ME 386P-2, Spring 2014 **Assigned:** January 30, 2014
Homework 3 **Due:** February 6, 2014

1. Consider an infinitesimally small rectangular parallelepiped element from a strained material. Define its initial dimensions as (a, b, c) along the (x, y, z) directions, respectively. When this element is strained, its dimensions elongate by $(\delta a, \delta b, \delta c)$ along the (x, y, z) directions. Answer the following.

(a) Calculate the dilatation of this element, $\Delta V/V_0$, in terms of its dimensions and the elongations $(\delta a, \delta b, \delta c)$.

(b) Prove that $\Delta V/V_0 = \epsilon_x + \epsilon_y + \epsilon_z$ when the δ terms of elongation are small, i.e., small strain theory is valid. Note that $\epsilon_x = \delta a/a$, for example.

(c) Consider the isotropic linear thermal expansion of this element using the linear thermal expansion coefficient $\alpha = (\delta L/L)/\Delta T$. (If thermal expansion were anisotropic, then the tensor form of the thermal expansion coefficient, α_{ij}, would be necessary.) The resulting displacement field is $\vec{u} = \alpha \Delta T (x\hat{\imath} + y\hat{\jmath} + z\hat{k})$, which can also be written as $u_i = \alpha \Delta T x_i$. Calculate the dilatation for temperature change ΔT using both the small-strain (ϵ_{ij}) and finite-strain (E_{ij}, which the book writes as G_{ij}) theories. Remark on the difference between these two predictions and calculate their difference for the case of $\alpha = 11.8 \times 10^{-6}$/K and $\Delta T = 25$ K. What would be the difference from dilatation calculated using the equation from (a) before simplification by assuming the δ terms to be small?

2. Prove the the Lamé coefficient Γ is undefined when $\nu = 0.5$. What implications does this have for the stiffness, c_{ij}, and compliance, s_{ij}, tensors? What implications does this have for calculating stresses from strains and *visa versa*?

3. Plot the Young's modulus for all possible directions in the (100), (110) and (123) planes for MgO (magnesium oxide). It is suggested that you make polar plots, in which radius is proportional to modulus and θ is the direction within the plane. The elastic moduli of MgO are $E_{\langle 100\rangle} = 247$ GPa and $E_{\langle 111\rangle} = 343$ GPa. (An example Octave script, that you may find useful, is available on the class secure web page.)

4. Consider an isotropic silica glass with $E = 80$GPa and $G = 31.5$ GPa and tungsten carbide (WC), which we will assume to be isotropic with $E = 530$ GPa and $G = 219$ GPa. Consider each material by itself and the two materials combined into a glass-matrix composite containing 50% WC particles, by volume, for the following.

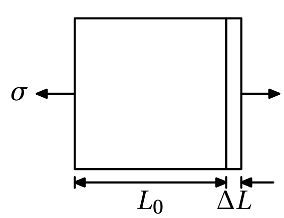
(a) Calculate the compliance matrix S_{ij} for each material alone.

(b) Calculate the effective compliance matrix for the composite, S_{ij}^{Reuss}, using the Reuss approximation.

(c) Calculate the effective stiffness matrix for the composite, C_{ij}^{Voigt}, using the Voigt approximation.

(d) Calculate the effective compliance, S_{ij}^{Voigt}, using the Voigt approximation and compare with the result of the Reuss approximation.

5. Consider a cube of rubber with initial, undeformed dimensions of $L \times L \times L$. This cube is subjected to a uniaxial stress σ, which elongates it by length ΔL in the direction of stress application. Answer the following, and remember that volume is conserved in deformation of rubber.



(a) Will stiffness of the rubber increase or decrease with increasing temperature? Give a brief, qualitative explanation as to why.

(b) Show that, for this problem, Finger's general constitutive equation ($\sigma_{ij} = GB_{ij} - p\delta_{ij}$) simplifies to the uniaxial constitutive model discussed in class,

$$\sigma = G\left(\lambda^2 - \frac{1}{\lambda}\right) \quad ,$$

where $\lambda = (L+\Delta L)/L_0$ is the stretch in the stressed direction. (*Hint:* Apply the condition of constant volume then solve for σ by eliminating p.)

(c) Convert your answer into engineering stress, s, to show that,

$$s = G\left(\lambda - \frac{1}{\lambda^2}\right) \quad ,$$