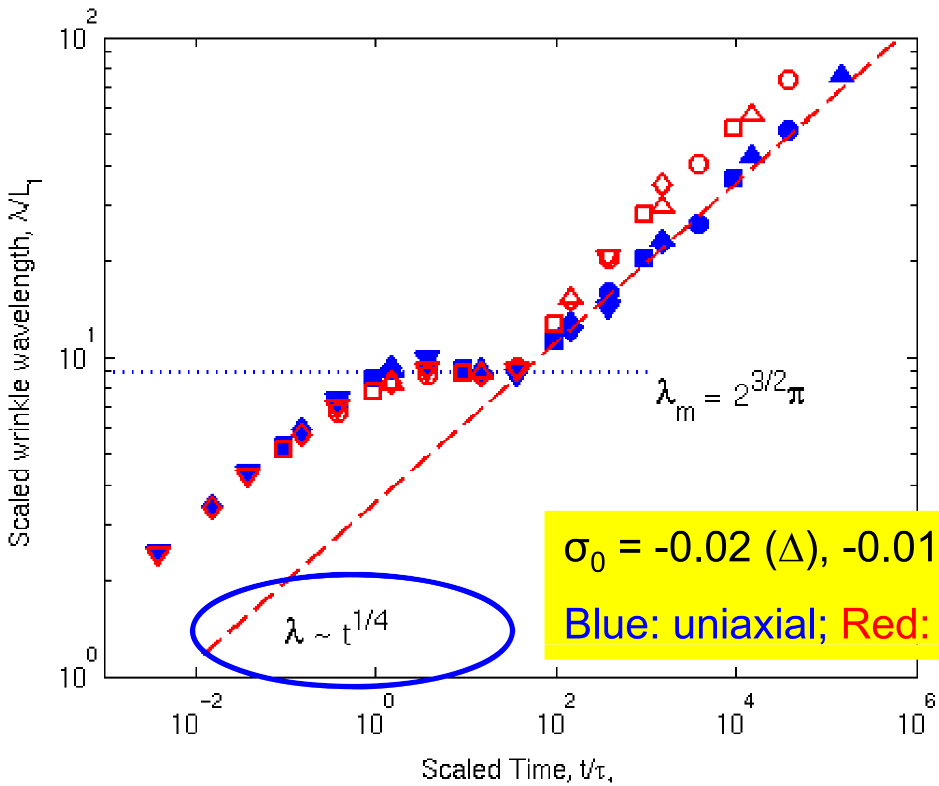
I: Exponential growth, dominated by the fastest growing mode

II: Coarsening ($\lambda_m \rightarrow \lambda_{eq}$, A↑)

III: Nearly equilibrium (with local ordering)

Im and Huang, JAM 2005 and PRE 2006.

Power-law Coarsening



Length and time scaling:

$$L_1 = \sqrt{-\frac{K}{F\sigma_0}}$$

$$\tau_1 = \frac{K}{(F\sigma_0)^2}$$

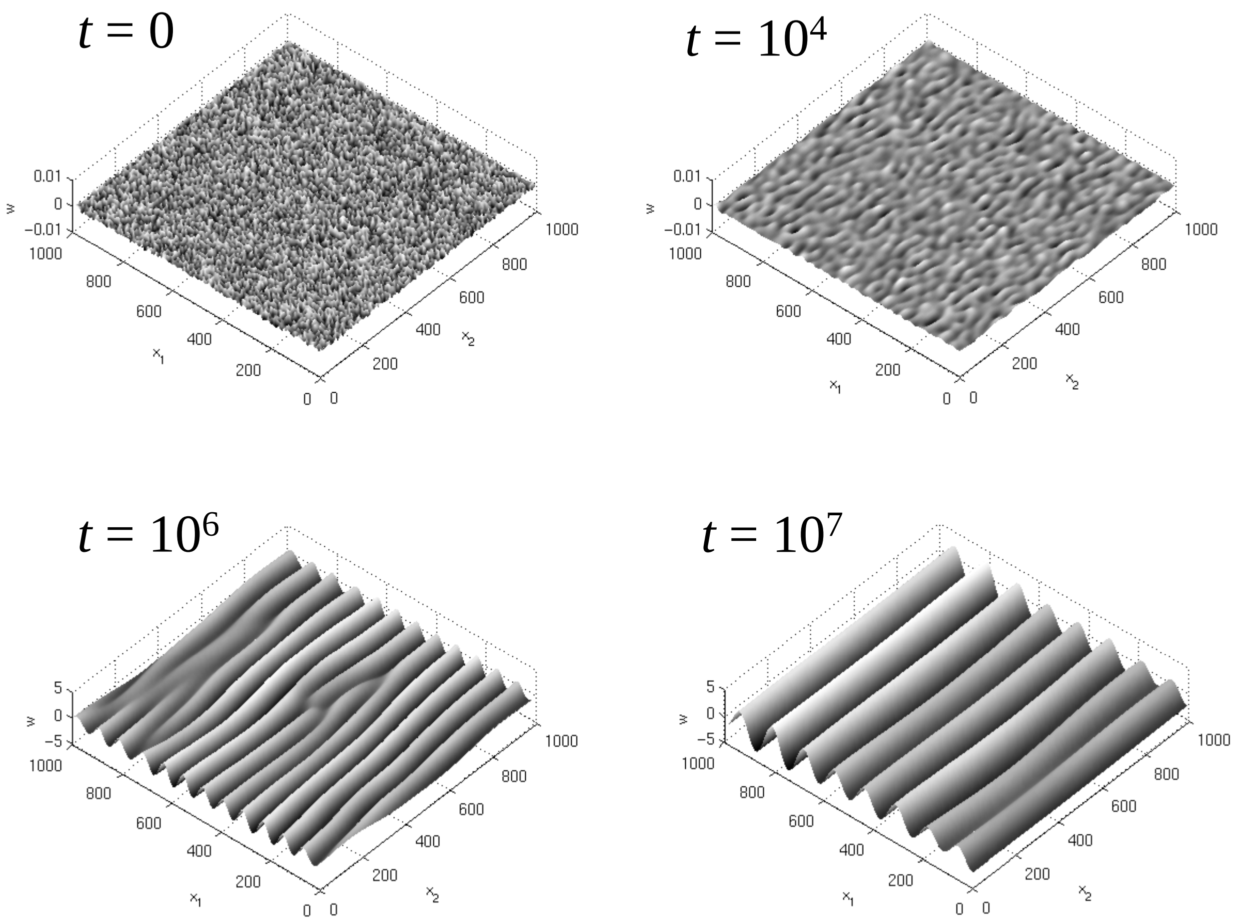
σ_0 = -0.02 (Δ), -0.01 (o), -0.005 (□), -0.002 (◊), -0.001 (∇)

Blue: uniaxial; Red: equi-biaxial

- On a compressible, thin viscoelastic substrate: $\lambda \sim t^{1/4}$;
- On an incompressible, thin viscoelastic substrate, $\lambda \sim t^{1/6}$;
- On a thick VE substrate, $\lambda \sim t^{1/3}$.
- The substrate elasticity slows down coarsening as it approaches the equilibrium state.

Huang and Im, PRE 2006.

Numerical simulation: uniaxial wrinkling

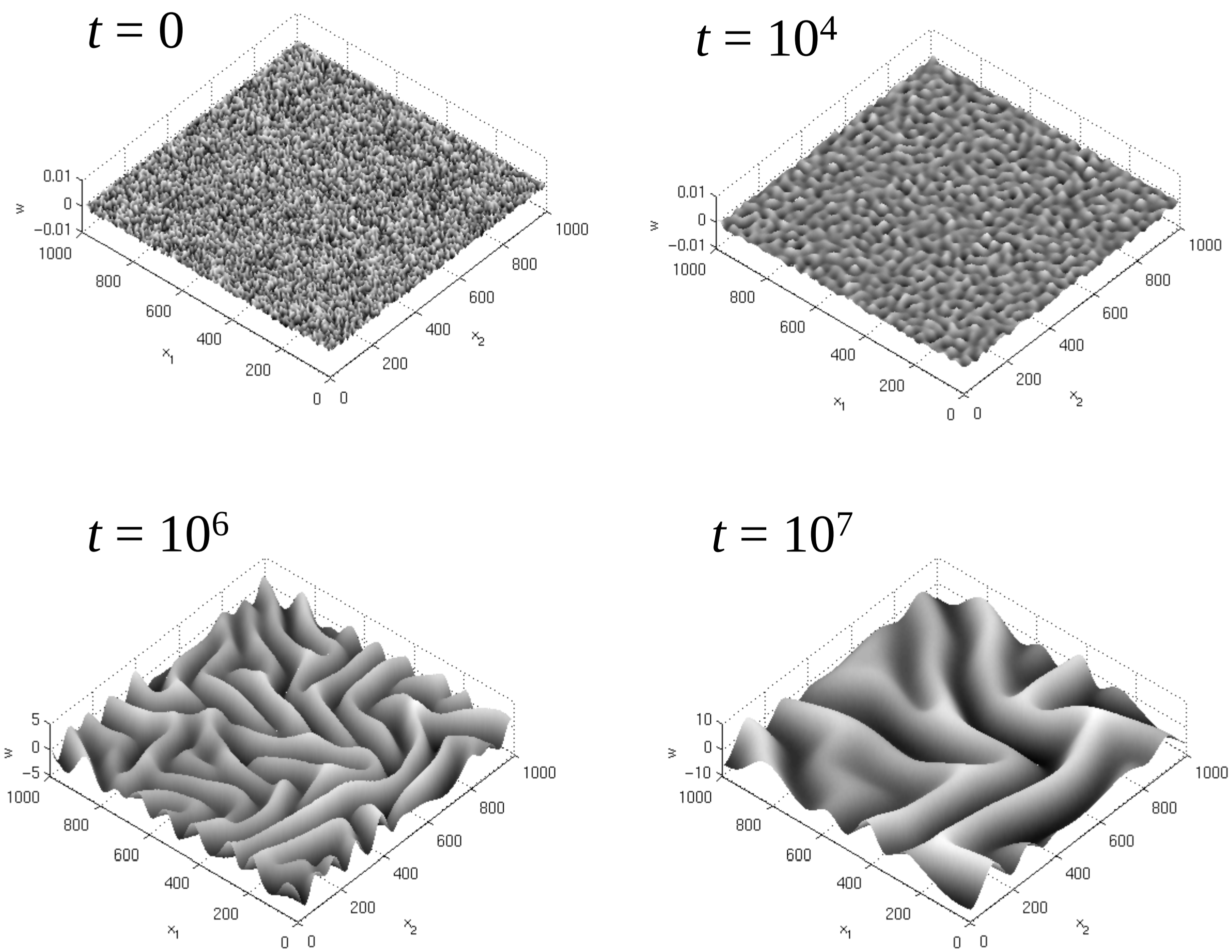


$$\sigma_{11}^{(0)} = -0.01, \sigma_{22}^{(0)} = 0$$

$$\frac{\mu_R}{\mu_f} = 0$$

Huang and Im, PRE 2006.

Numerical simulation: equi-biaxial wrinkling



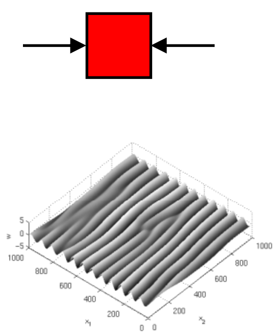
$$\sigma_{11}^{(0)} = \sigma_{22}^{(0)} = -0.01$$

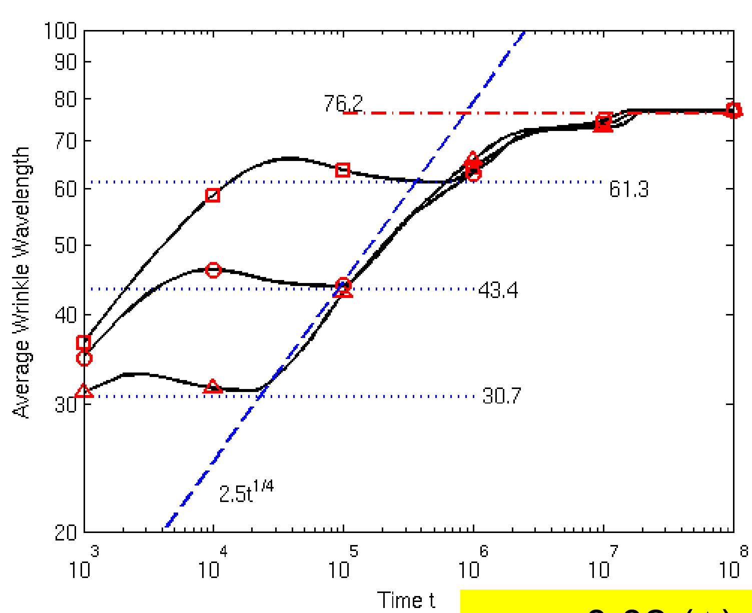
$$\frac{\mu_R}{\mu_f} = 0$$

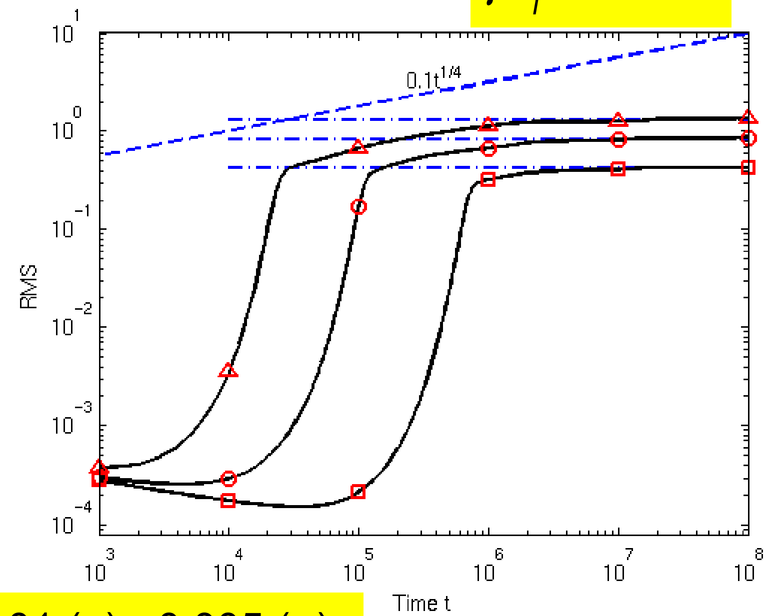
Huang and Im, PRE 2006.

Effect of Substrate Elasticity

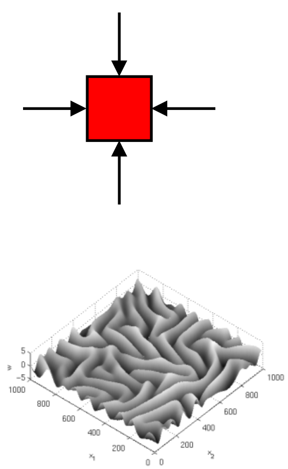
$$\frac{\mu_R}{\mu_f} = 10^{-5}$$

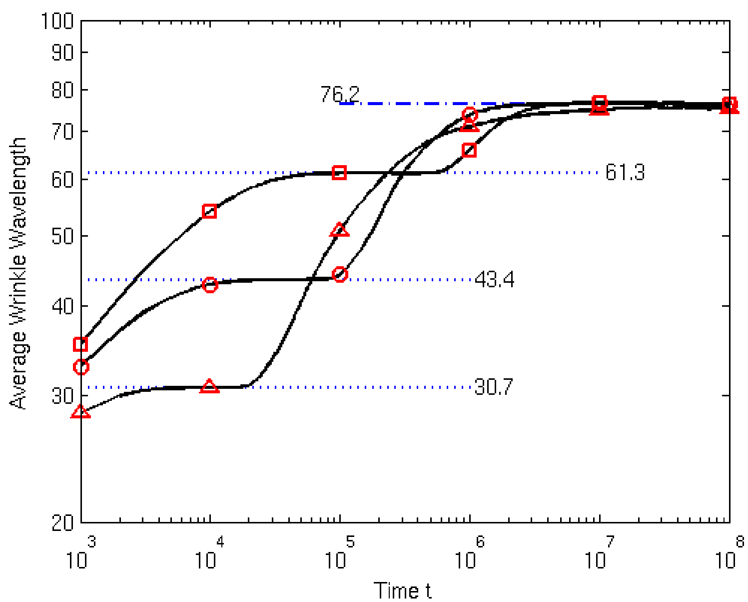


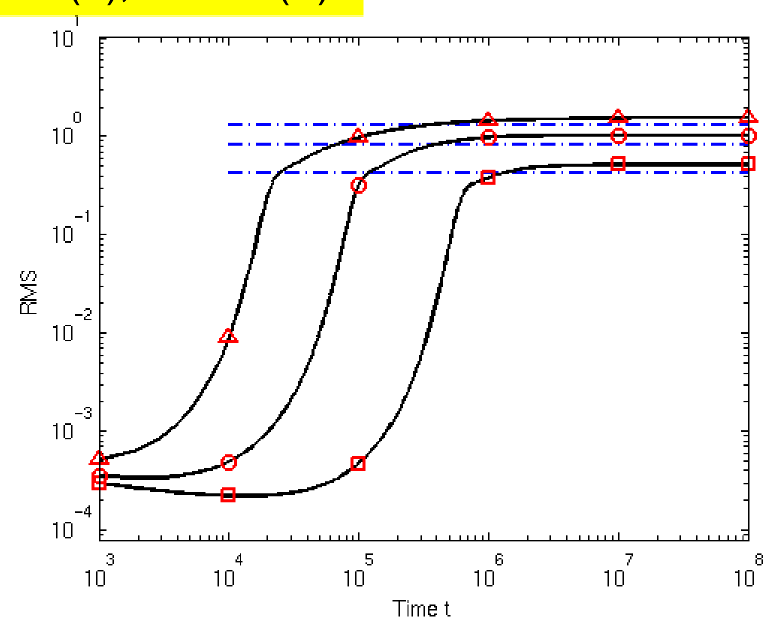




σ_0 = -0.02 (Δ), -0.01 (o), -0.005 (□)





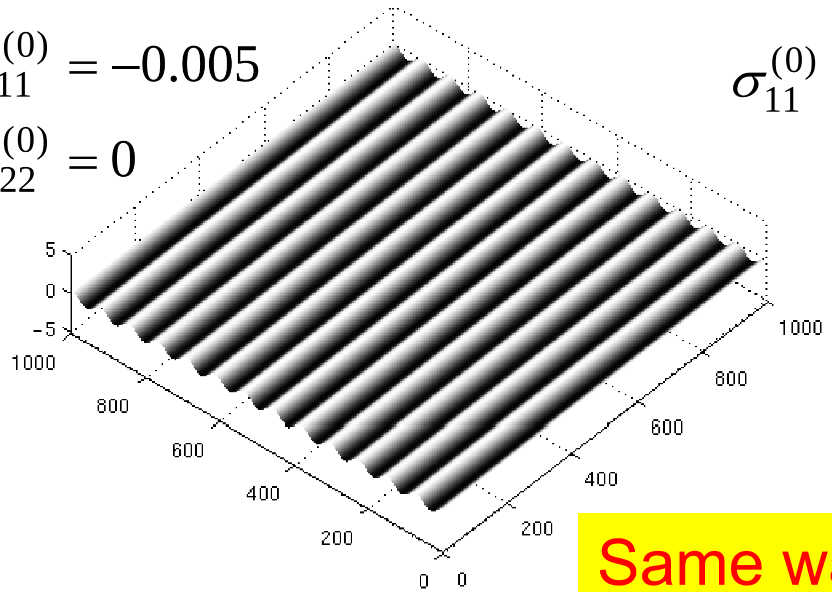


Near-Equilibrium Patterns

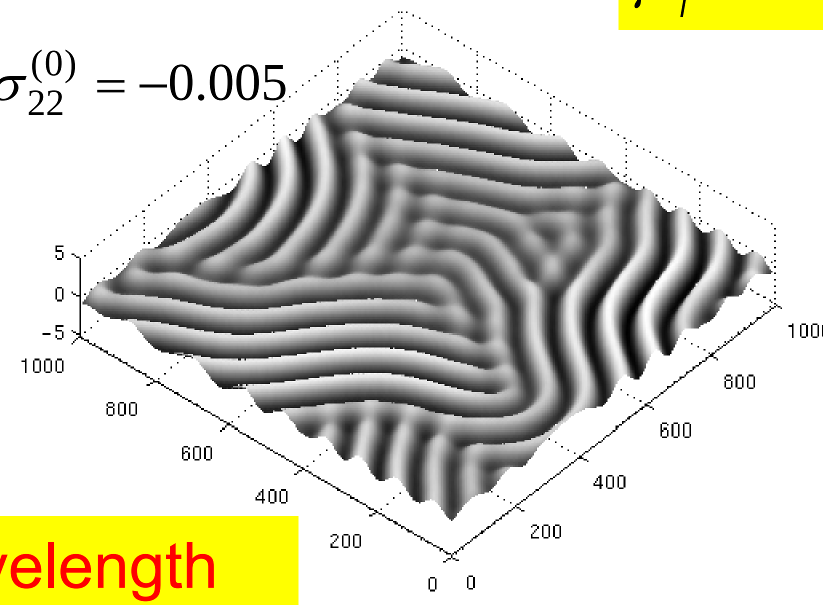
$$\frac{\mu_R}{\mu_f} = 10^{-5}$$

$\sigma_{11}^{(0)} = -0.005$

$\sigma_{22}^{(0)} = 0$



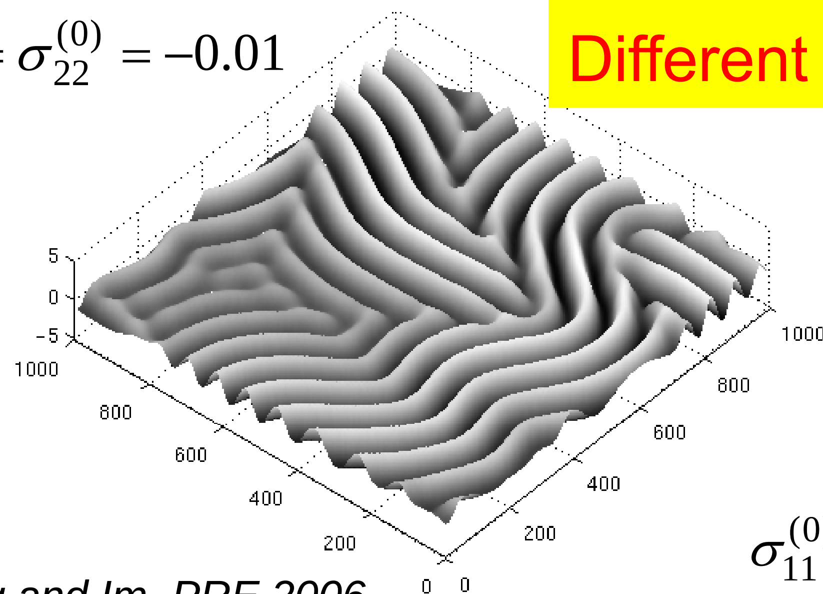
$\sigma_{11}^{(0)} = \sigma_{22}^{(0)} = -0.005$

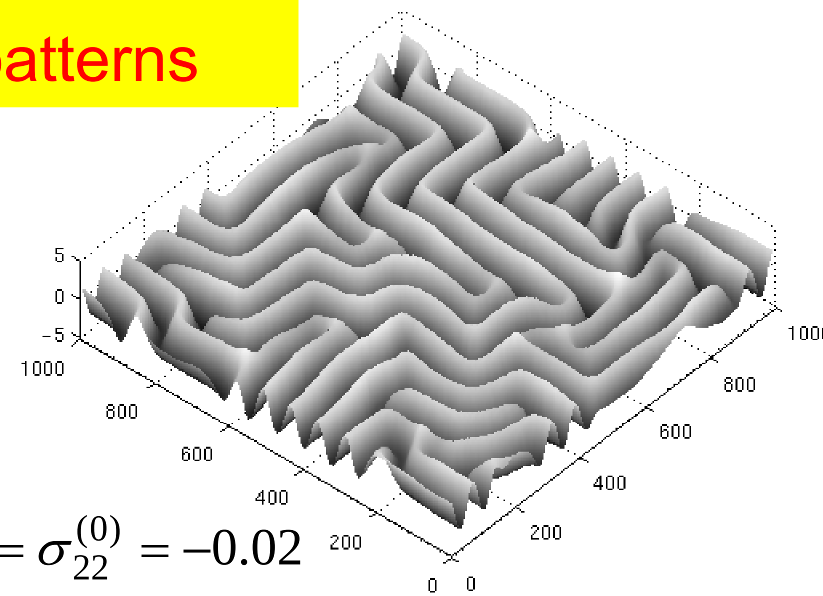


Same wavelength

Different patterns

$\sigma_{11}^{(0)} = \sigma_{22}^{(0)} = -0.01$

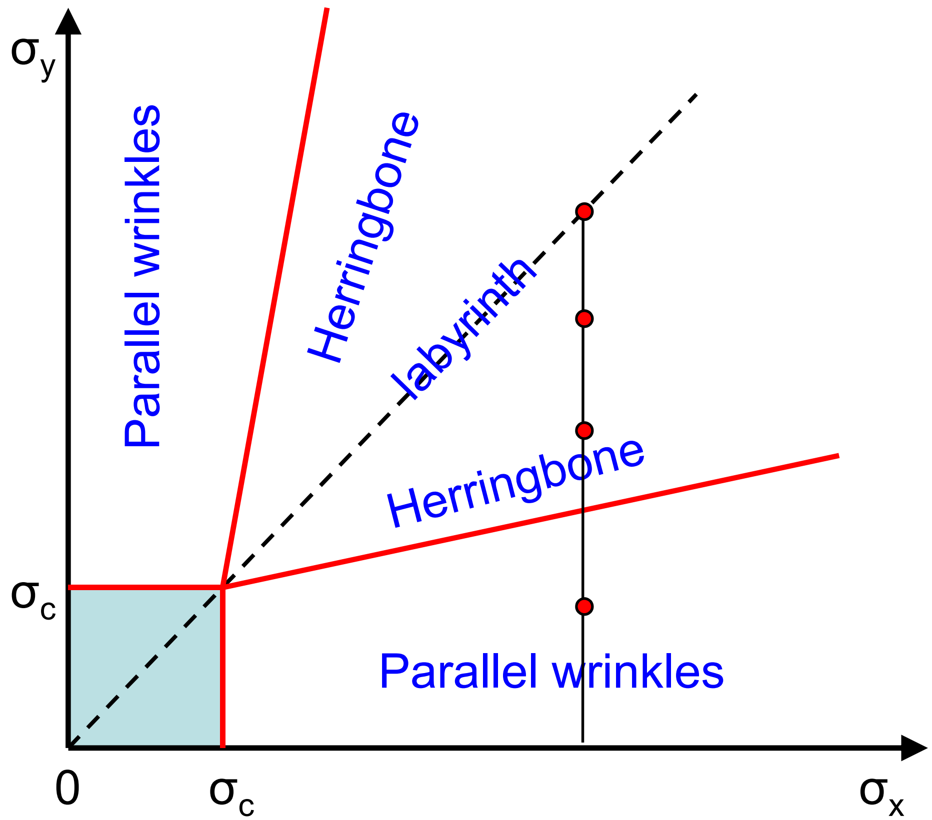


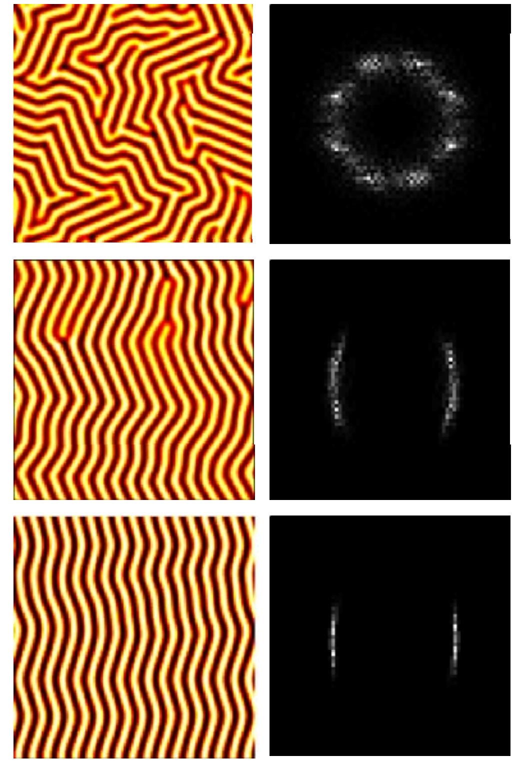


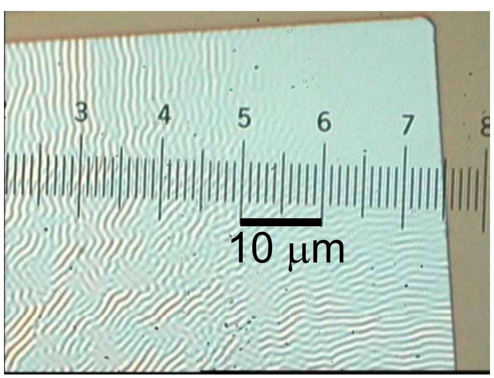
$\sigma_{11}^{(0)} = \sigma_{22}^{(0)} = -0.02$

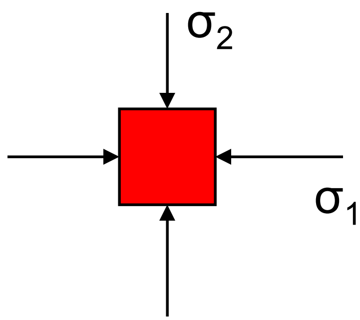
Huang and Im, PRE 2006.

Wrinkle patterns under biaxial stresses

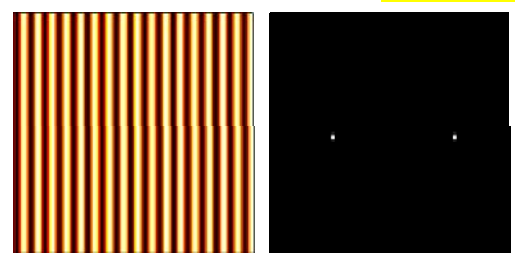




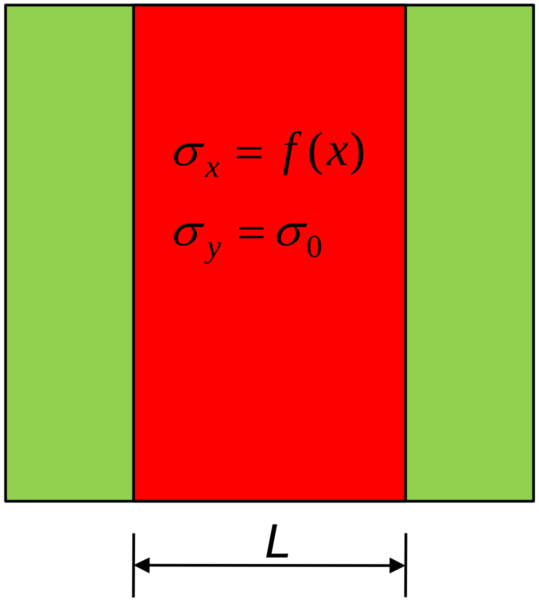




Transition stress: $\frac{\sigma_y - \sigma_c}{\sigma_x - \sigma_c} = \nu_f$



Wrinkle patterns under non-uniform stresses

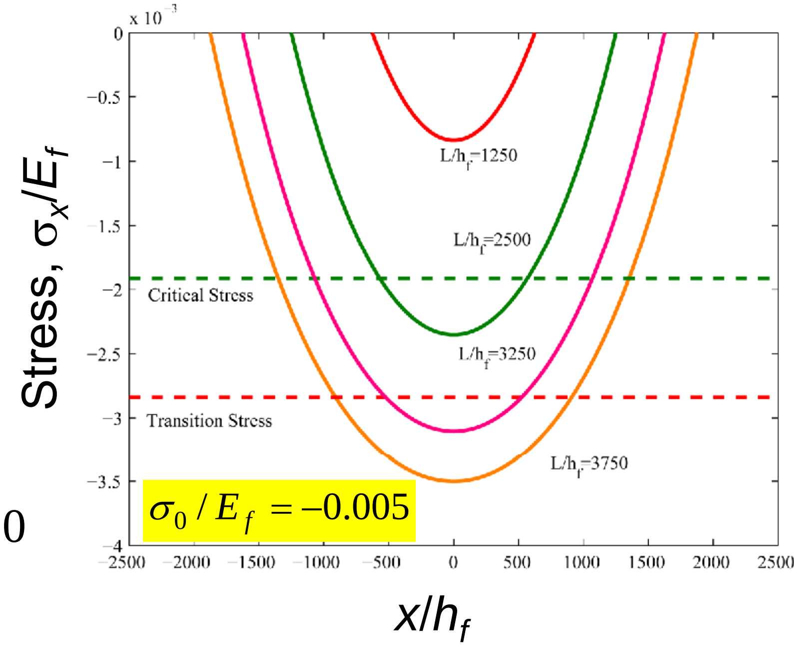


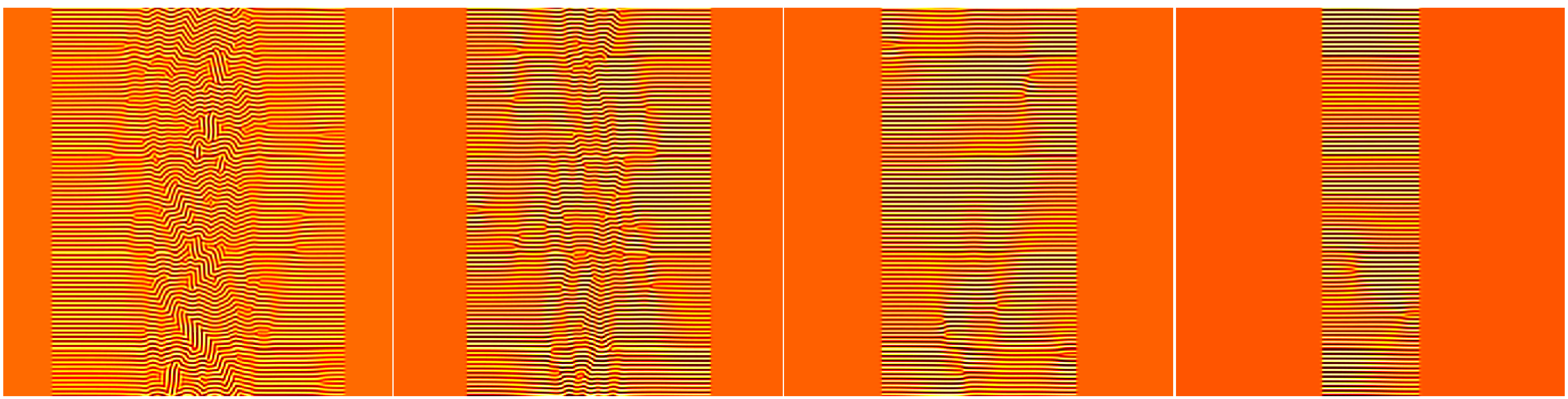
1D shear-lag model:

$$\sigma_x = \sigma_0\left[1 - \frac{\cosh(x/a)}{\cosh(L/2a)}\right]$$

$$a = \sqrt{\frac{\overline{E}_f h_s h_f}{\mu_R}}$$

outside (green): $\sigma_x = \sigma_y = 0$

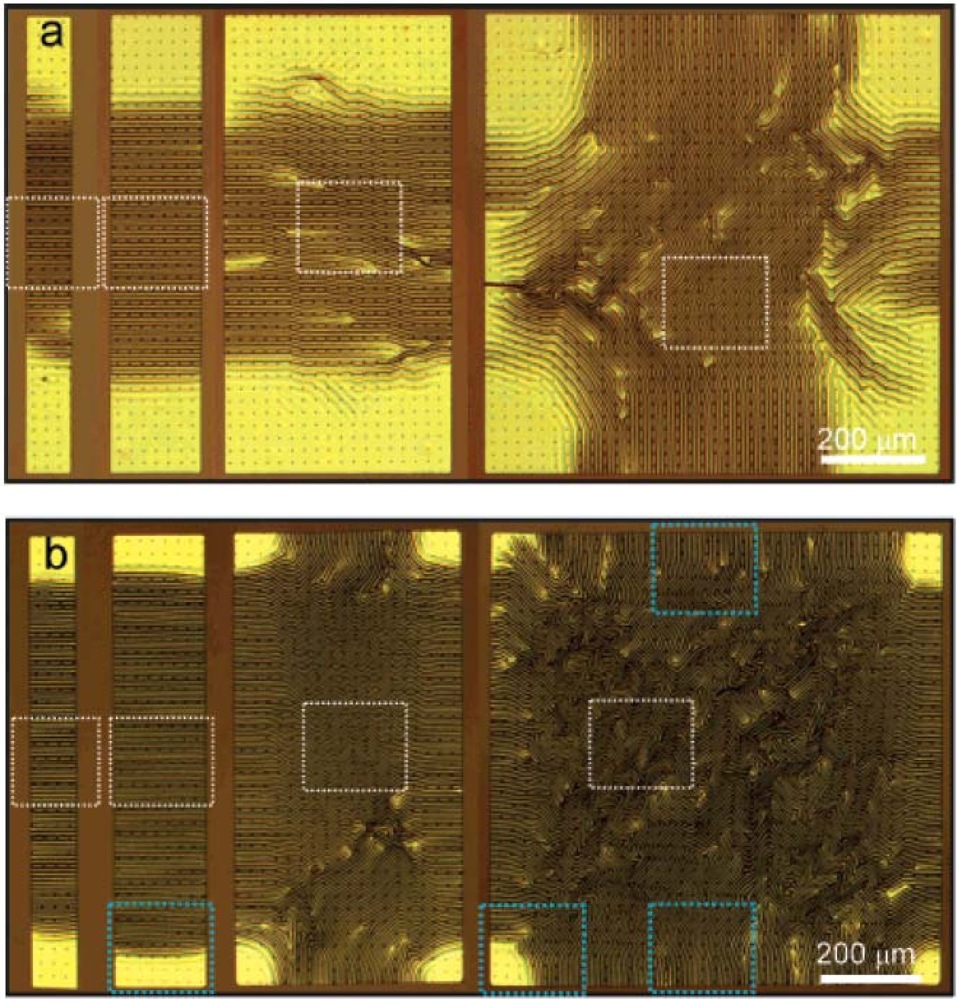




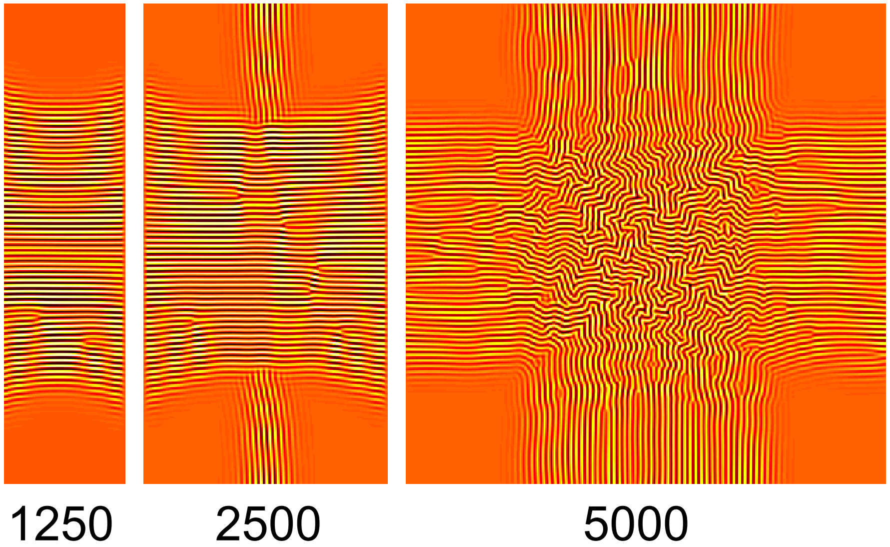
L = 3750 h_f L = 3250 h_f L = 2500 h_f L = 1250 h_f

Wrinkling of rectangular films

Experiments (Choi et al., 2007):



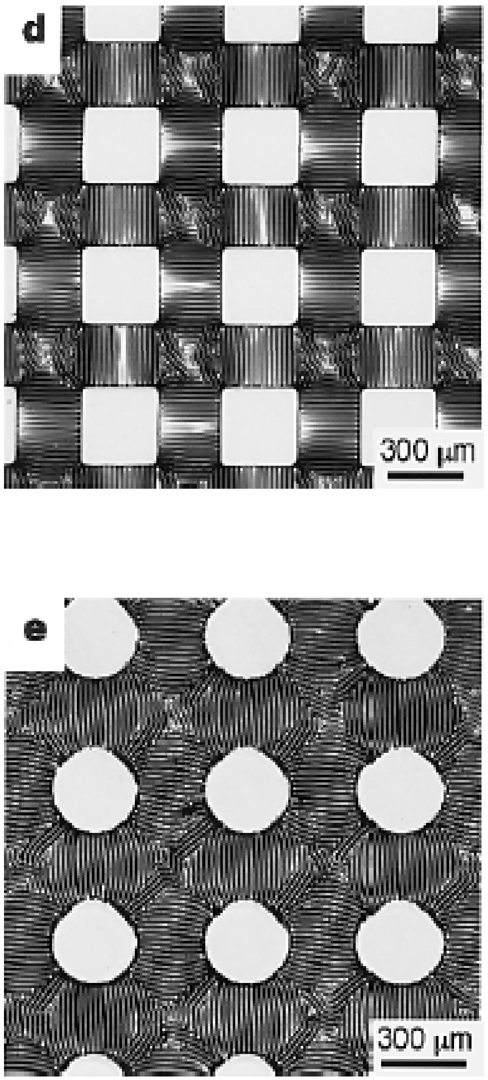
Simulations (Im and Huang, 2009):

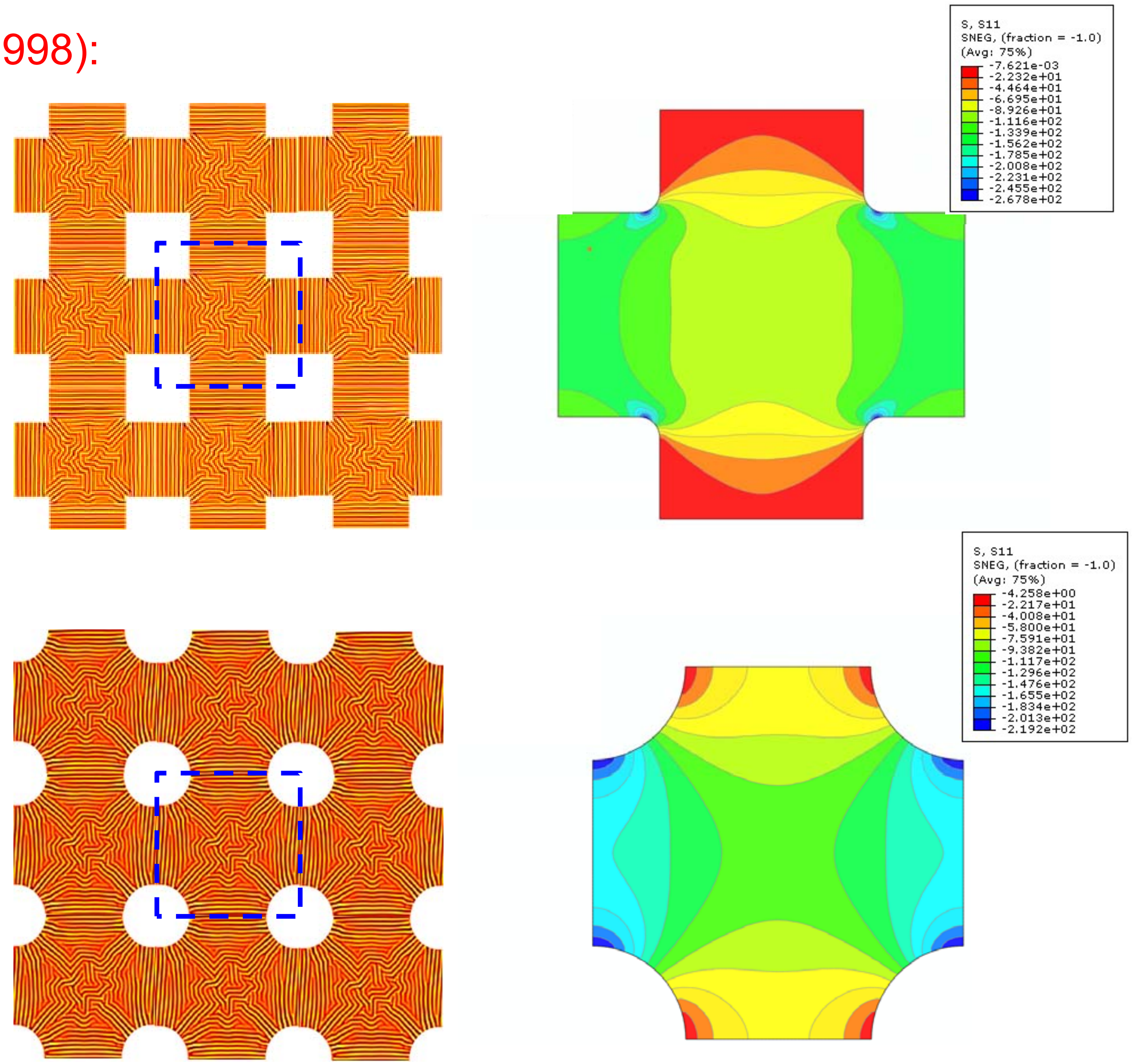


- Stress relaxation in both x and y directions from the edges.
- The length of edge relaxation depends on the elastic modulus of the substrate.
- No wrinkles at the corners, parallel wrinkles along the edges, and transition to zigzag and labyrinth in large films.

Wrinkling of periodically patterned films

Experiments (Bowden et al., 1998):



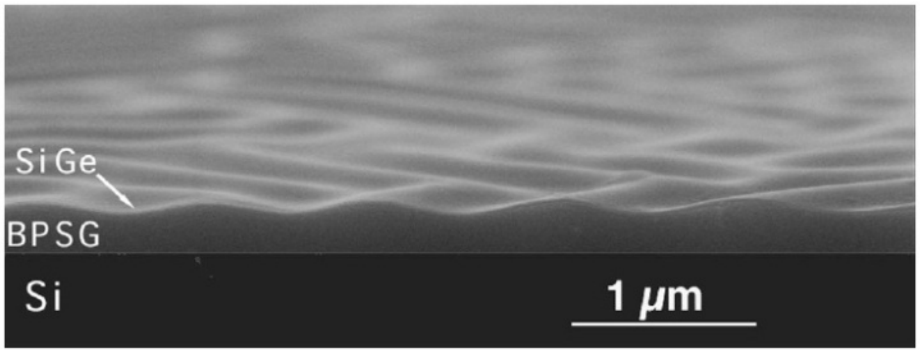


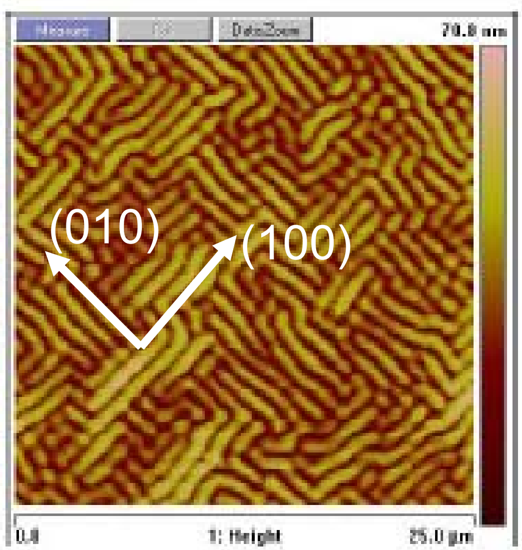
Simulations (Im and Huang, 2009)

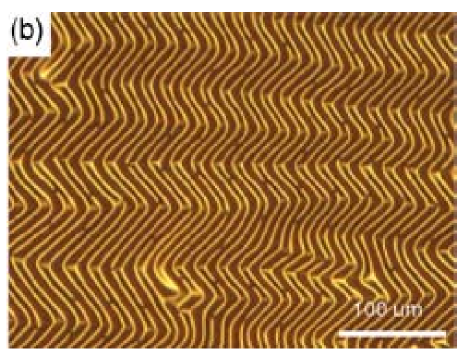
Wrinkling of Single-Crystal Thin Films

30 nm SiGe film on BPSG at 750°C
(Hobart et al., 2000; Yin et al., 2002 & 2003;
R.L Peterson, PhD thesis, Princeton, 2006)

100 nm Si on PDMS (*Song et al., 2008*)







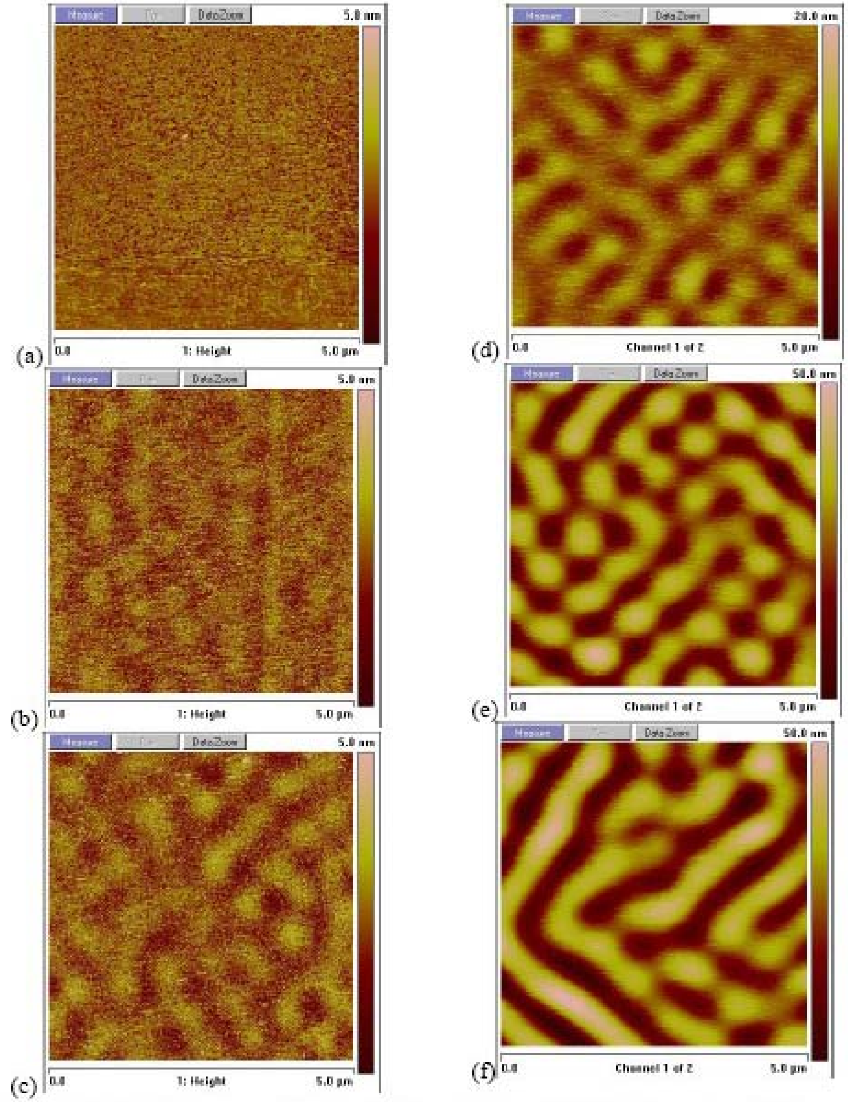
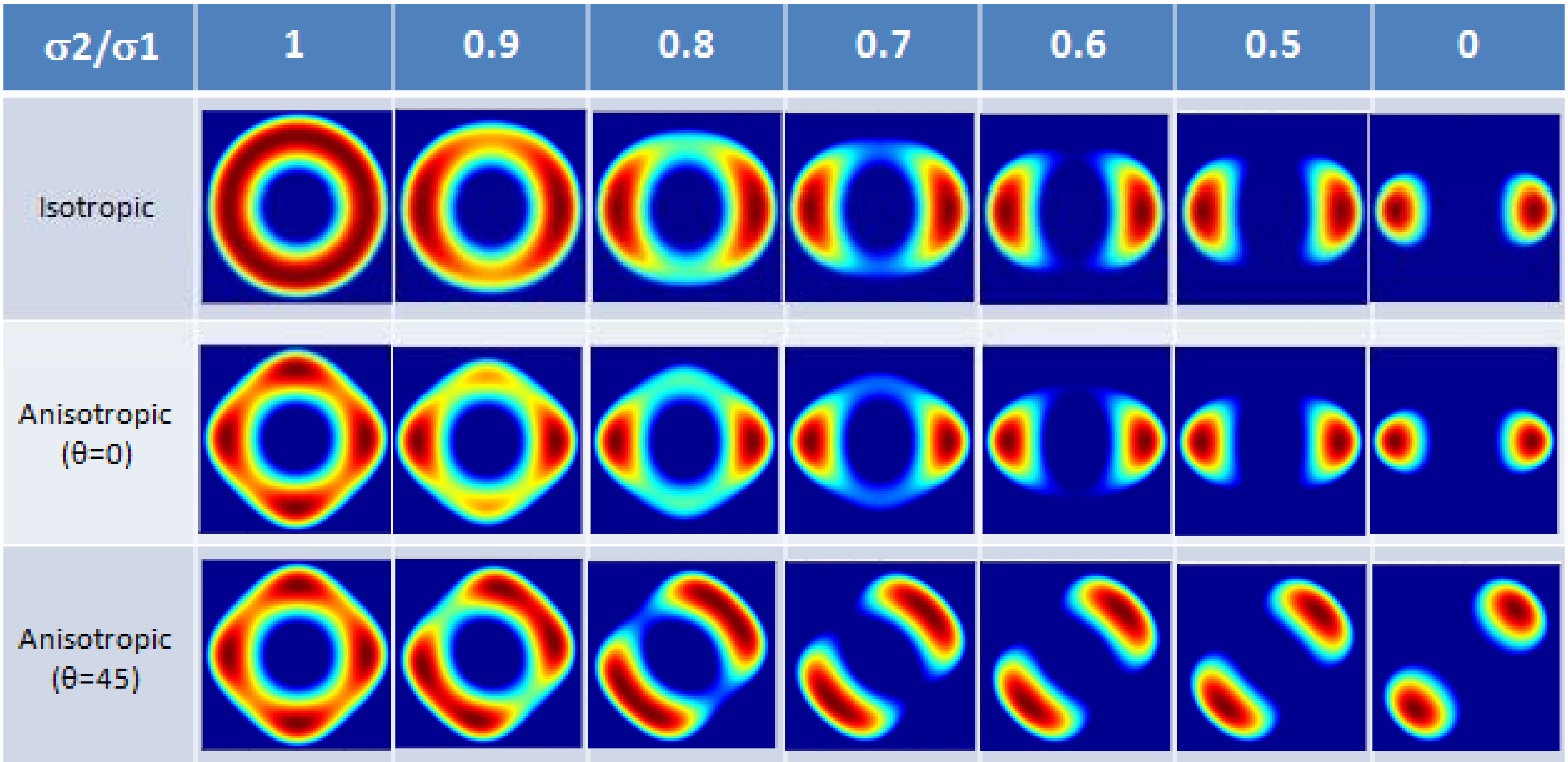
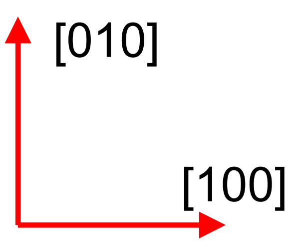
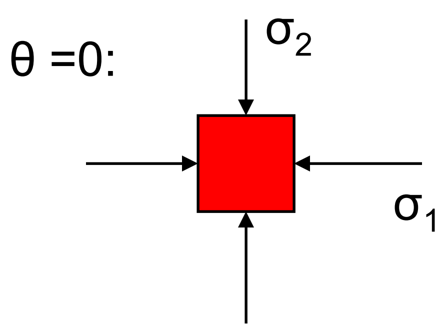


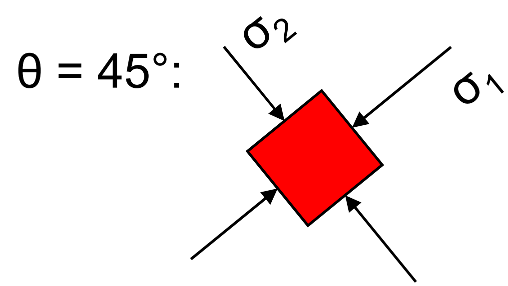
Figure 6.13: AFM images of buckled SiGe on 235-nm BPSG at the center of 150-μm x 150-μm islands after 750°C anneals of various lengths: (a) no anneal, (b) 25 min, (c) 50 min, (d) 105 min, (e) 253 min, and (f) 600 min. The AFM scan size is 5 μm x 5 μm, and the z-axis scale is indicated for each image. The scan edge is along a <110> crystal direction; buckles are along <100>, as before.

Spectra of initial growth rate



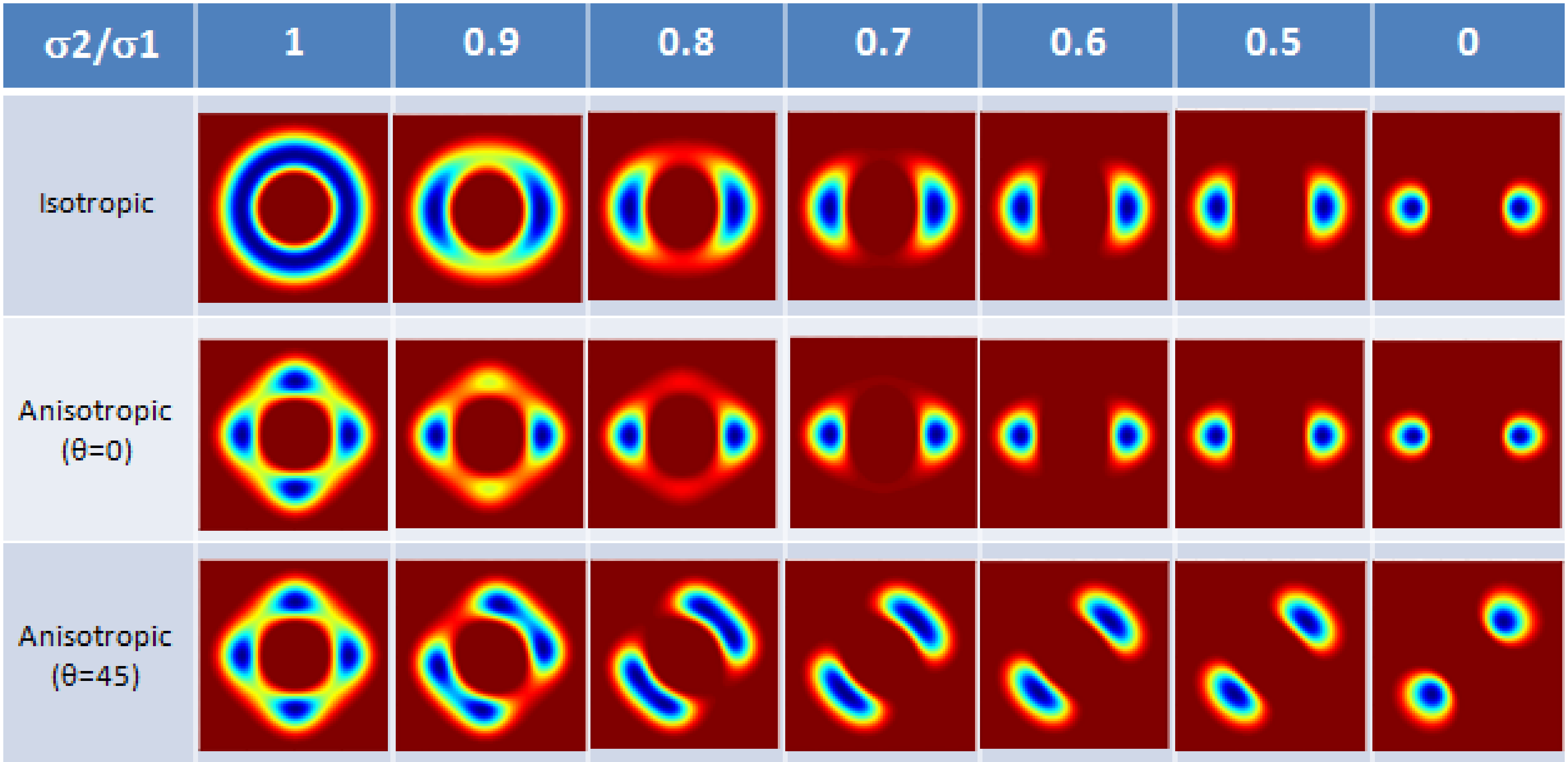


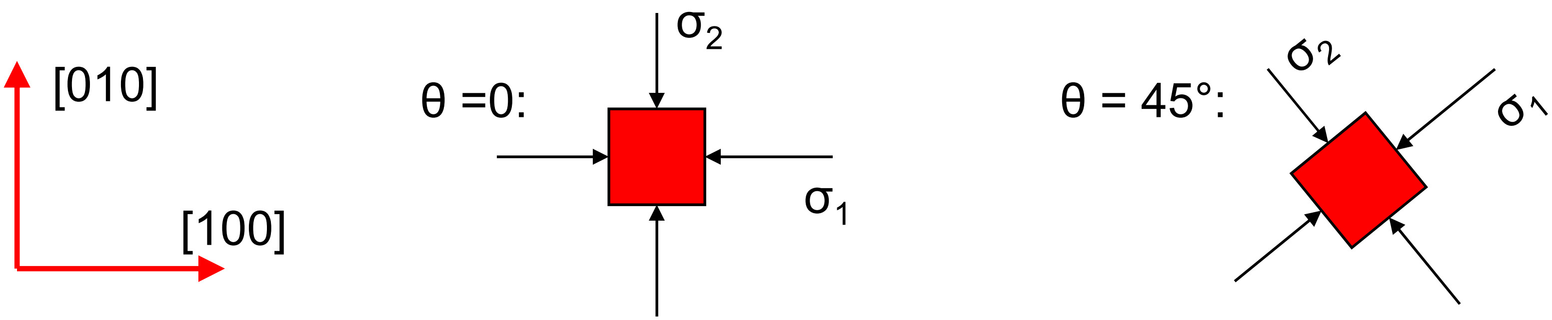




Im and Huang, JMPS 2008.

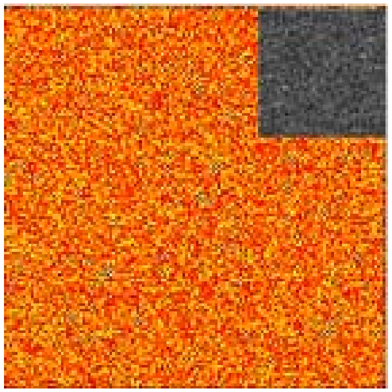
Energy Spectra of Equilibrium Wrinkles



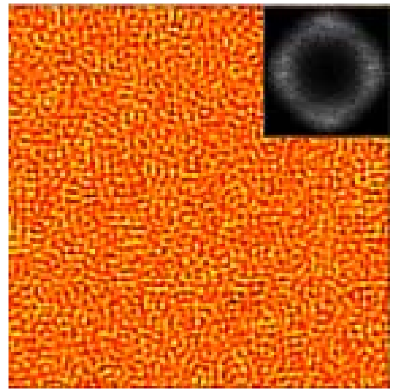


Evolution of orthogonal wrinkle patterns

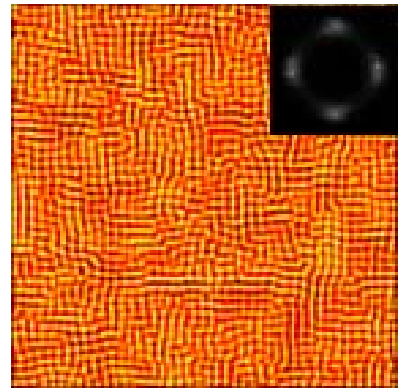
$\sigma_1 = \sigma_2 = -0.003$



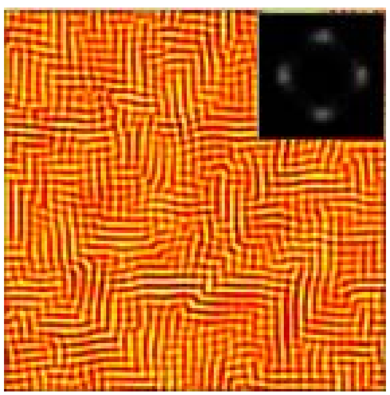
$t = 0$:
RMS = 0.0057
λ_{ave} = 38.8



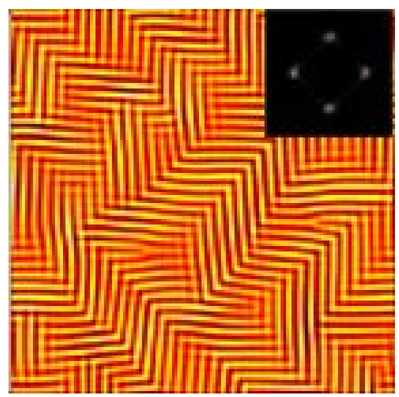
$t = 10^5$:
RMS = 0.0165
λ_{ave} = 44.8



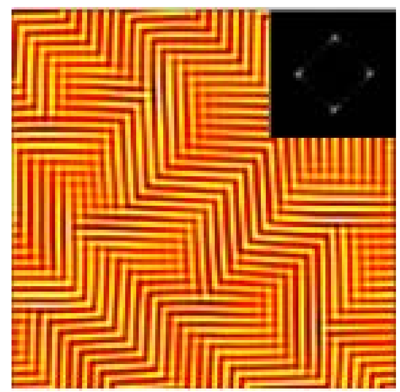
$t = 5 \times 10^5$:
RMS = 0.4185
λ_{ave} = 47.1



$t = 10^6$:
RMS = 0.4676
λ_{ave} = 50.8



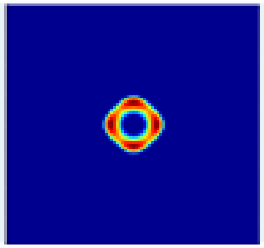
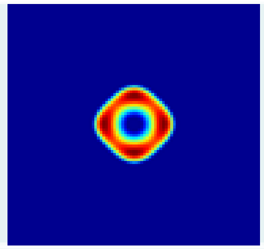
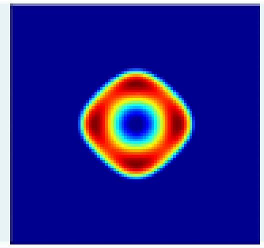
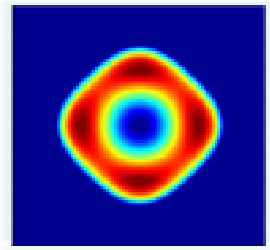
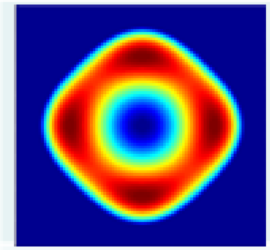
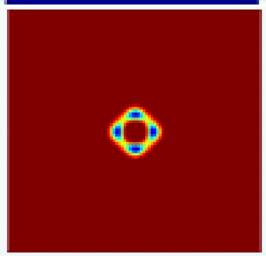
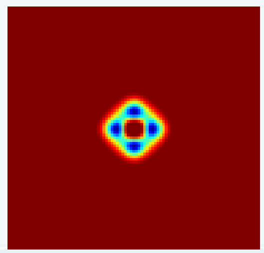
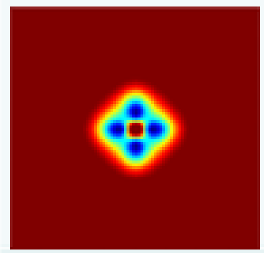
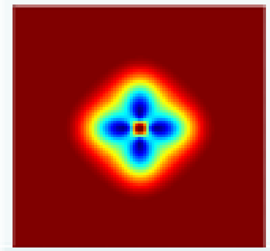
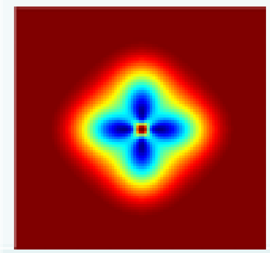
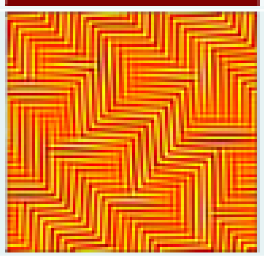
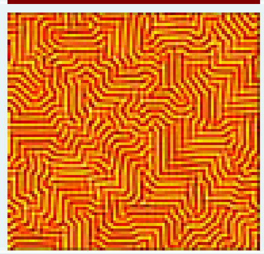
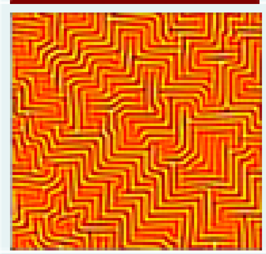
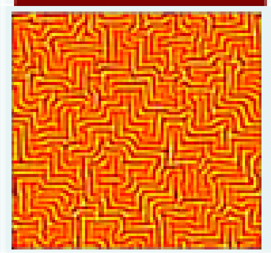
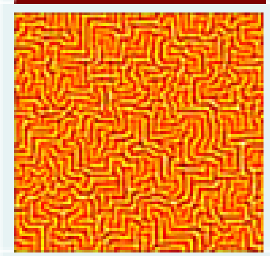
$t = 10^7$:
RMS = 0.5876
λ_{ave} = 56.4



$t = 10^8$:
RMS = 0.5918
λ_{ave} = 56.6

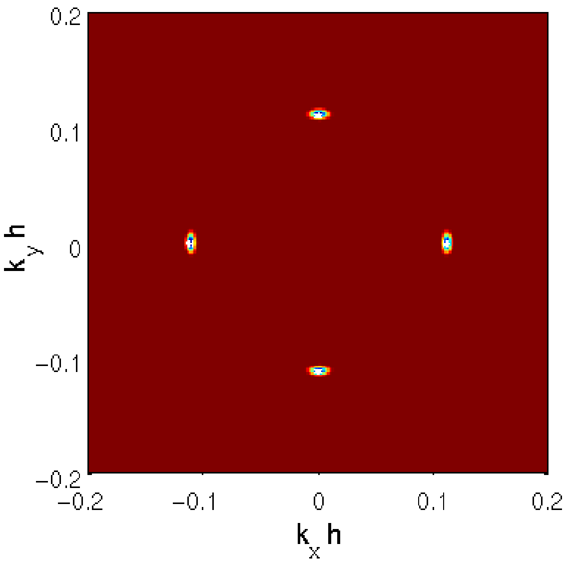
Im and Huang, JMPS 2008.

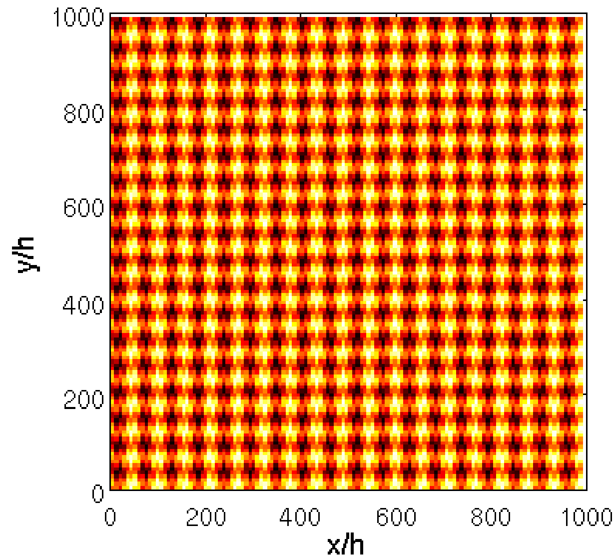
Effect of stress magnitude

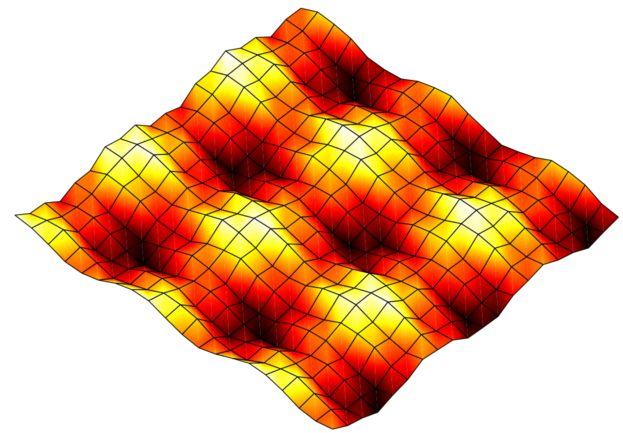
$\sigma_2 = \sigma_1$	-0.003	-0.005	-0.010	-0.020	-0.030
Growth rate					
Energy spectra					
Wrinkle patterns					
RMS	0.5918	0.9727	1.5845	2.3415	2.8829
$\bar{\lambda}$	56.63	57.03	57.90	57.30	56.60

Im and Huang, JMPS 2008.

Checkerboard wrinkle pattern





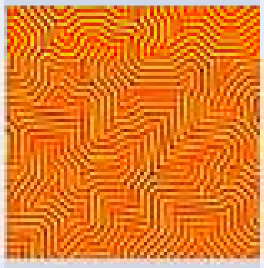
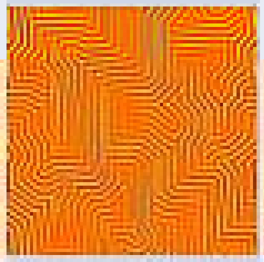
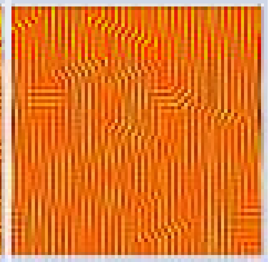
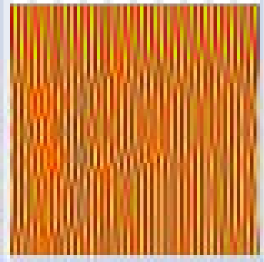
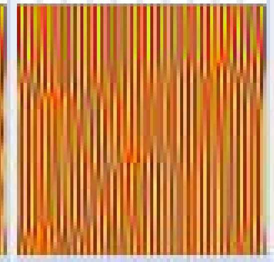
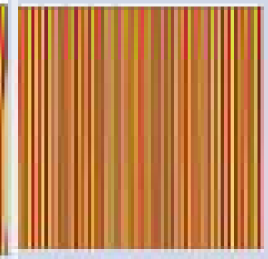
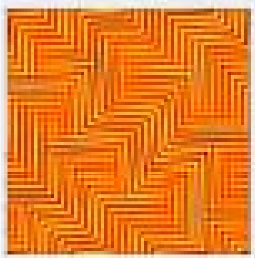
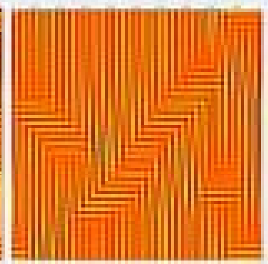
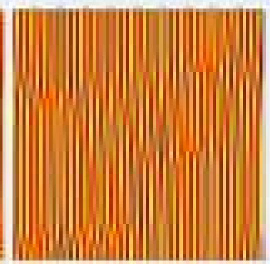
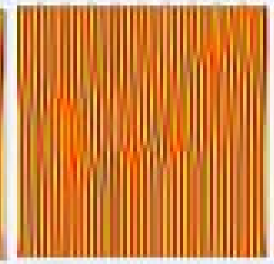
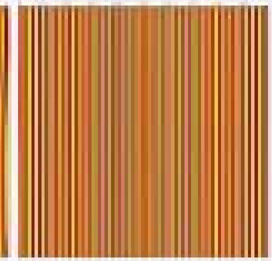
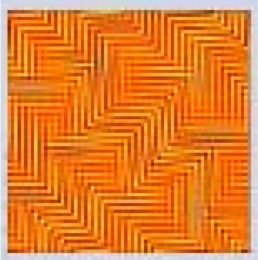
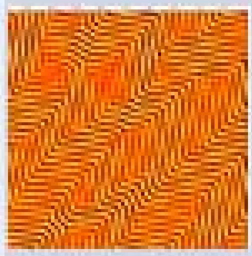
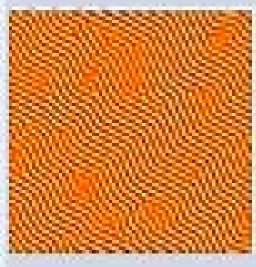
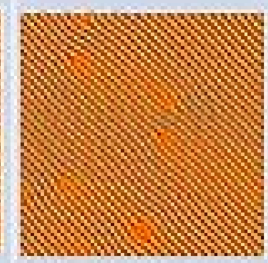
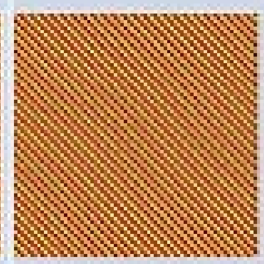
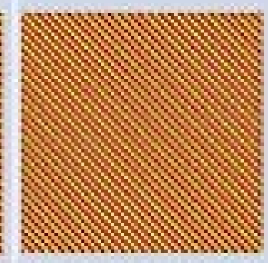


$\sigma_1 = \sigma_2 = -0.00178$ RMS = 0.05286 and λ_{ave}= 55.6

❖ Transition of local buckling mode from spherical to cylindrical as the compressive stress/strain increases and/or as the wrinkle amplitude increases.

Im and Huang, JMPS 2008.

Transition of wrinkle patterns

σ_2/σ_1	1	0.9	0.8	0.7	0.5	0
Isotropic						
Anisotropic (θ_p=0°)						
Anisotropic (θ_p=45°)						

Im and Huang, JMPS 2008.

Summary

- **Viscoelastic wrinkling**
  - Three-stage evolution
  - Power-law coarsening
  - Near-equilibrium patterns

- **Diverse wrinkle patterns**
  - Anisotropic patterns under biaxial stresses
  - Nonuniform patterns
  - Wrinkling of anisotropic crystal thin films