Prophet Inequality on I.I.D. Distributions: Beating $1 - 1/e$ with a Single Query*

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Abstract. In this work, we study the single-choice prophet inequality problem, where a gambler faces a sequence of n online i.i.d. random variables drawn from an unknown distribution. When a variable reveals its value, the gambler needs to decide irrevocably whether or not to accept the value. The goal is to maximize the competitive ratio between the expected gain of the gambler and that of the maximum variable. It is shown by Correa et al. (EC 2019) that when the distribution is unknown or only $o(n)$ uniform samples from the distribution are given, the best an algorithm can do is $1/e$-competitive. In contrast, when the distribution is known (Correa et al., EC 2017) or $\Omega(n)$ uniform samples are given (Rubinstein et al., ITCS 2020), the optimal competitive ratio 0.7451 can be achieved. In this paper, we study a new model in which the algorithm has access to an oracle that answers quantile queries about the distribution and investigate to what extent we can use a small number of queries to achieve good competitive ratios. We first use the answers from the queries to implement the threshold-based algorithms and show that with two thresholds our algorithm achieves a competitive ratio of 0.6786. Motivated by the two-threshold algorithm, we design the observe-and-accept algorithm that requires only a single query. This algorithm sets a threshold in the first phase by making a query and uses the maximum realization from the first phase as the threshold for the second phase. It can be viewed as a natural combination of the single-threshold algorithm and the algorithm for the secretary problem. By properly choosing the quantile to query and the break-point between the two phases, we achieve a competitive ratio of 0.6718, beating the benchmark of $1 - 1/e$.

Keywords: Prophet Inequality · IID Distributions · Quantile Query.

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