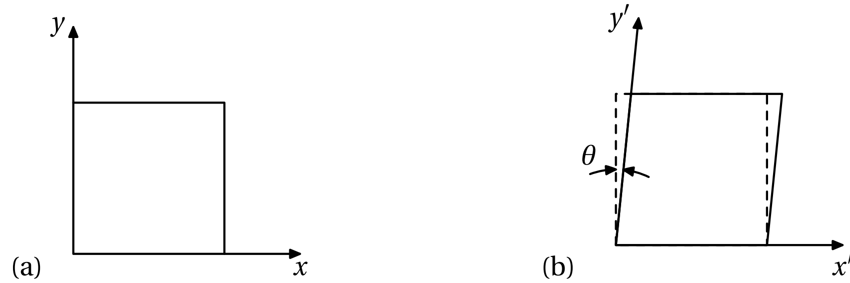
ME 386P-2, Spring 2011 **Assigned:** February 6, 2014
Homework 4 **Due:** February 11, 2014

1. Consider the simple shear deformation of a block of rubber, as discussed in lecture. This is illustrated below as a deformation from the undeformed configuration x_i in (a) to the deformed configuration x'_i in (b). The normal stresses σ_{yy} and σ_{zz} are zero.



The shear strain is defined as $\gamma = \tan\theta$. The deformed axes can be described in terms of the original axes as,

$$x' = x + \gamma\, y\,; \qquad y' = y\,; \qquad z' = z$$

The vector u_i describing the displacement of any point p to new position p' after deformation is described by,

$$u_x = \gamma\, y\,; \qquad u_y = 0\,; \qquad u_z = 0$$

Answer the following using this information.

(a) Calculate the Finger tensor (left Cauchy-Green tensor) using the definition

$$B_{ij} = \frac{\partial x'_i}{\partial x_k}\frac{\partial x'_j}{\partial x_k}$$

(b) Use Finger's constitutive relation to calculate the remaining unknown stresses.

(c) Calculate the finite-strain tensor, E_{ij}, and then use the small-strain assumption to calculate the small-strain tensor, ϵ_{ij} from E_{ij}.

2. Damping arises in α-Fe from the motion of C atoms between octahedral interstitial sites in the α-Fe lattice. This is the Snoek effect. The spacing between octahedral sites is approximately $x = a/\sqrt{2}$, where $a = 2.87 \times 10^{-10}$ m is the lattice parameter of α-Fe. The approximate distance through which an atom can diffuse is $x \approx \sqrt{D\,t}$, where D is the diffusion coefficient,

$$D = D_0 \exp\left(-\frac{Q}{R\,T}\right)$$

and t is the time for diffusion. For α-Fe, $D_0 = 4 \times 10^{-7}$ m^2/s and $Q = 80.1$ kJ/mole. (The universal gas constant is 8.314 J/mol·K.) Assume that the frequency for maximum anelastic damping in this system is approximately $f_r = 1/t$, where t is the time for a C atom to move between interstitial sites at room temperate, approximately 27 °C. The actual frequency for maximum damping at this temperature is 0.27 Hz. How closely does your calculation agree with this experimental result?

3. Answer the following using the given model for a viscoelastic material.

(a) For the Kelvin-Voigt model, assume a constant stress of σ_0 and an initial strain of zero. Derive an equation for strain as a function of time, assuming zero initial strain, and make a sketch of strain versus time.

(b) For the Maxwell model, assume a constant, positive strain and an initial stress of σ_0. Derive and equation for the stress as a function of time and make a sketch of stress versus time.