ME 386P-2, Spring 2014 **Assigned:** January 14, 2014
Homework 1 **Due:** January 21, 2014

1. Define each of the following terms: (a) uniaxial stress, (b) biaxial stress, (c) balanced biaxial stress, (d) triaxial stress, (e) plane stress, (f) plane strain, and (g) pure shear.

2. Prove that the SI unit of stress is equivalent to energy per unit volume.

3. Calculate the following for the stress tensor given below in units of MPa.

(a) The principal stresses σ_1, σ_2, and σ_3.
(b) The corresponding principal directions, in terms of unit vectors $\hat{v}_1$, $\hat{v}_2$, and $\hat{v}_3$.
(c) The hydrostatic stress, σ_{H}.

$$\sigma = \begin{bmatrix} 5 & 1 & 3 \\ 1 & 8 & 1 \\ 3 & 1 & 5 \end{bmatrix} \text{ MPa}$$

4. Consider an isotropic block of steel ($E = 210$ GPa and $\nu = 0.2$) which is subjected to elastic strains $\epsilon_x = 0.05$ and $\epsilon_y = -0.05$. Calculate: (a) the volume change and (b) the stress state. Are these stresses and strains realistic for elastic deformation?

5. Prove that for linear, isotropic elasticity:

$$G = \frac{E}{2(1+\nu)} \quad .$$

The typical route for this proof is to consider the following stress tensor,

$$\sigma_{ij} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & -\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ,$$

and its equivalent tensor in an orientation rotated by 45° about the z-axis. Both tensors describe a state of pure shear stress. Apply Hooke's law for normal stress in the first orientation and Hooke's law for shear stress in the rotated orientation. Use the results make the proof.

6. Calculate the rotation matrix which rotates the following stress tensor into principal orientation and prove that it does so.

$$\sigma = \begin{bmatrix} 4.7 & -7.5 & 3.7 \\ -7.5 & -7.8 & 2.8 \\ 3.7 & 2.8 & 6.5 \end{bmatrix} \text{ MPa}$$