

Energy-Trapping Torsional-Mode Resonators for Liquid Sensing

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Abstract—Thickness-shear mode quartz crystal microbalance (QCM) has been widely used as liquid-phase sensors, such as viscometers and bio-detectors. However, due to coupling between the in-plane shear motion and the out-of-plane flexure, when used in contact with or immersed in a liquid, the out-of-plane motion generates compressional waves in the liquid that reflect off the liquid surface and return to the crystal. This interference effect causes depth-sensitive perturbations in the sensor response, often undesirable. In this study, we show that torsional-mode resonators may be used for liquid sensing without the depth effect. Samples in form of stepped plates, circular decals, and convex contoured faces are machined in elastic plates (e.g., cast aluminum, stainless steel, and brass). A non-contact electromagnetic acoustic transducer (EMAT) was employed to drive torsional-mode vibrations. Efficient energy trapping was observed for first-order torsional modes, leading to high quality factors. When placed in contact with water, the resonance frequency of the torsional mode was found to be independent of the water depth, in contrast to depth-dependent frequency oscillation for the thickness-shear mode. Finite element analyses are conducted to understand the torsional-mode vibrations as well as the effect of material anisotropy.

I. INTRODUCTION

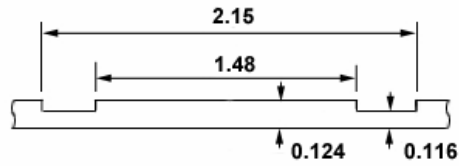
Used as a liquid-phase sensor, the resonance frequency of a thickness-shear mode (TSM) quartz crystal resonator changes as a function of the liquid properties [1-5]. The frequency change is proportional to the square root of the liquid density-viscosity product, assuming that a simple thickness-shear mode operates in the crystal plate and the motion is uniform across the crystal surface so that only a shear wave is generated in the viscous liquid. However, it has been observed that the frequency change in a TSM resonator oscillates periodically as a function of the liquid thickness [6-8]. It was found that, in a finite-sized crystal plate, the shear motion is non-uniform and typically couples to the out-of-plane motion of flexure, which generates a compressional wave in the liquid. Unlike the shear wave, the

compressional wave (i.e., sound waves) propagates in the viscous liquid with much less decaying. Consequently, it reflects from the liquid surface and back to the crystal, causing interference with the resonator. By coupling thickness-shear vibrations of a finite crystal plates and wave propagations in a Newtonian liquid (linearly viscous and compressive), Lee and Huang [9] developed a refined formula for the frequency change that effectively accounts for the depth-dependent frequency oscillation observed in experiments.

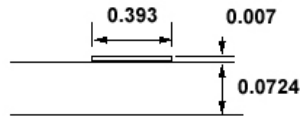
In practice, it is often undesirable to have the sensor response dependent upon the liquid thickness. To eliminate this effect, we propose to use torsional-mode resonators for liquid-phase sensing. In an elastically isotropic plate, torsional-mode vibrations do not coupled with flexural mode, thus no out-of-plane motion at the surface. Consequently, no compressional waves are generated in the adjacent liquid, and the sensor response can be related to the liquid properties without concerning the liquid thickness. Previously we have demonstrated that energy-trapping torsional-mode vibrations can be excited in elastic plates by using an electromagnetic acoustic transducer (EMAT) [10,11]. In this paper, experiments are conducted to study effect of liquid thickness on the torsional-mode resonators, in comparison with thickness-shear mode.

II. EXPERIMENTAL METHOD

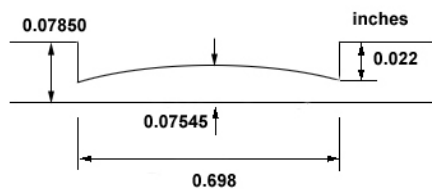
To achieve energy trapping, elastic plate samples with stepped thickness, attached decals, and convex contoured faces were used in this study, as illustrated in Fig. 1. First, the samples are made from cast aluminum, as an example of elastically isotropic materials. In this case, the stepped or contoured samples (A and C in Fig. 1) were machined in a 6000 series cast aluminum by removing the metal in the form of a moat, and for sample B a circular aluminum decal is bonded on top of an aluminum plate of uniform thickness. To study the effect of material anisotropy, stainless steel and brass samples were also used in the experiments.



(a) Sample A, a stepped cast aluminum plate



(b) Sample B, a uniform cast aluminum plate with a circular decal



(c) Sample C, a contoured cast aluminum plate

Fig. 1: Schematic illustration of aluminum sample plates (cross sectional view, with dimensions in inches).

Fig. 2 schematically shows the experimental set-up for a torsional-mode resonator. An EMAT, consisting of two permanent magnets and a spiral coil, was employed to set the samples into vibrations. A function generator was used to control the frequency of excitation signal. Upon termination of the excitation waveform, an oscilloscope read the decay of output voltage. From the initial voltage amplitude and its decay envelop, the resonance frequency and the quality factor of the resonator can be determined.

The working principle of EMAT is depicted in Fig. 3. First, the input oscillatory current in the spiral coil generates a magnetic field around the coil. This magnetic field is time variant and therefore generates eddy currents in the conductive sample plate (metals for the present study). By Lenz's law, the direction of the eddy currents flow in the opposite direction to the input current in the coil. Meanwhile, the two permanent magnets are placed directly under the coil such that the direction of their magnetic field flips 180° from one side to the other. Under such a permanent magnetic field and the eddy currents, the sample is subjected to the Lorentz force and thus vibrates in the same frequency as the input current. The direction of the resultant force depends on the relative placement of two magnets. By changing the direction and the location of EMAT, various vibration modes can be excited. Of interest in the present study are torsional modes and thickness-shear modes. The former is excited by placing the EMAT away from the center of circular plate samples and orienting to produce a resultant force in the circumferential direction.

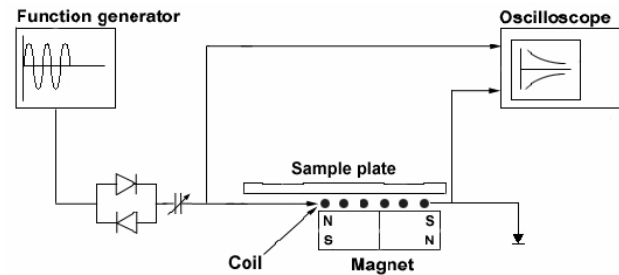


Fig. 2: Schematic of the experimental set-up.

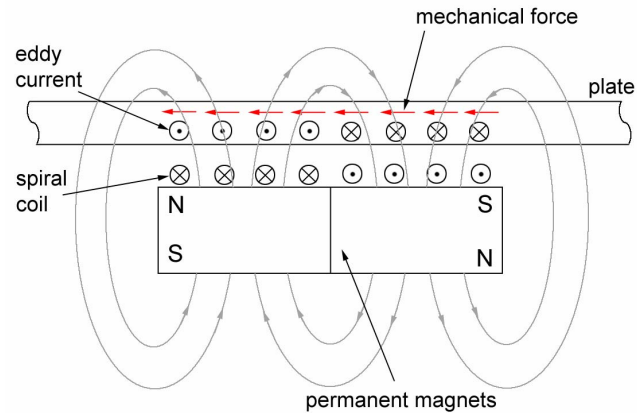


Fig. 3: The working principle of EMAT.

The latter is excited by placing the EMAT right at the center of circular plates, thus completely eliminating circumferential component. In a previous study [12], by using one permanent magnet, radially symmetric thickness-shear modes were excited in copper-coated silicon membrane resonators.

A series of tests were conducted to measure the surface motion at resonances in order to distinguish between torsional modes and thickness-shear modes. First, taking advantage of its non-contact property, the EMAT-induced traction was shifted along an arbitrary radial line of the sample, and we observed that the initial voltage amplitude varied with one or more peaks. For torsional modes, the peaks occur at radially symmetric locations corresponding to displacement maxima of the mode contour. Next, a stylus tipped with an absorbing material was used to probe the plate surface. We observed a sharp minimum in the output amplitude when the stylus touched the surface at locations with displacement maxima. When the stylus probe was moved along a set of radial lines through 360 degrees, the radial distances of the displacement maxima did not change, indicating radially symmetric motion which is characteristic of torsional modes. On the other hand, probing the surface outside the stepped or contoured region has no effect on the amplitude, confirming that the motion is trapped. The absorbing stylus technique, though useful, cannot measure the direction and relative amplitudes of the displacement. We

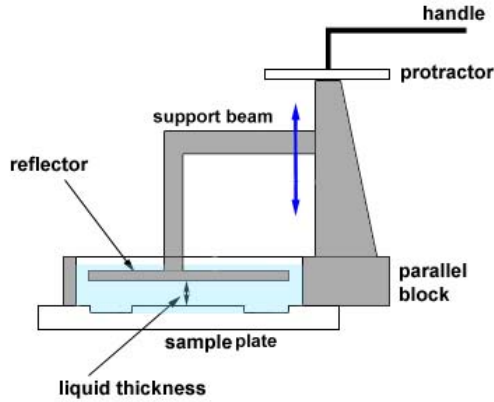


Fig. 4: Set-up for liquid effect experiments.

used a pick-up coil to probe the direction. Vibration of the metal plate in the presence of a magnetic field will induce electrical currents normal to the displacement direction. A small pick-up coil with windings adjacent and parallel to the top surface of the sample will have an induced voltage proportional to the current in the plate. The maximum voltage occurs when the coil windings are aligned along the current direction (normal to the displacement direction) and positioned at a displacement maximum. Torsional modes have circumferential displacements in the plane of the plate. These modes were confirmed as the pick-up coil had maximum output when it was aligned in a radial direction (normal to the motion), and the amplitude and phase of the coil output were independent of the radial angle. For thickness-shear modes, the displacement maximum occurs at the center, and the radial symmetry is lost.

The effect of liquid was measured for both torsional and thickness-shear modes. The test fixture system is illustrated in Fig. 4. For the present study, distilled water is used as the liquid. The thickness of water is defined by the distance between the sample surface and a parallel reflector plate. By revolving the handle, the support beam of the reflector moves vertically to change the water thickness. One revolution of the handle corresponds to 317.5 μm . Measurements were made at every 30° of revolution, which is equivalent to a resolution of 26.5 μm for the water thickness. Note that the wavelength of compressional wave in water is about 3 mm at 500 kHz and 1.5 mm at 1MHz at room temperature [9]. Within this frequency range, the resolution is sufficient to reveal the effect of compressional waves in the water. Instead of directly measuring the change of resonant frequency due to water, we measured the output amplitude as a function of the water thickness while keep frequency fixed at the resonance of dry state. The amplitude change indirectly reflects the change of resonant frequency.

III. EXPERIMENTAL RESULTS

Resonant frequencies were determined by adjusting the pulse train frequency for peak initial amplitude as observed with the oscilloscope across the EMAT coil. Figure 5 shows

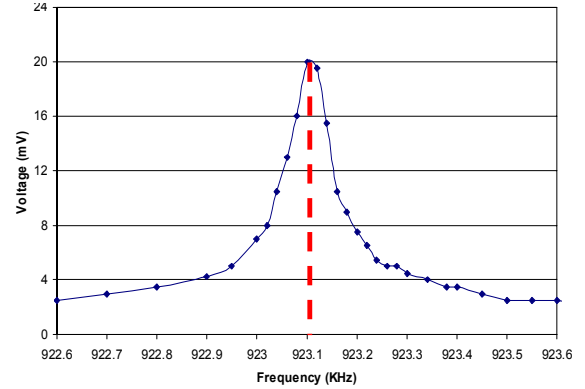


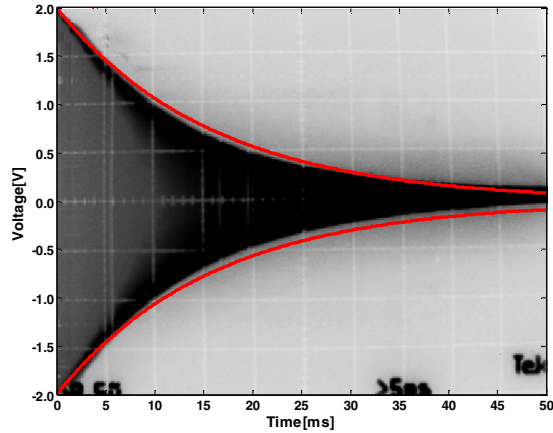
Fig. 5: Measured initial voltage amplitude as a function of frequency, with a resonance at 923.1 kHz for a torsional mode in a stepped aluminum sample.

an example of the measured initial amplitude as a function of frequency for stepped aluminum sample, which gives a resonance frequency at 923.1 kHz. Probing of the displacement maxima and directions confirmed that it was a torsional mode. For cast aluminum samples sketched in Fig. 1, with the EMAT placed at different locations, both torsional and thickness-shear modes were detected, and the measured resonance frequencies are listed in Table I. Note that all the frequencies are close to the corresponding cut-off frequencies of the first-order thickness modes, with frequencies of the torsional modes consistently higher than those of thickness-shear modes.

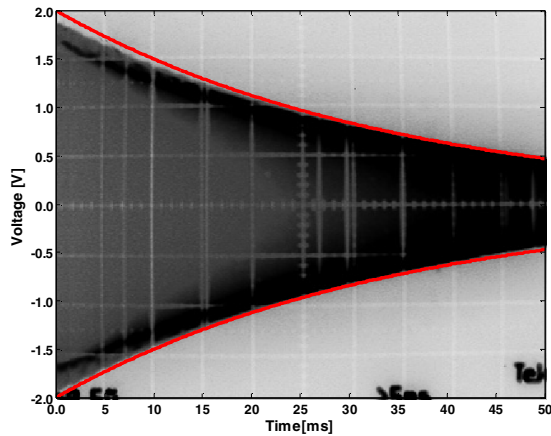
TABLE I. MEASURED RESONANCE FREQUENCIES FOR ALUMINUM SAMPLES SKETCHED IN FIG. 1

Mode Types	Samples		
	A	B	C
Torsional	506 kHz	810 kHz	933 kHz
Thickness-shear	503 kHz	808 kHz	904 kHz

The quality factors (Q-factor) of the resonators can be determined from the half width of the measured spectra such as that shown in Fig. 5. However, the frequency resolution of the equipment we had on hand limited us to relatively low Q. For this reason, a small section of absorbing tape on the sample was used in this measurement to suppress Q, with negligible effect on the resonance frequency. The Q-factor in this case is 12,300. Much higher Q-factors can be determined from the decay envelope observed in the oscilloscope upon termination of the excitation waveform at resonance frequencies. Figure 6 shows the decay curves for thickness-shear mode and torsional mode of an identical sample plate. The Q-factor is determined by fitting the envelop with a function, $V = V_0 \exp(-\pi f t / Q)$, where f is the resonance frequency and t is the time. For the thickness-shear mode in Fig. 6(a), the resonance frequency is 931 kHz, and the Q is determined to be 46,416. For the torsional mode in Fig. 6(b), the resonance frequency is 960 kHz and the Q is 104,425, much higher than the thickness-shear mode. For all tested



(a) Thickness-shear mode at 931 kHz.

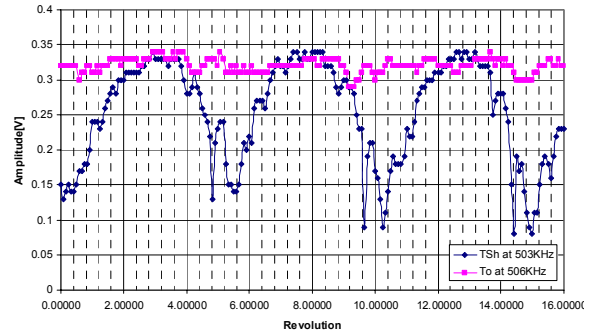


(b) Torsional mode at 960 kHz.

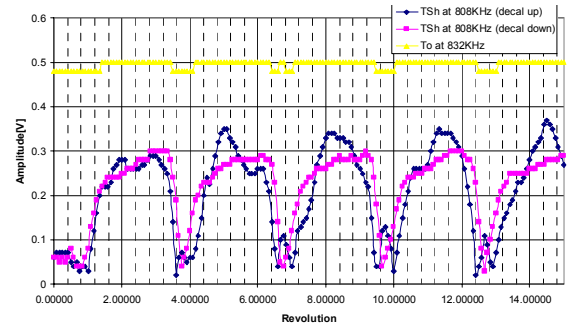
Fig. 6: Comparison of decay envelopes and quality factors for thickness-shear and torsional modes of an identical sample.

samples in this study, the Q-factors of torsional modes are higher than that of thickness-shear mode. As will be discussed later, the high-Q of the torsional modes is a result of better energy trapping in the axisymmetric samples.

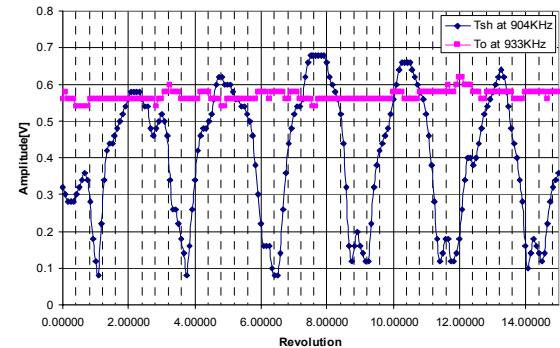
The measured effect of water thickness is shown in Fig. 7 for the cast aluminum samples. For each sample, effects on both thickness-shear mode and torsional mode were measured for comparison. The resonance frequencies were first determined at the dry condition, as described earlier. Then, each sample was brought in contact with water as illustrated in Fig. 4. With the frequency fixed at the corresponding resonances, the initial output magnitude was recorded as the water thickness increases. The initial gap between the sample surface and the reflector was not measured to a sufficient accuracy, and thus not reported. Rather, the relative change of the amplitude with respect to the water thickness was plotted in terms of revolutions. For all three samples, the amplitude changes periodically for thickness-shear modes, reflecting the periodical change in resonance frequencies. On the other hand, the amplitude of torsional modes is essentially independent of the water



(a) Sample A, stepped aluminum plate



(b) Sample B, aluminum plate with decal



(c) Sample C, contoured aluminum plate

Fig. 7: Effect of water thickness on thickness-shear and torsional modes of cast aluminum samples sketched in Fig. 1.

thickness, indicating thickness-insensitive resonance frequencies for torsional-mode resonators. As discussed earlier, the difference between thickness-shear mode and torsional mode in responses to water thickness is attributed to the effect of compressional waves in the liquid. The thickness-shear mode couples with flexural motion, thus strongly affected by compressional waves. The period in the amplitude changes in Fig. 7 roughly equals the half wavelength of the compressional wave in water at the given frequency, in agreement with theoretical predictions [9]. For torsional modes, the effect of compressional wave is negligible due to pure in-plane surface motion of the plate. We also observed double peaks in the responses of thickness-shear modes. The distance between these two

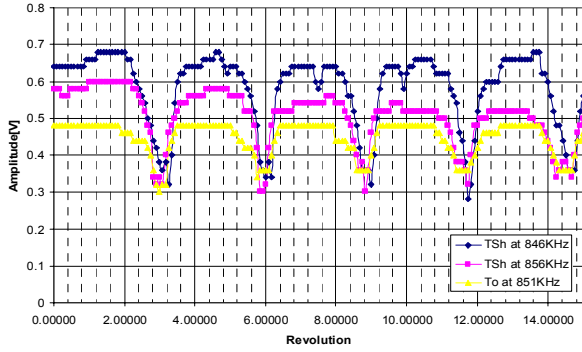


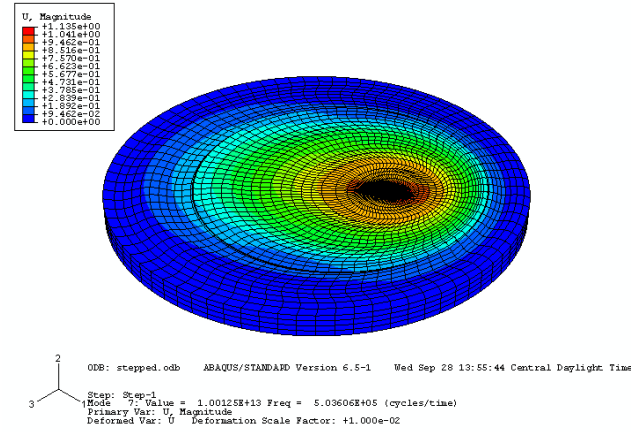
Fig. 8: Effect of water thickness on thickness-shear modes and torsional modes of a stepped plate of stainless steel.

peaks is roughly the thickness change for the stepped samples, as a result of different reflections of the compressional waves due to the thickness step. To confirm this conjecture, the sample with a circular decal was flipped over so that the flat surface is now in contact with water. Consequently, the change of water thickness is eliminated, and no double peaks was observed, as shown in Fig. 7(b). For contoured sample in Fig. 7(c), double peaks were also observed. Here, the edge of the contoured region has a thickness step, which could be the cause for the double peaks.

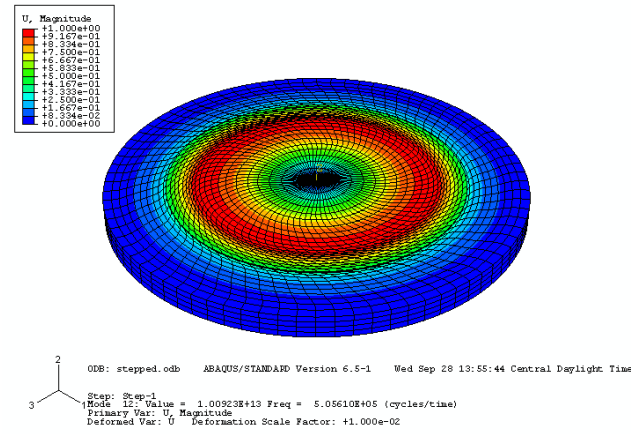
In addition to cast aluminum, which can be reasonably considered as elastically isotropic, anisotropic materials such as extruded stainless steel and brass were also used to make the sample plates. Figure 8 shows the effect of water thickness on a stepped stainless steel plate. Due to anisotropy, two different thickness-shear modes were detected at 846 kHz and 856 kHz, respectively, with the EMAT orientated in orthogonal directions. For both thickness-shear modes, the output amplitudes changed periodically as the water thickness increases. We also detected a torsional-like mode at 851 kHz. However, the output amplitude of this mode also changed periodically with respect to the water thickness, but with a relatively smaller magnitude compared to the two thickness-shear modes. It is believed that, for anisotropic materials, torsional mode in general couples with other modes including flexural, leading to cyclic responses due to the effect of compressional waves in the liquid. It may be possible to design the geometry of the step or contouring to reduce the coupling and thus the thickness-sensitivity.

IV. FINITE ELEMENT ANALYSIS

Finite element analyses (FEA) were performed for both torsional and thickness-shear modes. The commercial FEA package, ABAQUS, was used. Three-dimensional brick elements of quadratic order were employed in the analyses. Figure 9 shows displacement contours of a stepped aluminum plate (Sample A in Fig. 1) at its fundamental thickness modes. In the analysis, the shear modulus of aluminum was taken to be 26 GPa and the mass density is 2700 kg/m³. The resonance frequency of the fundamental thickness-shear mode is found to be 503.6 kHz, and the first torsional mode is found at 505.6 kHz, both in close



(a) Thickness shear mode at 503.6 kHz.



(b) Torsional mode at 505.6 kHz

Fig. 9: Finite element analyses of Sample A.

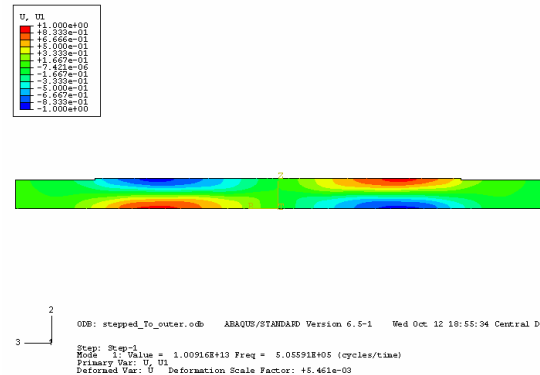
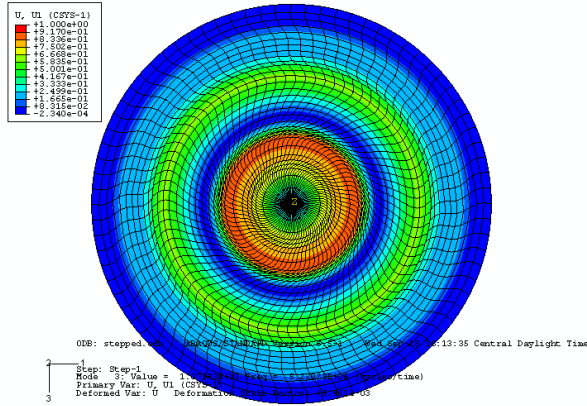
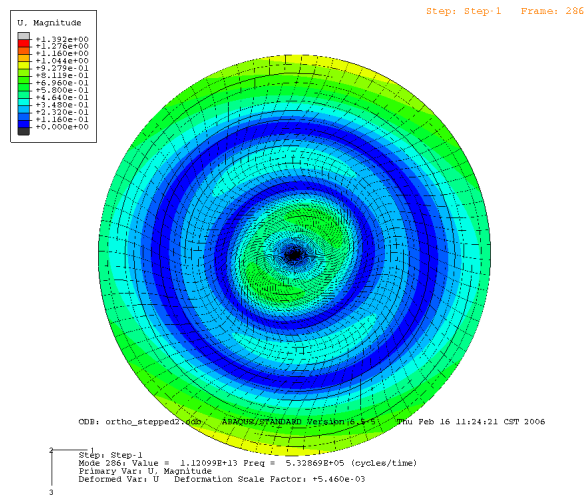


Fig. 10: Cross-sectional view of the circumferential displacement for the torsional mode at 505.6 kHz from FEA (Sample A).

agreement with the measurements (Table I). The displacement contours illustrate the radial symmetry for the torsional mode as opposed to the translational motion for the thickness-shear mode. Figure 10 shows the cross-sectional view of the circumferential displacement for the torsional mode. The 1st order thickness variation, the location of



(a) Isotropic material: first overtone of torsional mode.



(b) Anisotropic material: oval shaped “quasi-torsional” mode.

Fig. 11: FEA results for isotropic and anisotropic materials.

displacement maximum, and the effect of energy trapping are clearly revealed in this mode plot.

Furthermore, the effect of anisotropy was investigated by FEA. Figure 11 shows two mode plots, again, for Sample A, with (a) for the 1st overtone of the fundamental torsional mode and (b) for an anisotropic material of the same dimension. The elastic properties of the anisotropic material were generated arbitrarily by perturbing the elastic properties of aluminum, for purpose of comparison. The elastic modulus exhibits an orthotropic type. As a result, instead of circular displacement contours of the torsional mode in (a), an oval shaped “quasi-torsional” mode was obtained in (b).

V. SUMMARY

In this paper, EMAT-driven, energy-trapping torsional-mode resonators are demonstrated, with higher quality factors than thickness shear mode. When it is in contact with liquids, the responses of torsional-mode resonators show no dependence on the water thickness, as opposed to the cyclic responses for thickness-shear modes. The characteristics of energy-trapped torsional modes were revealed by finite element analyses. The effect of material anisotropy was investigated by both experiments and FEA.

REFERENCES

- [1] K.K. Kanazawa and J.G. Gordon, II, “Frequency of a quartz microbalance in contact with liquid.” *Anal. Chem.*, vol. 57, pp. 1771-1772, 1985.
- [2] F. Josse, Z.A. Shana, D.E. Radtke, and D.T. Haworth, “Analysis of piezoelectric bulk-acoustic-wave resonators as detectors in viscous conductive liquids.” *IEEE Trans. Ultrason. Ferroelect. Freq. Contr.*, vol. 37, pp. 359-368, 1990.
- [3] S.J. Martin, G.C. Frye, and K.O. Wessendorf, “Sensing liquid properties with thickness-shear mode resonators.” *Sensors and Actuators A*, vol. 44, pp. 209-218, 1994.
- [4] R. Thalhammer, S. Braun, B. Devcic-Kuhar, M. Gröschl, F. Trampler, E. Benes, H. Nowotny, and M. Kostal, “Viscosity sensor utilizing a piezoelectric thickness shear sandwich resonator.” *IEEE Trans. Ultrason. Ferroelect. Freq. Contr.*, vol. 45, pp. 1331-1340, 1998.
- [5] C. Behling, R. Lucklum, and P. Hauptmann, “Response of quartz-crystal resonators to gas and liquid analyte exposure.” *Sensors and Actuators A*, vol. 68, pp. 388-398, 1998.
- [6] L. Tessier, F. Patat, N. Schmitt, G. Feuillard, and M. Thomson, “Effect of the generation of compressional waves on the response of the thickness-shear mode acoustic wave sensor in liquids.” *Anal. Chem.*, vol. 66, pp. 3569-3574, 1994.
- [7] Z. Lin and M. D. Ward, “The role of longitudinal waves in quartz crystal microbalance applications in liquids.” *Anal. Chem.*, vol. 67, pp. 685-693, 1995.
- [8] T.W. Schneider and S.J. Martin, “Influence of compressional wave generation on thickness-shear mode resonator response in a fluid.” *Anal. Chem.*, vol. 67, pp. 3324-3335, 1995.
- [9] P.C.Y. Lee and R. Huang, “Effects of a liquid layer on thickness-shear vibrations of rectangular AT-cut quartz plates.” *IEEE Trans. Ultrason. Ferroelect. Freq. Contr.*, vol. 49, pp. 604-611, 2002.
- [10] T. Knowles, M.K. Kang, and R. Huang, “Trapped torsional vibrations in elastic plates.” *Appl. Phys. Lett.*, vol. 87, pp. 201911, 2005.
- [11] M.K. Kang, R. Huang, and T. Knowles, “Torsional vibrations of circular elastic plates with thickness steps.” *IEEE Trans. Ultrason. Ferroelect. Freq. Contr.*, vol. 53, pp. 349-359, 2006.
- [12] F. Lucklum, P. Hauptmann, N. Rooij, “Magnetic direct generation of acoustic resonances in silicon membranes.” *Meas. Sci. Technol.*, vol. 17, pp. 719-726, 2006.