ME 386P-2, Spring 2014 **Assigned:** January 23, 2014
Homework 2 **Due:** January 30, 2014

1. Consider the following as cases of linear elasticity.

(a) How many independent elastic constants does an isotropic material have?

(b) Materials with a cubic crystal structure, such as aluminum and iron, generally have three independent elastic constants. Yet, polycrystalline forms of these materials often exhibit essentially isotropic elastic behavior. Explain how this is possible.

2. Name the stress state described by these two stress tensors, both given in units of MPa, and explain how these two tensors are related to each other:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 5 \\ 0 & 5 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -5 \end{bmatrix} .$$

3. Calculate the following for the stress tensor given below in units of MPa.

(a) The stress deviator tensor (deviatoric stress tensor).

(b) The first invariant of the stress deviator, J_1.

(c) The second invariant of the stress deviator, J_2.

$$\sigma = \begin{bmatrix} 5 & 1 & 3 \\ 1 & 8 & 1 \\ 3 & 1 & 5 \end{bmatrix} \text{MPa}$$

4. Our Engineering Library has numerous materials standards available on-line, including the ASTM (American Society for Testing and Materials) standards for mechanical testing. These standards include geometries and dimensions of typical specimens and standards for test conditions to use when measuring particular material properties. Locate the designations and titles of appropriate ASTM standards for the following:

(a) Tensile testing of metallic materials

(b) Flexural strength testing of ceramics

(c) Plane-strain fracture toughness testing of metals

(d) Fracture toughness testing of ceramics

Remember to log out of your session after you are done so that our University license to the standards web site will be made available for the next user. (You might also check the homework section of the class web page.)

5. Consider the cylindrical rod shown below. It is subjected to deformation with time, t, such that the following displacement field results,

$$\begin{aligned} u_x &= 0.1\,t\,x \\ u_y &= -0.05\,t\,y \\ u_z &= -0.05\,t\,z \end{aligned}$$

as written in component of the displacement vector, u_i.

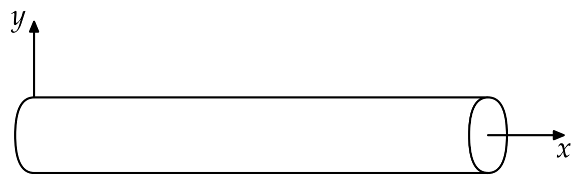
(a) Calculate the components of the resulting small-strain tensor.

(b) Calculate the components of the resulting Eulerian finite-strain tensor.

(c) Plot for $0 \le t \le 4$ the x and y normal strains from both small-strain and finite-strain theories.

(d) Calculate the time, t, at which E_{xx} deviates from ϵ_{xx} by 1%, and calculate both these strains at that point.

(e) Describe what is special about this deformation field. (Hint: Consider Poisson's ratio.)



6. Consider the case of an elastically anisotropic crystal of α-Fe. The elastic stiffness constants of this crystal are as follows, in units of GPa.

$E_{\langle 111 \rangle}$	$E_{\langle 100 \rangle}$	c_{11}	c_{12}	c_{44}
276	129	237	141	116

The 1, 2 and 3 directions correspond to the edges of the BCC unit cell of iron, and are taken to be the same as the x, y and z directions, respectively, in defining this problem. The following stress is applied to this crystal, in terms of this reference frame,

$$\sigma_{ij} = \begin{bmatrix} 15 & 15 & 0 \\ 15 & 15 & 0 \\ 0 & 0 & 30 \end{bmatrix} \text{ MPa.}$$

(a) Calculate and report the principal stress values and the corresponding principal directions. This is one of the special stress states considered in HW 1. Which is it and why?

(b) Use the elastic constants provided to calculate the strain tensor which results from this stress state. Report the strain tensor in the same form as that given above for the stress tensor.

(c) Calculate and report the principal strains and the corresponding principal directions.

  i. Are the principal directions for stress and strain the same? Comment on your result.

  ii. Are the number of independent principal stresses and strains the same? If not, then why?

(d) Calculate and report i. the hydrostatic stress, ii. the stress deviator and iii. the dilatation.