Single-Crystal Plasticity

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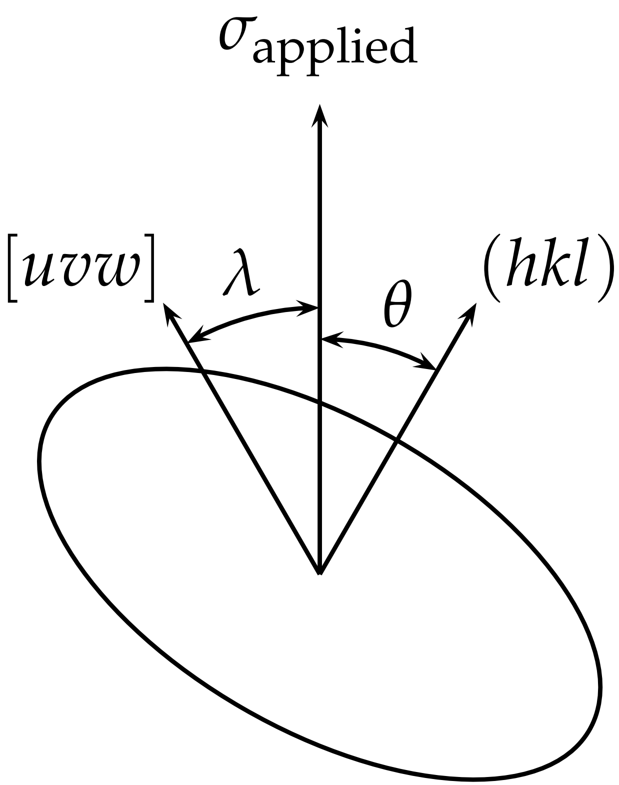
Schmid Factor



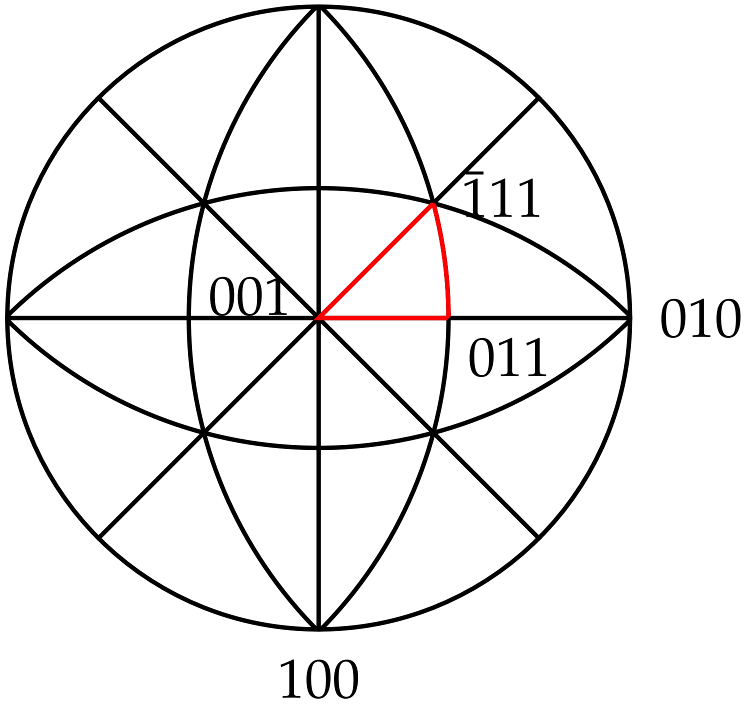
The relationship between applied uniaxial normal stress and the resolved shear stress on slip system $(hkl)[uvw]$ is given by the Schmid factor, m:

$$\tau_{\text{RSS}} = \sigma \cos\theta \cos\lambda = \sigma\, m$$

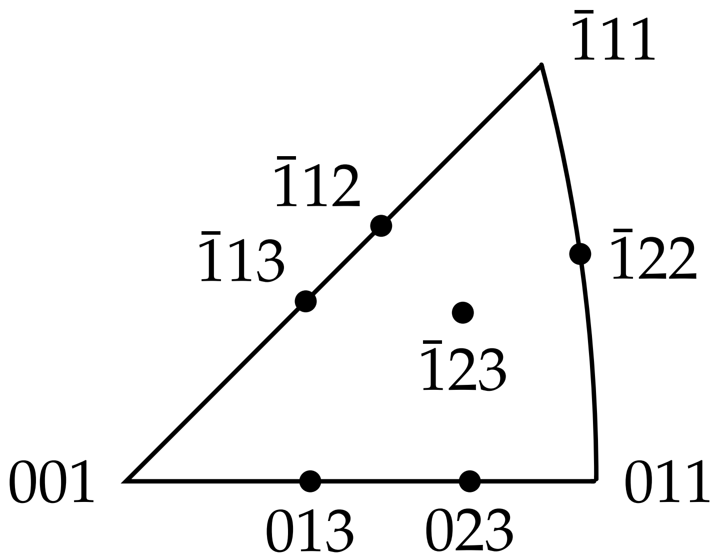
Slip occurs preferentially on the slip system with the largest m.

Sterographic Projection

For cubic crystals, the *standard triangle* contains all the information necessary to produce the *standard projection*.



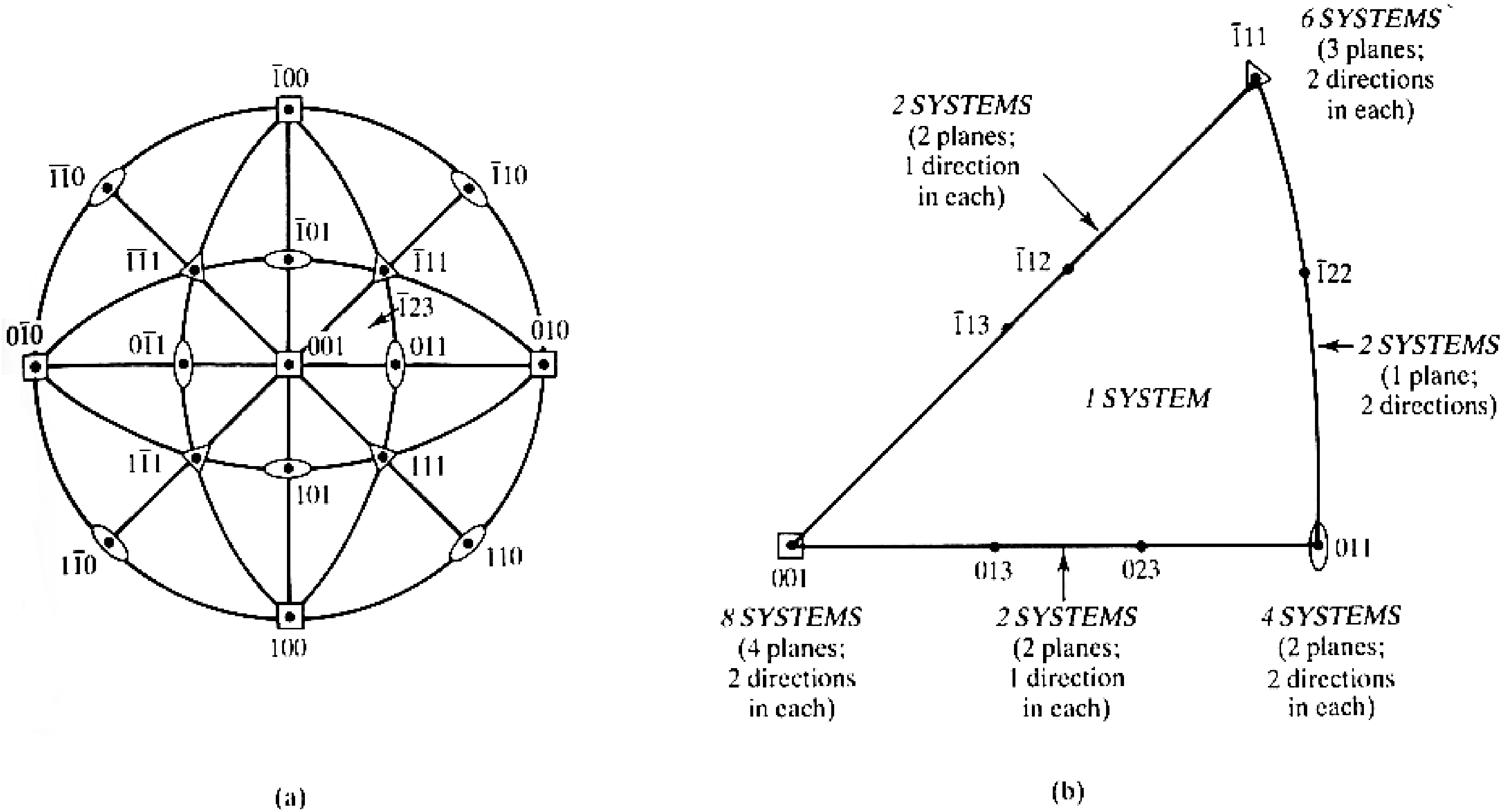
001 Stereographic Projection



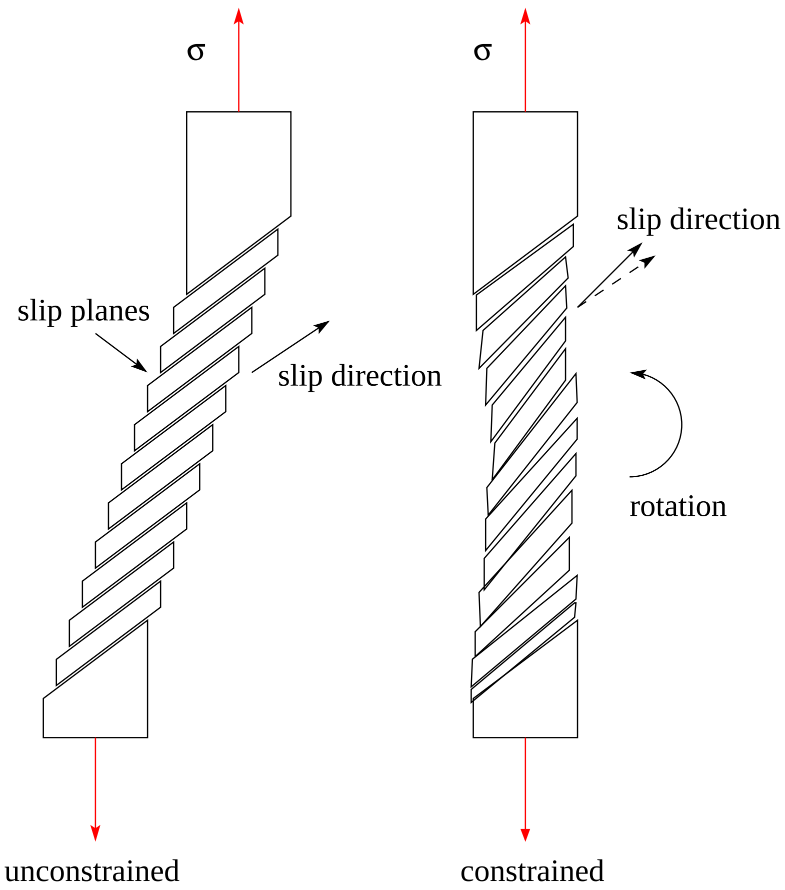
Standard Triangle

The Standard Triangle

Slip systems for FCC crystals described on the standard triangle.



Single Crystal Rotation



Tension: The slip direction rotates *toward* the tensile axis, while the slip plane normal rotates away.
Compression: The slip direction rotates *away* from the tensile axis.

Example:

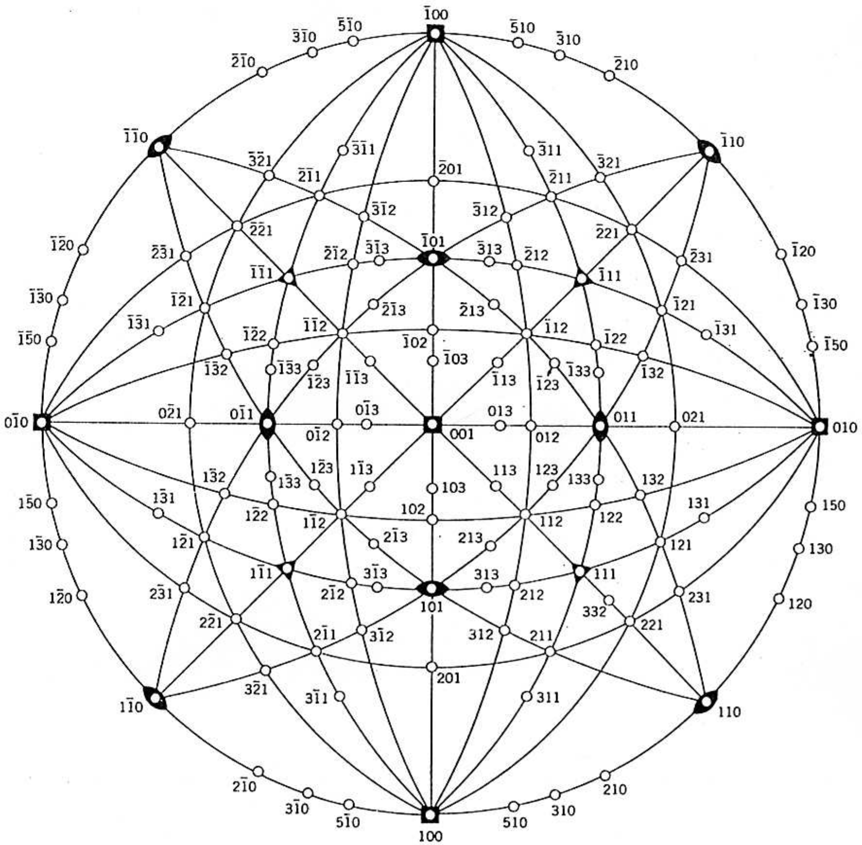
Consider deformation of a BCC single crystal which slips on the $\{110\}\langle 111\rangle$ systems and is initially loaded in tension along the $[123]$ direction.

Solution Steps:

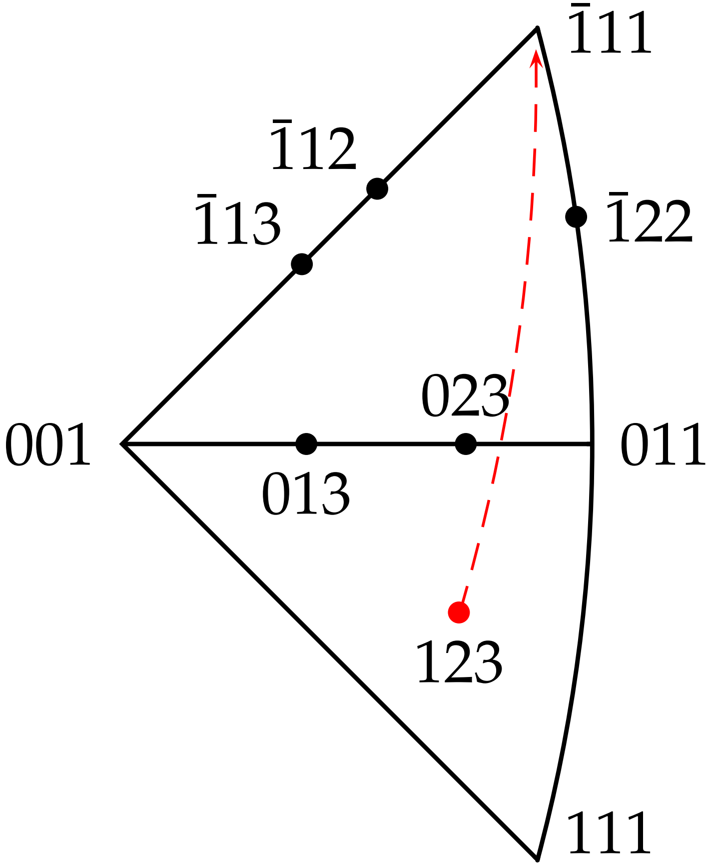
- Calculate Schmid factors to determine the preferred slip system.
- Use the standard stereographic projection to determine the rotation of the tensile axis.
- Follow the tensile axis rotation to determine the introduction of *conjugate slip systems*.

Slip Plane	Slip Direction	$\cos\theta$	$\cos\lambda$	m
(110)	$[\bar{1}11]$	0.56695	0.61721	0.34993
	$[1\bar{1}1]$	0.56695	0.30861	0.17496
$(1\bar{1}0)$	$[111]$	−0.18898	0.92582	−0.17496
	$[\bar{1}\bar{1}1]$	−0.18898	0.00000	0.00000
(011)	$[1\bar{1}1]$	0.94491	0.30861	0.29161
	$[\bar{1}\bar{1}1]$	0.94491	0.00000	0.00000
$(0\bar{1}1)$	$[111]$	0.18898	0.92582	0.17496
	$[\bar{1}11]$	0.18898	0.61721	0.11664
(101)	$[\bar{1}11]$	0.75593	0.61721	**0.46657**
	$[\bar{1}\bar{1}1]$	0.75593	0.00000	0.00000
$(\bar{1}01)$	$[111]$	0.37796	0.92582	0.34993
	$[1\bar{1}1]$	0.37796	0.30861	0.11664

The *only* initially active slip system is the $(101)[\bar{1}11]$. Use the stereographic projection to visualize the rotation of the tensile axis toward the slid direction.

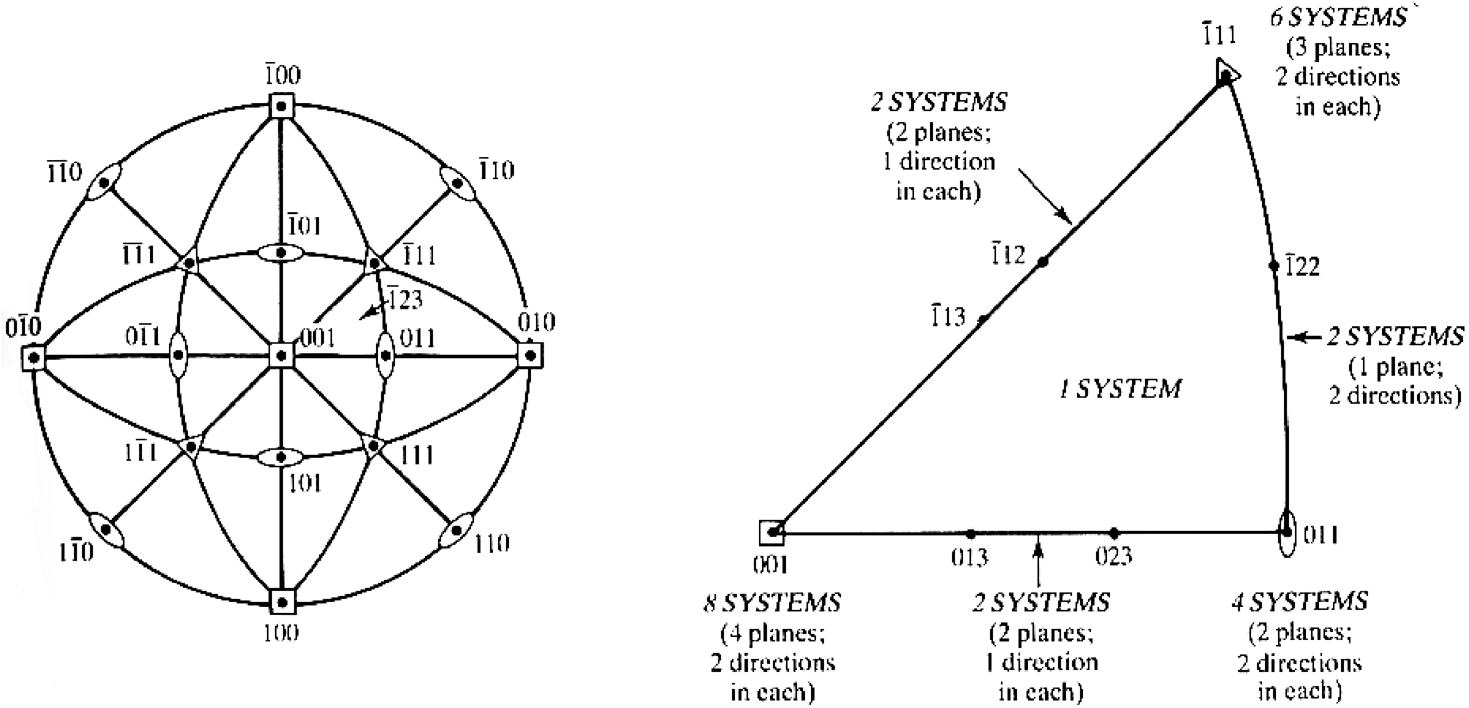


Note that [123] lies below the standard triangle. The tensile axis, originally along [123], will rotate toward the slip direction, $[\bar{1}11]$.



The tensile axis rotates from the [123] direction toward the $[\bar{1}11]$ direction on the standard stereographic projection. The great circle on which it rotates crosses the edge of the standard triangle. As it crosses this line, a *conjugate* slip system can activate!

Two slip systems occur on the bottom edge of the standard triangle. This is also true for BCC systems, although this figure is for FCC.



(a) (b)

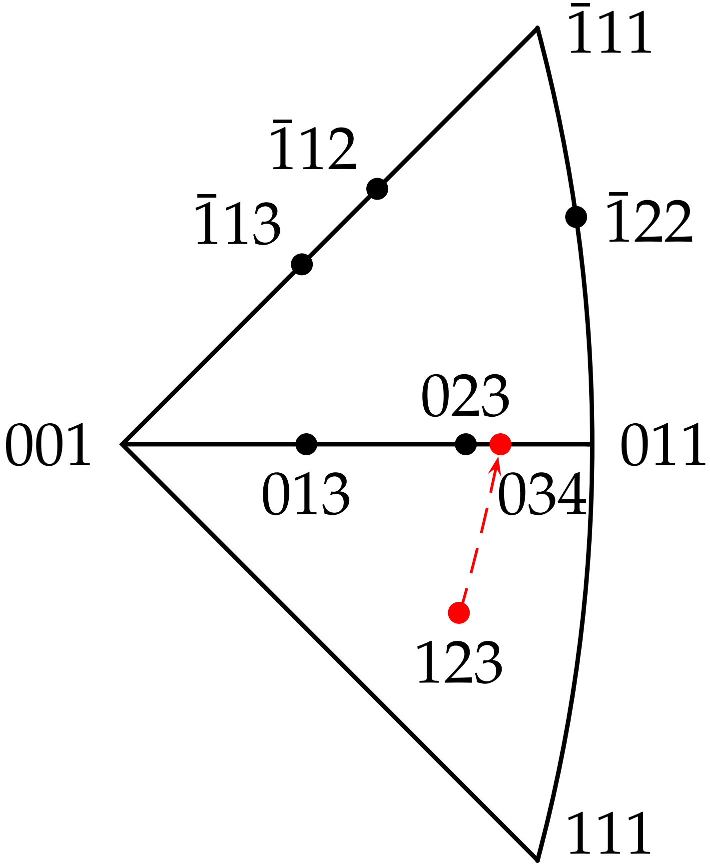
Direction on edge of standard triangle: To find the direction which the tensile axis rotates into along the edge of the standard triangle, first define the axis $\vec{a}$ about which it rotates. This axis is perpendicular to the plane in which both $[123]$ and $[\bar{1}11]$ lie.

$$\vec{a} = [123] \times [\bar{1}11] = [\bar{1}\bar{4}3]$$

The directions along the bottom edge of the standard triangle are of the form $[0kl]$. Since the tensile axis must lie in the same plane,

$$[0kl] \cdot [\bar{1}\bar{4}3] = 0$$

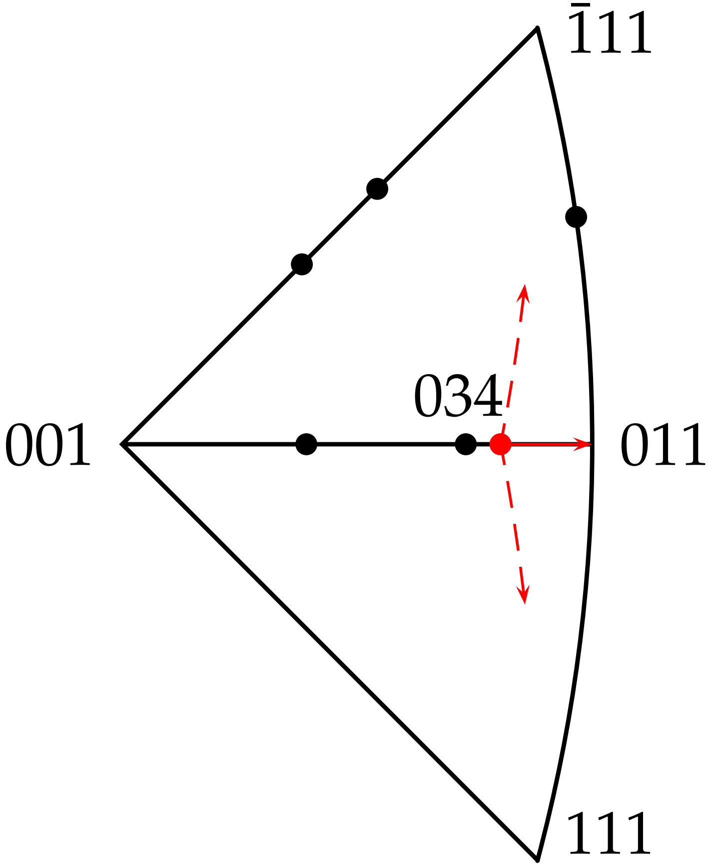
Solving for k and l yields the new tensile axis direction along the edge of the standard triangle: $[034]$.



Now the preferred slip systems for the new [034] orientation of the tensile axis must be calculated.

Calculate Schmid factors for a tensile axis along $[034]$

Slip Plane	Slip Direction	$\cos\theta$	$\cos\lambda$	m
(110)	$[\bar{1}11]$	0.42426	0.80829	0.34293
	$[1\bar{1}1]$	0.42426	0.11547	0.04899
$(1\bar{1}0)$	$[111]$	−0.42426	0.80829	−0.34293
	$[\bar{1}\bar{1}1]$	−0.42426	0.11547	−0.04899
(011)	$[1\bar{1}1]$	0.98995	0.11547	0.11431
	$[\bar{1}\bar{1}1]$	0.98995	0.11547	0.11431
$(0\bar{1}1)$	$[111]$	0.14142	0.80829	0.11431
	$[\bar{1}11]$	0.14142	0.80829	0.11431
(101)	$[\bar{1}11]$	0.56569	0.80829	**0.45724**
	$[\bar{1}\bar{1}1]$	0.56569	0.11547	0.06532
$(\bar{1}01)$	$[111]$	0.56569	0.80829	**0.45724**
	$[1\bar{1}1]$	0.56569	0.11547	0.06532



New slip systems: The old system, $(101)[\bar{1}11]$ still operates, but the new conjugate system $(\bar{1}01)[111]$ also now operates. This the tensile axis tries to rotate toward the $[\bar{1}11]$ and $[111]$ directions simultaneously. If both rotations occur equally, the net rotation is toward the $[011]$ direction.