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Poroelastic Effect on Fracture of Gels

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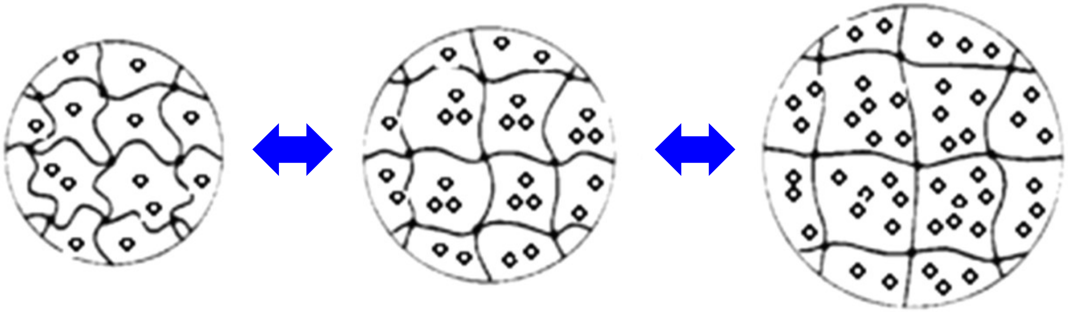
Funding from National Science Foundation is acknowledged.

November 2019

Hydrogel is Poroelastic

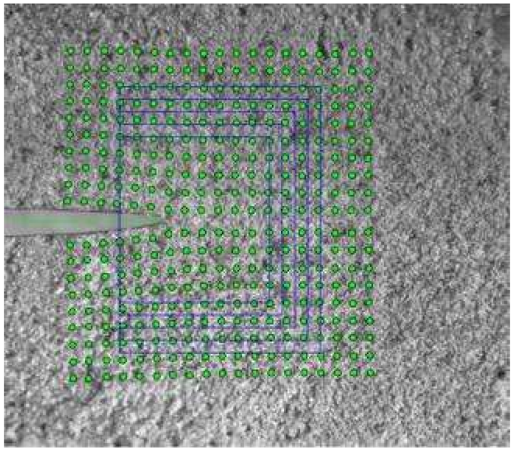


- **Nonlinear Poroelasticity**: large and reversible deformation of polymer network coupled with migration (diffusion) of solvent molecules, in response to mechanical, chemical, and other environmental stimuli (temperature, pH, light, etc)

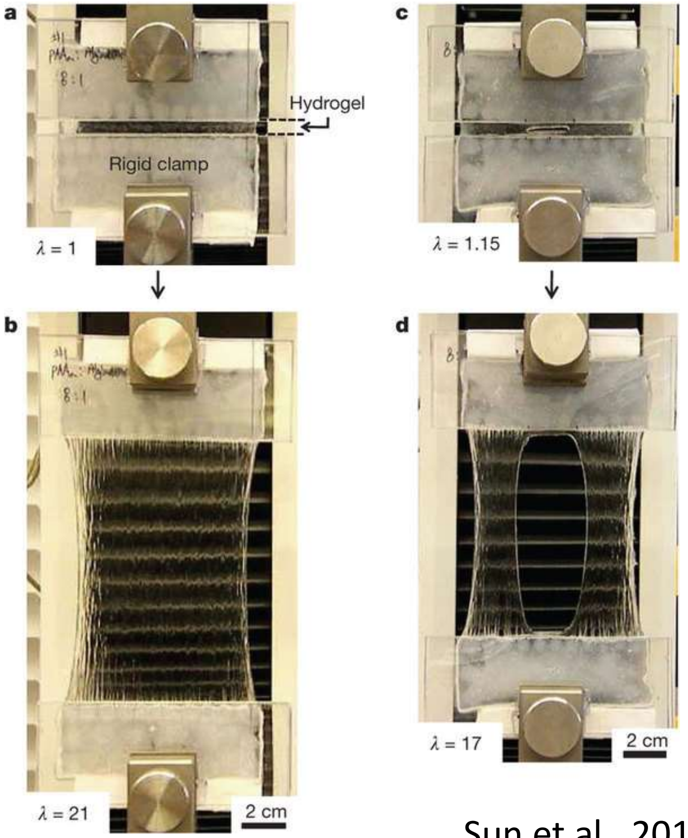
- Applications: biomedical, soft machines…

Fracture Toughness of Gels

- Reported values of fracture toughness range from 1 to 10^4 J/m^2
- Fracture mechanism may vary over different types of gels
- Potential rate dependence may relate to viscoelasticity or solvent diffusion (poroelasticity) or …



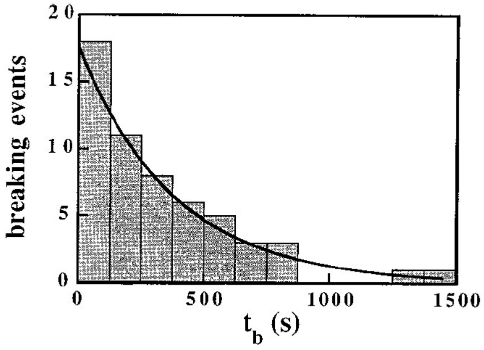
Kwon et al., 2011.



Sun et al., 2012.

Delayed Fracture of Gels

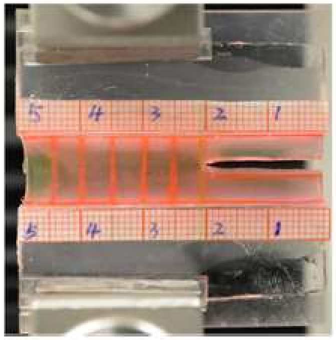
Bonn et al., 1998

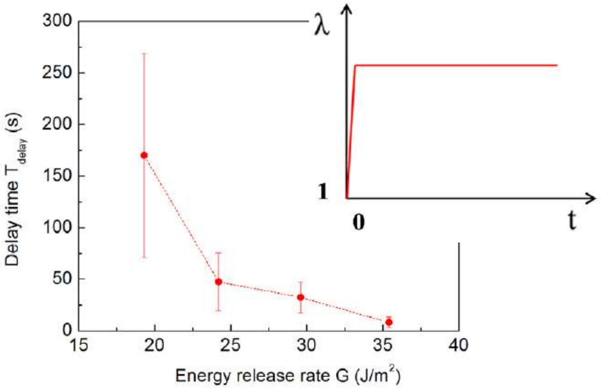


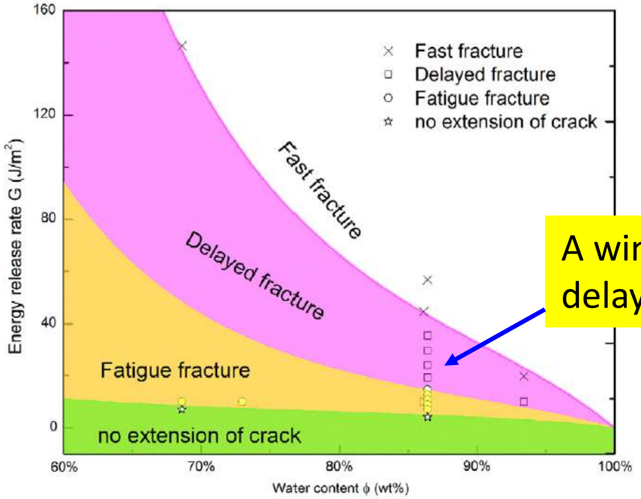
Time to fracture (nucleation):

$$\ln t_b \sim \sigma^{-2d+2}$$

Tang et al., 2017



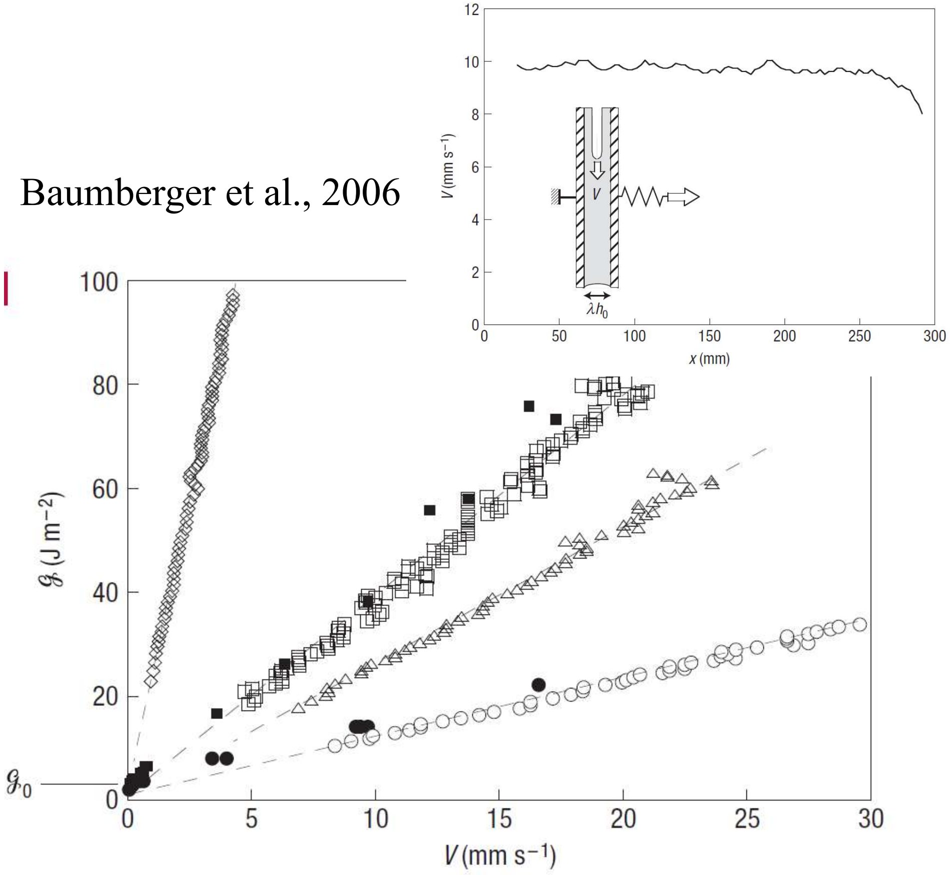




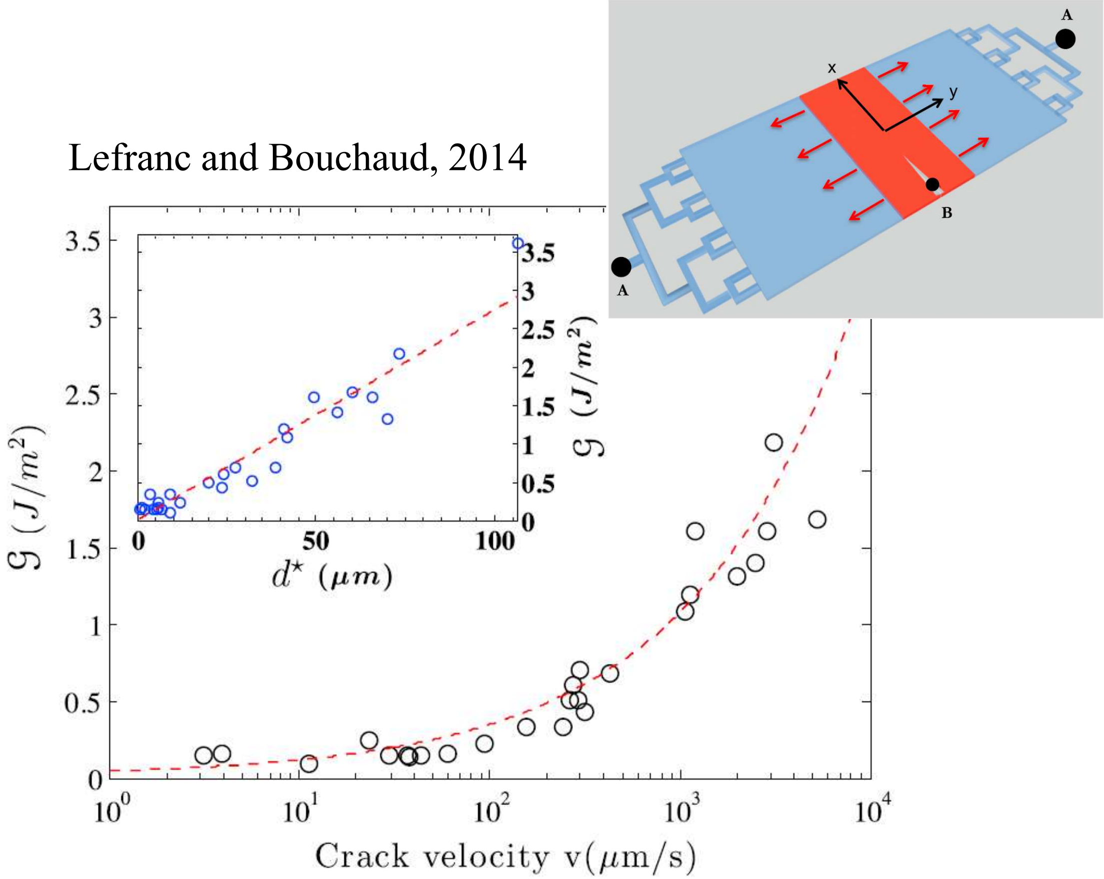
A window of delayed fracture

Rate-dependent Fracture of Gels

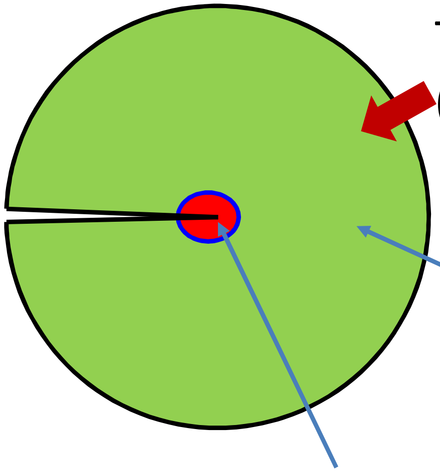
Baumberger et al., 2006



Lefranc and Bouchaud, 2014



Energetics of Fracture (J-integrals)



Total energy flux (remote loading)

$$J_{remote} = J_{elastic} + J_{inelastic}$$

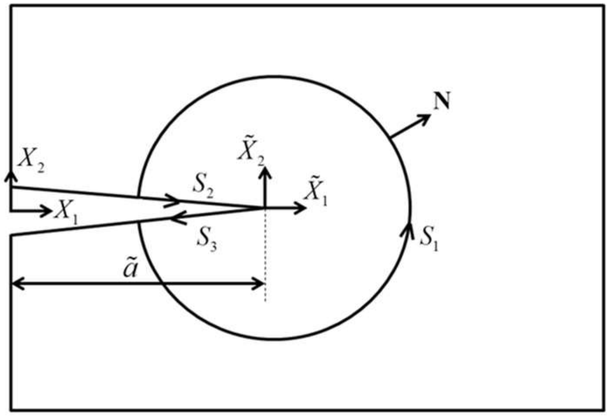
Large-scale processes (energy dissipation by plasticity, viscoelasticity, diffusion, etc)

Small-scale processes (fracture)

$$\Gamma_{fracture} = J_{elastic} = J_{remote} - J_{inelastic}$$

- The elastic energy is stored and (partly) released upon fracture.
- The elastic energy release rate is the driving force for fracture.

Energy release rate for gels: a modified J-integral



$$J^* = \int_{S_1}\left(UN_1 - s_{iJ}N_J\frac{\partial x_i}{\partial X_1}\right)dS - \int_{V_0}\mu\frac{\partial C}{\partial X_1}dV$$

$$J^* = \int_{S_1}\left(\hat{U}N_1 - s_{iJ}N_J\frac{\partial x_i}{\partial X_1}\right)dS + \int_{V_0}\frac{\partial \mu}{\partial X_1}C\,dV$$

$$\hat{U}(\mathbf{F},\mu) = U(\mathbf{F},C) - \mu C$$

- J* is path independent.
- The second form is more convenient for numerical calculations.
- A domain integral method can be used to calculate J*.

Bouklas et al., J. Appl. Mech. 82, 081007 (2015).

Proposed Fracture Criteria for Gels

- For a stationary pre-existing crack:

$$J^*(t) = \Gamma_0$$ → **Delayed fracture**

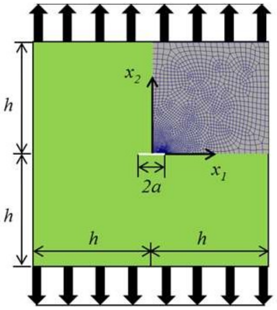
- For steady-state crack growth:

$$J^*(\dot{a}) = \Gamma_{SS}$$ → **Rate-dependent fracture**

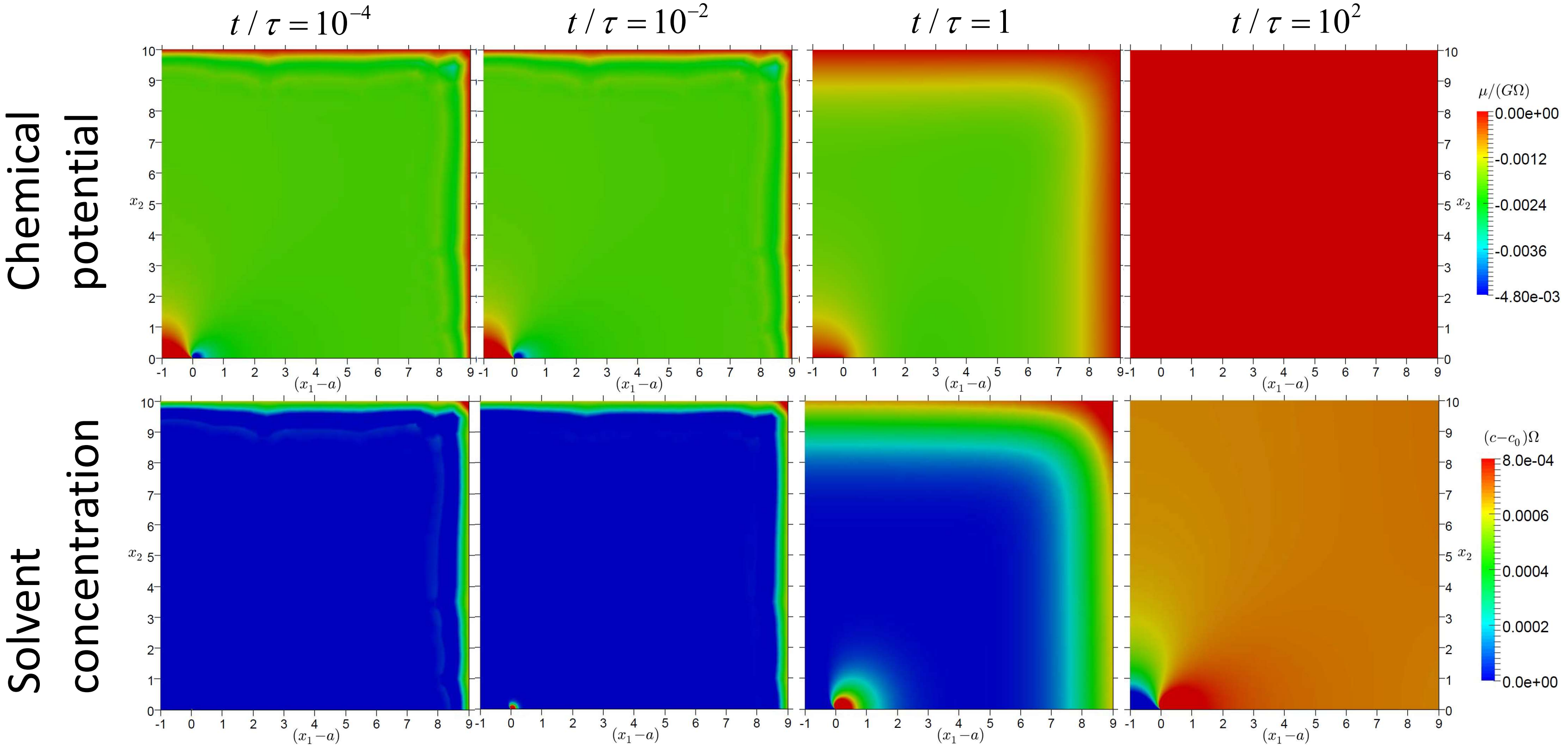
“Division of labor”:

- Calculate the crack-tip energy release rate
- Measure the *intrinsic* fracture toughness

Transient full-field finite element analysis

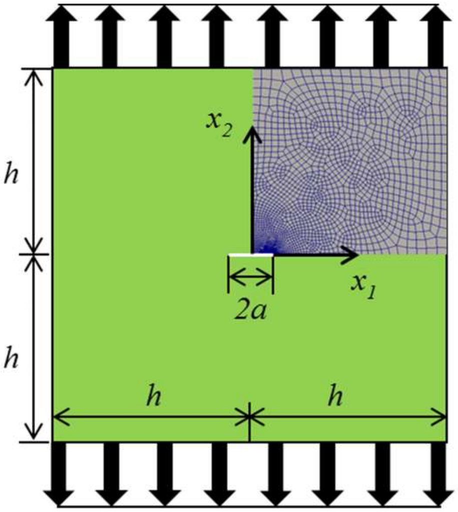


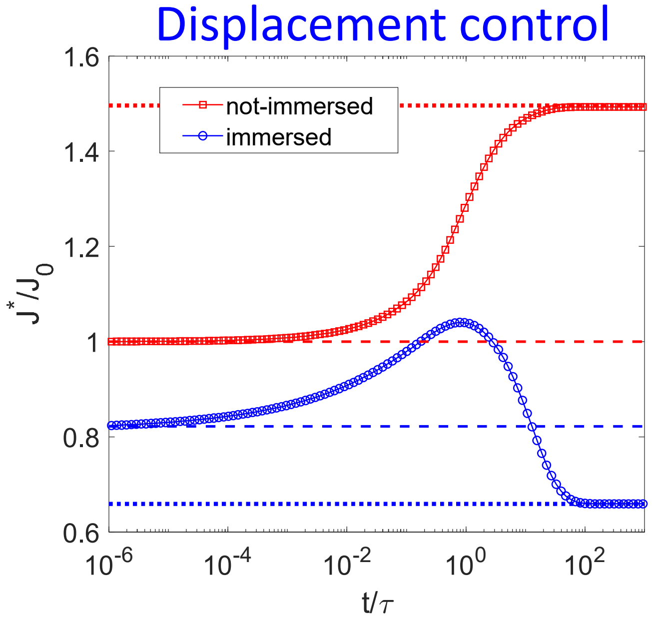
$\tau = a^2 / D^*$

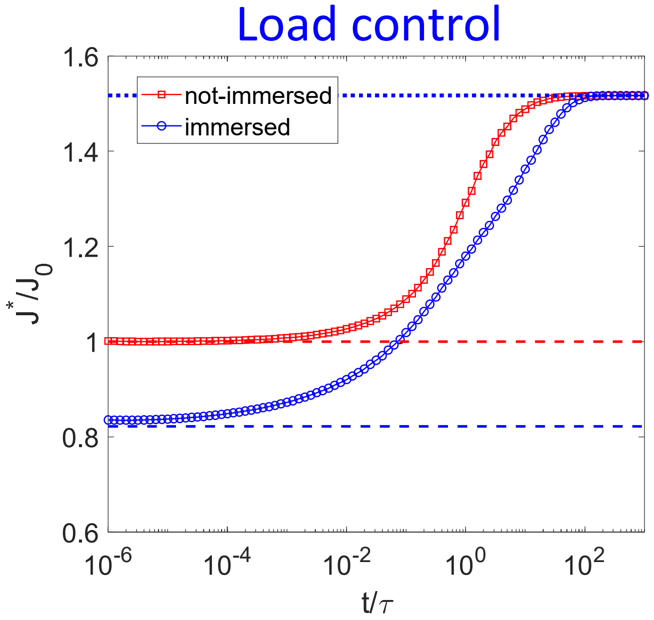


Yu, et al., J. Appl. Mech. 85, no.111011 (2018).

Time-dependent Energy Release Rate




Displacement control


Load control

$$\frac{J^*}{J_0}=\Lambda\left(\frac{t}{\tau},\frac{h}{a},\nu\right)$$

$$J_0 \sim \sigma_h^2 a/G$$

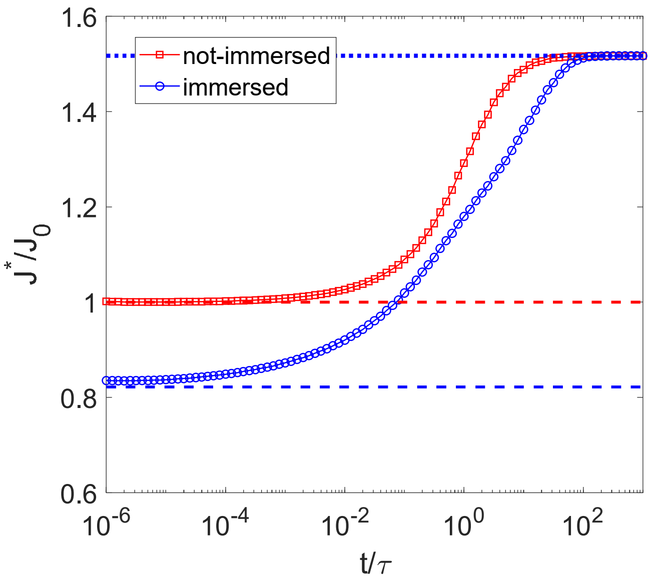
- The short-time limits are different for immersed and not-immersed specimens.
- The long-time limits are different for immersed specimens under displacement and load control.

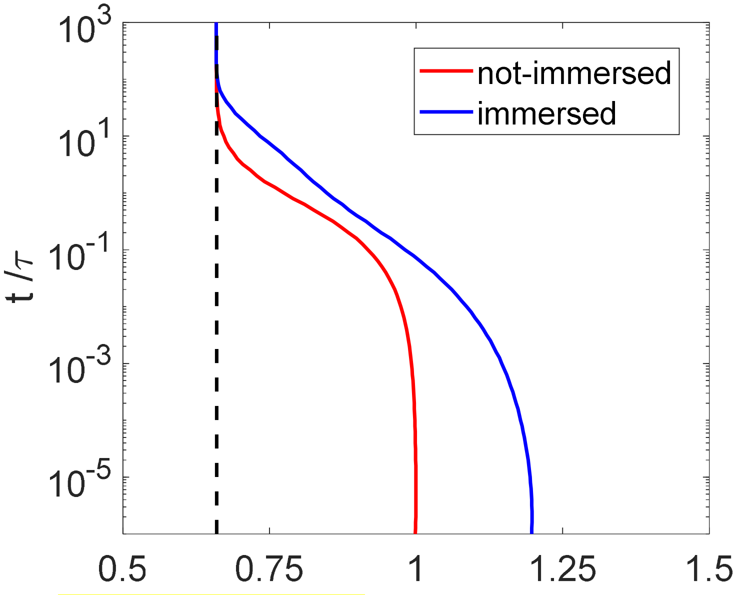
Yu, et al., J. Appl. Mech. 85, no.111011 (2018).

Delayed Fracture by Poroelasticity

$$J^*(t) = J_0 \Lambda\left(\frac{t}{\tau}\right) = \Gamma$$

$$t_b = \frac{a^2}{D^*} f\left(\frac{J_0}{\Gamma}, \nu\right)$$





$$J_0 \sim \sigma_h^2 a / G$$

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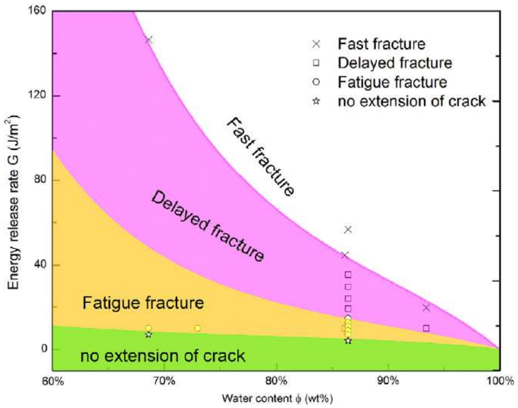


Critical condition for instantaneous fracture:

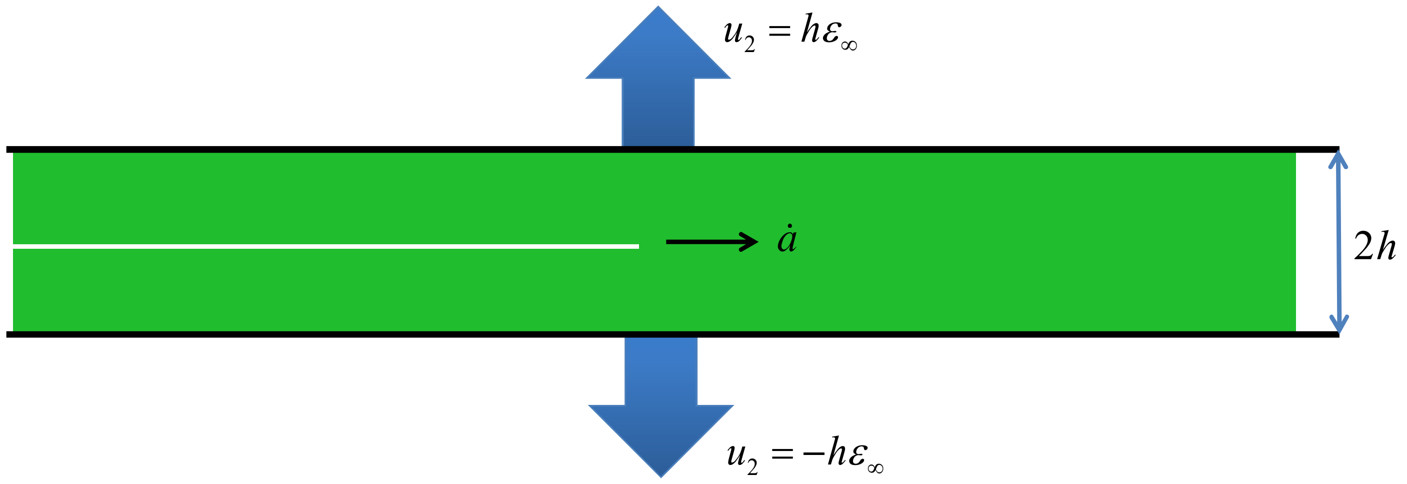
$$J^*(t \to 0^+) > \Gamma$$

Threshold for delayed fracture:

$$J^*(t \to \infty) > \Gamma$$

Same threshold but different critical loads for immersed and not-immersed specimens.

Steady-state crack growth model



Peclet number

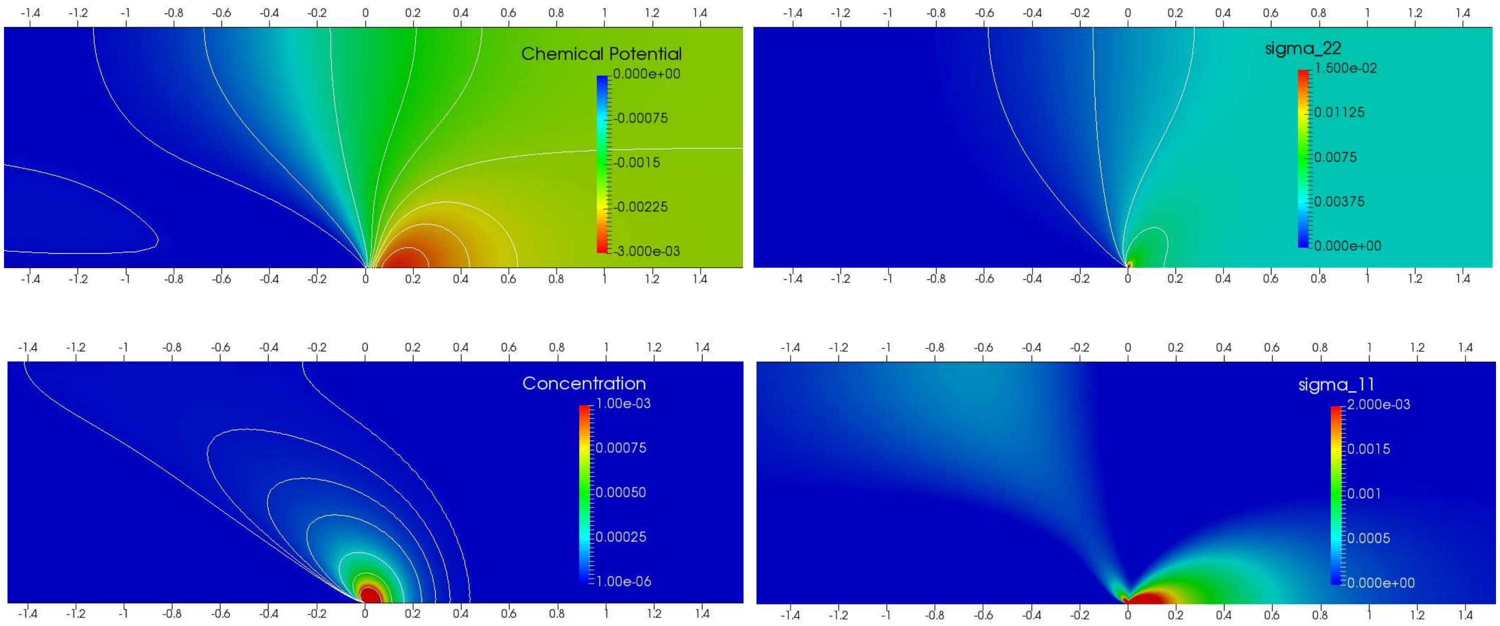
$$Pe = \frac{h}{l_{SS}} = \frac{\dot{a}h}{D^*}$$

- A semi-infinite crack in an infinitely long strip made of a **linearly poroelastic** material;
- Uniform vertical displacements are applied at the top and bottom, while the crack grows in a **steady state**.
- Ignore inertia for quasi-static crack growth.

Yu, et al., J. Mech. Phys. Solids 118, 15-39 (2018).

Numerical Results

$$Pe = \dot{a}h / D^* = 10,\ \ \nu = 0.2414$$



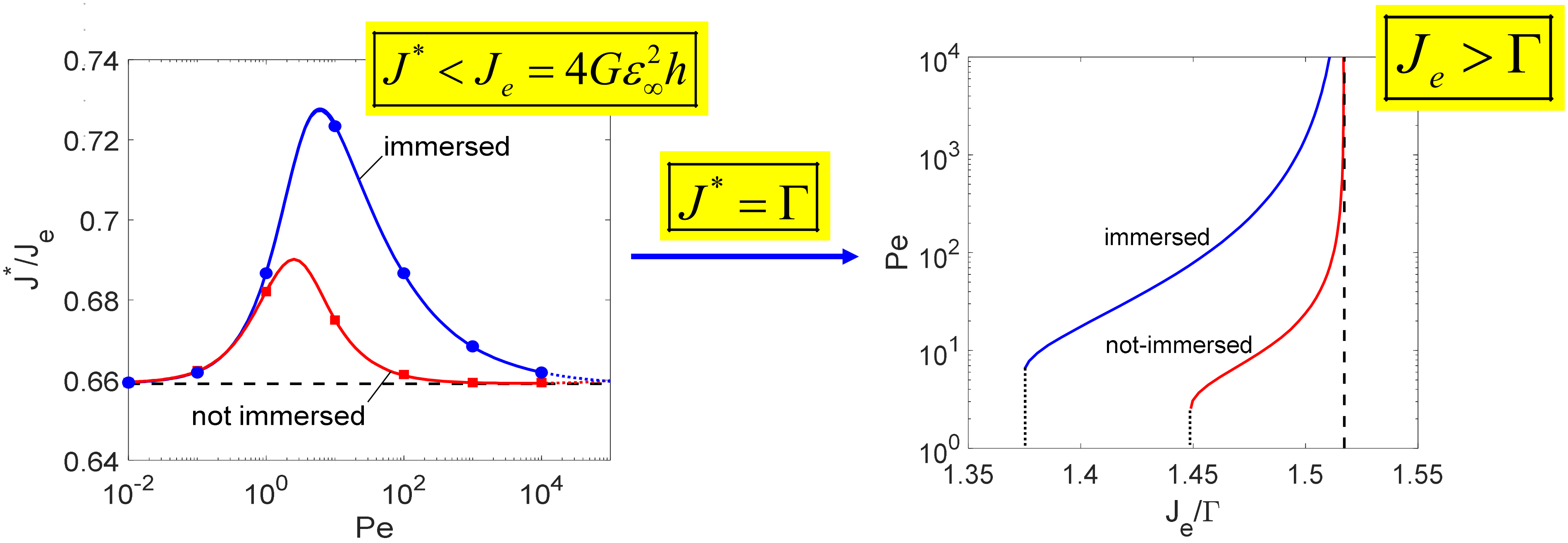
Poroelastic shielding: the crack-tip stress intensity factor is lower than the elastic case.

$$\frac{K_{tip}}{K_e} = f\left(Pe, \nu\right)$$

$$K_{tip} < K_e = 4G\varepsilon_\infty \sqrt{h}$$

Yu, et al., J. Mech. Phys. Solids 118, 15-39 (2018).

Poroelastic Toughening



$J^* < J_e = 4G\varepsilon_\infty^2 h$

$J^* = \Gamma$

$J_e > \Gamma$

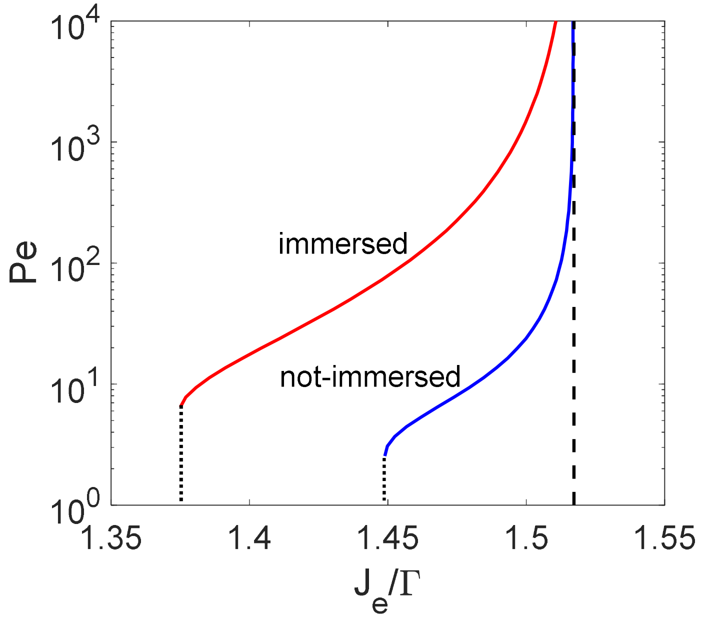
The apparent energy release rate is greater than the intrinsic fracture energy (toughness) because energy dissipation by solvent diffusion around the crack tip.

Yu, et al., J. Mech. Phys. Solids 118, 15-39 (2018).

Experimental implications

Measure intrinsic fracture toughness as a function of crack speed:

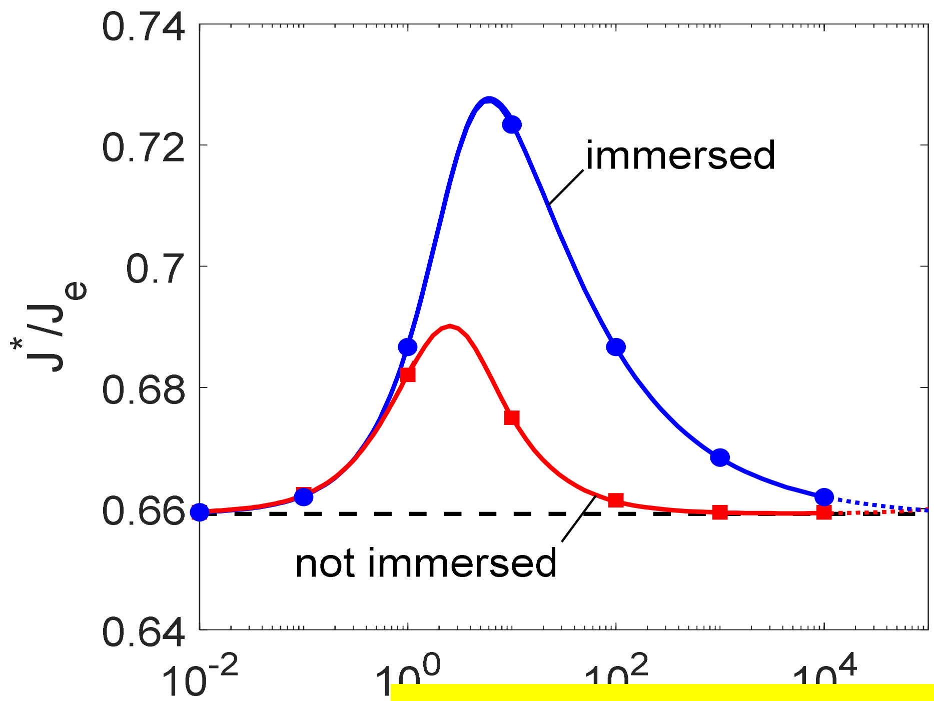
$$\Gamma = J^* = 4G\varepsilon_\infty^2 h\Lambda\left(\frac{\dot{a}h}{D^*}, \nu\right)$$

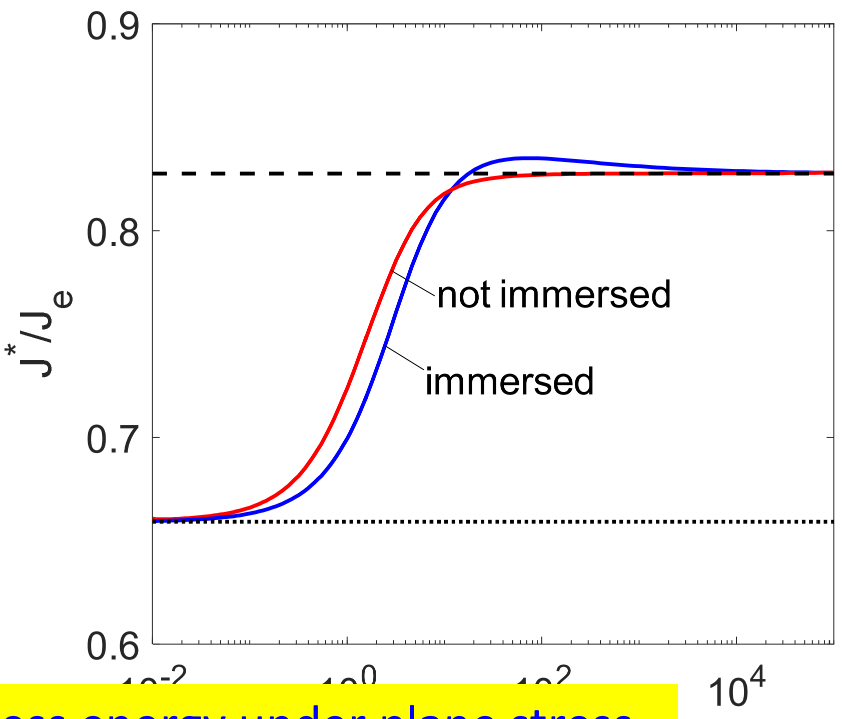


- "velocity toughening": the apparent fracture energy increases with crack speed.
- Effect of solvent viscosity: high viscosity -> low diffusivity -> low crack speed
- Effect of crack-tip soaking: immersed (wet) versus not-immersed (dry)

Yu, et al., J. Mech. Phys. Solids 118, 15-39 (2018).

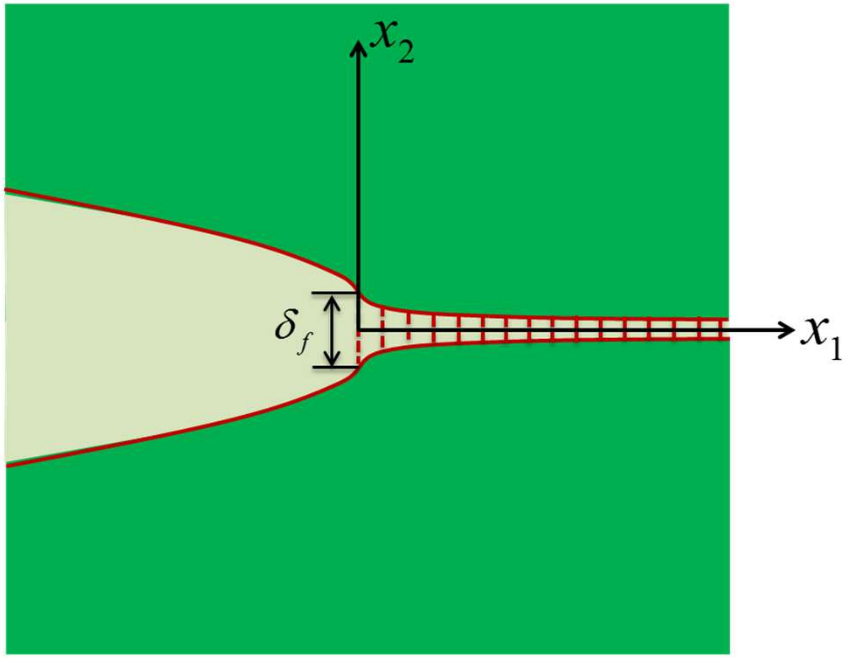
Plane Strain versus Plane Stress

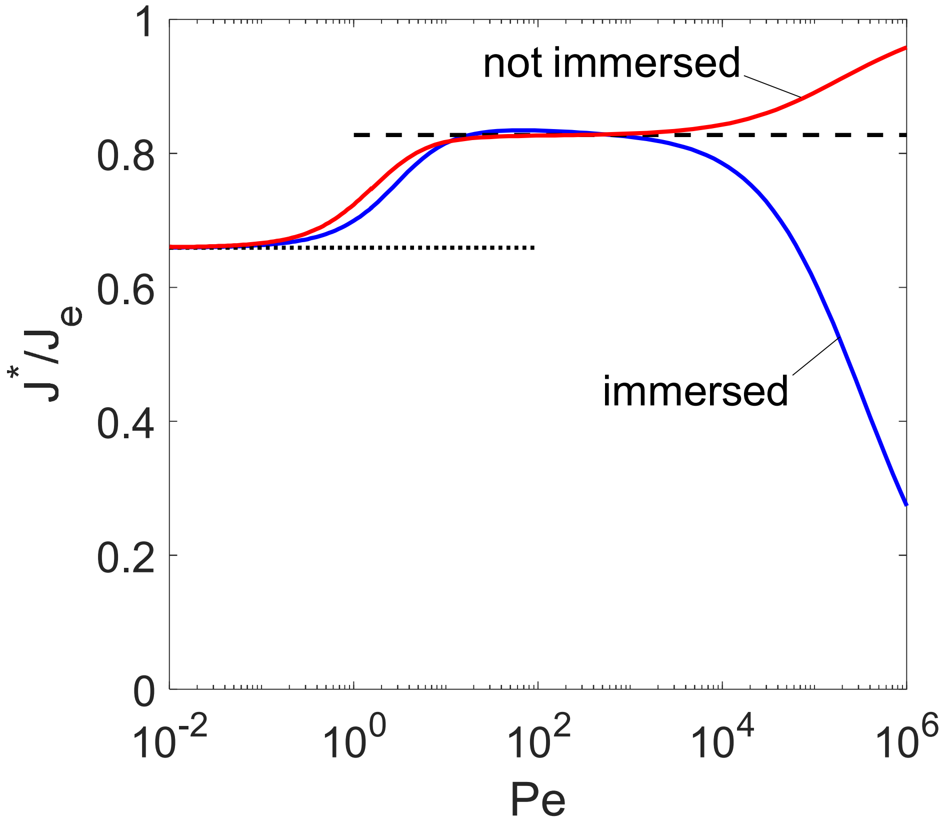




Solvent diffusion dissipates less energy under plane stress, opposite to the plastic dissipation in metals.

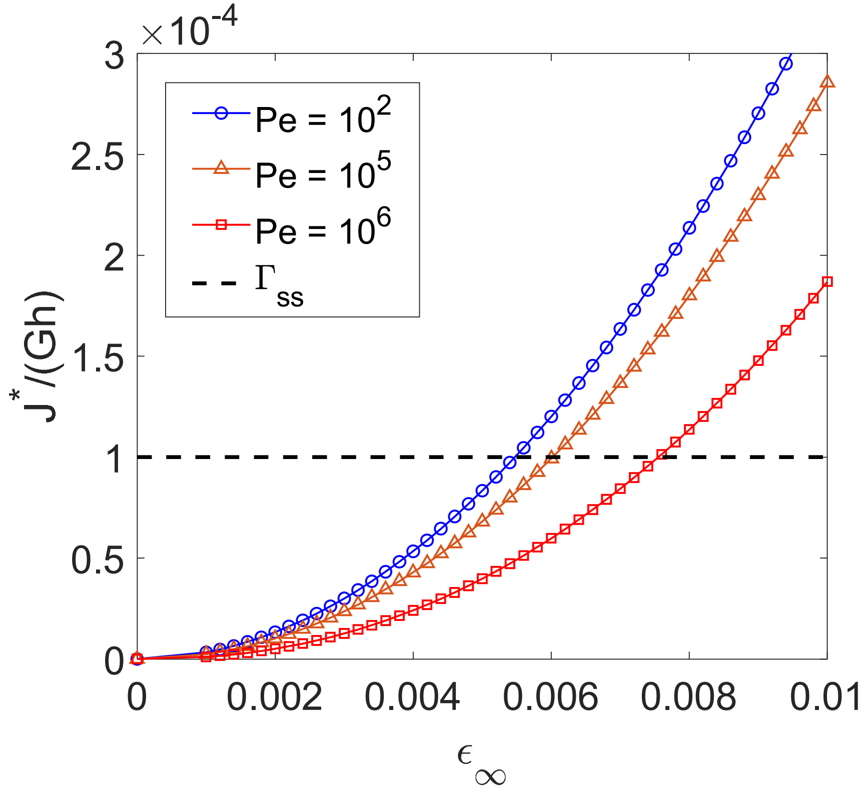
A poroelastic cohesive zone model

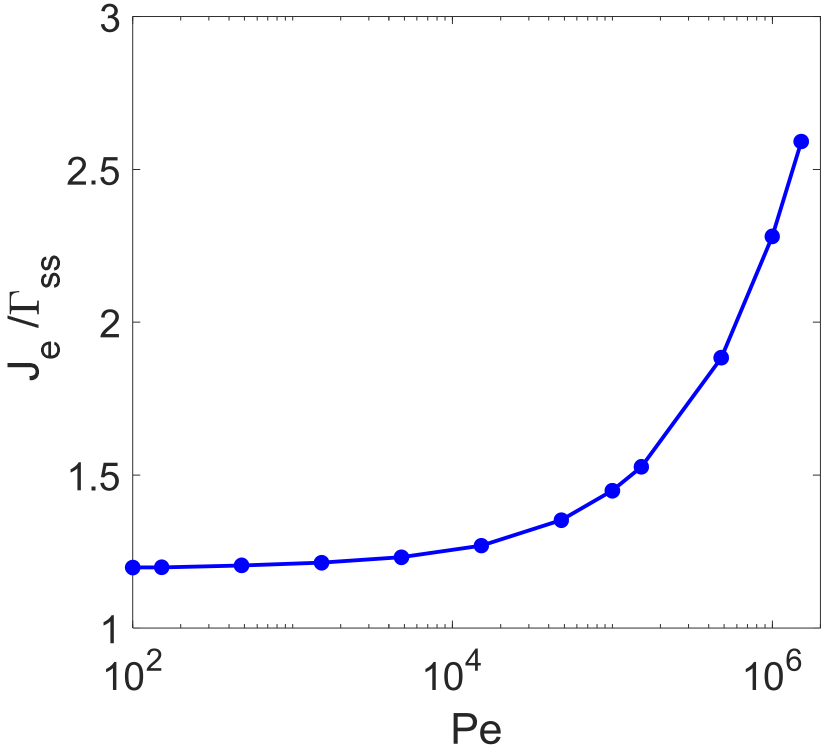




Yu, et al., J. Appl. Mech. (2019).

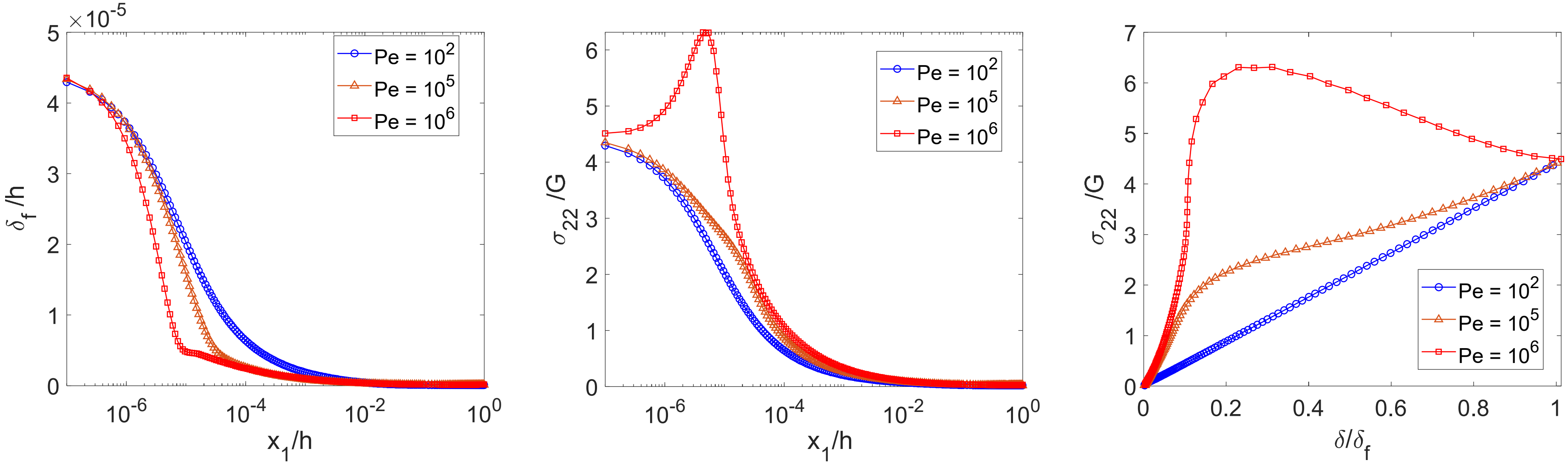
Rate-dependent fracture toughness





Yu, et al., J. Appl. Mech. (2019).

Rate-dependent traction-separation



Solvent diffusion within the cohesive introduces addition toughening effect, leading to rate-dependent fracture.

Yu, et al., J. Appl. Mech. (2019).

Summary: Fracture Criteria for Gels

- For a stationary pre-existing crack:

$$\boxed{J^*(t) = \Gamma_0}$$

→ **Delayed fracture**

- For steady-state crack growth:

$$\boxed{J^*(\dot{a}) = \Gamma_{SS}}$$

→ **Rate-dependent fracture**

- A poroelastic cohesive zone model predicts rate-dependent traction-separation relations and additional toughening.