ary of the feasible polyhedron, repeatedly increasing the objective function $c_1x_1 + \ldots + c_nx_n$ until either an optimal solution is found or it is established that no solution exists. In principle, the time required might be an exponential function of the number of variables, and this can occur in some contrived cases. In practice, however, the simplex method is highly efficient, typically requiring a number of steps which is just a small multiple of the number of variables. Linear programs in thousands or even millions of variables are routinely solved using the simplex method on modern computers. Efficient, highly sophisticated implementations are widely available in the form of computer software packages.

Interior-point Methods. In 1979 Leonid Khachian presented the ellipsoid method, guaranteed to solve any linear program in a number of steps, which is a polynomial function of the amount of data defining the linear program. Consequently, the ellipsoid method is faster than the simplex method in contrived cases where the simplex method performs poorly. In practice, however, the simplex method is far superior to the ellipsoid method. In 1984 mathematician Narendra Karmarkar introduced an interior-point method for linear programming, combining the desirable theoretical properties of the ellipsoid method and the practical advantages of the simplex method. Its success initiated an explosion in the development of interior-point methods. These do not pass from vertex to vertex but instead pass only through the interior of the feasible region. Though this property is easy to state, the analysis of interior-point methods is a subtle subject that is much less easily understood than the behavior of the simplex method. Interior-point methods are now generally considered competitive with the simplex method in most, though not all, applications, and sophisticated software packages implementing them are now available. Whether they will ultimately replace the simplex method in industrial applications is not clear.

An essential component of both the simplex method and interior-point methods is the solution of systems of linear equations, using techniques developed by Carl F. Gauss and A. L. Cholesky in the 19th century. (See MATHEMATICS; MATRIX.) Linear programming is part of a larger field known as mathematical programming, which includes disciplines such as integer programming, quadratic programming, nonlinear programming, and stochastic programming. (See OPERATIONS RESEARCH.)

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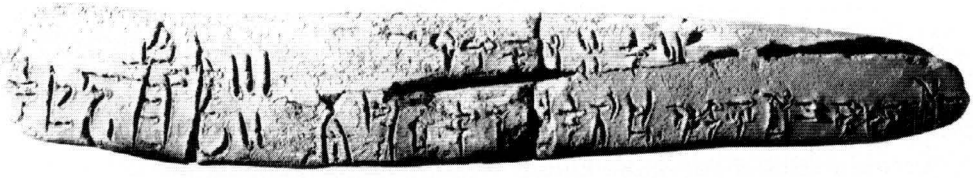
LINEAR SCRIPTS, scripts used by the Minoans and Mycenaeans in the Aegean area, primarily for economic record keeping. Linear A (c. 1850–1450 B.C.) and Linear B (c. 1450–1200 B.C.) are syllabic scripts with signs composed of straight or curving "linear" strokes. Linear B is inscribed on clay tablets and sealings and painted on transport pottery. Linear A occurs mainly on Minoan forms of tablets, sealings, and pottery, and on stone libation tables and miniature gold and silver double-axes and pins. The Mycenaeans devised the leaf-shaped tablet for single-topic transactions and the full-page-shaped tablet for longer lists and compilations.

Approximately 5,000 Linear B tablets have been found from the main centers of Thebes, Mycenae, Midea, Tiryns, Pylos, Knossos, and Khaniá (Canea). The Linear A tablets and sealings come from major and minor centers and town and sanctuary sites, but the tablets are many fewer in number (approximately 320 total) and contain much less information on average. Outside of Crete, Linear A is attested on the islands of Kíthera (Cythera), Thera, Melos, and Kéa, and at the site of Miletus.

The language of the texts in Linear A is unidentified. The language of the Linear B texts was deciphered in 1952 as an early form of Greek, although the records contain many non-Greek proper names (for persons, places, and deities), ethnic adjectives, and technical terms.

Linear B uses some 87 signs for sounds (phonograms). Linear A has just over 100 phonetic

Linear B leaf-shaped tablet, found in Pylos, c. 1200 B.C. (actual size). Reading from the left, the text is a record of five pairs of sets of body armor. The notation describes the armor phonetically (in the three leftmost characters) as "old" (*pa-ra-jo*); shows *armor* ideographically (a helmet and breastplate, fourth from left); uses a phonetic sign to qualify *armor* as a pair; and indicates *five* with vertical hatch marks (three over two).



signs. The Linear B phonograms consist of open syllabic signs (that is, signs for syllables ending in a vowel) for the essential Greek vowels (*a*, *e*, *i*, *o*, and *u*) and combinations of single or modified consonants plus vowels. An individual phonogram may represent a range of consonantal values, for example, sign *44 = *ke* or *ge* or *khe*. This tactic keeps the number of phonetic signs manageable. More than 70% of the Linear B phonograms are borrowed from Linear A. But 7 of 13 basic signs with the *o* vowel and 2 out of 13 with the *e* vowel had to be invented. Such features provide clues to the structure of Linear A, which is thought to be a three-vowel syllabary.

Both Linear A and Linear B use signs for objects or the words for objects (ideograms or logograms) and other signs for increments or units of measure and numbers (metrograms). Linear A and Linear B use the same ideograms for basic agricultural commodities (olives, grain, figs, wine, and olive oil) and livestock (goat, sheep, cow, pig). Linear B has a far fuller repertory of ideograms for chariots, armor, weapons, furniture, and ritual vessels. The two scripts are distinctive in their character systems and document typology. The idea of writing may have come from elsewhere, but the scripts were invented in the Aegean and evolved in response to the particular needs of Minoan-Mycenaean palatial civilization.

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LINEN, a strong, highly durable, absorbent cloth made from the bast fibers of the flax plant. Unbleached linen varies in color from gray-brown to creamy white.

Linen absorbs moisture more quickly than cotton and soils less readily. However, it is also more sensitive to chemicals than cotton and dyes less readily. Linen has a natural stiffness and luster, while cotton is a soft, dull fiber.

Leading flax growers include Ukraine, France, Belgium, the Netherlands, and Germany. Major linen cloth producers include Belgium, Great Britain and Northern Ireland, Italy, France, Austria, Germany, Poland, the Czech and Slovak republics, and Romania.

History. One of the oldest known textiles, linen was first produced during the Neolithic Period. It is mentioned in the Bible. The Egyptians grew high-quality flax, and pieces of linen that they used to wrap mummies are still in existence. The Greeks and Romans imported flax from Egypt, although the Greeks did not make use of flax for its fiber but for its seeds, from which they obtained oil for cooking. Flax was grown in northern Italy and its fibers used for cloth, ropes, and lampwicks. The Spanish grew fine flax and made wide use of linen for clothing. By the Middle Ages flax was cultivated throughout Europe for both its fiber and its seed. Until the development of automation, the treatment of flax for the production of linen changed very little over the centuries. (See FLAX.)

Uses. There is a wide variety of linen fabrics, ranging from very heavy canvas and carpet-backing material to fine damask. Linen fabrics also include bird's-eye; butcher linen; cambric, which is used for handkerchiefs and dress goods; and huckaback, which is a durable, very absorbent type of linen used for towels. Linen damask, which is woven on jacquard looms, is used for tablecloths, napkins, doilies, runners, and similar products. (See DAMASK.) Other linen fabrics are used for clothing, sheets and pillowcases, screen and stencil printing, and upholstery and drapery. Linen thread is used for sewing leather, and linen is also used in the manufacture of industrial belts and fire hoses.

Manufacture. Flax is harvested either by hand pulling, in which the entire plant is pulled up by the roots, or by machine, where it is cut off as close to the ground as possible. It is tied in bundles and left in the field to cure and dry. Seeds and leaves are then removed from the stalks by combing. This process is known as rippling. Next the bundles are untied, and the stalks undergo retting, in which they are partially rotted, or fermented, to loosen the fibers from the woody portions of the stem.

In dew retting, the stems are laid out on the ground and turned regularly. The retting is accomplished over two to three weeks by the action of dew, rain, sun, and bacteria from the soil. Pool retting is done in stagnant water, where bacterial action greatly increases the speed of the process. Stream retting is done in a flowing stream and takes from 5 to 15 days for the action on the fibers. The finest fibers are produced by this method. Tank retting, done in concrete tanks, is the quickest way to ret flax, but the results are often variable. However, tank retting can be done in six to eight days, and the fibers produced are light in color and not stained as they may be from other retting processes. Chemical retting is done with chemicals, such as soda ash, caustic soda, and oxalic acid. The reactions involved are difficult to control, and the process must be closely watched at all times.

Following retting, the stalks are thoroughly dried, either in fields or in special drying rooms. Drying the flax stalks stops the fermentation process of retting.

Next the fibers are separated from the woody portions of the stalk by breaking and scutching. The breaking of the woody parts of the stem is done by passing the stems through fluted metal rollers, which break the wood into small pieces called shives. In scutching, machines remove the shives from the fibers.

The fibers are then hackled, or combed, and short, tangled tow fibers, which have been broken during scutching, are separated from the long, choice line fibers. Hackling also begins the separation of the line fibers from one another. Hackling is done with a series of iron or steel combs. Each successive set of combs has a greater number of teeth per square inch than its predecessor.

The tow fibers are used for padding or rope. They are also carded to form slivers and spun into a coarse yarn that is used in the manufacture of heavier linen products, such as canvas.

After hackling, the line fibers are spread, either by hand or by machine, to form sliver, which is a continuous thin sheet that is then spun into yarn. Before spinning, the slivers are condensed and twisted in a process called roving.