ME 386P-2, Spring 2014 Assigned: February 18, 2014
Homework: 5 Due: February 27, 2014

1. For the case of $\sigma_3 = 0$, plot the yield loci for the Tresca and von Mises yield criteria on axes of σ_2 versus σ_1, as shown in Figure 3.17 of your text. Indicate on your plot the positions corresponding to yielding in each of the following stress states for both the Tresca and von Mises yield criteria. Also, calculate the required value of the applied stress (either σ or τ) for yielding according to each of these yielding criteria. (Put your answers in terms of the uniaxial yield stress, s_y. If a stress is not listed, assume that it is zero.)

(a) Pure shear: $\tau_{xy} = \tau$
(b) Uniaxial tension: $\sigma_x = \sigma$
(c) Plane strain: $\sigma_x = 2\sigma_y = \sigma$, $\sigma_z = 0$

2. Consider a thin-walled pipe with an average radius of $R = 10$ cm and a thickness of $t = 1$ cm. This pipe is subjected to torsion by a torque, T, and a varying axial force, F. The torque causes a shear stress, τ, and the axial force causes a normal stress, σ, which can be approximated as follows,

$$\begin{aligned} \tau &= \frac{T}{2\pi R^2 t} \\ \sigma &= \frac{F}{2\pi R t} \end{aligned}$$

The uniaxial yield strength of the pipe was measured to be $s_y = 340$ MPa. Calculate the torque, T, required for yielding for the following cases of axial force: (a) $F = 0$, (b) $F = 534$ kN and (c) $F = -534$ kN. Use the von Mises yield criterion (J_2 flow theory) for your calculations.

3. Consider the definition of effective stress and a case of pure shear. The pure shear stress for plastic flow is τ_f. The tensile flow stress under uniaxial tension is σ_f. Assume that the effective stress, $\overline{\sigma}$, for plastic flow is equal for both conditions to prove that the conversion between shear and normal flow stresses is given by,

$$\tau_f = \frac{1}{\sqrt{3}}\sigma_f \quad .$$

4. The second invariant of the incremental strain tensor, $d\epsilon_{ij}$, is defined as,

$$I_2 = \frac{1}{2}\left(d\epsilon_{ij}\, d\epsilon_{ij} - d\epsilon_{ii}\, d\epsilon_{jj}\right) \quad .$$

For plastic deformation of metals, conservation of volume can be reasonably assumed. Define effective incremental strain as,

$$d\bar{\epsilon}_{ij} = \sqrt{2}\, C\, I_2^{1/2} \quad ,$$

where C is a constant. Complete the following.

(a) Use the above to prove that the effective incremental strain for metal plasticity can be written in the following form,

$$d\bar{\epsilon} = C\,(d\epsilon_1^2 + d\epsilon_2^2 + d\epsilon_3^2)^{1/2} \quad ,$$

where $d\epsilon_1$, $d\epsilon_2$ and $d\epsilon_3$ are the principal incremental strains.

(b) Assume that the incremental work of the effective stress and effective incremental strain is equivalent to that of the principal stresses and incremental strains, i.e.

$$\overline{\sigma}\, d\bar{\epsilon} = \sigma_1\, d\epsilon_1 + \sigma_2\, d\epsilon_2 + \sigma_3\, d\epsilon_3 \quad .$$

Use the case of uniaxial tension in an isotropic material to prove that $C = \sqrt{2/3}$

(c) How useful would such a definition of effective strain be in describing the plastic deformation of a solid which changes volume during plastic flow?

(d) Write the definition of effective strain shown in (a) in terms of general strains in x-y-z, including shear components. Then prove that the strain rate in pure shear, $\dot{\gamma} = 2\dot{\epsilon}_{xy}$ is equivalent to a normal strain rate of $\dot{\gamma} = \sqrt{3}\dot{\epsilon}$.

5. Tensile coupons are taken from a sheet of rolled aluminum alloy with the tensile axis in one of two different orientations. The first orientation is parallel the rolling direction, and the second orientation is perpendicular to the rolling direction, ie. along the long transverse direction in the sheet. The yield stress found from tensile tests parallel to the rolling direction is s_y. The yield stress in the long transverse direction is $0.9\,s_y$. In addition to these tests, a test in balanced biaxial tension was also conducted, ie. $\sigma_x = \sigma_y = \sigma$, which found yielding to occur when the biaxial stress reached a value of $\sigma = 0.95\,s_y$. From these data, use the Hill formulation to calculate the stress along the rolling direction, σ_x in terms of s_y, required to yield a sample of the sheet for the case of plane strain with $\epsilon_y = 0$ in the long transverse direction and $\sigma_z = 0$ perpendicular to the sheet; assume that $\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$.

6. Plot the 2-D yield locus, assuming $\sigma_3 = 0$, using Hill's formulation for a materials of normal anisotropy with R values of 0.5, 1.0, 1.5, and 2.0, respectively. To make this process easier, you may assume that the uniaxial yield stress is equal to unity, $s_y = 1$.

7. The case of plane-strain plasticity is often presented as a stress state for which $\sigma_2 = \sigma_1/2$. This holds for plastic deformation when $\epsilon_2 = 0$ and $\sigma_3 = 0$. Use the Levy-Mises equations to prove that this stress relationship holds for plane-strain plasticity.