Technical Paper by J.P. Giroud, J.G. Zornberg, and J.F. Beech

HYDRAULIC DESIGN OF GEOSYNTHETIC AND GRANULAR LIQUID COLLECTION LAYERS COMPRISING TWO DIFFERENT SLOPES

ABSTRACT: Liquid collection layers used in landfills often comprise two sections with different slopes. Typically, one of the slopes is much steeper than the other one. The steeper slope is generally the downstream slope in a landfill cover and the upstream slope in a leachate collection layer. The liquid collection material is generally a geosynthetic (such as a geonet) on the steep slope and is either a geosynthetic or a granular material (such as sand or gravel) on the other slope. Design methods are available for the case where there is a drain that promptly removes the liquid at the toe of each of the two sections. This paper provides a method to design the liquid collection layer for the case where there is no drain at the connection between the two sections, i.e. when the only drain is at the toe of the downstream section. The method consists of analytical expressions for calculating the maximum thickness of liquid under steady-state conditions in each of the two sections of the liquid collection layer. Design examples are presented and practical guidance is provided for the use of a transition zone between the two sections when needed.

KEYWORDS: Liquid collection layer, Drainage, Landfill, Slope, Leachate collection, Landfill cover, Geosynthetic, Geonet, Geocomposite, Analytical, Design.

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1 INTRODUCTION

1.1 The Need for a Methodology

Calculating the thickness of liquid in a liquid collection layer is an important design step because one of the design criteria for a liquid collection layer is that the maximum thickness of liquid must be less than an allowable thickness (Giroud et al. 2000b). It should be noted that the term "thickness" is used instead of the more familiar term "depth", because thickness (measured perpendicular to the liquid collection layer slope), and not depth (measured vertically), is actually used in design.

The thickness of liquid in a liquid collection layer depends on the rate of liquid supply. A typical case of liquid supply is that of liquid impinging onto the liquid collection layer. Two examples of liquid collection layers with such a type of liquid supply can be found in landfills: (i) the drainage layer of the cover system (Figure 1a), where the liquid that impinges onto the liquid collection layer is the precipitation water that has percolated through the soil layer overlying the drainage layer; and (ii) the leachate collection layer (Figure 1b), where the liquid that impinges onto the liquid collection layer is the leachate that has percolated through the waste and through the protective soil layer overlying the leachate collection layer. The terminology "liquid impingement rate" is often used in the case of landfills to designate the rate of liquid supply.

Equations are available (Giroud et al. 2000a) to calculate the maximum thickness of liquid in a liquid collection layer that meets the following conditions:

- the liquid supply rate is uniform (i.e. it is the same over the entire area of the liquid collection layer) and is constant (i.e. it is the same during a period of time that is long enough that steady-state flow conditions can be reached);
- the liquid collection layer is underlain by a geomembrane liner without defects and, therefore, liquid losses are negligible;
- the slope of the liquid collection layer is uniform (a situation referred to herein as "single slope"); and
- there is a drain at the toe of the slope that promptly removes the liquid.

The last two conditions are not met in cases where the liquid collection layer comprises two sections with different slopes, with no drain removing the liquid at the connection between the two sections; in those cases, the only drain is at the toe of the downstream section. The equations for the case of a single slope cannot be readily used for the case of a liquid collection layer comprising two sections with different slopes. Therefore, a methodology is needed for the case of liquid collection layers comprising two sections with different slopes.

1.2 Description of Liquid Flow in a Liquid Collection Layer on Two Slopes

Two examples of liquid collection layers that comprise two sections with different slopes are presented in Figure 2: a landfill cover system and a landfill leachate collection system. The two sections of a liquid collection layer are designated as the upstream section and the downstream section. The subscript "*up*" is used for the characteristics

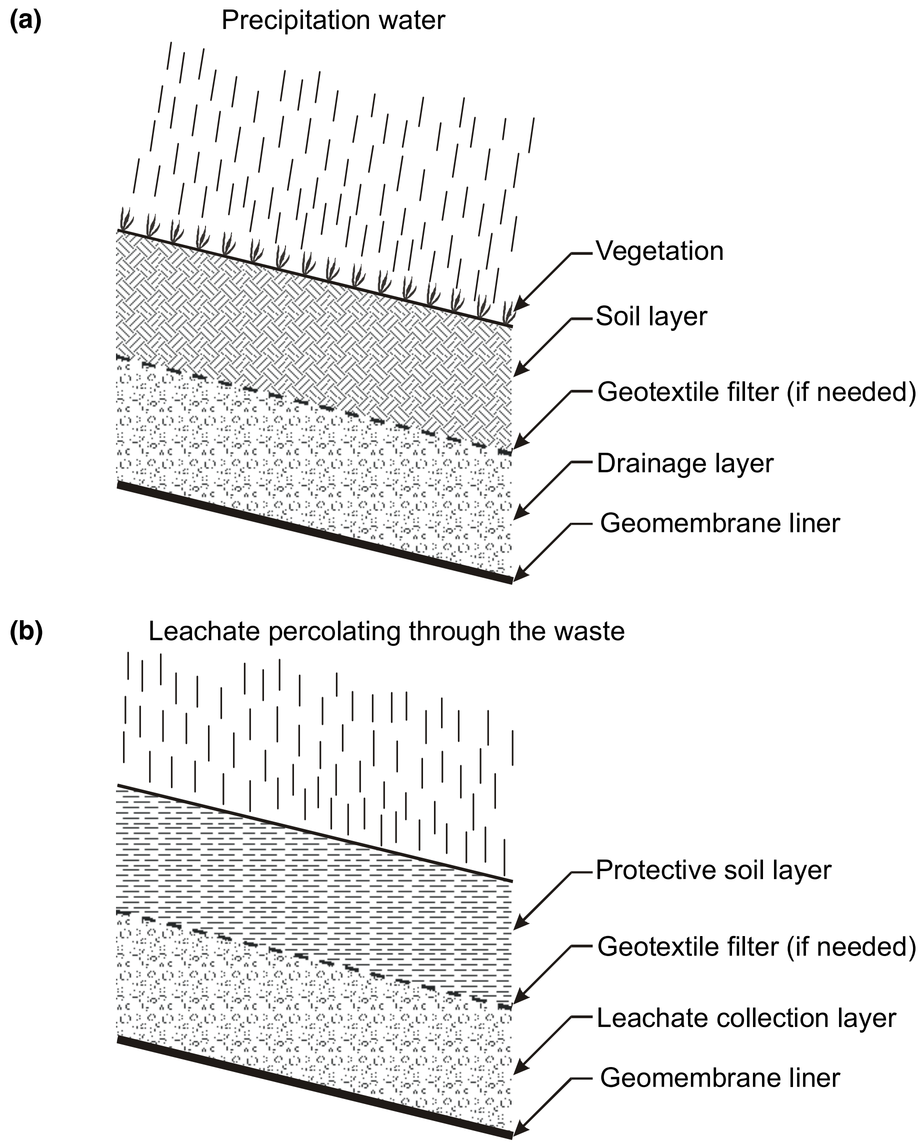

Figure 1. Examples of liquid collection layers subjected to a uniform supply of liquid in a landfill: (a) drainage layer in a cover system; (b) leachate collection layer.

Note: The drainage layer and the leachate collection layer, represented above as consisting of a granular material, may also consist of a geosynthetic drainage material, such as a geonet.

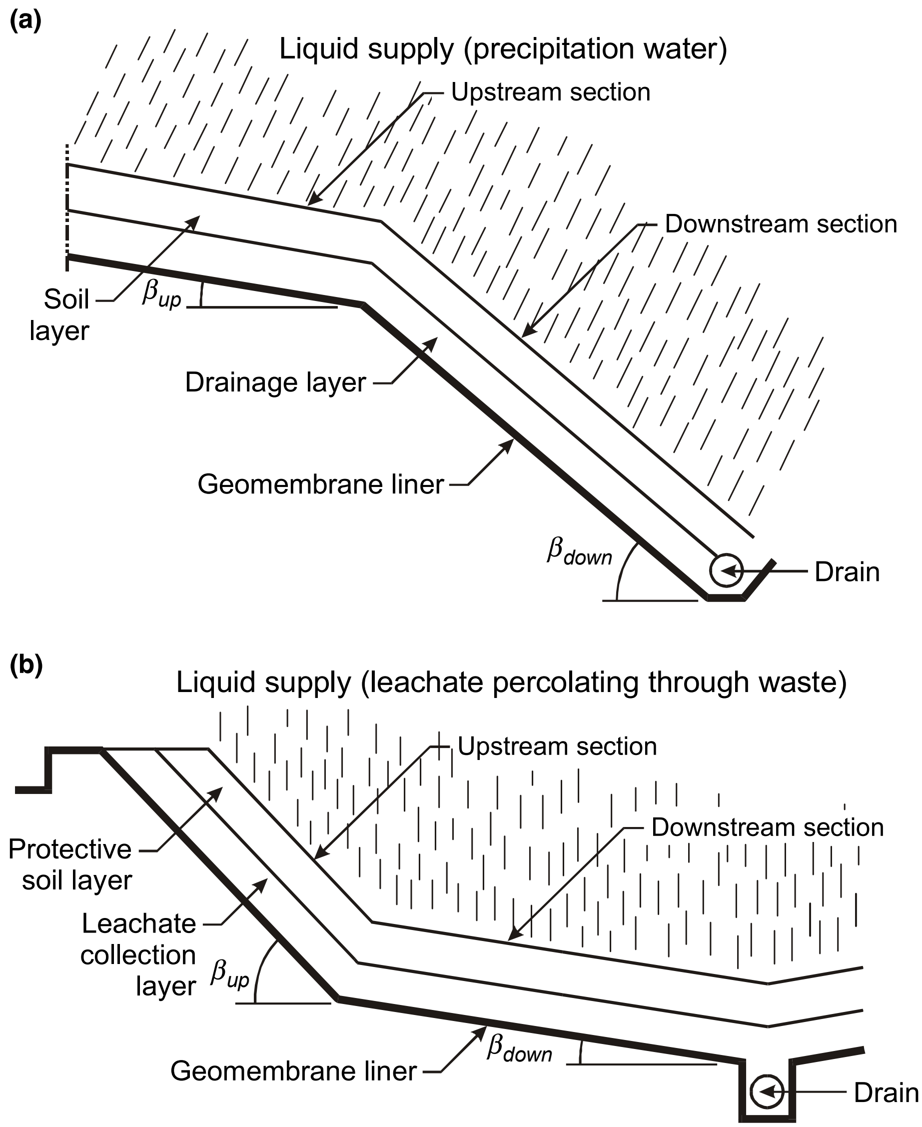

Figure 2. Examples of liquid collection layers located on two different slopes with no drain at the connection between the two slopes: (a) drainage layer in landfill cover system; (b) leachate collection layer in a landfill.

Notes: The liquid collection layer material can be a geonet or a granular material (sand, gravel) and can be different on the upstream slope and on the downstream slope. Typically, a geonet is used on steep slopes, and any type of permeable material is used on slopes that are not steep.

of the upstream section of the liquid collection layer and the subscript "*down*" for the characteristics of the downstream section of the liquid collection layer.

Liquid flow in the upstream section of the liquid collection layer *results only from liquid that impinges onto that section*, whereas liquid flow in the downstream section of the liquid collection layer *results from liquid that impinges onto both the upstream and downstream sections* (since it has been assumed that there is no drain removing the liquid at the connection between the two sections). The liquid surface in each section is affected by the characteristics of both the upstream and downstream sections. Particularly important is what happens at the connection between the two sections.

The above description can be summarized as follows for each section:

- In the upstream section, the flow rate is affected only by the characteristics of the upstream section (assuming steady-state conditions), whereas the liquid thickness is affected by the characteristics of both sections.
- In the downstream section, both the flow rate and the liquid thickness are affected by the characteristics of both sections.

1.3 Cases Considered

When a liquid collection layer comprises two sections, different liquid collection materials may be used in the two sections; for example, a geonet may be used on the steep slope and gravel may be used on the other slope. However, there are many applications where the same material is used in both sections; for example, a geonet may be used as the liquid collection layer in the various slopes of a landfill cover.

It would be impractical to consider all possible cases. The cases considered in the present paper are summarized in Figure 3. These cases are consistent with the state of practice. In particular, a geosynthetic liquid collection layer is considered on steep slopes, which is consistent with the current practice in landfill design.

1.4 Purpose and Scope of the Present Paper

The purpose of the present paper is to provide a method for calculating the maximum thickness of liquid in each of the two sections of a liquid collection layer that comprises two sections with different slopes, with no drain removing the liquid at the connection between the two sections. However, prior to presenting the method, it is necessary to review available information on analytical methods for evaluating the thickness of liquid in liquid collection layers placed on a single slope. This is done in Section 2. Then, the development of a method for calculating the maximum liquid thickness in the downstream section of the liquid collection layer is presented in Section 3, whereas the development of a method for calculating the maximum liquid thickness in the upstream section is presented in Section 4. The method is summarized in Section 5 where examples are presented.

The assumptions used in the analyses presented herein (e.g. laminar flow, steady-state flow conditions, uniform liquid supply) are the same as those made in the paper by Giroud et al. (2000a). Therefore, assumptions are not discussed herein. Also, the use of reduction factors and factors of safety is not discussed herein. Detailed guidance on the use of reduction factors and factors of safety is provided by Giroud et al. (2000a).

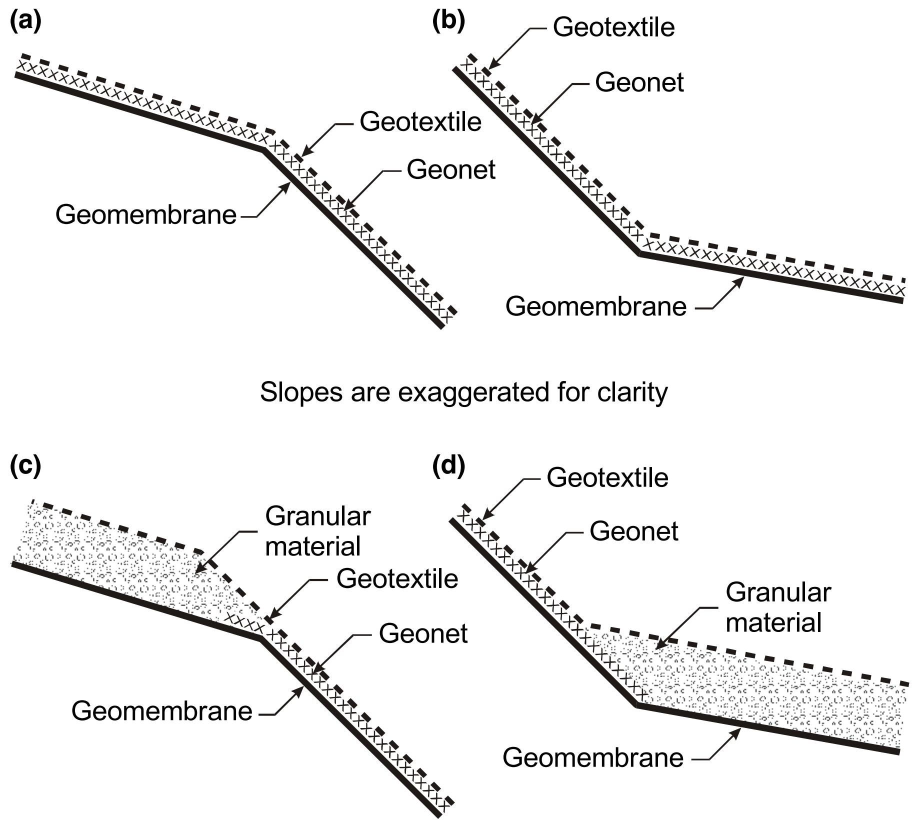

Figure 3. Schematic representation of the considered cases: (a) entirely geosynthetic liquid collection layer with steep downstream section; (b) entirely geosynthetic liquid collection layer with steep upstream section; (c) granular upstream section and geosynthetic downstream section; (d) geosynthetic upstream section and granular downstream section.

Note: A geosynthetic drainage material (designated as geonet for the sake of simplicity) is used on all steep slopes, which is consistent with the state of practice in landfill design in the United States.

2 REVIEW OF HYDRAULIC DESIGN OF LIQUID COLLECTION LAYERS WITH A SINGLE SLOPE

2.1 Overview

Section 2 presents information on liquid collection layers that meet the four conditions listed in Section 1.1. This information will be used in the analyses presented in Sections 3 and 4 to develop a methodology for liquid collection layers comprising two sections with different slopes.

Two cases are considered: the case where there is a perfect drain at the toe of the liquid collection layer, and the case where there is not a perfect drain at the toe of the liquid collection layer. The term "perfect drain" indicates that the elevation of liquid in the drain located at the toe of the liquid collection layer slope is below the bottom of the liquid collection layer. The liquid thickness is then zero at the toe of the liquid collection layer.

2.2 Maximum Liquid Thickness in a Liquid Collection Layer With a Perfect Drain at the Toe

A detailed study of liquid flow in a liquid collection layer located on a single slope with a perfect drain at the toe is presented by Giroud et al. (2000a).

2.2.1 *Shape of the Liquid Surface*

The shape of the liquid surface in the liquid collection layer when there is a perfect drain at the toe of the liquid collection layer is shown in Figure 4. The shape of the liquid surface depends on a dimensionless parameter, λ, called "characteristic parameter", and defined as follows:

$$\lambda = \frac{q_h}{k \tan^2 \beta} \tag{1}$$

where: q_h = liquid impingement rate (i.e. rate of liquid supply per unit horizontal area); k = hydraulic conductivity of the liquid collection material in the direction of the flow; and β = slope angle of the liquid collection layer with the horizontal.

2.2.2 *Maximum Liquid Thickness*

Regardless of the shape of the liquid surface, the maximum liquid thickness, t_{max}, in the liquid collection layer is given by the following equation, known as the modified Giroud's equation (Giroud et al. 2000a):

$$t_{max} = j \frac{\sqrt{\tan^2\beta + 4 q_h / k} - \tan\beta}{2 \cos\beta} L = j \frac{\sqrt{1 + 4\lambda} - 1}{2} \frac{\tan\beta}{\cos\beta} L \tag{2}$$

where L is the horizontal projection of the length of the liquid collection layer in the direction of the flow, and j is a dimensionless parameter called "modifying factor" and defined as follows:

$$j = 1 - 0.12 \exp\left\{-\left[\log\left(8\lambda / 5\right)^{5/8}\right]^2\right\} \tag{3}$$

Numerical values of the modifying factor, j, range between 0.88 and 1.00, as shown in Table 1. Therefore, a conservative approximation of Equation 2 is the following equation, which is known as the original Giroud's equation:

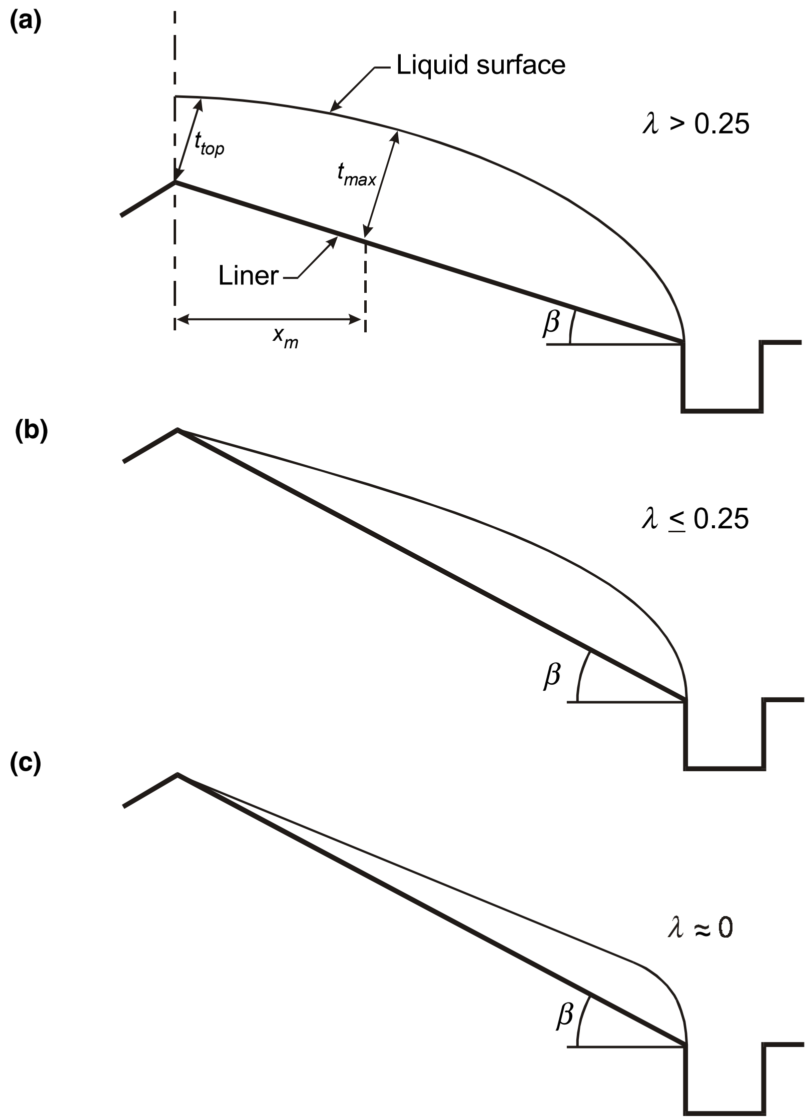

Figure 4. Shape of the liquid surface in a liquid collection layer as a function of the dimensionless characteristic parameter, λ: (a) $\lambda > 0.25$; (b) $\lambda \leq 0.25$; (c) λ very small (based on information from McEnroe (1993)).

Note: The maximum liquid thickness and the abscissa, x_m , at which it occurs are shown only in Figure 4a.

$$t_{max} = \frac{\sqrt{\tan^2\beta + 4\,q_h / k} - \tan\beta}{2\cos\beta} L = \frac{\sqrt{1+4\lambda} - 1}{2} \frac{\tan\beta}{\cos\beta} L \tag{4}$$

Table 1. Numerical values of *j* and x_m/L.

λ	j	x_m/L	λ	j	x_m/L
0	1.000	1.000	5	0.913	0.327
0.0001	1.000	0.999	10	0.932	0.252
0.001	0.994	0.993	15	0.943	0.214
0.01	0.966	0.956	20	0.950	0.190
0.05	0.925	0.883	30	0.960	0.160
0.10	0.906	0.830	50	0.971	0.128
0.15	0.897	0.792	100	0.982	0.093
0.20	0.891	0.761	200	0.990	0.068
0.25	0.887	0.735	500	0.996	0.044
0.50	0.880	0.645	1000	0.998	0.031
1	0.882	0.545	2000	0.999	0.022
2	0.891	0.446	5000	1.000	0.014

Note: The dimensionless parameter j was calculated using Equation 3 and x_m/L was calculated using Equation 7. The dimensionless parameter λ is defined by Equation 1.

When λ is very small (e.g. $\lambda < 0.01$), which occurs in many practical situations (Section 2.2.4), Equations 2 and 4 are equivalent to the following approximate equation (Giroud et al. 2000a):

$$t_{max} \approx t_{lim} = \frac{q_h}{k \sin\beta} L = \frac{q_h}{k \tan^2\beta} \frac{\tan\beta}{\cos\beta} L = \lambda \frac{\tan\beta}{\cos\beta} L \qquad (5)$$

where t_{lim} is the maximum liquid thickness in the limit case where q_h is small and β and k are large (Giroud et al. 2000a).

It should be noted that:

$$j \frac{\sqrt{1+4\lambda}-1}{2} < \frac{\sqrt{1+4\lambda}-1}{2} < \frac{\sqrt{1+4\lambda+4\lambda^2}-1}{2} = \frac{(1+2\lambda)-1}{2} = \lambda \qquad (6)$$

Therefore, regardless of the value of λ, Equation 5 provides a conservative value of the maximum liquid thickness (i.e. a value of the maximum liquid thickness greater than the value calculated more accurately using Equations 2 or 4).

Equation 5 is simpler than Equation 4, which in turn is simpler than Equation 2. A detailed discussion of the approximation made when Equations 4 or 5 are used is presented by Giroud et al. (2000a) who concluded that Equation 5 provides an acceptable approximation of t_{max} if the liquid thickness is less than one tenth of the height of the liquid collection layer (i.e. the difference in elevation between the top and the toe of the liquid collection layer slope). As a result, from a practical standpoint, Equation 5 is always valid in the case of geosynthetic liquid collection layers (and is then preferred to Equations 2 and 4 because it is simpler) and rarely valid in the case of granular liquid collection layers located on a slope that is not steep. Accordingly, in the present paper, Equation 5 will be used systematically for geosynthetic liquid collection layers, and

Equation 2 (or Equation 4, which is a conservative approximation of Equation 2) will be used systematically for granular liquid collection layers.

2.2.3 *Location of the Maximum Liquid Thickness*

As shown by Giroud et al. (2000a), the location of the maximum liquid thickness is given by the following equation:

$$\frac{x_m}{L} = \frac{t_{max}}{t_{lim}} = j\,\frac{\sqrt{\tan^2\beta \; + \; 4\,q_h\,/\,k} \; - \; \tan\beta}{2\,\left(q_h\,/\,k\right)\,/\,\tan\beta} = j\,\frac{\sqrt{1+4\lambda}-1}{2\lambda} \qquad (7)$$

where x_m is the horizontal distance from the top of the liquid collection layer slope to the location of the maximum liquid thickness (Figure 4a).

Numerical values of x_m/L calculated using Equation 7 are given in Table 1. It appears in Table 1 that: when λ is small (e.g. $\lambda < 0.1$), x_m/L is close to 1, and when λ is large (e.g. $\lambda > 0.25$), x_m/L is less than 0.735.

2.2.4 *Discussion*

Table 2 presents values of λ that are typical for landfill liquid collection layers. These values were obtained from a parametric study considering low and high values of:

- liquid impingement rate (i.e., in a landfill, the rate at which the fraction of precipitation water that percolates through the vegetative layer reaches the cover liquid collection layer, or the rate at which leachate reaches the leachate collection layer): 1 m/year and 0.1 m/day;
- hydraulic conductivity of the liquid collection layer material: 1×10^{-3} m/s (sand) and 0.1 m/s (geonet, gravel); and
- slope of the liquid collection layer: 2% and 1V:3H.

Table 2. Typical values of the dimensionless parameter λ.

		Liquid supply rate, q_h			
		1 m/year (3.2×10^{-8} m/s)		0.1 m/day (1.2×10^{-6} m/s)	
Hydraulic conductivity of the drainage layer material, *k* (m/s)		1×10^{-3}	1×10^{-1}	1×10^{-3}	1×10^{-1}
Slope $\tan\beta$	2% = 0.02	7.9×10^{-2}	7.9×10^{-4}	2.9	2.9×10^{-2}
	1V:3H = 0.33	2.9×10^{-4}	2.9×10^{-6}	1.0×10^{-2}	1.0×10^{-4}

Note: The tabulated values of λ were calculated using Equation 1. A hydraulic conductivity of 1×10^{-3} m/s is typical of sand whereas a hydraulic conductivity of 1×10^{-1} m/s is typical of gravel or geonet.

It appears in Table 2 that λ is rather small (i.e. less than 0.1) in all typical cases, except in the case of a liquid collection layer with a relatively low hydraulic conductivity (sand) placed on a slope that is not steep (2%) and that is subjected to a high liquid impingement rate (0.1 m/day). Furthermore, in the case of geosynthetic liquid collection layers, λ is very small because the maximum liquid thickness is very small compared to the length of the liquid collection layer. Indeed, Equation 5 shows that, if t_{max}/L is very small, λ is also very small. The shape of the liquid surface is then illustrated in Figure 4c. The thickness at the top is zero and the maximum liquid thickness (which occurs close to the toe) is small. Therefore, in the case of a geosynthetic liquid collection layer, the slope of the liquid surface is quasi parallel to the slope of the liquid collection layer and, as a result, the hydraulic gradient is approximately equal to the classical value for flow parallel to a slope, $\sin\beta$. In contrast, in the case of a granular liquid collection layer, the slope of the liquid surface (Figures 4a and 4b) increases from the top to the toe of the liquid collection layer. As a result, the hydraulic gradient increases from the top to the toe of the liquid collection layer, where it is significantly greater than $\sin\beta$. This comment will be useful in Section 4.3.

2.3 Maximum Liquid Thickness in a Liquid Collection Layer Without a Perfect Drain at the Toe

If there is not a perfect drain at the toe of the liquid collection layer, the liquid thickness at the toe of the slope is not zero, and the maximum liquid thickness in the liquid collection layer can be calculated using the following equations derived from McEnroe's equations (McEnroe 1993) using a transformation developed by Giroud et al. (2000a):

- for $\lambda < 0.25$

$$t_{max} = L\frac{\tan\beta}{\cos\beta}\left[\lambda - \frac{t_{toe}\cos\beta}{L\tan\beta} + \left(\frac{t_{toe}\cos\beta}{L\tan\beta}\right)^2\right]^{1/2}\left[\frac{(1-A'-2\lambda)\left(1+A'-\dfrac{2\,t_{toe}\cos\beta}{L\tan\beta}\right)}{(1+A'-2\lambda)\left(1-A'-\dfrac{2\,t_{toe}\cos\beta}{L\tan\beta}\right)}\right]^{1/(2A')} \tag{8}$$

- for $\lambda = 0.25$

$$t_{max} = \left(\frac{L\tan\beta}{2\cos\beta} - t_{toe}\right)\exp\left(\frac{4\,t_{toe} - L\tan\beta/\cos\beta}{L\tan\beta/\cos\beta - 2\,t_{toe}}\right) \tag{9}$$

- for $\lambda > 0.25$

$$t_{max} = L\,\frac{\tan\beta}{\cos\beta}\left[\lambda - \frac{t_{toe}\cos\beta}{L\tan\beta} + \left(\frac{t_{toe}\cos\beta}{L\tan\beta}\right)^2\right]^{1/2}\exp\left[\frac{1}{B'}\tan^{-1}\left(\frac{\dfrac{2\,t_{toe}\cos\beta}{L\tan\beta}-1}{B'}\right) - \frac{1}{B'}\tan^{-1}\left(\frac{2\lambda-1}{B'}\right)\right] \tag{10}$$

where t_{toe} is the liquid thickness at the toe of the liquid collection layer slope, and A' and B' are dimensionless parameters defined by:

$$A' = \sqrt{1-4\lambda} \qquad B' = \sqrt{4\lambda - 1} \tag{11}$$

A detailed discussion of McEnroe's equations is provided by Giroud et al. (2000a).

In the special case where t_{toe} is equal to t_{lim} given by Equation 5, Equations 8 to 10 show that $t_{max} = t_{lim}$. The same conclusion can be inferred from Equation 7. If the maximum liquid thickness occurs at the toe of the liquid collection layer (i.e. if $t_{max} = t_{toe}$), then, $x_m = L$, i.e. $x_m/L = 1.0$; hence, from Equation 7, $t_{max}/t_{lim} = 1.0$, i.e. $t_{max} = t_{toe} = t_{lim}$. It is important to note that this result is valid regardless of the value of the characteristic parameter, λ. This important result will be extensively used in the present paper.

The important result mentioned above becomes even more relevant in the case of a geosynthetic liquid collection layer. As indicated in Section 2.2.2, $t_{max} \approx t_{lim}$ in the case of a geosynthetic liquid collection layer with a perfect drain at the toe. Combining this and the important result mentioned above shows that, in the case of a geosynthetic liquid collection layer, $t_{max} \approx t_{lim}$ regardless of the value of t_{toe} , provided that $0 \leq t_{toe} \leq t_{lim}$. This will be useful in Section 4.3.2.

Further discussion of the case where there is not a perfect drain at the toe of the liquid collection layer may be found in the appendix.

3 MAXIMUM LIQUID THICKNESS IN THE DOWNSTREAM SECTION OF THE LIQUID COLLECTION LAYER

3.1 Approach

Liquid flow in the downstream section of the liquid collection layer is governed by a differential equation. This differential equation is complex because liquid flow in the downstream section depends on the characteristics of both the upstream and downstream sections (as indicated in Section 1.2). Instead of trying to solve a complex differential equation, it is proposed to use the superposition of two elementary components to obtain an approximate value of the maximum liquid thickness in the downstream section. The two elementary components that are superimposed are (Figure 5):

- liquid flow in the downstream section of the liquid collection layer due to *liquid flowing from the upstream section*, which is due to liquid impinging onto the upstream section only (Figure 5a); and
- liquid flow in the downstream section of the liquid collection layer due to *liquid impinging onto the downstream section* only (Figure 5b).

The subscript "*1*" will be used for the first elementary component and the subscript "*2*" for the second elementary component. It is assumed that the superposition of the above elementary components provides an acceptable approximation of the maximum liquid thickness for the actual case (Figure 5c) where liquid flow in the downstream section of the liquid collection layer results from *liquid flowing from the upstream section* (which results from liquid impinging onto the upstream section) plus *liquid impinging onto the downstream section*.

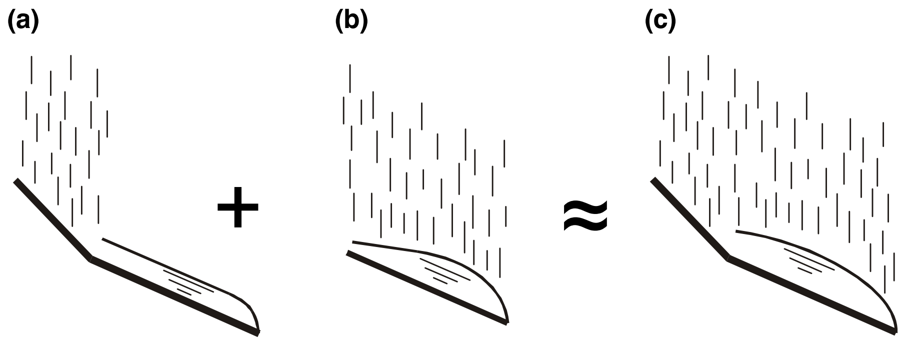

Figure 5. Use of superposition to determine the flow in the downstream section of a liquid collection layer layer that comprises two sections: (a) liquid impinging onto the upstream section only; (b) liquid impinging onto the downstream section only; (c) actual case.

Superposition is an approach that is often used to address flow problems. A discussion of the validity of this approach is presented in Section 3.3.1.

3.2 First Component of the Superposition: Liquid Flow in the Downstream Section of the Liquid Collection Layer Due to Liquid Impinging Onto, and Flowing From, the Upstream Section

Analysis of liquid flow in the downstream section of the liquid collection layer due to liquid impinging onto, and flowing from, the upstream section is the classical case of liquid flow in a liquid collection layer, located on a single slope with a perfect drain at the toe, due to a source of liquid at the top of the slope (Figure 6). In this case, the liquid thickness in the liquid collection layer is uniform and equal to the maximum liquid thickness along most of the slope. The liquid thickness only decreases near the toe of the slope due to the presence of a drain.

The relationship between the flow rate per unit width in the downstream section of the liquid collection layer due to liquid impinging onto, and flowing from, the upstream section, $Q^*_{down\,1}$, and the maximum liquid thickness (i.e. the liquid thickness away from the toe of the slope) in the downstream section of the liquid collection layer due to liquid impinging onto, and flowing from, the upstream section, $t_{down\,max\,1}$, is given as follows by Darcy's equation:

$$Q^*_{down\,1} = k_{down}\ i_{down\,1}\ t_{down\,max\,1} \tag{12}$$

where: k_{down} = hydraulic conductivity of the liquid collection layer material in the downstream section of the liquid collection layer; and i_{down1} = hydraulic gradient (related to the liquid impinging onto, and flowing from, the upstream section) in the portion of the downstream section of the liquid collection layer where the thickness of liquid flowing from the upstream section is uniform (i.e. the upper portion of the downstream section, as seen in Figure 6).

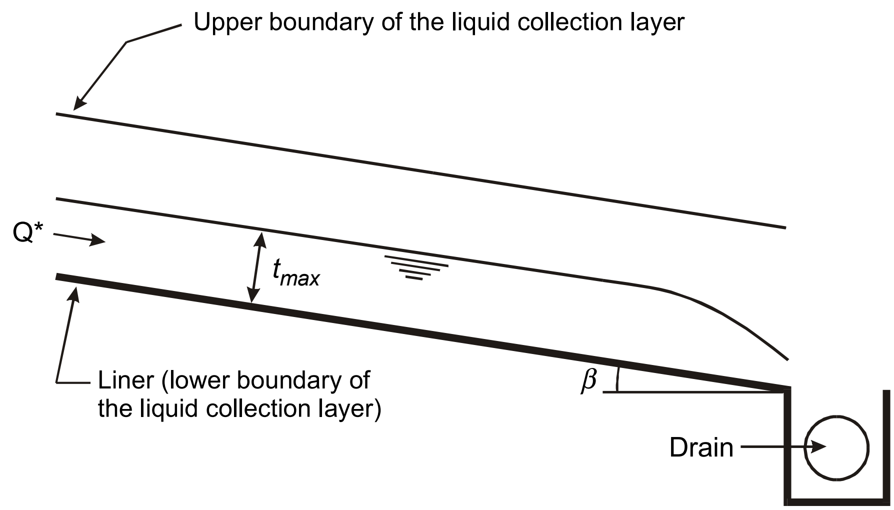

Figure 6. Liquid flow in a liquid collection layer located on a single slope due to a source of liquid at the top of the slope.

Note: The liquid surface is parallel to the slope, except near the toe of the slope, due to the presence of a drain. The hydraulic gradient is equal to sinβ in the portion where the liquid surface is parallel to the slope.

In the portion of the downstream section of the liquid collection layer where the thickness of liquid flowing from the upstream section is uniform (Figure 6), the hydraulic gradient (related to the liquid impinging onto, and flowing from, the upstream section) is given by the following classical equation:

$$i_{down\ 1} = \sin \beta_{down} \tag{13}$$

where β_{down} is the slope angle of the downstream section of the liquid collection layer (Figure 2).

Combining Equations 12 and 13 gives:

$$t_{down\ max\ 1} = \frac{Q^*_{down\ 1}}{k_{down}\ \sin\beta_{down}} \tag{14}$$

The liquid flow rate in the downstream section of the liquid collection layer due to liquid impinging onto, and flowing from, the upstream section, $Q^*_{down\ 1}$, can be calculated as follows:

$$Q^*_{down\ 1} = q_h\ L_{up} \tag{15}$$

where L_{up} is the horizontal length of the upstream section of the liquid collection layer.

Combining Equations 14 and 15 gives:

$$t_{down\ max\ 1} = \frac{q_h\ L_{up}}{k_{down} \sin \beta_{down}} \tag{16}$$

Equation 16 is applicable regardless of the type of liquid collection layer material (i.e. geosynthetic or granular).

3.3 Superposition

3.3.1 *Basic Equation and Discussion*

The superposition approach illustrated in Figure 5 is used as follows to calculate the maximum thickness of liquid in the downstream section of the liquid collection layer, $t_{down\ max}$:

$$t_{down\ max} = t_{down\ max\ 1} + t_{down\ max\ 2} \tag{17}$$

where $t_{down\ max\ 2}$ is the maximum liquid thickness in the downstream section of the liquid collection layer due to liquid impinging onto the downstream section.

It should be noted that Equation 17 implies that the maximum thickness $t_{down\ max\ 1}$ (case shown in Figure 5a) occurs at the same abscissa, x, as the maximum thickness $t_{down\ max\ 2}$ (case shown in Figure 5b). If the two maxima do not occur at the same abscissa, x, Equation 17 is conservative, i.e. it gives a value of $t_{down\ max}$ greater than the value calculated by adding up the two components of the liquid thickness shown in Figures 5a and 5b, for each value of x, and taking the maximum of the curve thus obtained. As shown in Figure 6, $t_{down\ max\ 1}$ occurs everywhere along the slope except near the toe. As indicated in Table 1, $t_{down\ max\ 2}$ occurs near the toe of the slope when λ_{down} is small (where λ_{down} is the value of λ calculated with the characteristics of the downstream section of the liquid collection layer). In contrast, as pointed out in Section 2.2.3, the abscissa where $t_{down\ max\ 2}$ occurs, x_m , is such that x_m/L is between 0.735 and 0 when λ_{down} is between 0.25 and infinity. Therefore, $t_{down\ max\ 1}$ and $t_{down\ max\ 2}$ are likely to occur at the same abscissa when λ_{down} is large (e.g. greater than approximately 0.25), and they are not likely to occur at the same abscissa when λ_{down} is small. Therefore, the use of Equation 17 is accurate when λ_{down} is large (which is generally the case of granular liquid collection layers) and is conservative when λ_{down} is small (which is the case of geosynthetic liquid collection layers). However, the fact that the use of Equation 17 is accurate or conservative does not necessarily mean that using superposition is accurate or conservative, respectively. In other words, a conservative method of implementing superposition does not necessarily mean that using superposition to calculate $t_{down\ max}$ is conservative. Indeed, in Section 3.1, it was assumed that the superposition approach provides an acceptable approximation, but it was not demonstrated whether or not superposition is conservative.

The only conclusion that can be drawn from the foregoing discussion is the following: since λ_{down} is small in the case of geosynthetic liquid collection layers and large in the case of granular liquid collection layers, the use of superposition is more conservative in the case of geosynthetic liquid collection layers than in the case of granular liquid collection layers.

The superposition discussed above and expressed by Equation 17 is used for geosynthetic liquid collection layers in Section 3.3.2 and for granular liquid collection layers in Section 3.3.3.

3.3.2 *Case of a Geosynthetic Liquid Collection Layer in the Downstream Section*

Based on information provided in Section 2.2.2, the maximum thickness of liquid in a geosynthetic downstream section of the liquid collection layer due to liquid impinging onto that section is given by the following equation derived from Equation 5 by using the subscripts "*down*" and "*2*" at appropriate locations:

$$t_{down\ max\ 2} = \frac{q_h\ L_{down}}{k_{down}\ \sin\beta_{down}} \tag{18}$$

where L_{down} is the horizontal length of the downstream section of the liquid collection layer.

Combining Equations 16 to 18 gives:

$$t_{down\ max} = \frac{q_h\ L_{up}}{k_{down}\sin\beta_{down}} + \frac{q_h\ L_{down}}{k_{down}\sin\beta_{down}} \tag{19}$$

hence:

$$t_{down\ max} = \frac{q_h\left(L_{up} + L_{down}\right)}{k_{down}\sin\beta_{down}} \tag{20}$$

Equation 20 gives the maximum liquid thickness in the case of a geosynthetic liquid collection layer in the downstream section. Comparing Equations 5 and 20 shows that, in the case of a geosynthetic downstream section, the maximum liquid thickness in the downstream slope is the same as if the entire liquid collection layer (from the top of the upstream slope to the toe in the downstream slope, i.e. a horizontal length $L_{up} + L_{down}$) had a hydraulic conductivity k_{down} and a slope angle β_{down} .

3.3.3 *Case of a Granular Liquid Collection Layer in the Downstream Section*

Based on information provided in Section 2.2.2, the maximum thickness of liquid in a granular downstream section of the liquid collection layer due to liquid impinging onto that section is given by the following equation derived from Equation 2 by using the subscripts "*down*" and "*2*" at appropriate locations:

$$t_{down\ max\ 2} = j_{down}\ \frac{\sqrt{\tan^2\beta_{down} + 4\,q_h / k_{down}} - \tan\beta_{down}}{2\cos\beta_{down}}\ L_{down} \tag{21}$$

where j_{down} is the value of the dimensionless modifying factor, j, calculated using the following equation derived from Equation 3:

$$j_{down} = 1 - 0.12\ \exp\left\{-\left[\log\left(8\lambda_{down}/5\right)^{5/8}\right]^2\right\} \tag{22}$$

where λ_{down} is calculated using the following equation derived from Equation 1:

$$\lambda_{down} = \frac{q_h}{k_{down}\tan^2\beta_{down}} \tag{23}$$

Combining Equations 16, 17, and 21 gives:

$$t_{down\,max} = j_{down} \frac{\sqrt{\tan^2\beta_{down} + 4\,q_h / k_{down}} - \tan\beta_{down}}{2\cos\beta_{down}} L_{down} + \frac{q_h\, L_{up}}{k_{down}\sin\beta_{down}} \tag{24}$$

Combining Equations 23 and 24 gives:

$$t_{down\,max} = \left(j_{down} \frac{\sqrt{1+4\lambda_{down}} - 1}{2} L_{down} + \lambda_{down}\, L_{up} \right) \frac{\tan\beta_{down}}{\cos\beta_{down}} \tag{25}$$

where λ_{down} is given by Equation 23 and j_{down} is given by Equation 22.

Equation 24 (or Equation 25, which is equivalent) gives the maximum liquid thickness in the case of a granular liquid collection layer in the downstream section. It should be noted that, if λ_{down} is very small (i.e. $\lambda_{down} < 0.001$), which may happen in the case of granular liquid collection layers made with very permeable gravel, Equations 24 and 25 tend toward Equation 20.

4 MAXIMUM LIQUID THICKNESS IN THE UPSTREAM SECTION OF THE LIQUID COLLECTION LAYER

4.1 Approach

4.1.1 *Cases Considered*

Two cases must be considered regarding the maximum liquid thickness in the upstream section of the liquid collection layer. These two cases depend on the material in the *downstream* section. The case where the downstream section of the liquid collection layer is made of a geosynthetic is analyzed in Section 4.2, and the case where the downstream section of the liquid collection layer is made of a granular material is analyzed in Section 4.3. First, a general relationship used in all analyses is presented in Section 4.1.2.

4.1.2 *General Relationship*

The following general relationship at the connection between two sections will be used in the analyses. The flow rate at the top of the downstream section is equal to the flow rate at the toe of the upstream section. This can be expressed as follows using Darcy's equation:

$$k_{up}\; i_{up\,toe}\; t_{up\,toe} = k_{down}\; i_{down\,top}\; t_{down\,top} \tag{26}$$

where: k_{up} = hydraulic conductivity of the material of the upstream section of the liquid collection layer; $i_{up\,toe}$ = hydraulic gradient at the toe of the upstream section; $t_{up\,toe}$ = liquid thickness at the toe of the upstream section; $i_{down\,top}$ = hydraulic gradient at the top of the downstream section; and $t_{down\,top}$ = liquid thickness at the top of the downstream section.

4.2 Case of a Geosynthetic Downstream Section

4.2.1 *Overview*

The case of a geosynthetic downstream section is illustrated in Figures 3a, 3b, and 3c. The analysis is performed in two steps. First, an equation giving the liquid thickness at the toe of the upstream section is established (Section 4.2.2). Then, this equation is used in an analysis that leads to the calculation of the maximum liquid thickness in the upstream section: first, in the case of a geosynthetic upstream section (Section 4.2.3, Figures 3a and 3b); and second, in the case of a granular upstream section (Section 4.2.4, Figure 3c).

4.2.2 *Determination of the Liquid Thickness at the Toe of the Upstream Section*

Section 4.2 presents the case of a downstream section that is made of a geosynthetic. There is a perfect drain at the toe of that section of the liquid collection layer. The liquid thickness in the downstream section is the sum of two components: (i) the liquid thickness due to *liquid impinging onto*, and *flowing from*, *the upstream section*; and (ii) the liquid thickness due to *liquid impinging onto the downstream section*.

Hydraulic Gradient at the Top of the Geosynthetic Downstream Section. The following comments can be made on the two components mentioned above, regarding hydraulic gradient:

- As pointed out in Section 2.2.4 for the case of flow due to liquid impinging onto a geosynthetic liquid collection layer on a single slope with a perfect drain at the toe, the hydraulic gradient is approximately equal to $\sin\beta$ along the entire slope (i.e. *the liquid surface is almost parallel to the slope*). Therefore, the hydraulic gradient in the downstream section due to the first component of the liquid thickness in the downstream section is approximately $\sin\beta_{down}$.
- As indicated in Section 3.2 (Figure 6), for the case of flow in the downstream section due to liquid impinging onto, and flowing from, the upstream section, the liquid surface is *parallel* to the slope in the upper part of the liquid collection layer. As a result, the hydraulic gradient in the upper part of the downstream section of the liquid collection layer due to the second component of the liquid thickness in the downstream section is equal to $\sin\beta_{down}$.

Therefore, when the two flow components are superimposed, the liquid surface continues to be *almost parallel* to the slope and, consequently:

$$i_{down\ top} \approx \sin\beta_{down} \tag{27}$$

Liquid Thickness at the Top of the Geosynthetic Downstream Section. There is a perfect drain at the toe of the downstream section. As indicated at the beginning of Section 4.2.2, the liquid thickness at the top of the downstream section is the sum of two components: (i) the liquid thickness due to liquid impinging onto, and flowing from, the upstream section; and (ii) the liquid thickness due to liquid impinging onto the downstream section. It has been shown in Section 2.2.4 that the liquid thickness at the top of a single-slope geosynthetic liquid collection layer with a perfect drain at the toe is zero (Figure 4c). Therefore, the liquid thickness at the top of a geosynthetic downstream section due to liquid impinging onto that section is zero. As a result, the liquid thickness at the top of a geosynthetic downstream section is only due to liquid impinging onto, and flowing from, the upstream section. It has been shown in Section 3.2 (Figure 6) that, in this case, the liquid thickness at the top of the downstream section is equal to the maximum liquid thickness. Therefore, based on Equation 16:

$$t_{down\ top} = t_{down\ max\ 1} = \frac{q_h\, L_{up}}{k_{down} \sin \beta_{down}} \tag{28}$$

Expression of the Liquid Thickness at the Toe of the Upstream Section. Combining Equations 26 to 28 gives:

$$t_{up\ toe} = \frac{q_h\, L_{up}}{k_{up}\ i_{up\ toe}} \tag{29}$$

Equation 29 will be used for the case of a geosynthetic upstream section in Section 4.2.3, and for the case of a granular upstream section in Section 4.2.4.

4.2.3 *Maximum Liquid Thickness in the Upstream Section of an Entirely Geosynthetic Liquid Collection Layer*

The case of an entirely geosynthetic liquid collection layer is illustrated in Figures 3a and 3b. As indicated at the end of Section 4.2.2, Equation 29 is applicable to this case. To use this equation, it is necessary to evaluate the hydraulic gradient at the toe of the upstream section of the liquid collection layer, $i_{up\ toe}$. Since the thickness of the geosynthetic is very small compared to the length of the slope, the liquid surface is almost parallel to the slope and, therefore:

$$i_{up\ toe} \approx i_{up} \approx \sin \beta_{up} \tag{30}$$

where β_{up} is the slope angle of the upstream section of the liquid collection layer (Figure 2).

Combining Equations 29 and 30 gives:

$$t_{up\ toe} \approx \frac{q_h\, L_{up}}{k_{up} \sin \beta_{up}} \tag{31}$$

Based on Equation 5:

$$t_{up\ lim} = \frac{q_h\, L_{up}}{k_{up} \sin \beta_{up}} \tag{32}$$

where $t_{up\ lim}$ is the value of t_{lim} in the upstream section of the liquid collection layer.

Comparing Equations 31 and 32 shows that $t_{up\ toe} \approx t_{up\ lim}$. Based on the result mentioned after Equation 11, it is then concluded that:

$$t_{up\,max} = t_{up\,lim} \approx t_{up\,toe} \tag{33}$$

where $t_{up\,max}$ is the maximum liquid thickness in the upstream section of the liquid collection layer.

Combining Equations 31 and 33 gives:

$$t_{up\,max} \approx \frac{q_h L_{up}}{k_{up} \sin \beta_{up}} \tag{34}$$

Equation 34 gives the maximum liquid thickness in the upstream section of an entirely geosynthetic liquid collection layer.

4.2.4 *Case of a Geosynthetic Liquid Collection Layer in the Downstream Section and a Granular Liquid Collection Layer in the Upstream Section*

Upper Boundary of the Maximum Liquid Thickness. The case of a geosynthetic liquid collection layer on the steep downstream slope and a granular liquid collection layer on the upstream slope that is not steep is illustrated in Figure 3c. As indicated at the end of Section 4.2.2, Equation 29 is applicable to this case. To use Equation 29, it is necessary to evaluate the hydraulic gradient at the toe of the upstream section of the liquid collection layer, $i_{up\,toe}$. As pointed out in Section 2.2.4 for the case of a granular liquid collection layer on a single slope with a perfect drain at the toe, the hydraulic gradient varies along the slope and is significantly greater than $\sin\beta$ at the toe of the slope. There is not a perfect drain at the toe of the upstream section. However, since a geosynthetic is used in the downstream section, the liquid thickness in the downstream section (and, in particular, at the top of the downstream section) is small. Therefore, it may be considered that there is almost a perfect drain at the toe of the upstream section, and, consequently, it may be assumed that (as in the case of a perfect drain) the hydraulic gradient at the toe of the upstream section is greater than $\sin\beta_{up}$, i.e.:

$$i_{up\,toe} > \sin \beta_{up} \tag{35}$$

Therefore, Equation 31, which was obtained for $i_{up\,toe} = \sin\beta_{up}$, can be used to calculate an upper boundary of the liquid thickness at the toe of the upstream section made of a granular material when the downstream section is made of a geosynthetic. Hence:

$$t_{up\,toe} < \frac{q_h L_{up}}{k_{up} \sin \beta_{up}} \tag{36}$$

Therefore, Equation 34, which is derived from Equation 31, can be used to calculate an upper boundary of the maximum liquid thickness in the upstream section made of a granular material when the downstream section is made of a geosynthetic. Hence:

$$t_{up\,max} < \frac{q_h L_{up}}{k_{up} \sin \beta_{up}} \tag{37}$$

Approximate Value of the Maximum Liquid Thickness. An approximate value of the maximum thickness of liquid in a granular upstream section when a geosynthetic is used in the downstream section can be calculated as follows. As indicated above, there is almost a perfect drain at the toe of the upstream section. Therefore, an approximate value of the maximum liquid thickness in the upstream section can be calculated using the following equation derived from Equation 2:

$$t_{up\,max} \approx j_{up} \frac{\sqrt{\tan^2\beta_{up} + 4q_h/k_{up}} - \tan\beta_{up}}{2\cos\beta_{up}} L_{up} = j_{up} \frac{\sqrt{1+4\lambda_{up}}-1}{2} \frac{\tan\beta_{up}}{\cos\beta_{up}} L_{up} \quad (38)$$

where λ_{up} is the value of the dimensionless characteristic parameter, λ, for the upstream section, calculated using the following equation derived from Equation 1:

$$\lambda_{up} = \frac{q_h}{k_{up}\tan^2\beta_{up}} \quad (39)$$

and j_{up} is the value of the dimensionless modifying factor, j, for the upstream section, calculated using the following equation derived from Equation 3:

$$j_{up} = 1 - 0.12\ \exp\left\{-\left[\log\left(8\lambda_{up}/5\right)^{5/8}\right]^2\right\} \quad (40)$$

The value of $t_{up\,max}$ calculated using Equation 38 is, strictly speaking, a lower boundary of $t_{up\,max}$ because Equation 38 assumes that the value $t_{up\,toe}$ is zero, whereas in reality it is not zero. However, the $\approx$ sign is used in Equation 38 (rather than the $<$ sign) to show that the value of $t_{up\,max}$ calculated using Equation 38 is an approximate value of $t_{up\,max}$ (in addition to being a lower boundary). This is because the approximately calculated value of $t_{up\,max}$ is likely to be close to the rigorously calculated value since the actual liquid thickness at the toe of the upstream section is close to zero.

Another way to calculate an approximate value of $t_{up\,max}$ consists of using Equations 8 to 10 with $\beta = \beta_{up}$ and $\lambda = \lambda_{up}$ given by Equation 39. However, to use Equations 8 to 10, it is necessary to know $t_{up\,toe}$. The value of $t_{up\,toe}$ should be calculated using Equation 29 (which is equivalent to saying that $t_{up\,toe}$ should be derived from the known value of $t_{down\,top}$ using Equation 26). However, since $i_{up\,toe}$ is unknown, Equation 26 (or Equation 29) cannot be used and, as a result, the value of $t_{up\,toe}$ is unknown. Tentatively, it may be assumed that $t_{up\,toe}$ is equal to $t_{down\,top}$ (which can be calculated using Equation 28). This assumption is arbitrary, but it makes it possible to calculate an approximate value/lower boundary for $t_{up\,max}$, which can be expected to be higher than the approximate value/lower boundary obtained with Equation 38. The use of Equations 8 to 10 requires lengthy calculations. These calculations can be avoided by using a graphical solution presented in the appendix. The use of this method is illustrated by Example 2.

Finally, an approximate value of $t_{up\,max}$ can be obtained by using the original Giroud's equation (Equation 4), i.e. by using the following equation, which is identical to Equation 38 without the modifying factor, j_{up}:

$$t_{up\,max} \approx \frac{\sqrt{\tan^2\beta_{up} + 4\,q_h / k_{up}} - \tan\beta_{up}}{2\cos\beta_{up}} L_{up} = \frac{\sqrt{1+4\lambda_{up}} - 1}{2} \frac{\tan\beta_{up}}{\cos\beta_{up}} L_{up} \qquad (41)$$

The value of $t_{up\,max}$ calculated using Equation 41 is larger than the value calculated using Equation 38 because numerical values of j_{up} range between 0.88 and 1.0 (see Section 2.2.2). The use of Equation 41 is illustrated by Example 2.

4.3 Case of a Granular Downstream Section and Geosynthetic Upstream Section

4.3.1 *Overview*

The case of a granular downstream section considered herein is illustrated in Figure 3d: the slope of the granular downstream section is not steep and a geosynthetic is used on the steep upstream slope. Two cases must be considered depending on the value of the dimensionless parameter λ_{down} . The case where λ_{down} is less than or equal to 0.25 is addressed in Section 4.3.2, and the case where λ_{down} is greater than 0.25 is addressed in Section 4.3.3.

4.3.2 *Case $\lambda_{down} \leq 0.25$*

In the case of a granular downstream section with $\lambda_{down} \leq 0.25$, the approach is similar to the approach used for a geosynthetic downstream section (Section 4.2) because the liquid thickness at the top of the downstream section is similar (i.e. the liquid thickness component due to liquid impinging onto the downstream section is zero).

Liquid Thickness at the Top of the Granular Downstream Section. There is a perfect drain at the toe of the downstream section. The liquid thickness at the top of the downstream section is the sum of two components: (i) the liquid thickness due to liquid impinging onto, and flowing from, the upstream section; and (ii) the liquid thickness due to liquid impinging onto the downstream section. It has been shown in Section 2.2.4 that the liquid thickness at the top of a single-slope liquid collection layer with a perfect drain at the toe is zero if $\lambda \leq 0.25$ (Figure 4b). Therefore, the liquid thickness at the top of a granular downstream section due to liquid impinging onto that section is zero if $\lambda_{down} \leq 0.25$. As a result, the liquid thickness at the top of a granular downstream section with $\lambda_{down} \leq 0.25$ is only due to liquid impinging onto, and flowing from, the upstream section. Therefore, Equation 28 is applicable to this case.

Liquid Thickness at the Toe of the Geosynthetic Upstream Section. Combining Equations 26 and 28 gives:

$$t_{up\,toe} = \frac{q_h \; L_{up} \; i_{down\,top}}{k_{up} \; i_{up\,toe} \; \sin\beta_{down}} \qquad (42)$$

The values of the hydraulic gradients, $i_{down\,top}$ and $i_{up\,toe}$, are discussed below.

Hydraulic Gradients. The liquid thickness at the top of the downstream section is the sum of two components: (i) the liquid thickness due to liquid impinging onto, and flowing from, the upstream section; and (ii) the liquid thickness due to liquid impinging onto the downstream section. Based on Figure 6, the hydraulic gradient at the top of the downstream section related to the first component is $\sin\beta_{down}$, i.e. the liquid surface is parallel to the slope. Based on Figure 4b, the hydraulic gradient at the top of the downstream section related to the second component is less than $\sin\beta_{down}$, i.e. the slope of the liquid surface is less than the slope of the liquid collection layer. Therefore, when the two components are superimposed, the slope of the liquid surface continues to be less than the slope of the liquid collection layer. As a result, the hydraulic gradient for the flow resulting from these two components is:

$$i_{down\ top} < \sin\beta_{down} \tag{43}$$

Since the thickness of the geosynthetic drainage material used in the upstream section is very small compared to the length of the slope, the liquid surface is almost parallel to the slope and, therefore:

$$i_{up\ toe} \approx i_{up} \approx \sin\beta_{up} \tag{44}$$

Maximum Liquid Thickness in the Upstream Section. Combining Equations 42 to 44 gives:

$$t_{up\ toe} < \frac{q_h L_{up}}{k_{up} \sin\beta_{up}} \tag{45}$$

Comparing Equations 32 and 45 shows that $t_{up\ toe} < t_{up\ lim}$. From this, it results that $t_{up\ max} \approx t_{up\ lim}$, based on a comment made on geosynthetic liquid collection layers at the end of Section 2.3. Therefore, in the case of a granular downstream section with $\lambda_{down} \le 0.25$, the maximum liquid thickness in the geosynthetic upstream section (Figure 3d) is given by Equation 34. This case is illustrated by Example 3.

4.3.3 *Case* $\lambda_{down} > 0.25$

In the case of a granular downstream section with $\lambda_{down} > 0.25$, the liquid thickness at the top of the downstream section due to liquid impinging onto that section is not zero (Figure 4a). Therefore, the approach used for the case of a granular downstream section with $\lambda_{down} \le 0.25$ (Section 4.3.2), or for the case of a geosynthetic downstream section (Section 4.2), cannot be used.

Table 2 shows that λ_{down} is large if k_{down} is small; λ_{down} can be greater than 0.25 only if the downstream section is made with a material having a relatively low hydraulic conductivity, such as sand. If $\lambda_{down} > 0.25$, the liquid thickness at the top of the downstream section is large, and there is a risk that the liquid thickness at the toe of the upstream section exceeds $t_{up\ lim}$, which would lead to a maximum liquid thickness in the upstream section that is greater than $t_{up\ lim}$. Such a large liquid thickness might exceed the capacity of the upstream section; furthermore, there is no available method to evaluate this

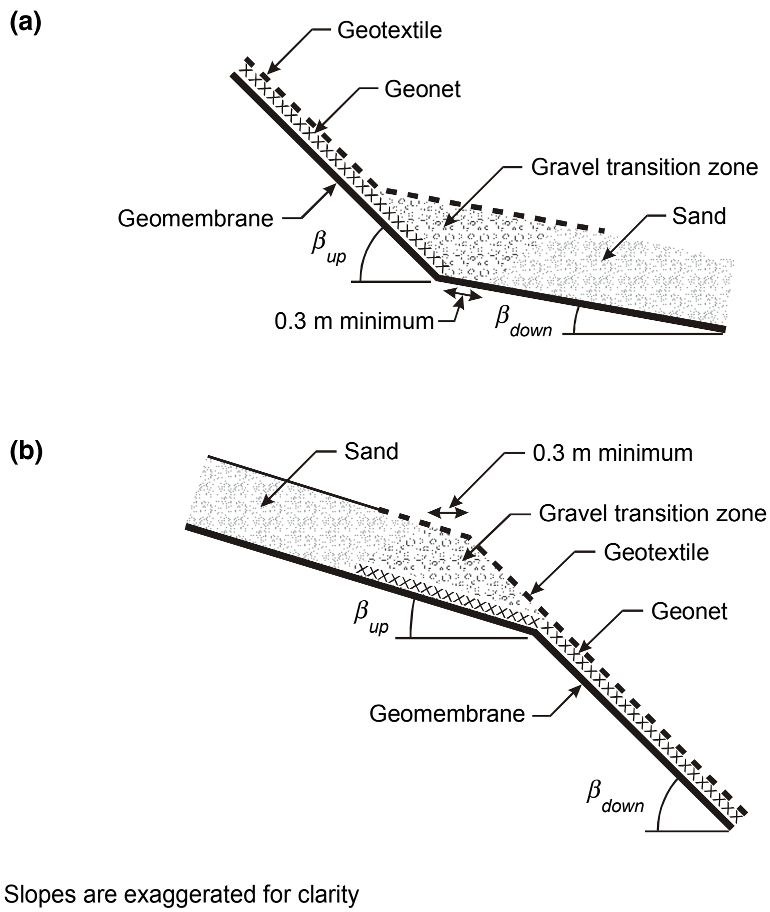

Figure 7. Gravel transition zone.

Note: There should be no filter between the geosynthetic drainage material and gravel.

liquid thickness, which is an important consideration from a design standpoint. Therefore, this situation must be avoided. Since this situation results from the fact that the hydraulic conductivity, k_{down} , is small, the solution consists of using, between the sand and the geosynthetic drainage material, a transition zone constructed with a material having a hydraulic conductivity greater than that of the granular material used in the downstream section (Figure 7a). The material used in the transition zone (gravel) should have a hydraulic conductivity that leads to a value of λ smaller than 0.25, hence, from Equation 1:

$$\frac{q_h}{k_{transition} \ \tan^2 \beta_{down}} < 0.25 \tag{46}$$

where $k_{transition}$ is the minimum required hydraulic conductivity of the material used in the transition zone. Hence:

$$k_{transition} > \frac{4\ q_h}{\tan^2 \beta_{down}} \tag{47}$$

Due to the high hydraulic conductivity of the transition zone material, the liquid surface in the transition zone remains at the same level as at the top of the downstream section, thereby preventing the development of excessive liquid thickness at the toe of the upstream section. As a result, the liquid thickness at the toe of the upstream section is equal to or less than $t_{up\ lim}$. This leads to a maximum liquid thickness in the upstream section that is equal to $t_{up\ lim}$ (based on a comment on geosynthetic liquid collection layers made at the end of Section 2.3). Thus, in this case, and provided that there is a transition zone, the maximum liquid thickness in the upstream section is given by Equation 34.

5 PRACTICAL APPLICATIONS

5.1 Summary of the Methodology

When a liquid collection layer comprises two sections with different slopes, the liquid thickness must be calculated separately for each of the two sections.

5.1.1 *Maximum Liquid Thickness in the Downstream Section*

The maximum thickness of liquid in the downstream section of the liquid collection layer depends on the material used in that section of the liquid collection layer, geosynthetic or granular. Equations can be found in Table 3.

Geosynthetic Downstream Section. The maximum liquid thickness in a geosynthetic downstream section can be calculated using Equation 20, as indicated in Section 3.3.2.

Granular Downstream Section. The maximum liquid thickness in a granular downstream section can be calculated using Equation 24 (or 25, which is equivalent), as indicated in Section 3.3.3. These equations are valid for any value of λ_{down} defined by Equation 23. If λ_{down} is small (e.g. less than 0.001), which may happen in the case of a granular liquid collection layer made with a very permeable gravel, Equations 24 and 25 tend toward Equation 20.

5.1.2 *Maximum Liquid Thickness in the Upstream Section*

The maximum thickness of liquid in the upstream section of the liquid collection layer depends on the material used in both the downstream and upstream sections. Equations can be found in Table 4. The following cases should be considered:

Table 3. Maximum liquid thickness in the downstream section.

Type of liquid collection layer in the downstream section	Maximum liquid thickness in the downstream section
Geosynthetic (Figures 3a, 3b, 3c)	$t_{down\ max} = \dfrac{q_h \left(L_{up} + L_{down} \right)}{k_{down} \sin \beta_{down}}$ (20)
Granular (Figure 3d)	$t_{down\ max} = j_{down} \dfrac{\sqrt{\tan^2 \beta_{down} + 4 q_h / k_{down}} - \tan \beta_{down}}{2 \cos \beta_{down}} L_{down} + \dfrac{q_h L_{up}}{k_{down} \sin \beta_{down}}$ (24) which is equivalent to: $t_{down\ max} = \left(j_{down} \dfrac{\sqrt{1 + 4\lambda_{down}} - 1}{2} L_{down} + \lambda_{down} L_{up} \right) \dfrac{\tan\beta_{down}}{\cos\beta_{down}}$ (25) where: $j_{down} = 1 - 0.12 \exp\left\{ -\left[\log \left(8\lambda_{down} / 5 \right)^{5/8} \right]^2 \right\}$ (22) $\lambda_{down} = \dfrac{q_h}{k_{down} \tan^2 \beta_{down}}$ (23)

- If both the downstream and upstream sections are made with geosynthetic (Figures 3a and 3b), the maximum liquid thickness in the upstream section can be calculated using Equation 34, as indicated in Section 4.2.3.
- In the case of a geosynthetic liquid collection layer in the downstream section and a granular liquid collection layer in the upstream section (Figure 3c), an upper boundary and an approximate value (which is also a lower boundary) of the maximum liquid thickness in the upstream section can be calculated as follows, as indicated in Section 4.2.4: (i) the upper boundary can be calculated using Equation 37; (ii) the approximate value (which is also a lower boundary) can be calculated using Equation 38; and (iii) alternatively, the approximate value (which is also a lower boundary) can be calculated using Equations 8 to 10, with $\beta = \beta_{up}$, $\lambda = \lambda_{up}$ (given by Equation 39), and $t_{up\ toe} = t_{down\ top}$ (given by Equation 28). Finally, an approximate value of the maximum liquid thickness in the upstream section can be calculated using Equation 41, which is simpler than the equations mentioned above.
- In the case of a granular liquid collection layer on a downstream slope that is not steep and a geosynthetic liquid collection layer on the upstream slope that is steep (Figure 3d), the maximum liquid thickness in the upstream section can be calculated using Equation 34, as indicated in Sections 4.3.2 and 4.3.3. If λ_{down} (defined by Equation 23) is greater than 0.25, a transition zone made of gravel must be used between the sand and the geosynthetic (Figure 7a), as indicated in Section 4.3.3.

Table 4. Maximum liquid thickness in the upstream section.

Figure	Downstream section	Upstream section	Maximum liquid thickness in the upstream section
3a, 3b	Geosynthetic	Geosynthetic	$t_{up\,max} \approx \dfrac{q_h\,L_{up}}{k_{up}\sin\beta_{up}}$ (34)
3d	Granular $\lambda_{down} \leq 0.25$		
3d	Granular $\lambda_{down} > 0.25$	Geosynthetic	$t_{up\,max} \approx \dfrac{q_h\,L_{up}}{k_{up}\sin\beta_{up}}$ (34) In this case, a transition zone is required with: $k_{transition} > \dfrac{4\,q_h}{\tan^2\beta_{down}}$ (47)
3c	Geosynthetic	Granular	Upper boundary: $t_{up\,max} < \dfrac{q_h\,L_{up}}{k_{up}\sin\beta_{up}}$ (37) Approximate value/lower boundary: $t_{up\,max} \approx j_{up}\,\dfrac{\sqrt{1+4\lambda_{up}}-1}{2}\,\dfrac{\tan\beta_{up}}{\cos\beta_{up}}\,L_{up}$ (38) where: $\lambda_{up} = \dfrac{q_h}{k_{up}\tan^2\beta_{up}}$ (39) $j_{up} = 1-0.12\,\exp\left\{-\left[\log\left(8\lambda_{up}/5\right)^{5/8}\right]^2\right\}$ (40) Simple approximate value: $t_{up\,max} \approx \dfrac{\sqrt{1+4\lambda_{up}}-1}{2}\,\dfrac{\tan\beta_{up}}{\cos\beta_{up}}\,L_{up}$ (41)

5.2 Practical Recommendation

A gravel transition zone is always recommended between a sand liquid collection layer and a geosynthetic liquid collection layer (Figure 7). Sand particles tend to penetrate into the large openings of geosynthetic drainage media, such as geonets, thereby creating a small zone where the hydraulic conductivity is less than in both the upstream and downstream sections. The presence of such a low hydraulic conductivity zone slows down the flow at the connection between the two sections of the liquid collection layer. It is, of course, not possible to use a filter between the sand and the geonet: such a filter would prevent the penetration of the geosynthetic by the sand particles (which is beneficial), but would also slow down the flow to unacceptable rates.

It should be noted that the transition zone is not connected to an outlet. If it were connected to an outlet, the transition zone would act as a drain between the two sections of the liquid collection layer, which would be contrary to the basic assumption of the present paper.

The transition zone has the same thickness as the adjacent granular liquid collection layer. Its sizing is dictated only by practical considerations such as a minimum width of approximately 0.3 m to facilitate placement and ensure continuity.

Finally, it is important to note that, in addition to being recommended when sand is used, a transition zone is required to justify the use of Equation 34 when λ_{down} is greater than 0.25. In this case, Equation 47 gives the minimum required hydraulic conductivity of the material to be used in the transition zone.

5.3 Design Examples

Three examples are presented to illustrate typical configurations of liquid collection layers with two slopes: two examples of landfill cover liquid collection layers; and one example of a landfill leachate collection layer. In both cases, a geosynthetic (e.g. a geonet) is used on the steep slope, which is consistent with the state of practice in landfill design in the United States. As indicated in Section 1.4, the use of reduction factors and factors of safety is not discussed in the present paper. Therefore, the examples that follow do not include reduction factors and factors of safety. Examples of the use of reduction factors and factors of safety are given by Giroud et al. (2000a).

Example 1. The liquid collection layer in a landfill cover comprises an upstream section with a 4% slope and a horizontal length of 25 m, and a downstream section with a 1V:4H slope and a horizontal length of 40 m (Figure 3a). The liquid collection material for both sections is a 6 mm-thick geonet with a hydraulic conductivity of 0.2 m/s for a hydraulic gradient less than 0.05, and 0.1 m/s for a hydraulic gradient of the order of 0.25. The landfill cover is designed for an infiltration rate of 5 mm/hour. Calculate the maximum thickness of water in the liquid collection layer.

First, the impingement rate must be expressed in SI units as follows:

$$q_h = \frac{5 \times 10^{-3}}{3600} = 1.39 \times 10^{-6} \text{ m/s}$$

Then, the maximum liquid thickness in the downstream section can be calculated using Equation 20 (Table 3) as follows:

$$t_{down\,max} = \frac{\left(1.39 \times 10^{-6}\right)\left(25 + 40\right)}{\left(0.1\right)\left[\sin\left(\tan^{-1} 0.25\right)\right]} = 0.00373 \text{ m} = 3.73 \text{ mm}$$

The calculated maximum liquid thickness in the downstream section of the liquid collection layer (3.73 mm) is less than the geonet thickness (6 mm).

Then, the maximum liquid thickness in the upstream section is calculated using Equation 34 (Table 4) as follows:

$$t_{up\ max} \approx \frac{\left(1.39\times10^{-6}\right)(25)}{(0.2)\left[\sin\left(\tan^{-1} 0.04\right)\right]} = 0.00435 \text{ m} = 4.35 \text{ mm}$$

The calculated maximum liquid thickness in the upstream section of the liquid collection layer (4.35 mm) is less than the thickness of the geonet (6 mm).

END OF EXAMPLE 1

Example 2. A liquid collection layer used in a landfill cover, that is schematically illustrated in Figure 3c, comprises two sections. The upstream section consists of a 0.3 m-thick sand layer placed on a 4% slope that is 25 m long, measured horizontally. The downstream section consists of a 5 mm-thick geonet placed on a 1V:4H slope that is 40 m long, measured horizontally. The hydraulic conductivity of the sand is 5×10^{-3} m/s and that of the geonet is 1×10^{-1} m/s. The landfill cover is designed for an infiltration rate of 5 mm/hour. Calculate the maximum thickness of water in the liquid collection layer.

First, the impingement rate must be expressed in SI units as follows:

$$q_h = \frac{5\times10^{-3}}{3600} = 1.39\times10^{-6} \text{ m/s}$$

Then, the maximum liquid thickness in the downstream section can be calculated using Equation 20 (Table 3) as follows:

$$t_{down\ max} = \frac{\left(1.39\times10^{-6}\right)(25+40)}{(0.1)\left[\sin\left(\tan^{-1} 0.25\right)\right]} = 0.00373 \text{ m} = 3.73 \text{ mm}$$

The calculated maximum liquid thickness in the downstream section of the liquid collection layer (3.73 mm) is less than the geonet thickness (6 mm).

Then, an upper boundary of the maximum liquid thickness in the upper section is calculated using Equation 37 (Table 4) as follows:

$$t_{up\ max} < \frac{\left(1.39\times10^{-6}\right)(25)}{\left(5\times10^{-3}\right)\left[\sin\left(\tan^{-1} 0.04\right)\right]} = 0.174 \text{ m}$$

A lower boundary (which is also an approximate value) of the maximum liquid thickness in the upper section can be calculated. Two methods can be used, as indicated in Section 4.2.4.

The first method for obtaining an approximate value of the maximum liquid thickness in the upstream section consists of using the modified Giroud's equation (Equation 38). To use Equation 38, it is necessary to calculate λ_{up} using Equation 39 (Table 4) as follows:

$$\lambda_{up} = \frac{1.39 \times 10^{-6}}{\left(5 \times 10^{-3}\right)\left(0.04\right)^2} = 0.17375$$

To use Equation 38, it is also necessary to calculate j_{up} using Equation 40 (Table 4) as follows:

$$j_{up} = 1 - 0.12 \ \exp\left\{-\left[\log\left(8 \times 0.17375/5\right)^{5/8}\right]^2\right\} = 0.89365$$

Then, Equation 38 (Table 4) can be used as follows:

$$t_{up\ max} \approx \left(0.89365\right)\frac{\sqrt{1+\left(4\right)\left(0.17375\right)} - 1}{2}\ \frac{0.04}{\cos\left(\tan^{-1} 0.04\right)}\left(25\right) = 0.1350 \text{ m}$$

The second method for obtaining an approximate value of the maximum liquid thickness in the upstream section consists of using Equations 8 to 10, with $\beta = \beta_{up}$ and $\lambda = \lambda_{up}$, assuming that $t_{up\ toe}$ is equal to $t_{down\ top}$, as suggested in Section 4.2.4. Therefore, $t_{down\ top}$ should be calculated first, using Equation 28 as follows:

$$t_{down\ top} = \frac{\left(1.39 \times 10^{-6}\right)\left(25\right)}{\left(0.1\right)\left[\sin\left(\tan^{-1} 0.25\right)\right]} = 0.00143 \text{ m} = 1.43 \text{ mm}$$

Since λ_{up} is less than 0.25, Equation 8 must be used to calculate $t_{up\ max}$. To use Equation 8, it is necessary to calculate A' using Equation 11 as follows:

$$A' = \sqrt{1-\left(4\right)\left(0.17375\right)} = 0.55227$$

Then, Equation 8 can be used as follows:

$$t_{up\,max} \approx \left(25\right)\frac{0.04}{\cos\left(\tan^{-1} 0.04\right)}\left\{0.17375 - \frac{\left(0.00143\right)\cos\left(\tan^{-1} 0.04\right)}{\left(25\right)\left(0.04\right)} + \left[\frac{\left(0.00143\right)\cos\left(\tan^{-1} 0.04\right)}{\left(25\right)\left(0.04\right)}\right]^2\right\}^{1/2}$$

$$\left\{\frac{\left[1-0.55227-\left(2\right)\left(0.17375\right)\right]\left[1+0.55227-\frac{\left(2\right)\left(0.00143\right)\cos\left(\tan^{-1} 0.04\right)}{\left(25\right)\left(0.04\right)}\right]}{\left[1+0.55227-\left(2\right)\left(0.17375\right)\right]\left[1-0.55227-\frac{\left(2\right)\left(0.00143\right)\cos\left(\tan^{-1} 0.04\right)}{\left(25\right)\left(0.04\right)}\right]}\right\}^{\frac{1}{\left(2\right)\left(0.55227\right)}}$$

hence:

$$t_{up\ max} \approx 0.1354 \text{ m}$$

Alternatively, the graph presented in Figure A-1 of the appendix could have been used. To use this graph, it is necessary to calculate $t_{up\ lim}$ using Equation 32 as follows:

$$t_{up\ lim} = \frac{\left(1.39\times10^{-6}\right)(25)}{\left(5\times10^{-3}\right)\left[\sin\left(\tan^{-1}0.04\right)\right]} = 0.174\ \text{m}$$

Then, $t_{up\ toe}$, which is assumed to be equal to $t_{down\ top}$, is divided by $t_{up\ lim}$ as follows:

$$\frac{t_{up\ toe}}{t_{up\ lim}} = \frac{t_{down\ top}}{t_{up\ lim}} = \frac{0.00143}{0.174} = 0.0082$$

For λ_{up} = 0.17375 and $t_{up\ toe}/t_{up\ lim}$ = 0.008, the graph in Figure A-1 gives $t_{max}/t_{lim} \approx$ 0.77 to 0.78, hence $t_{up\ max} \approx$ 0.77 to 0.78 $\times$ 0.174 $\approx$ 0.134 to 0.136 mm, which is very close to the values calculated above (0.1350 and 0.1354).

Finally, an approximate value of the maximum liquid thickness in the upstream section can be calculated using Equation 41 (Table 4) as follows:

$$t_{up\ max} \approx \frac{\sqrt{1+(4)(0.17375)}\ -1}{2}\ \frac{0.04}{\cos\left(\tan^{-1}0.04\right)}(25) = 0.1510\ \text{m}$$

In conclusion, the following values were calculated for the maximum liquid thickness in the upstream section:

- upper boundary = 0.174 m;
- lower boundary/approximate value = 0.1350 m (using Equation 37), 0.1354 m (using Equation 8, which requires extensive calculations); and 0.134 to 0.136 m (using the graphical method presented in Figure A-1); and
- approximate value = 0.1510 m (using Equation 41, which is straightforward).

It appears that there is not much difference, in this example, between the two calculated values of the lower boundary/approximate value. As shown by the graph in Figure A-1, there may be a significant difference between the two calculated values of the lower boundary/approximate value if the characteristic parameter λ_{up} has a large value, such as 10. However, this is unlikely. Indeed, simple calculations performed using Equation 5 show that, if λ_{up} = 10 and if the maximum liquid thickness is not allowed to be greater than 0.3 m, then the length of the upstream section cannot exceed approximately one meter, which is not a realistic situation. This is confirmed by Table 2, which shows that, in typical situations, λ rarely exceeds 1.0, and is generally less than 0.1. Clearly, in realistic situations, λ_{up} is not large, and it is recommended to avoid the lengthy calculations associated with Equations 8 to 10. Instead, it is recommended to use Equation 38, or Equation 41, which is simpler.

In conclusion, a prudent design engineer will use the upper boundary calculated using Equation 37 (0.174 m, in the above example), and if more precision is needed (e.g. for a performance analysis or a forensic analysis) then Equation 38 or 41 can be used.

END OF EXAMPLE 2

Example 3. A landfill leachate collection layer that is schematically illustrated in Figure 3d comprises two sections. The upstream section consists of a 5 mm-thick geonet placed on a 1V:3H slope that is 12 m high. The downstream section consists of a 0.3 m-thick sand layer placed on a 3% slope. The horizontal distance between the toe of the upstream slope and the drain that removes the leachate at the toe of the downstream slope is 30 m. The hydraulic conductivity of the geonet is 2×10^{-1} m/s and that of the sand is 5×10^{-3} m/s. The rate of leachate impingement onto the leachate collection layer is 4×10^{-7} m/s. Calculate the maximum thickness of liquid in the leachate collection layer.

The thickness of liquid in the downstream section of the leachate collection layer will be calculated using Equation 24. To use this equation, it is first necessary to calculate the horizontal length of the upstream slope as follows:

$$L_{up} = 3 \times 12 = 36\,\text{m}$$

To use Equation 24, it is also necessary to know the dimensionless modifying factor, j_{down}. To calculate j_{down}, using Equation 22, it is necessary to calculate the dimensionless characteristic parameter, λ_{down}, using Equation 23 (Table 3) as follows:

$$\lambda_{down} = \frac{4 \times 10^{-7}}{\left(5 \times 10^{-3}\right)\left(0.03\right)^2} = 0.089$$

Then, j_{down} is calculated using Equation 22 (Table 3) as follows:

$$j_{down} = 1 - 0.12\ \exp\left\{-\left[\log\left(8 \times 0.089/5\right)^{5/8}\right]^2\right\} = 0.909$$

Then, the maximum thickness of leachate in the downstream section of the leachate collection layer is calculated using Equation 24 (Table 3) as follows:

$$t_{down\ max} = \left(0.909\right)\frac{\sqrt{\left(0.03\right)^2 + \left(4\right)\left(4 \times 10^{-7}\right)/\left(5 \times 10^{-3}\right)} - 0.03}{2\ \cos\left(\tan^{-1} 0.03\right)}\left(30\right) + \frac{\left(4 \times 10^{-7}\right)\left(36\right)}{\left(5 \times 10^{-3}\right)\sin\left(\tan^{-1} 0.03\right)}$$

hence:

$$t_{down\ max} = 0.067 + 0.096 = 0.163\ \text{m}$$

The calculated maximum leachate thickness in the downstream section of the liquid collection layer (0.163 m) is less than the sand layer thickness (0.3 m).

Then, an approximate value of the maximum liquid thickness in the upstream section can be calculated using Equation 34 (Table 4) as follows, since λ_{down} is less than 0.25, as calculated above:

$$t_{up\ max} \approx \frac{\left(4 \times 10^{-7}\right)\left(36\right)}{\left(2 \times 10^{-1}\right)\ \sin\left(\tan^{-1} 0.333\right)} = 2.28 \times 10^{-4} = 0.23\,\text{mm}$$

The calculated approximate value of the maximum liquid thickness in the upstream section of the liquid collection layer (0.23 mm) is less than the geonet thickness (5 mm).

As indicated in Section 5.2, a transition zone is recommended between the geonet and the sand. If the hydraulic conductivity of the sand had been lower, λ_{down} could have been greater than 0.25. In such a case, the use of a transition zone would not only be recommended for the reasons given in Section 5.2, but would also be required to justify the use of Equation 34 (as indicated in Section 4.3.3). For example, if k_{down} had been equal to 1×10^{-3} m/s (instead of 5×10^{-3} m/s), the value of λ_{down} would have been:

$$\lambda_{down} = \frac{4 \times 10^{-7}}{\left(1 \times 10^{-3}\right)\left(0.03\right)^2} = 0.444$$

The minimum required hydraulic conductivity for the material to be used in the transition zone would then be obtained as follows using Equation 47:

$$k_{transition} > \frac{(4)\left(4 \times 10^{-7}\right)}{\left(0.03\right)^2} = 1.8 \times 10^{-3} \text{ m/s}$$

END OF EXAMPLE 3

6 CONCLUSIONS

There are many cases, particularly in landfills, when a liquid collection layer comprises two sections with different slopes. If there is a drain between the two sections, each section can be treated as a liquid collection layer on a single slope, using the method presented by Giroud et al. (2000a). However, there are cases where there is no drain removing the liquid at the connection between the two sections. Those cases are addressed in the present paper. A method (which is provided in the present paper) was developed to calculate the maximum thickness of liquid in each of the two sections of the liquid collection layer. The determination of the maximum thickness of liquid is an essential design step because the maximum liquid thickness must be less than an allowable thickness.

The maximum liquid thickness in the downstream section of a two-slope liquid collection layer can be calculated using equations that account for both the *liquid impinging onto the downstream section* and the *liquid impinging onto*, and *flowing from*, the *upstream section*. The maximum liquid thickness in the upstream section of a two-slope liquid collection layer can be calculated using equations that depend on the material used in the upstream section and in the downstream section. In some cases, a transition zone is needed between the upstream and downstream sections.

The present paper should facilitate the design of liquid collection layers comprising two sections with different slopes, with no drain removing the liquid between the two sections. However, it should not be construed from the present paper that the authors do not encourage the use of drains at the toe of each slope. Indeed, using such drains is often the best solution. The present paper is only intended to help engineers design an alternative solution.

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NOTATIONS

The Subscript "*up*" is used for the upstream section of the liquid collection layer and the Subscript "*down*" for the downstream section of the liquid collection layer. Basic SI units are in parentheses.

A' = parameter defined by Equation 11 (dimensionless)

B' = parameter defined by Equation 11 (dimensionless)

i = hydraulic gradient (dimensionless)

$i_{down\ 1}$ = hydraulic gradient (related to liquid impinging onto, and flowing from, upstream section) in the portion of downstream section of liquid collection layer where thickness of liquid flowing from upstream section is uniform (dimensionless)

$i_{down\ top}$ = hydraulic gradient at top of downstream section (dimensionless)

$i_{up\ toe}$ = hydraulic gradient at toe of upstream section (dimensionless)

j = modifying factor used in Equation 1 (dimensionless)

j_{down} = value of j calculated with characteristics of downstream section of liquid collection layer (dimensionless)

j_{up} = value of j calculated with characteristics of upstream section of liquid collection layer (dimensionless)

k = hydraulic conductivity of liquid collection layer material (m/s)

k_{down} = hydraulic conductivity of the material of downstream section of liquid collection layer (m/s)

$k_{transition}$ = minimum required hydraulic conductivity of material used in transition zone (m/s)

k_{up} = hydraulic conductivity of the material of upstream section of liquid collection layer (m/s)

L = horizontal projection of length (i.e. "horizontal length") of liquid collection layer in direction of flow (m)

L_{down} = horizontal length of downstream section of liquid collection layer (m)

L_{up} = horizontal length of upstream section of liquid collection layer (m)

Q^* = flow rate per unit width (m^2/s)

$Q^*_{down\ 1}$ = flow rate per unit width in downstream section of liquid collection layer due to liquid flowing from upstream section (m^2/s)

q_h = rate of liquid supply ("liquid impingement rate") (m/s)

$t_{down\ max}$ = maximum liquid thickness in downstream section of liquid collection layer (m)

$t_{down\ max\ 1}$ = maximum liquid thickness in downstream section of liquid collection layer due to liquid impinging onto, and flowing from, upstream section (m)

$t_{down\ max\ 2}$ = maximum liquid thickness in downstream section of liquid collection layer due to liquid impinging onto that section (m)

$t_{down\ top}$ = liquid thickness at top of downstream section of liquid collection layer (m)

t_{lim} = maximum liquid thickness in limit case where q_h is small and β and k are large (m)

t_{max} = maximum liquid thickness (m)

t_{toe} = liquid thickness at toe of slope (m)

t_{top} = liquid thickness at top of slope (m)

$t_{up\ lim}$ = value of t_{lim} in upstream section of liquid collection layer (m)

$t_{up\ max}$ = maximum liquid thickness in upstream section of liquid collection layer (m)

$t_{up\ toe}$ = liquid thickness at toe of upstream section of liquid collection layer (m)

x = horizontal distance measured from top of slope (abscissa) (m)

x_m = horizontal distance between top of slope and the location of maximum liquid thickness (m)

β = slope angle of the liquid collection layer (°)

β_{down} = slope angle of downstream section of liquid collection layer (°)

β_{up} = slope angle of upstream section of liquid collection layer (°)

λ = parameter defined by Equation 1 (dimensionless)

λ_{down} = value of λ in downstream section of liquid collection layer (dimensionless)

λ_{up} = value of λ in upstream section of liquid collection layer (dimensionless)

APPENDIX

A.1 Graphical Solution

Figure A-1 gives values of t_{max}/t_{lim} as a function of t_{toe}/t_{lim} , using the following equations derived from Equations 5 and 8 to 10:

- for $\lambda < 0.25$

$$\frac{t_{max}}{t_{lim}} = \left[\frac{1 - t_{toe}/t_{lim}}{\lambda} + \left(t_{toe}/t_{lim}\right)^2\right]^{1/2} \left\{\frac{(1 - A' - 2\lambda)\left[1 + A' - 2\lambda\left(t_{toe}/t_{lim}\right)\right]}{(1 + A' - 2\lambda)\left[1 - A' - 2\lambda\left(t_{toe}/t_{lim}\right)\right]}\right\}^{\frac{1}{2A'}} \quad \text{(A-1)}$$

- for $\lambda = 0.25$

$$\frac{t_{max}}{t_{lim}} = \left(2 - t_{toe}/t_{lim}\right) \exp\left[\frac{2\left(t_{toe}/t_{lim} - 1\right)}{2 - t_{toe}/t_{lim}}\right] \quad \text{(A-2)}$$

- for $\lambda > 0.25$

$$\frac{t_{max}}{t_{lim}} = \left[\frac{1 - t_{toe}/t_{lim}}{\lambda} + \left(t_{toe}/t_{lim}\right)^2\right]^{1/2} \exp\left\{\frac{1}{B'}\left[\tan^{-1}\left(\frac{2\lambda\left(t_{toe}/t_{lim}\right) - 1}{B'}\right) - \tan^{-1}\left(\frac{2\lambda - 1}{B'}\right)\right]\right\}$$

(A-3)

where λ is defined by Equation 1, and A' and B' by Equation 11.

It is important to note that normalizing t_{max} and t_{toe} (by dividing them by t_{lim}) leads to equations (i.e. Equations A-1 to A-3) that are independent of β. As a result, the curves presented in Figure A-1 are independent of β. It should, however, be remembered that both λ and t_{lim} depend on β.

Figure A-1 shows that:

- t_{lim} is a good approximation of t_{max} for any value of t_{toe}/t_{lim} if λ is small (e.g. $\lambda < 0.01$);
- $t_{max}/t_{lim} = 1$ for $t_{toe}/t_{lim} = 1$, i.e. $t_{max} = t_{lim}$ if $t_{toe} = t_{lim}$, which is consistent with a comment made in Section 2.3; and
- t_{max}/t_{lim} tends toward t_{toe}/t_{lim} if λ tends toward infinity (e.g. the curve for $t_{toe}/t_{lim} = 0$ tends toward 0, the curve for $t_{toe}/t_{lim} = 0.1$ tends toward 0.1, etc.).

A.2 Location of Maximum Liquid Thickness

The first part of Equation 7 in the main text of the present paper is as follows:

$$\frac{x_m}{L} = \frac{t_{max}}{t_{lim}} \quad \text{(A-4)}$$

Equation A-1 provides the location of the maximum liquid thickness, as demonstrated by Giroud et al. (2000a). The demonstration is based on the assumption that, at the location of the maximum, the tangent to the liquid surface is parallel to the slope.

This is true regardless of the value of t_{toe} . Therefore, Figure A-1 also gives the normalized value of the location of the maximum liquid thickness as a function of the normalized liquid thickness at the toe of the liquid collection layer.

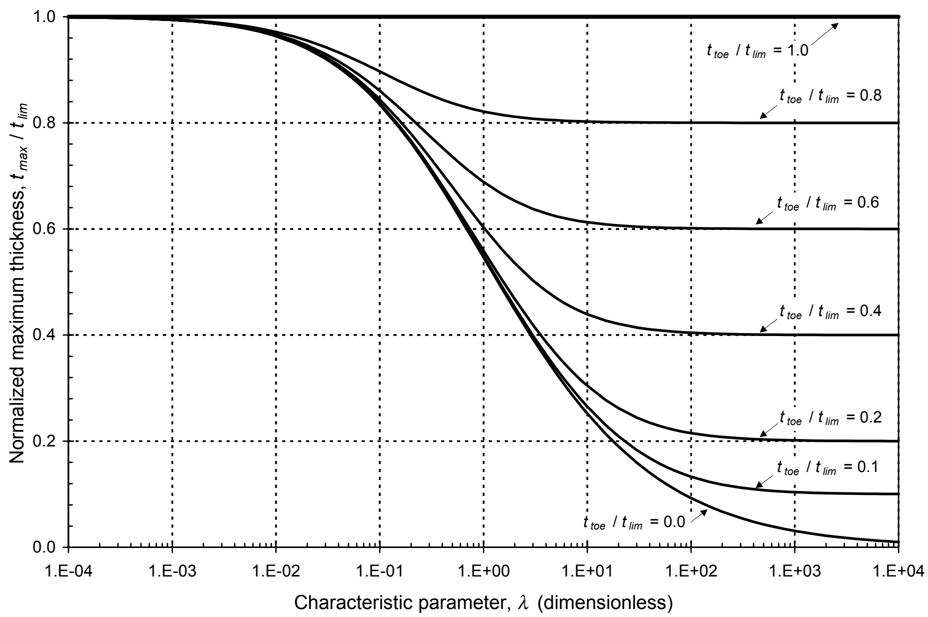

Figure A-1. Values of t_{max}/t_{lim} as a function of t_{toe}/t_{lim} .

Note: Figure A-1 was obtained using Equations A-1 to A-3. The curve for $t_{toe}/t_{lim} = 0$ can also be obtained using Equation 2 divided by t_{lim} . Figure A-1 also gives the normalized location of the maximum liquid thickness, x_m/L, which is equal to t_{max}/t_{lim} .