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SUCTION PROFILES AND SCALE FACTORS FOR UNSATURATED FLOW UNDER INCREASED GRAVITATIONAL FIELD

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ABSTRACT

Scale factors for centrifuge modelling have been traditionally defined using dimensional analysis concepts. This is the case, for example, of centrifuge modelling of unsaturated water flow. However, scale factors governing suction, discharge velocity, and time obtained using dimensional analysis have often differed from those obtained from methodologies not based on dimensionless groups. In this paper, a consistent framework is developed for analytic determination of suction profiles for steady-state unsaturated flow under both natural and increased gravitational fields. This framework allows deduction of the scale factors, which emerge from direct comparison of the analytic solutions for model and prototype without the need to use dimensionless groups. For centrifuge conditions leading to an approximately uniform acceleration field, the suction profile in the prototype is found to be the same as that in the model, while the discharge velocity is found to be properly scaled by $1/N$ and time by N^2, where N is the average acceleration ratio between model and prototype. If acceleration field is not uniform, the scale factors should be defined as a function of the centrifuge radius and model length. In addition, evaluation of the effect of different test conditions allows identification of the suction profiles and test setup best suited for hydraulic conductivity determination using centrifuge techniques.

Key words: centrifuge modeling, model tests, partial saturation, suction, water flow (**IGC**: D7/E14)

INTRODUCTION

The use of alternative earthen covers such as evapotranspirative cover systems or capillary barriers has been proposed for waste containment in arid and semiarid regions. This has led to increased need for proper understanding of the mechanisms governing the water flow in unsaturated soils. Field monitoring programs and numerical simulations have provided invaluable insight into the significance of the parameters that dominate the behaviour of earthen cover systems (e.g. Khire et al., 1999, 2000; Morris and Stormont, 1997). However, difficulties in field monitoring have prevented full validation of numerical tools. In order to gain further understanding into the complex unsaturated processes taking place in soil covers, this study seeks the use of physical modelling using a geotechnical centrifuge as an extra source of geotechnical data. The laboratory centrifuge environment favours a systematic control of variables governing the hydraulic behaviour of earthen systems and facilitates data collection for validation of unsaturated flow numerical simulations.

Centrifuge modelling represents a feasible alternative to full scale prototype monitoring, since the stress levels in the model equals those in the prototype, the cost of testing is comparatively small, and the long-term behaviour of the geotechnical model can be obtained within a reduced time frame. In addition, the use of centrifuge testing has proven useful to accelerate determination of the hydraulic conductivity-moisture content relationship of soils (Nimmo et al., 1987, 1992; Conca and Wright, 1990). These investigations have shown the feasibility of centrifuge testing of unsaturated soils, defined procedures for measurement of unsaturated hydraulic conductivity, and validated the use of Darcy's law under increased gravitational fields.

The principle of centrifuge modelling is based upon the requirement of similitude between model and prototype. If a model of a prototype structure is built with dimensions reduced by a factor $1/N$, then an acceleration field of N times the acceleration of gravity, g, will generate stresses by self-weight in the model that are the same as those in the prototype structure. Scale factors for unsaturated flow in soils have been investigated using dimensional analysis concepts (Cargill and Ko, 1983; Goodings, 1982; Arulandanan et al., 1988; Cooke and Mitchell, 1991; Barry et al., 2001; Butterfield, 2000). These investigations concluded that unsaturated flow problems, governed by the dimensionless "capillary effects number", can be analysed using the same scaling relations as

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(saturated) laminar flow problems, governed by the dimensionless "advection number". While use of dimensional analysis has led to scale factors for unsaturated flow equal to those used for saturated flow, different scale factors have often been obtained in studies that have avoided the use of dimensional analysis (e.g. by focusing on equations governing unsaturated flow in model and prototype). For example, based on the analysis of flow differential equations, Goforth et al. (1991) concluded on the impossibility of scaling unsaturated flow when suction gradients dominate water flow. In addition, using Poiseuille's equation for capillary flow, Lord (1999) arrived at scale factors for capillary flow different to those governing saturated flow.

Buckingham's 'Pi' theorem (Buckingham, 1914) has been recognized to provide necessary, but not sufficient conditions for solution of a problem. This is because Buckingham's theorem provides an incomplete algorithm for precisely identifying the dimensionless groups governing a problem (e.g. if some of the key variables have either identical dimensions or are dimensionless), for deciding what variables might or might not be used to form the groups, and for evaluating the consequences of incorporating too many (or too few) dimensions in the analysis (Butterfield, 1999). Algorithms imposing additional conditions have been proposed to overcome the perceived limitations of Buckingham's 'Pi' theorem (Butterfield, 2000). However, the perceived skepticism of relying solely on dimensional analysis and discrepancies with past studies involving demonstrations using equations for unsaturated flow has made the important task of validating scale factors for unsaturated flow, at best, incomplete.

The overall objective of this paper is to provide a consistent framework of analytical solutions for steady-state, one-dimensional unsaturated flow for natural and increased gravitational fields. The solutions are obtained for a generic hydraulic conductivity function (k-function). The scale factors for suction, discharge velocity, flow rate, and time are then shown to emerge directly from comparison of the solutions obtained under natural and increased gravitational fields, avoiding altogether the use of dimensional analysis and the determination of dimensionless groups. The unsaturated flow solutions obtained using a generic k-function are then applied using a specific k-function (Gardner, 1958). This provides insight into limitations for the applicability of the deduced scale factors and into optimisation of centrifuge modelling for the determination of the soil unsaturated hydraulic conductivity.

BASIC FRAMEWORK

Basic Framework for Unsaturated Flow in a Prototype

Figure 1 shows a schematic representation of one-dimensional flow taking place through a control volume in a prototype (i.e. a system under natural gravitational field). Flow is driven by a gradient in fluid potential (i.e. energy per unit mass of fluid). The fluid potential in the prototype control volume equals:

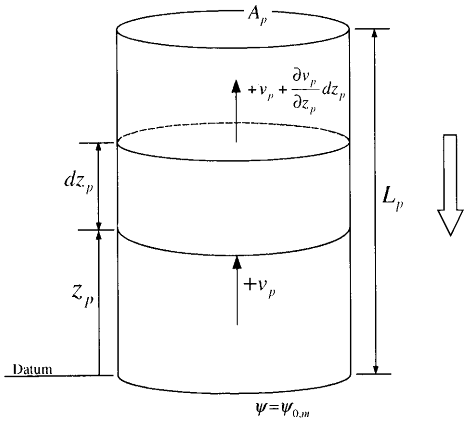

Fig. 1. Flow through the control volume in a vertical prototype

$$\Phi_p = gz_p + \frac{1}{2}\left(\frac{v_p}{n}\right)^2 - \frac{\psi_p}{\rho_w} \tag{1}$$

where Φ_p is the fluid potential, g the acceleration of gravity, z_p the elevation from a datum, v_p the discharge velocity, n the soil porosity, ψ_p is the total suction (using atmospheric pressure as reference), and ρ_w the fluid density. The subscript p denotes "prototype." The terms in Eq. (1) correspond to the potential energy, kinetic energy of the fluid, and energy due to fluid pressure. The seepage velocity (ratio between discharge velocity and soil porosity) is generally small, leading to a negligible component due to kinetic energy. In this case, the fluid potential becomes:

$$\Phi_p = gz_p - \frac{\psi_p}{\rho_w} \tag{2}$$

Water flow in the prototype control volume is estimated by Darcy's law, which can be expressed as:

$$Q_p = -\frac{k(\psi)}{g}\frac{\partial \Phi_p}{\partial z_p}A_p \tag{3}$$

where Q_p is the flow rate, A_p is the cross-sectional area of the control volume and $k(\psi)$ is the unsaturated hydraulic conductivity described by a generic k-function (expressing the hydraulic conductivity as a function of the soil suction). The discharge velocity can be defined as the flow rate per unit of area, as follows:

$$v_p = -\frac{k(\psi)}{g}\frac{\partial \Phi_p}{\partial z_p} \tag{4}$$

where v_p is the discharge velocity in the prototype control volume. Combining Eqs. (2) and (4), the discharge velocity can be expressed by:

$$v_p = -\frac{k(\psi)}{\rho_w g}\left(\rho_w g - \frac{\partial \psi_p}{\partial z_p}\right) \tag{5}$$

The soil k-function and the suction profile across the prototype are needed to estimate the discharge velocity.

Considering the prototype control volume shown in Fig. 1, the principle of continuity leads to:

$$\frac{\partial v_p}{\partial z_p} = -\frac{\partial \theta}{\partial t} \tag{6}$$

where θ is the volumetric water content. Assuming the validity of Darcy's law (Eq. (4)), the continuity principle for flow in the prototype (Eq. (6)) can be expressed as:

$$\frac{\partial}{\partial z_p}\left[\frac{k(\psi)}{g}\frac{\partial \Phi_p}{\partial z_p}\right] = \frac{\partial \theta}{\partial t} \tag{7}$$

For a homogeneous saturated medium that does not undergo volume changes with time, and considering that steady-state condition has been reached, Eq. (7) takes the form of Laplace's equation:

$$\frac{\partial^2 \Phi_p}{\partial z_p^2} = 0 \tag{8}$$

Considering Eq. (2) into Eq. (8), the differential equation for saturated one-dimensional steady-state flow can be stated as:

$$\frac{\partial^2 \psi_p}{\partial z_p^2} = 0 \tag{9}$$

In the case of unsaturated flow, the derivative $\partial\theta/\partial t$ in Eq. (7) equals zero if the unsaturated medium does not undergo volume changes with time and a steady-state condition has been reached. In this case, considering Eq. (5) into Eq. (6), the continuity principle can be expressed as the Richards' equation for steady-state:

$$\frac{\partial}{\partial z_p}\left[\frac{k(\psi)}{\rho_w g}\left(\rho_w g - \frac{\partial \psi_p}{\partial z_p}\right)\right] = 0 \tag{10}$$

Solution of an unsaturated flow problem using Eq. (10) involves determination of the k-function of the soil and precising the boundary conditions of the problem. By further developing Eq. (10), the governing equation for steady-state unsaturated flow can also be expressed as:

$$\frac{\partial^2 \psi_p}{\partial z_p^2} = \rho_w g v_p \frac{\partial[1/k(\psi)]}{\partial \psi_p}\frac{\partial \psi_p}{\partial z_p} \tag{11}$$

Under steady-state condition, the suction profile through the soil prototype does not change with time and, consequently, the suction profile is only a function of z_p.

Basic Framework for Unsaturated Flow in a Centrifuge Model

Figure 2 shows a schematic representation of one-dimensional flow taking place through a control volume in a centrifuge model (i.e. a system under increased gravitational field). The centripetal acceleration is a function of the angular velocity and the radial distance, as follows:

$$a_c = \omega^2 r = N_r g \tag{12}$$

where a_c is the centripetal acceleration, ω is the angular velocity, r is the radial distance from the centrifuge axis to the control volume, and N_r is the ratio between the centripetal acceleration and the acceleration of gravity at a distance r from the centrifuge axis.

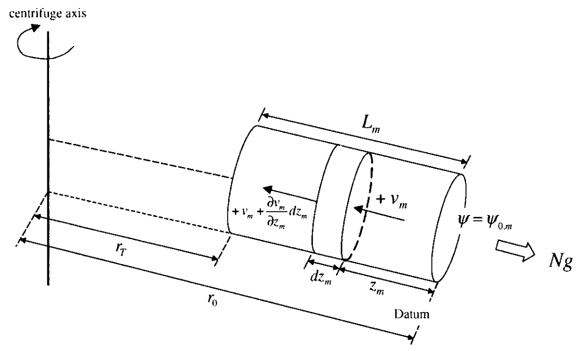

Fig. 2. Flow through the control volume in a centrifuge model

The coordinate z_m depicted in Fig. 2 is defined as:

$$z_m = r_0 - r \tag{13}$$

where r_0 is the distance from the centrifuge axis to the datum used to define the potential energy of the fluid (i.e. $z_m = 0$). In the analyses presented herein, the datum is located at the base of the model. From Eqs. (12) and (13), the acceleration ratio can be expressed as:

$$N_r = \frac{\omega^2}{g}(r_0 - z_m) \tag{14}$$

Water flow in the model is also driven by a gradient in fluid potential. The component of the fluid potential that corresponds to the potential energy in a centrifuge model will differ from that in a prototype under natural gravity. Accordingly, the fluid potential in the model control volume equals:

$$\Phi_m = -\frac{1}{2}\omega^2(r_0 - z_m)^2 + \frac{1}{2}\left(\frac{v_m}{n}\right)^2 - \frac{\psi_m}{\rho_w} \tag{15}$$

where Φ_m is the fluid potential, v_m the discharge velocity, and ψ_m the total suction. The subscript m denotes "model". The terms of the fluid potential stated by Eq. (15) correspond to the potential energy, kinetic energy of the fluid, and energy due to pressure. The first term is negative because potential energy increases in opposite direction to the centripetal acceleration, which acts in the direction of the radius. The discharge velocity to be obtained using the fluid potential defined by Eq. (15) will be positive in the direction of z_m, that is, in the opposite direction of the radius.

As in the case of the prototype, the seepage velocity is negligible and the kinetic energy component in Eq. (15) can be disregarded if turbulent flow does not occur during centrifuge testing. In this case, Eq. (15) becomes:

$$\Phi_m(z_m) = -\frac{1}{2}\omega^2(r_0 - z_m)^2 - \frac{\psi_m}{\rho_w} \tag{16}$$

Water flow in the centrifuge model can also be estimated by Darcy's law. The discharge velocity (positive in the direction of z_m) is given by:

$$v_m = -\frac{k(\psi)}{g}\frac{\partial \Phi_m}{\partial z_m} \tag{17}$$

where v_m is the discharge velocity in the model control volume.

Adopting the atmospheric pressure as reference, and considering Eq. (16) into Eq. (17):

$$v_m = -\frac{k(\psi)}{\rho_w g}\left(\rho_w \omega^2 (r_0 - z_m) - \frac{\partial \psi_m}{\partial z_m}\right) \tag{18}$$

Considering a model control volume, the principle of continuity leads to:

$$\frac{\partial v_m}{\partial z_m} = -\frac{\partial \theta}{\partial t} \tag{19}$$

Assuming the validity of Darcy's law (Eq. (17)), the continuity principle for flow in the centrifuge (Eq. (19)) can be expressed as:

$$\frac{\partial}{\partial z_m}\left(\frac{k(\psi)}{g}\frac{\partial \Phi_m}{\partial z_m}\right) = \frac{\partial \theta}{\partial t} \tag{20}$$

For a homogeneous saturated medium that does not undergo volume changes with time, considering that steady-state condition has been reached, and using the fluid potential stated in Eq. (16), the governing equation (Eq. (20)) can be expressed as:

$$\frac{\partial}{\partial z_m}\left[-\frac{k_{sat}}{\rho_w g}\left(\rho_w \omega^2 (r_0 - z_m) - \frac{\partial \psi_m}{\partial z_m}\right)\right] = 0 \tag{21}$$

where k_{sat} is the saturated hydraulic conductivity. Rearranging the terms, Eq. (21) can be expressed as:

$$\frac{\partial^2 \psi_m}{\partial z_m^2} = -\rho_w \omega^2 \tag{22}$$

In an unsaturated flow problem, the governing equation (Eq. (20)) is based on the same assumptions as Richards' equation (Eq. (10)). The derivative $\partial\theta/\partial t$ in Eq. (20) equals zero if the unsaturated medium does not undergo volume changes with time and the seepage regime has reached steady-state. In this case, the continuity principle can be expressed as:

$$\frac{\partial}{\partial z_m}\left[-\frac{k(\psi)}{\rho_w g}\left(\rho_w \omega^2 (r_0 - z_m) - \frac{\partial \psi_m}{\partial z_m}\right)\right] = 0 \tag{23}$$

Equation (23) describes the one-dimensional, steady-state flow through unsaturated soils under an increased gravitational field. By further developing Eq. (23), steady-state unsaturated flow in the centrifuge can also be expressed as:

$$\frac{\partial^2 \psi_m}{\partial z_m^2} = \rho_w g v_m \frac{\partial[1/k(\psi)]}{\partial \psi_m}\frac{\partial \psi_m}{\partial z_m} - \rho_w \omega^2 \tag{24}$$

SUCTION PROFILES FOR STEADY-STATE UNSATURATED FLOW

Suction Profiles for Unsaturated Flow in a Prototype

The suction profile in a prototype is obtained herein by solving the equations governing unsaturated flow through a soil having a generic k-function. The boundary conditions considered for the problem are an imposed suction, $\psi_{0,p}$, at the base of the prototype (i.e. at $z_p = 0$) and an imposed discharge velocity, v_p, at the top of the prototype. Since the problem is solved for steady-state conditions, v_p is constant for the entire prototype length, L_p. Although the hydraulic conductivity varies along the length of the soil sample, it does not vary with time. Consequently, the k-function can also be expressed as a function of z_p. The suction gradient can be defined from the equation for discharge velocity (Eq. (6)), as:

$$\frac{\partial \psi_p}{\partial z_p} = \frac{\rho_w g v_p}{k(\psi)} + \rho_w g \tag{25}$$

For the particular case of saturated flow, the hydraulic conductivity of the soil is constant (i.e. $k(\psi) = k_{sat}$). In this case, integrating Eq. (25), and considering that the integration constant can be defined using the boundary conditions (i.e. $\psi_p = \psi_{0,p}$ at $z_p = 0$), the suction profile is defined by:

$$\psi_p = \rho_w g z_p + \frac{\rho_w g v_p}{k_{sat}} z_p + \psi_{0,p} \tag{26}$$

For a generic unsaturated flow problem, integration of Eq. (25) for a generic coordinate z_p leads to:

$$\psi_p = \rho_w g z_p + \rho_w g v_p K_p(z_p) + C \tag{27}$$

where C is the integration constant and $K_p(z_p)$ is the k-function factor for the prototype, which is defined as (Dell'Avanzi and Zornberg, 2002a):

$$K_p(z_p) = \int_0^{z_p} \frac{1}{k(\psi)} dz_p \tag{28}$$

As for the saturated case, the integration constant can be defined using the boundary conditions of the problem. Considering that at $z_p = 0$ the suction equals $\psi_{0,p}$ and the k-function factor equals zero (i.e. $K_p(0) = 0$), then $C = \psi_{0,p}$. Consequently, the suction profile in a prototype with a generic k-function is given by the following solution:

$$\psi_p = \rho_w g z_p + \rho_w g v_p K_p(z_p) + \psi_{0,p} \tag{29}$$

The k-function factor (Eq. (28)) can be redefined as follows:

$$K_p(z_p) = \int_{\psi_{0,p}}^{\psi_p} \frac{1}{k(\psi)}\frac{dz_p}{d\psi_p} d\psi_p \tag{30}$$

The inverse of the suction gradient can be obtained from Eq. (5), leading to:

$$K_p(z_p) = \frac{1}{\rho_w g}\int_{\psi_{0,p}}^{\psi_p} \frac{d\psi_p}{v_p + k(\psi)} \tag{31}$$

The k-function factor $K_p(z_p)$ can be defined for any k-function in the form $k(\psi)$. Integration in Eq. (31) can be performed either analytically or numerically depending on the k-function used in a given problem. The analytic determination of the k-function factor is illustrated herein using the k-function proposed by Gardner (1958), which is defined as:

$$k(\psi) = k_{sat} e^{-\alpha\psi} \tag{32}$$

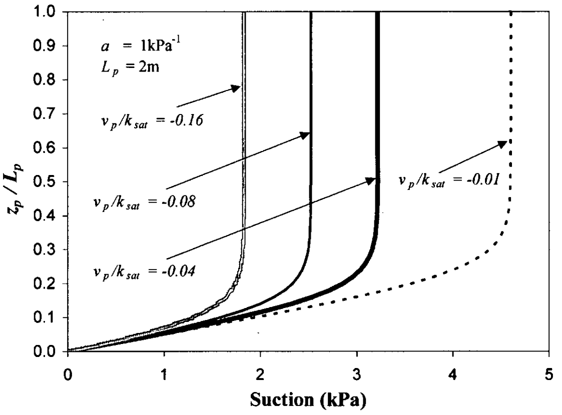

Fig. 3. Suction profiles for unsaturated flow in a prototype

where e is the natural base of logarithms, and a is an exponential parameter. The advantage of adopting the k-function defined by Eq. (32) is that analytical solutions can be easily obtained for transient (Srivastava and Yet, 1991) and steady state flow conditions (Gardner, 1958). The k-function factor (Eq. (31)) for Gardner's k-function can be obtained analytically as (Dell'Avanzi and Zornberg, 2002b):

$$K_{\mathrm{p}}(z_{\mathrm{p}})=\frac{1}{a\rho_{\mathrm{w}}gv_{\mathrm{p}}}\left(a(\psi_{\mathrm{p}}-\psi_{0,\mathrm{p}})+\ln\frac{\left|\frac{v_{\mathrm{p}}}{k_{\mathrm{sat}}}+e^{-a\psi_{\mathrm{p}}}\right|}{\left|\frac{v_{\mathrm{p}}}{k_{\mathrm{sat}}}+e^{-a\psi_{0,\mathrm{p}}}\right|}\right) \tag{33}$$

The suction profile in a prototype, considering Gardner's k-function, can then be obtained by substituting Eq. (33) into Eq. (29). After rearranging the terms, the suction profile is defined as:

$$\psi_{\mathrm{p}}=-\frac{1}{a}\ln\left(e^{(\ln|(v_{\mathrm{p}}/k_{\mathrm{sat}})+e^{-a\psi_{0,\mathrm{p}}}|-a\rho_{\mathrm{w}}gz_{\mathrm{p}})}-\frac{v_{\mathrm{p}}}{k_{\mathrm{sat}}}\right)\quad \text{if}\quad \left(\frac{v_{\mathrm{p}}}{k_{\mathrm{sat}}}+e^{-a\psi_{0,\mathrm{p}}}\right)>0 \tag{34a}$$

$$\psi_{\mathrm{p}}=-\frac{1}{a}\ln\left(-e^{(\ln|(v_{\mathrm{p}}/k_{\mathrm{sat}})+e^{-a\psi_{0,\mathrm{p}}}|-a\rho_{\mathrm{w}}gz_{\mathrm{p}})}-\frac{v_{\mathrm{p}}}{k_{\mathrm{sat}}}\right)\quad \text{if}\quad \left(\frac{v_{\mathrm{p}}}{k_{\mathrm{sat}}}+e^{-a\psi_{0,\mathrm{p}}}\right)<0 \tag{34b}$$

Figure 3 shows the suction profiles obtained in a 2 m long prototype for varying values of discharge velocities. This analysis considered that the water level is positioned at the base (i.e. $\psi_{0,\mathrm{p}}=0$), and that Gardner's parameter a equals 1 kPa^{-1}. The value of 1 kPa^{-1} for parameter a is within the range of values reported in the technical literature (Choo and Yanful, 2000) for homogeneous soils. A parametric evaluation conducted using values of a ranging from 0.5 to 2.5 kPa^{-1} leads to similar observations to those reported herein for $a=1$ kPa^{-1}. The results shown in the figure indicate that part of the soil sample is under approximately zero suction gradient (i.e. approximately unity total gradient). The imposed discharge velocity defines the suction magnitude in the upper region of the model, where the suction is approximately constant. From Eq. (34a), it can be shown that:

$$\lim_{z_{\mathrm{p}}\to\infty}\psi_{\mathrm{p}}=\psi_{\mathrm{lim,p}}=-\frac{1}{a}\ln\left(-\frac{v_{\mathrm{p}}}{k_{\mathrm{sat}}}\right) \tag{35}$$

Inspection of Eq. (35) and of the results in Fig. 3 indicate that the magnitude of the suction obtained towards the surface of the prototype for a sufficiently large prototype length depends only on the imposed discharge velocity and the soil k-function, but it is independent of the suction imposed at the base of the prototype.

Suction Profiles for Unsaturated Flow in a Centrifuge Model

The suction profile in a centrifuge model is obtained herein by solving the equations governing unsaturated flow through a soil having a generic k-function. The boundary conditions considered for the problem are an imposed suction, $\psi_{0,\mathrm{m}}$, at the base of the model (i.e. at $z_{\mathrm{m}}=0$) and an imposed discharge velocity, v_{m}, at the top of the model. Since the problem is solved for steady-state conditions, v_{m} is constant for the entire model length, L_{m}. The suction gradient can be defined from the equation for discharge velocity (Eq. (18)) as follows:

$$\frac{\partial\psi_{\mathrm{m}}}{\partial z_{\mathrm{m}}}=\frac{\rho_{\mathrm{w}}gv_{\mathrm{m}}}{k(\psi)}+\rho_{\mathrm{w}}\omega^2r_0-\rho_{\mathrm{w}}\omega^2z_{\mathrm{m}} \tag{36}$$

For the particular case of saturated flow, the hydraulic conductivity of the soil is constant (i.e. $k(\psi)=k_{\mathrm{sat}}$). In this case, integrating Eq. (36), and considering that the integration constant can be defined using the boundary condition (i.e. $\psi_{\mathrm{m}}=\psi_{0,\mathrm{m}}$ at $z_{\mathrm{m}}=0$), the suction profile is defined by:

$$\psi_{\mathrm{m}}=\rho_{\mathrm{w}}\omega^2z_{\mathrm{m}}\left(r_0-\frac{z_{\mathrm{m}}}{2}\right)+\frac{\rho_{\mathrm{w}}gv_{\mathrm{m}}}{k_{\mathrm{sat}}}z_{\mathrm{m}}+\psi_{0,\mathrm{m}} \tag{37}$$

For a generic unsaturated flow problem in the centrifuge, integration of Eq. (36) for a generic coordinate z_{m} leads to:

$$\psi_{\mathrm{m}}=\rho_{\mathrm{w}}z_{\mathrm{m}}\omega^2\left(r_0-\frac{z_{\mathrm{m}}}{2}\right)+\rho_{\mathrm{w}}gv_{\mathrm{m}}K_{\mathrm{m}}(z_{\mathrm{m}})+C' \tag{38}$$

where C' is the integration constant and $K_{\mathrm{m}}(z_{\mathrm{m}})$ is the k-function factor for the model, which is defined as:

$$K_{\mathrm{m}}(z_{\mathrm{m}})=\int_0^{z_{\mathrm{m}}}\frac{1}{k(\psi)}dz_{\mathrm{m}} \tag{39}$$

The integration constant can be defined using the boundary conditions of the problem. Considering that at $z_{\mathrm{m}}=0$ the suction equals $\psi_{0,\mathrm{m}}$ and the k-function factor equals zero ($K_{\mathrm{m}}(0)=0$), then $C'=\psi_{0,\mathrm{m}}$. Consequently, the suction profile in a model with a generic k-function is given by the following solution:

$$\psi_{\mathrm{m}}=\rho_{\mathrm{w}}z_{\mathrm{m}}\omega^2\left(r_0-\frac{z_{\mathrm{m}}}{2}\right)+\rho_{\mathrm{w}}gv_{\mathrm{m}}K_{\mathrm{m}}(z_{\mathrm{m}})+\psi_{0,\mathrm{m}} \tag{40}$$

The k-function factor (Eq. (39)) can be redefined as follows:

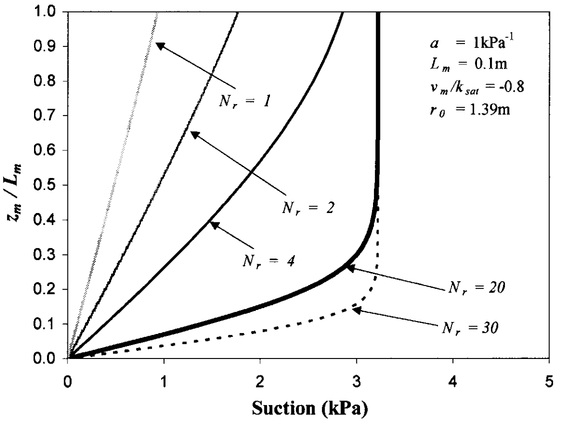

Fig. 4. Suction profiles for unsaturated flow in a centrifuge model (Note. N_r is defined as the acceleration ratio at the center of the model ($z_m = L_m/2$))

$$K_m(z_m) = \int_{\psi_{0,m}}^{\psi_m} \frac{1}{k(\psi)} \frac{dz_m}{d\psi_m} d\psi_m \tag{41}$$

The inverse of the suction gradient can be obtained from Eq. (18), leading to:

$$K_m(z_m) = \frac{1}{\rho_w g N_r} \int_{\psi_{0,m}}^{\psi_m} \frac{d\psi_m}{\left(\frac{v_m}{N_r} + k(\psi)\right)} \tag{42}$$

As previously mentioned, the k-function factor (Eq. (42)) can be defined for any k-function in the form $k(\psi)$. For example, adopting Gardner's k-function (Eq. (32)), the k-function factor can be obtained analytically as (Dell'Avanzi and Zornberg, 2002):

$$K_m(z_m) = \frac{1}{a\rho_w g v_m} \left(a(\psi_m - \psi_{0,m}) + \ln \frac{\left| \frac{v_m}{N_r k_{sat}} + e^{-a\psi_m} \right|}{\left| \frac{v_m}{N_r k_{sat}} + e^{-a\psi_{0,m}} \right|} \right) \tag{43}$$

The suction profile in a centrifuge model, considering Gardner's k-function, can then be obtained by substituting Eq. (43) into Eq. (40). After rearranging the terms, the suction profile is defined as:

$$\psi_m = -\frac{1}{a} \ln \left[e^{(\ln |v_m/N_r k_{sat} + e^{-a\psi_{0,m}}| - a\rho_w z_m (r_0 - z_m/2))} - \frac{v_m}{N_r k_{sat}} \right] \quad \text{if} \quad \left(\frac{v_m}{N_r k_{sat}} + e^{-a\psi_{0,m}} \right) > 0 \tag{44a}$$

$$\psi_m = -\frac{1}{a} \ln \left[-e^{(\ln |v_m/N_r k_{sat} + e^{-a\psi_{0,m}}| - a\rho_w z_m \omega^2 (r_0 - z_m/2))} - \frac{v_m}{N_r k_{sat}} \right] \quad \text{if} \quad \left(\frac{v_m}{N_r k_{sat}} + e^{-a\psi_{0,m}} \right) < 0 \tag{44b}$$

Figure 4 shows the suction profiles obtained in a 0.1 m long model for varying values of acceleration ratio N_r. This analysis considered that the water level is positioned at the model base (i.e. $\psi_{0,m} = 0$), that Gardner's parameter a equals 1 kPa^{-1} and that the constant discharge velocity equals $-0.8k_{sat}$. The results shown in the figure indicate that increasing acceleration ratios induce a gradual change in the suction profile. Beyond a certain acceleration ratio, the suction gradient becomes negligible towards the top of the model. The imposed discharge velocity defines the suction magnitude in the upper portion of the model, where the suction is approximately constant. From Eq. (44a), it can be shown that:

$$\lim_{z_m \to \infty} \psi_m = \psi_{\lim,m} = -\frac{1}{a} \ln \left(-\frac{v_m}{N_r k_{sat}} \right) \tag{45}$$

Inspection of Eq. (45) and of the results in Fig. 4 indicate that the magnitude of suction obtained towards the surface of the model for a sufficiently large model length depends on the imposed discharge velocity, the soil k-function, and the acceleration ratio, but it is independent of the suction imposed at the base of the model.

SCALE FACTORS

Scale factors relating variables in a model with their equivalent in a prototype should be defined in order to infer the response of a prototype based on the monitored response of a reduced scale model. The prototype and model are assumed to be composed of the same material and permeated by the fluids with same density. Geometric similarity requires a constant length ratio between homologous points in the model and prototype. That is:

$$z_p = \alpha_z z_m \tag{46}$$

where α_z is the geometric scale factor.

Similarity in an unsaturated flow problem requires that, in addition to geometric similarity, the suction and discharge velocity in model and prototype be related by constant scale factors. That is:

$$\psi_p = \alpha_\psi \psi_m \tag{47}$$

$$v_p = \alpha_v v_m \tag{48}$$

where α_ψ and α_v are the scale factors for suction and discharge velocity, respectively.

The scale factors for suction and discharge velocity are expected to be related to the geometric scale factor α_z and the acceleration ratio N_r. Substituting Eqs. (46), (47) and (48) into the suction profile solution for a prototype with a generic k-function (Eq. (29)) leads to:

$$\psi_m = \frac{1}{\alpha_\psi} [\rho_w g \alpha_z z_m + \rho_w g \alpha_v v_m K_p(z_p) + \alpha_\psi \psi_{0,m}] \tag{49}$$

The suction profile solution in a centrifuge model with a generic k-function for unsaturated steady-state flow (Eq. (40)) can be rearranged as:

$$\psi_m = \rho_w g N_r z_m \chi + \rho_w g v_m K_m(z_m) + \psi_{0,m} \tag{50}$$

where χ is the uniformity factor of the acceleration field, defined as:

$$\chi = 1 + \frac{z_m/r_0}{2(1 - z_m/r_0)} \tag{51}$$

The uniformity factor describes the geometric

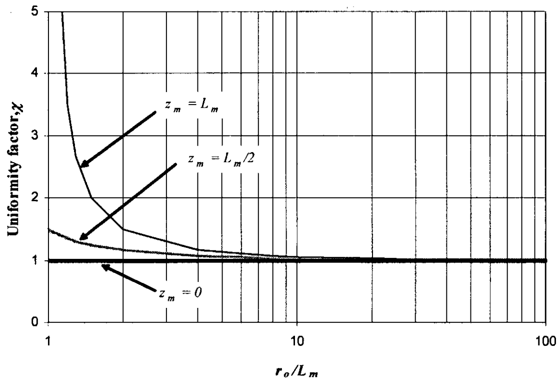

Fig. 5. Uniformity factor χ in a centrifuge model

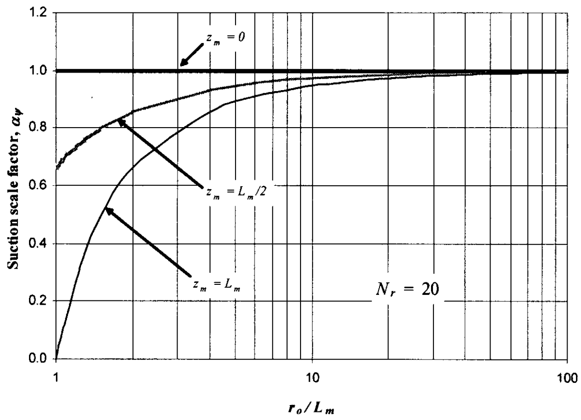

Fig. 6. Suction scale factor as a function of the ratio centrifuge radius/model length

conformance between model and centrifuge equipment. Figure 5 illustrates the sensitivity of the uniformity factor χ to the ratio between the centrifuge arm length and the model length (r_0/L_m). The results are presented for z_m equal to zero, $L_m/2$, and L_m (i.e. for the base, center, and top of the model). As shown in the figure, the uniformity factor equals one at the location where the suction is imposed (i.e. at $z_m=0$). The figure also shows that, for model of sufficiently small size (e.g. r_0/L_m larger than 10), the uniformity factor equals approximately one for any location within the model.

By equating the expressions for suction defined by Eqs. (49) and (50), the scale factor α_ψ is obtained as:

$$\alpha_\psi = \frac{\alpha_z z_m + \alpha_v v_m K_p(z_p)}{N_r z_m \chi + v_m K_m(z_m)} \tag{52}$$

Consistent with experimental results presented by Nimmo et al. (1987), the k-function of the soil is assumed to be independent of the applied g-level. Consequently, the relationship between the k-function factors for model and prototype (i.e. between $K_m(z_m)$ and $K_p(z_p)$) can be obtained by substituting the geometric scale factor (Eq. (47)) into Eq. (39), as follows:

$$K_m(z_m) = \frac{1}{\alpha_z} \int_0^{z_p} \frac{1}{k(\psi)} dz_p \tag{53}$$

Substituting Eq. (28) into Eq. (53):

$$K_m(z_m) = \frac{1}{\alpha_z} K_p(z_p) \tag{54}$$

Substituting Eq. (54) into Eq. (52):

$$\alpha_\psi = \frac{\alpha_z z_m + \alpha_v v_m \alpha_z K_m(z_m)}{N_r \chi z_m + v_m K_m(z_m)} \tag{55}$$

The discharge velocity in the model can be obtained by substituting Eqs. (46), (47) and (48) into the equation for discharge velocity in the prototype (Eq. (5)):

$$v_m = -\frac{k(\psi)}{\alpha_v \rho_w g}\left(\rho_w g - \frac{\alpha_\psi}{\alpha_z}\frac{\partial \psi_m}{\partial z_m}\right) \tag{56}$$

By equating the discharge velocity expressions defined by Eqs. (56) and (18), the scale factor α_v is obtained as:

$$\alpha_v = \frac{\rho_w g - \dfrac{\alpha_\psi}{\alpha_z}\dfrac{\partial \psi_m}{\partial z_m}}{\rho_w g N_r - \dfrac{\partial \psi_m}{\partial z_m}} \tag{57}$$

Substituting Eq. (57) into Eq. (55), using Eqs. (14) and (18) in the resulting expression, and rearranging leads to the following suction scale factor:

$$\alpha_\psi = \frac{\alpha_z \dfrac{z_m}{k(\psi)} - \alpha_z K_m(z_m)}{N_r \chi \dfrac{z_m}{k(\psi)} - N_r K_m(z_m)} \tag{58}$$

Inspection of Eq. (58) indicates that the suction scale factor is a function of the geometric scale factor α_z, the acceleration ratio N_r, and the uniformity factor χ.

If the length of the centrifuge arm is significantly large than the model length, N_r is approximately constant throughout the model. That is:

$$N_r = N \tag{59}$$

where N is a constant acceleration ratio representative of the entire centrifuge model.

It has been common practice in geotechnical modelling to specify the geometric scale factor, α_z, as equal to the average acceleration ratio N (e.g. Cargill and Ko, 1983). In this case:

$$\alpha_z = N \tag{60}$$

Introducing Eqs. (59) and (60) into Eq. (58):

$$\alpha_\psi = \frac{\dfrac{z_m}{k(\psi)} - K_m(z_m)}{\chi \dfrac{z_m}{k(\psi)} - K_m(z_m)} \tag{61}$$

Figure 6 illustrates the sensitivity of α_ψ as a function of the ratio between centrifuge arm length and model length (r_0/L_m). The results are presented for z_m equal to zero, $L_m/2$, and L_m. The results were obtained using the k-function and boundary conditions used in Fig. 4 for the curve obtained using $N_r=20$. Parametric evaluations indicated that the results shown in Fig. 6 are not very sensitive to the k-function and boundary conditions when r_0/L_m is comparatively large. As observed in the figure, the suction scale factor equals approximately one for sufficiently large r_0/L_m ratios (e.g. $r_0/L_m>10$, which leads to $\chi\approx 1$ as shown in Fig. 5). Consequently, if the ratio r_0/L_m is sufficiently large, the uniformity factor is approximately 1.0, and Eq. (61) becomes:

$$\alpha_\psi=1 \tag{62}$$

If the ratio r_0/L_m is sufficiently large, the discharge velocity scale factor for unsaturated flow can be obtained by substituting Eqs. (59), (60) and (62) into Eq. (57), leading to:

$$\alpha_v=\frac{1}{N} \tag{63}$$

which shows that for sufficiently large r_0/L_m ratios (e.g. ratios larger than 10), the discharge velocity for unsaturated flow in the prototype scales by $1/N$ with respect to that in the model.

Inspection of Figs. 2 and 3 indicates that the suction profile obtained for a 2 m long prototype with $v_p/k_{sat}=-0.04$ is the same as that obtained for a 0.1 m long model with $N=20$ and $v_m/k_{sat}=-0.8$. This is consistent with scale factors defined by Eqs. (60), (62) and (63) (i.e. $\alpha_z=20$, $\alpha_\psi=1$, and $\alpha_v=1/20$).

The flow rate in the model, Q_m, is defined as:

$$Q_m=v_mA_m \tag{64}$$

where A_m is the model cross-sectional area. Substituting the scale factors for discharge velocity (Eq. (63)) and length (Eq. (46)) into Eq. (64):

$$Q_m=Nv_p\frac{1}{N^2}A_p \tag{65}$$

Noting that the expression v_pA_p in Eq. (65) is the prototype flow rate Q_p, the scale factor for flow rate, α_Q, can then be obtained as:

$$\alpha_Q=\frac{Q_p}{Q_m}=N \tag{66}$$

The transit time required for water to travel from top to bottom of mode, t_m, is defined as:

$$t_m=\frac{L_m}{v_m/n} \tag{67}$$

If the same soil is used in the model and the prototype, their porosities are the same. Consequently, substituting the scale factors for discharge velocity (Eq. (63)) and for length (Eq. (46)) into Eq. (67):

$$t_m=\frac{1}{N^2}\frac{nL_p}{v_p} \tag{68}$$

Noting that the expression nL_p/v_p in Eq. (68) is the prototype transit time, t_p, the time scale factor, α_t, can then be obtained as:

$$\alpha_t=\frac{t_p}{t_m}=N^2 \tag{69}$$

The scale factors governing unsaturated flow under increased gravitational field were shown to emerge directly from the comparison of analytical solutions, without the need of relying on dimensional analysis concepts. Although past studies that avoided the use of dimensional analysis have often led to different results, the scale factors obtained herein are in agreement with those obtained using dimensional analysis concepts.

ADDITIONAL CONSIDERATIONS

Solution of the governing equations for unsaturated flow led to the determination of scale factors for suction, discharge velocity, flow rate, and time, which are relevant in centrifuge testing programs aimed at evaluating the behaviour of full-scale prototypes. In addition, analysis of the suction profiles for unsaturated flow is relevant in centrifuge testing programs aimed at determination of the soil unsaturated hydraulic conductivity. Centrifuge testing represents an appealing alternative to conventional techniques for direct measurement of unsaturated hydraulic conductivity. This is not only because the time required to achieve steady-state is significantly minimized (e.g. Nimmo et al., 1987; Conca and Wright, 1990), but also because the centrifuge testing setup may be optimised to obtain suction profiles that are particularly suitable for experimental measurements. Analysis of the suction profiles under an increased gravitational field is presented herein using the k-function proposed by Gardner (1958). As shown in Fig. 4, which illustrates the influence of the acceleration ratio on the suction profile obtained for a test performed using a constant discharge velocity, the suction profile shows an approximately constant value towards the top of the specimen beyond a certain acceleration ratio (N_r approximately 20 in this case).

The length $z_{\beta,m}$ within a centrifuge model where the suction equals a certain percentage, β, of the limit suction (Eq. (45)) can be estimated using Eq. (44). That is the coordinate $z_{\beta,m}$ at which $\psi_m=\beta$. $\psi_{lim,m}$ can be estimated as follows (for the conditions for which Eq. (44a) is valid):

$$z_{\beta,m}=\frac{1}{a\rho_w gN_r\chi}\ln\left[\frac{\dfrac{v_m}{N_rk_{sat}}+e^{-a\psi_{0,m}}}{\dfrac{v_m}{N_rk_{sat}}+\left(-\dfrac{v_m}{N_rk_{sat}}\right)^\beta}\right] \tag{70}$$

For a percentage β sufficiently high (e.g. 99%), the length $(L_m-z_{\beta,m})$ represents the portion of the centrifuge model over which the suction gradient is negligible. For example, the limit suction defined by Eq. (45) for the suction profile shown in Fig. 4 using $N_r=20$ equals $\psi_{lim,m}=3.22$ kPa. For a value $\beta=0.99$, the length ratio $(z_{\beta,m}/L_m)$ estimated using Eq. (70) equals 0.33. That is, as can be observed in Fig. 4, 66% of the model length has a negligible suction gradient (i.e. an approximately con-

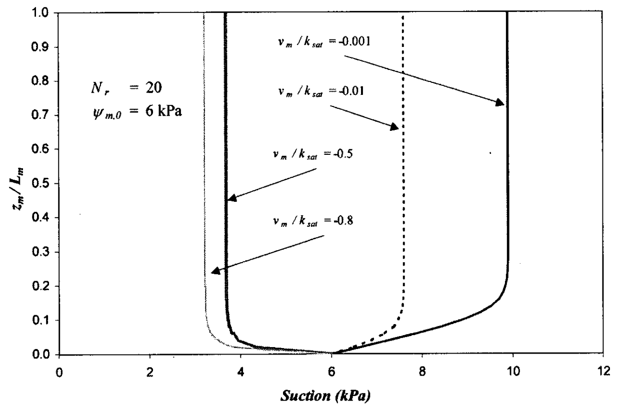

Fig. 7. Influence of the discharge velocity on the suction profiles in a centrifuge model

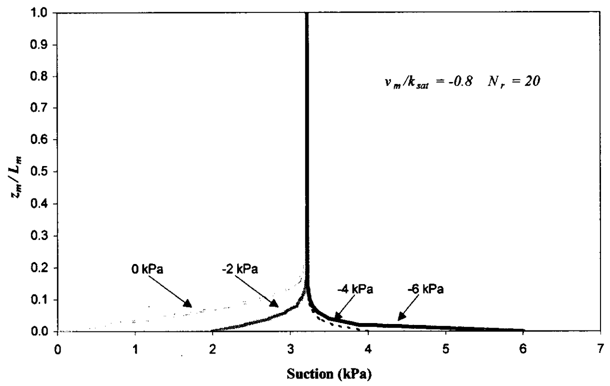

Fig. 8. Influence of the suction imposed at the base of the sample on the suction profiles in a centrifuge model

stant suction value). The existence of a portion of the suction profile with negligible suction gradient under steady-state constitutes a favourable condition for the determination of the unsaturated hydraulic conductivity. This is not only because this profile can be achieved within a relatively short period of time, but also because experimental measurement of suction in this region is not significantly affected by the precise location of the suction measurement device. Equation (71) can also be used to define the length of the specimen so that a minimum portion of its length has an approximately constant suction value at a given acceleration ratio.

The experimentally measured suction value can be used to estimate an unsaturated hydraulic conductivity value in order to define points of the soil k-function. For the suction measured along the portion of the specimen with negligible suction gradient, the corresponding unsaturated hydraulic conductivity can be obtained directly from the imposed discharge velocity, v_m, by considering a negligible suction gradient in Eq. (18). That is, the unsaturated hydraulic conductivity associated to the measured suction value can be determined by:

$$k(\psi) = -\frac{v_m}{N_r} \tag{71}$$

Figure 7 shows the influence of varying values of imposed discharge velocity, v_m, on the suction profiles obtained for a constant suction imposed at the base of the specimen ($\psi_{0,m} = 6$ kPa in this case). As shown in the figure, a different value of approximately constant suction is obtained for varying discharge velocity values. Consequently, different points of the soil k-function can be obtained by simply imposing different discharge velocities, which will lead to different hydraulic conductivity values estimated using Eq. (71). Figure 8 shows the influence on the suction profiles obtained as a function of the suction imposed at the base of the model, $\psi_{0,m}$, for a constant value of imposed discharge velocity. As shown in the figure, the value of the approximately constant suction obtained towards the top of the specimen is independent of the imposed suction at the base of the model. The trends observed in these analytical results are

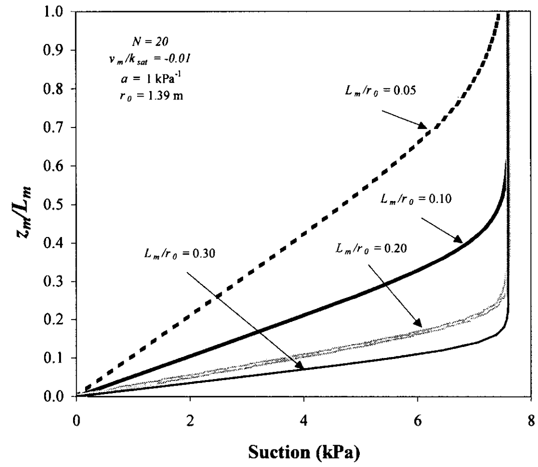

Fig. 9. Influence of sample length on the suction profiles in a centrifuge model

in agreement with data reported by Nimmo et al. (1987, 1992). Figure 9 shows the influence of the specimen length on the suction profile obtained considering $N = 20$, discharge velocity $v_m = -0.01k_{sat}$, and a suction $\psi_{0,m} =$ 0 kPa imposed at the base of the model. The obtained suction profiles indicate that a well-defined region of negligible suction gradient can be obtained for comparatively large specimens in relation to the centrifuge arm length. That is, while comparatively small specimens are desirable if the objective of centrifuge testing is to simulate the behavior of full-scale prototypes, comparatively large specimens are desirable if the objective of centrifuge testing is to determine the soil unsaturated hydraulic conductivity.

CONCLUSIONS

A consistent framework was developed for analytic determination of suction profiles for steady-state unsaturated flow under both natural and increased gravitational fields. Since Buckingham's 'Pi' theorem provides necessary, but not sufficient conditions for solution of a problem, an important objective of this study was to define the scale factors governing unsaturated centrifuge water flow without adopting dimensional analysis concepts. The differential equations governing unsaturated flow were deduced assuming the validity of Darcy's law and of Richards' equation, no volume changes within the soil, and the independence of the soil k-function with increased gravitational fields. While the study of scale factors was made for a generic k-function, the k-function proposed by Gardner (1958) was also used to evaluate the sensitivity of the suction profiles to different boundary conditions. The main conclusions drawn from this investigation are:

(a) The scale factors governing unsaturated flow under increased gravitational field emerge directly from comparison of analytical solutions, without the need of relying on dimensional analysis concepts. These scale factors are independent of the k-function selected to represent the soil unsaturated hydraulic conductivity.

(b) For conditions leading to an approximately uniform acceleration field, the suction profile in the prototype is found to be the same as that in the model, while the discharge velocity is found to be properly scaled by $1/N$, the flow rate by N, and the time by N^2. These scale factors, obtained without using dimensional analysis, are consistent with those obtained in past studies using dimensionless groups.

(c) For conditions leading to a non-uniform acceleration field, the scale factors governing centrifuge unsaturated flow are no longer only a function of the acceleration ratio, but also a function of the uniformity factor, which depends on the relative dimensions of centrifuge arm and model length.

(d) Adequate centrifuge testing setup allows control of the suction profile to be induced within the model. In particular, the portion of the model length over which the suction gradient is negligible can be estimated. If the objective of centrifuge testing is the determination of the soil unsaturated hydraulic conductivity, comparatively large specimens (in relation to the centrifuge arm length) are desirable.

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NOTATION

a: Exponential parameter in Gardner (1958) k-function
a_c: Centrifugal acceleration
A: Cross section area
e: Natural base of logarithms
g: Acceleration of gravity
k: Hydraulic conductivity
k_{sat}: Saturated hydraulic conductivity
L: Length
n: Soil porosity
N: Constant acceleration ratio
N_r: Acceleration ratio
Q: Flow rate
r: Centrifuge radius
r_0: Distance from the centrifuge axis to the origin of coordinate system z_m
t: Time
v: Discharge velocity
z: Coordinate
α_ψ, α_v, α_Q, α_t, α_z: Suction, discharge velocity, flow rate, time, and geometric scale factors
β: Factor of proportionality
χ: Uniformity factor
K: k-function factor
ρ: Density
ψ: Suction
$\psi_{0,m}$, $\psi_{0,p}$: Suction at the base of the centrifuge model and prototype
Φ: Fluid potential
ω: Angular velocity

Subscripts

m: Model
p: Prototype
w: Water

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