Technical Paper by J.P. Giroud, J.G. Zornberg, and A. Zhao

HYDRAULIC DESIGN OF GEOSYNTHETIC AND GRANULAR LIQUID COLLECTION LAYERS

ABSTRACT: The present paper provides equations for the hydraulic design of liquid collection layers. A first series of equations gives the maximum thickness of the liquid collected in a liquid collection layer. These equations are used in design to check that the maximum liquid thickness is less than an allowable thickness. Some of the equations make it possible to rigorously calculate the liquid thickness, whereas other equations, which are simpler, give an approximate value of the liquid thickness. A second series of equations makes it possible to calculate the required hydraulic conductivity of the liquid collection layer material and the required hydraulic transmissivity of the liquid collection layer. These equations are useful for selecting the material used to construct the liquid collection layer. The equations provided in the present paper include reduction factors to quantify the decrease in flow capacity of liquid collection layers due to thickness reduction (caused by the applied stresses) and hydraulic conductivity reduction (caused by clogging). Practical recommendations and design examples are presented for both geosynthetic and granular liquid collection layers.

KEYWORDS: Liquid collection layer, Leachate collection layer, Leakage detection and collection layer, Drainage layer, Thickness, Granular, Geosynthetic.

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1 INTRODUCTION

1.1 Liquid Collection Layers

Liquid collection layers are used to collect liquid and to convey the collected liquid by gravity to a low point such as a sump. Liquid collection layers are used in a variety of geotechnical and geoenvironmental structures: (i) they are used as drainage blankets in dams, embankments, roads, landslide repair, etc.; (ii) they are used as leachate collection layers in landfills (where they are also called "primary leachate collection layers"); and (iii) they are used, underneath a liner, as leakage detection and collection layers in landfills (where they are also called "secondary leachate collection layers") and in liquid containment structures such as ponds, canals, reservoirs, dams, etc. Based on these examples, it appears that there are two categories of liquid collection layers: (i) the drainage layers (or "primary liquid collection layers") that collect liquid that percolate through a mass of soil or waste; and (ii) the leakage collection layers (or "secondary liquid collection layers") that collect liquid that leaks through a liner.

1.2 Liquid Collection Layer Materials

Liquid collection layers are constructed with materials that can convey liquids, i.e. materials that can perform the liquid transmission function (referred to, hereafter, as the transmission function). Materials that can perform the transmission function include granular materials and thick geosynthetics.

Granular materials include gravel, sand, and mixtures of these two materials. Liquid collection layers constructed with gravel are generally protected from clogging by migrating particles using a filter. The filter may be a layer of sand or a geotextile.

Geosynthetics that can perform the transmission function are thick geosynthetics having a high hydraulic conductivity in the direction of their plane. These geosynthetics are characterized by a high hydraulic transmissivity, which is the product of the hydraulic conductivity by the thickness. Geosynthetics having a high hydraulic transmissivity include thick needle-punched nonwoven geotextiles and a variety of geosynthetics sometimes generically called "geospacers", such as geonets, geomats, cuspated polymeric plates, embossed polymeric plates, formed plastic products, etc. Geospacers have a much greater hydraulic transmissivity than thick needle-punched nonwoven geotextiles; therefore, geospacers, in particular geonets and geomats, are used more often than needle-punched nonwoven geotextiles as liquid collection layers. However, there are numerous examples of geotechnical structures, including dams, where thick needle-punched nonwoven geotextiles have been successfully used for drainage. Geonets, geomats, and other geospacers are generally in contact with a geotextile on one side or on both sides; this geotextile serves as a filter between the geospacer and soil, or as a friction layer (bonded to the geospacer) to increase interface shear strength between the geospacer and a geomembrane. When a geospacer is bonded to one or two geotextiles in a factory, the product thus obtained is referred to as a geocomposite. The layer that performs the transmission function in a geocomposite is referred to as the transmissive component of the geocomposite, or transmissive core. In the context of the present paper, the term "geocomposite" is used in all cases where a polymeric transmissive material is associated with a geotextile, whether the transmissive material and the geotextile were bonded together in a factory or sequentially installed against each other in the field.

1.3 Purpose and Organization of the Present Paper

The purpose of the present paper is to provide a complete methodology for the hydraulic design of liquid collection layers. In the context of the present paper, the terminology "hydraulic design" refers to the design steps required to check that the liquid collection layer being designed has sufficient flow capacity to ensure that the thickness of liquid in the liquid collection layer is less than an allowable thickness (Section 1.6). Therefore, the hydraulic design of liquid collection layers requires the calculation of the maximum liquid thickness. To that end, the present paper provides in Section 2 a variety of relationships between the maximum liquid thickness, the liquid supply, the characteristics of the liquid collection layer, and the geometry of the slope on which the liquid collection layer is located. Some of the relationships are relatively complex and give accurate solutions; others are relatively simple and give approximate solutions. The validity of all approximations is assessed, and practical guidance is given for the use of the equations.

The hydraulic design of a liquid collection layer can be done by following one of two approaches: the "thickness approach" and the "hydraulic characteristic approach". With the thickness approach, a given liquid collection material is considered and the design engineer checks that the maximum liquid thickness is less than the allowable thickness. The thickness approach is described in Section 3. With the hydraulic characteristic approach, the design engineer determines the hydraulic characteristics of the liquid collection material needed to ensure that the liquid thickness is less than the allowable thickness. The hydraulic characteristic approach is described in Section 4. Design examples are used to illustrate both approaches.

The equations presented in Section 2 (and the application of these equations presented in Sections 3 and 4) are not valid for vertical liquid collection layers. The case of vertical (and quasi-vertical) liquid collection layers will be addressed in a future paper.

1.4 Definitions and Assumptions

1.4.1 *Description and Geometry of Liquid Collection Layers*

The liquid collection layer is assumed to be underlain by an impermeable liner. Therefore, no liquid is lost by infiltration into or through the material underlying the liquid collection layer.

The liquid collection layer is located on a slope with an angle β (Figure 1). The slope angle is assumed to be less than 90° ($\beta < 90°$). The case of vertical liquid collection layers is not addressed in the present paper.

There is an effective drain at the toe of the slope and the liquid level in the drain is always significantly lower than the elevation of the liner at the toe of the slope. Therefore, the flow of liquid in the liquid collection layer is not impeded at the toe of the slope.

For the development of the equations presented in the present paper, the liquid collection layer is assumed to be rectangular with a length, L, measured in the direction of the flow (i.e. the direction of the slope), and a width, B, perpendicular to the direction

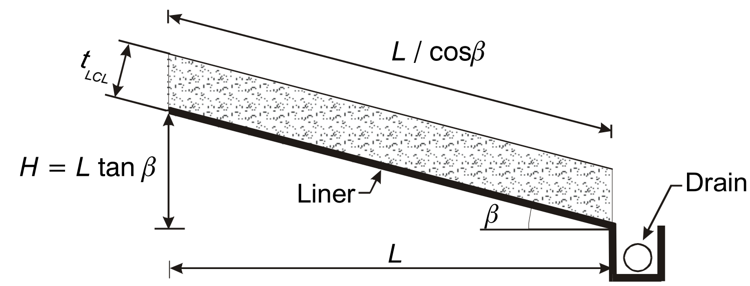

Figure 1. Liquid collection layer.

of the flow. The length, L, is measured horizontally (i.e. L is the length of the horizontal projection of the slope), whereas the width, B, is horizontal.

The thickness of the liquid collection layer is t_{LCL} measured perpendicular to the slope (Figure 1). It is important to note that, when a geocomposite is used as a liquid collection layer, t_{LCL} is the thickness of the transmissive component of the geocomposite (i.e. the core of the geocomposite), not the total thickness of the geocomposite including the geotextile (Section 1.2).

1.4.2 *Hydraulic Characteristics of Liquid Collection Layers*

The liquid collection layer material is characterized by its hydraulic conductivity, k. The liquid collection layer is characterized by its hydraulic transmissivity, θ, which is defined by the following equation:

$$\theta = k \, t_{LCL} \tag{1}$$

where t_{LCL} is the thickness of the transmissive component of the geocomposite. When the liquid collection layer is a geosynthetic, the hydraulic transmissivity of the geosynthetic is measured using a hydraulic transmissivity test. The hydraulic conductivity of the geosynthetic is then derived from the hydraulic transmissivity using the following equation derived from Equation 1:

$$k = \frac{\theta}{t_{LCL}} \tag{2}$$

Equation 2 is very useful because in a number of the equations provided in the present paper, the relevant property of the liquid collection layer material is the hydraulic conductivity, whereas the given properties in the case of a geosynthetic liquid collection layer are generally the hydraulic transmissivity and the thickness.

It should be noted that hydraulic transmissivity and hydraulic conductivity of geosynthetics are not constant material properties, as they are functions of the hydraulic gradient (and, consequently, of the slope of the liquid collection layer). Hydraulic transmissivity and hydraulic conductivity values should, therefore, be obtained from tests performed under a range of hydraulic gradients representative of field conditions.

1.4.3 *Liquid Thickness, Depth, and Head*

The flow is characterized by the liquid thickness, t, which is measured in the direction perpendicular to the slope (Figure 2). The liquid thickness is different from the liquid depth, which is measured vertically. When liquid flows parallel to the slope, which is approximately the case in all liquid collection layers (and which is exactly the case at the location where liquid thickness is maximum), the following classical relationships exist:

$$t = D \cos \beta \tag{3}$$

$$t = h / \cos \beta \tag{4}$$

$$h = D \cos^2 \beta \tag{5}$$

where: D = liquid depth; β = slope angle; and h = hydraulic head. The hydraulic head (or, more accurately, the hydraulic head above the liner located at the base of the liquid collection layer) is the difference in elevation between two points located on the same equipotential surface, one being on the liquid surface, the other being on the liner (see the note below the Figure 2 caption).

1.4.4 *Liquid Supply and Flow*

The amount of liquid supplied to the liquid collection layer is defined by the rate of liquid supply, which is the volume of liquid per unit area and unit time supplied to the

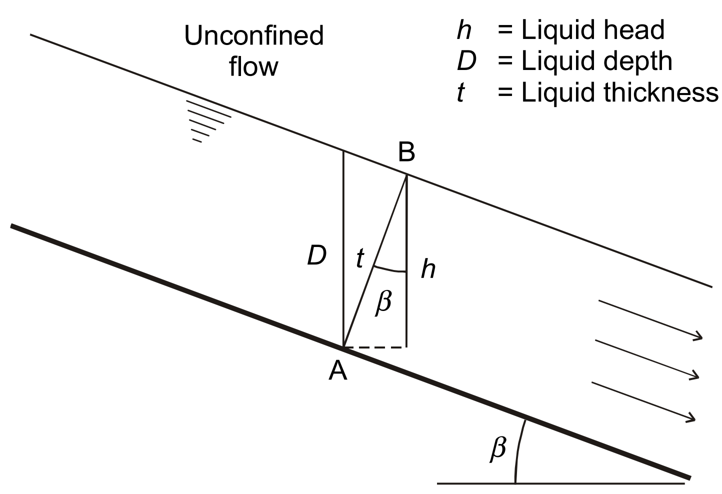

Figure 2. Thickness, depth, and head of liquid on top of the liner in the case of unconfined flow parallel to a slope.

Notes: AB is an equipotential line because it is perpendicular to the flow lines. The head above the liner, h, is the difference in elevation between Points B and A.

liquid collection layer. Theoretically, any orientation may be selected for the unit area used in the definition of the rate of liquid supply, provided that this orientation is properly taken into account in the calculations. For the sake of simplicity, a horizontal unit area is used in the definition of the rate of liquid supply for the case of liquid collection layers that are not vertical (i.e. the cases addressed in the present paper). The rate of liquid supply is then noted q_h and is defined by the following equation:

$$q_h = \frac{Q_h}{A_h} \tag{6}$$

where Q_h is the rate of liquid flow through a horizontal area A_h .

The rate of liquid flow through a horizontal area, A_h , can be measured using a horizontal pan with a surface area A_h (Figure 3):

$$Q_h = A_h \, D_L / \Delta t^* \tag{7}$$

where D_L is the depth of liquid collected in the pan during the time Δt^*.

Combining Equations 6 and 7 gives:

$$q_h = \frac{D_L}{\Delta t^*} \tag{8}$$

Equations 6, 7, and 8 can be used with any set of coherent units. The basic SI units are: Q_h (m^3/s), A_h (m^2), D_L (m), Δt^* (s), and q_h (m/s). For example, if the liquid supply is rainwater, and 100 mm of rain is collected in one day, D_L = 0.1 m and Δt^* = 1 day = 86,400 s. Equation 8 then gives q_h = 0.1/86,400 = 1.157 × 10^{-6} m/s.

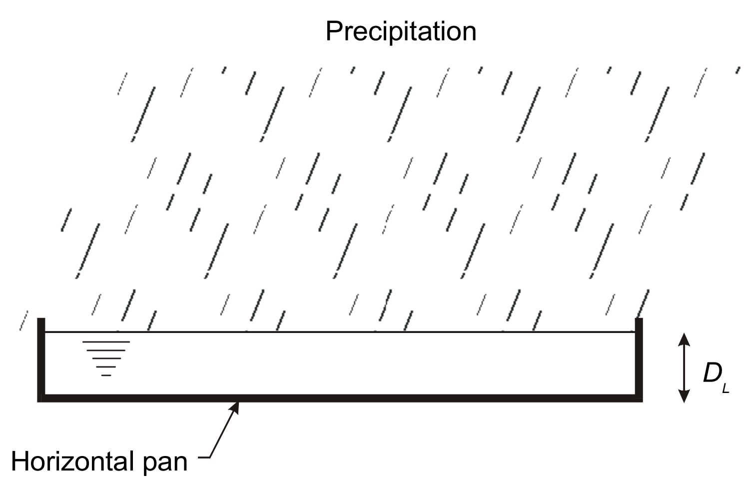

Figure 3. Measurement of the rate of liquid flow through a horizontal area.

It should be noted that no assumption is required regarding the orientation of the rainfall; i.e., whether the rain falls vertically or at an angle, the definition of q_h remains the same. In the present paper, it is assumed that the rate of liquid supply is uniform over the entire area of the liquid collection layer and is constant over time (i.e. steady-state flow conditions are assumed).

Water and aqueous liquids are incompressible; therefore, mass conservation results in volume conservation. Since steady-state flow conditions are assumed, the principle of mass conservation results not only in volume conservation, but also in flow rate conservation. Also, the flow is assumed to be unconfined (Section 1.3), i.e. the considered equations are valid only if the maximum liquid thickness, t_{max}, is less than the thickness of the liquid collection layer, t_{LCL}.

Capillarity affects essentially unsaturated flow (i.e. flow in zones where the liquid collection layer material is not saturated) and flow under transient conditions that precede the establishment of steady-state flow conditions. In the present paper, the liquid collection layer material is assumed to be saturated in the entire zone comprised below the liquid surface, and steady-state flow conditions are assumed, as mentioned above. Therefore, capillarity has no effect, or a negligible effect, on the type of flow discussed in the present paper. Consequently, capillarity is not considered in the present paper.

1.5 Description of Flow in Liquid Collection Layers

In the case of unconfined flow and under steady-state flow conditions, the profile of the liquid surface in a liquid collection layer is as shown in Figure 4 (from Giroud and Houlihan 1995). The liquid profile depends on a dimensionless parameter, λ, defined by:

$$\lambda = \frac{q_h}{k \tan^2\beta} \tag{9}$$

The dimensionless parameter λ is extensively used in the present paper, where it is sometimes referred to as the "characteristic parameter" because it characterizes the liquid collection layer and the liquid supply.

In the general case ($\lambda \neq 0$), the slope of the liquid surface increases from the top to the toe of the liquid collection layer slope. As a result, the hydraulic gradient increases from the top to the toe of the liquid collection layer slope. The maximum value of the liquid thickness, t_{max}, occurs at a certain point between the top and the toe of the slope (Figures 4a and 4b). The location of this point is discussed in Section 2.8.

Another dimensionless parameter, R, is also used in the present paper. This parameter is defined as follows:

$$R = \frac{q_h}{k \sin^2\beta} \tag{10}$$

Combining Equations 9 and 10 gives:

$$\lambda = R \cos^2\beta \tag{11}$$

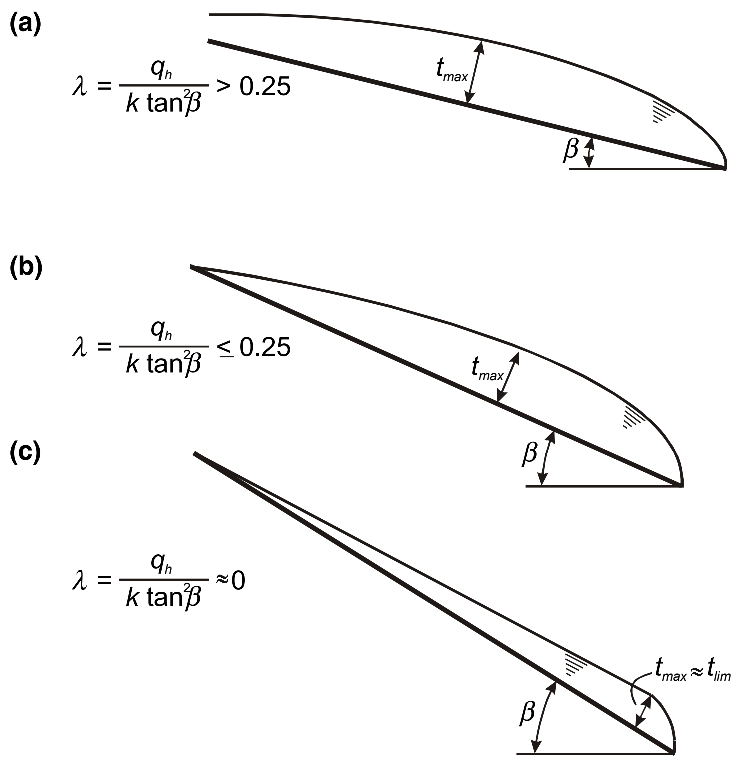

Figure 4. Profile of flow: (a) in the general case, for $\lambda > 0.25$; (b) in the general case, for $\lambda \leq 0.25$; (c) in the case where $\lambda \approx 0$.

Notes: The dimensionless parameter λ is defined by Equation 9. At the toe of the liquid collection layer slope, the liquid thickness t_{toe} is very small. At the scale of the figure, it appears to be zero. A very small liquid thickness is possible at the toe of the slope because the liquid surface is vertical at the toe of the slope and, as a result, the hydraulic gradient is very high (Appendix B). In the case where $\lambda \approx 0$, the maximum liquid thickness is approximately equal to t_{lim} and occurs near the toe of the liquid collection layer slope.

In the special case defined by $\lambda \approx R \approx 0$, the slope of the liquid surface is constant and approximately equal to β because, in this case, the liquid thickness is almost zero. In this case, the maximum liquid thickness occurs approximately at the toe of the slope (Figure 4c).

1.6 Allowable Liquid Thickness

The allowable liquid thickness, t_{allow}, is the thickness that the maximum liquid thickness should not exceed (Section 1.3). The allowable liquid thickness is the lesser of: (i) a maximum liquid thickness prescribed by regulation (if any regulation is applicable to the considered case), for example 0.3 m, as it is often the case in landfills in the United States; and (ii) the thickness of the liquid collection layer. It is important that the liquid thickness be less than the liquid collection layer thickness to ensure that there is no pressure buildup in the liquid collection layer. In other words, the flow must be "unconfined". A detailed discussion of the rationale for the unconfined flow requirement is provided by Giroud et al. (2000a).

In the case of a geocomposite, the thickness of the liquid collection layer is the thickness of the transmissive component of the geocomposite (i.e. the core of the geocomposite). The thickness of geosynthetics currently used as liquid collection layers is virtually always less than the maximum liquid thickness prescribed by regulations. Therefore, in the case of a geosynthetic liquid collection layer, the allowable liquid thickness is virtually always the thickness of the liquid collection layer. A regulatory requirement such as a maximum liquid thickness of 0.3 m is essentially intended for granular liquid collection layers, since these layers may be thicker than 0.3 m.

Regulatory requirements regarding maximum liquid thickness exist essentially in the case of geoenvironmental structures that contain liquids likely to contaminate the ground or the ground water if they leak through the liner underlying the liquid collection layer. In contrast with the case of geoenvironmental structures, there are generally no liquid thickness requirements in the case of liquid collection layers typically used in geotechnical structures where the liquid is water. In this case, the allowable liquid thickness is the thickness of the liquid collection layer.

1.7 Long-Term-In-Soil Performance

1.7.1 *Decrease in Flow Capacity*

As indicated Section 1.3, a liquid collection layer must have sufficient flow capacity to ensure that there is no pressure buildup in the liquid collection layer. Therefore, to ensure long-term performance, the hydraulic design of a liquid collection layer must ensure that the liquid collection layer has sufficient flow capacity under the conditions that exist in the field during the entire design life of the liquid collection layer. The flow capacity under those conditions is referred to as "long-term-in-soil flow capacity". In other words, the design engineer must check that the long-term-in-soil flow capacity is adequate.

The long-term-in-soil flow capacity is likely to be less than the "virgin" flow capacity, i.e. the flow capacity of the liquid collection layer under ideal conditions, before it has been subjected to any stress or mechanism that could decrease its hydraulic characteristics. The decrease from the virgin flow capacity to the long-term-in-soil flow capacity results from instantaneous and time-dependent mechanisms that take place in a drainage medium located in the soil. These mechanisms are discussed in Section 1.7.2 for geosynthetic liquid collection layers and in Section 1.7.3 for granular liquid collection layers.

1.7.2 *Flow Capacity Reduction in the Case of Geosynthetic Liquid Collection Layers*

On a given slope (characterized by the slope angle and the slope length), the flow capacity of a liquid collection layer depends on the thickness of the liquid collection layer and the hydraulic conductivity of the liquid collection layer material. Geosynthetic liquid collection layers are typically constructed using geocomposites (Section 1.2). Therefore, the discussion presented below is essentially related to geocomposites. It should be remembered that, as indicated in Section 1.4.1, when a geocomposite is used to construct a liquid collection layer, the thickness of the liquid collection layer is the thickness of the transmissive component of the geocomposite (i.e. the core of the geocomposite), not the total thickness of the geocomposite including the geotextile.

The flow capacity of a geocomposite in the field can be reduced by a variety of mechanisms that depend on the following parameters: applied load, time, contact with adjacent materials, and environmental conditions (e.g. presence of chemicals, biological activity, and temperature). More specifically, the thickness and/or the hydraulic conductivity of the transmissive core of a geocomposite may be reduced by instantaneous compression of the transmissive core, intrusion of the geotextile filter into the transmissive core, time-dependent compression (i.e. creep) of the transmissive core, and additional intrusion of the geotextile due to time-dependent deformation of the geosynthetic; these four mechanisms are caused by the applied stresses. In addition, chemical degradation of the polymeric compound(s) used to make the geocomposite may reduce its effective thickness and/or its hydraulic conductivity. Finally, clogging of the transmissive core may reduce its effective thickness and/or its hydraulic conductivity. Clogging results from physical, chemical, and biological mechanisms; biological clogging is typically caused by the growth of micoorganisms (Giroud 1996), but an extreme case is that of clogging due to root penetration in the drainage medium. A given mechanism (e.g. compression or clogging) may result in (or may be interpreted as) a reduction in effective thickness and/or a reduction in hydraulic conductivity. Therefore, to evaluate the decrease in flow capacity of a geocomposite, it is simpler to use the hydraulic transmissivity, which is the product of thickness and hydraulic conductivity (Equation 1). Accordingly, from a practical standpoint, the decrease in flow capacity due to the mechanisms described above is expressed by using reduction factors on the hydraulic transmissivity as follows:

$$\theta_{LTIS} = \frac{\theta_{measured}}{\prod(RF)} = \frac{\theta_{measured}}{RF_{IMCO} \times RF_{IMIN} \times RF_{CR} \times RF_{IN} \times RF_{CD} \times RF_{PC} \times RF_{CC} \times RF_{BC}} \quad (12)$$

where: θ_{LTIS} = long-term-in-soil hydraulic transmissivity of the considered geosynthetic, i.e. the minimum hydraulic transmissivity calculated for the geosynthetic subjected to the maximum stress anticipated in the soil during the design life of the liquid collection layer and subjected to all mechanisms likely to reduce its hydraulic transmissivity; $\theta_{measured}$ = value of hydraulic transmissivity measured in a laboratory test; $\Pi(RF)$ = product of all reduction factors; RF_{IMCO} = reduction factor for immediate compression, i.e. decrease of hydraulic transmissivity due to compression of the transmissive core immediately following the application of stress; RF_{IMIN} = reduction factor for immediate intrusion, i.e. decrease of hydraulic transmissivity due to geotextile intrusion into the transmissive core immediately following the application of stress; RF_{CR} = reduction factor for creep, i.e. time-dependent hydraulic transmissivity reduction due to creep of the transmissive core under the applied stress; RF_{IN} = reduction factor for delayed intrusion, i.e. decrease of hydraulic transmissivity over time due to geotextile intrusion into the transmissive core resulting from time-dependent deformation of the geotextile; RF_{CD} = reduction factor for chemical degradation, i.e. decrease of hydraulic transmissivity due to chemical degradation of the polymeric compound(s) used to make the geocomposite; RF_{PC} = reduction factor for particulate clogging, i.e. decrease of hydraulic transmissivity due to clogging by particles migrating into the transmissive core; RF_{CC} = reduction factor for chemical clogging, i.e. decrease of hydraulic transmissivity due to chemical clogging of the transmissive core; and RF_{BC} = reduction factor for biological clogging, i.e. decrease of hydraulic transmissivity due to biological clogging of the transmissive core.

The following comments can be made:

- θ_{LTIS} is sometimes called θ_{allow}, i.e. allowable hydraulic transmissivity. The terminology "long-term-in-soil hydraulic transmissivity" is preferred in the present paper because it lends more clarity to discussions, in particular discussions on the factor of safety.
- Each reduction factor corresponds to a mechanism that reduces the hydraulic transmissivity of the considered material in the field. If one of these mechanisms occurs during hydraulic transmissivity testing in the laboratory, to the same extent as in the field, then the corresponding reduction factor is equal to 1.0. (It is important to understand that a reduction factor equal to one does not necessarily mean that the related mechanism affecting the hydraulic transmissivity of a virgin material does not exist; it simply means that the effect of this mechanism is already incorporated in the value of $\theta_{measured}$.) An ideal hydraulic transmissivity test would perfectly simulate in the laboratory all the mechanisms that reduce the hydraulic transmissivity in the field. In this ideal case, all reduction factors would be equal to 1.0. However, such a test is not achievable from a practical standpoint because it would be extremely complex and would require a very long time.
- RF_{IMCO} and RF_{IMIN} correspond to instantaneous mechanisms (i.e. mechanisms that take place as soon as the stress is applied), whereas the other reduction factors correspond to time-dependent mechanisms.
- RF_{IMCO}, RF_{IMIN}, RF_{CR}, and RF_{IN} result from mechanical mechanisms, i.e. they are directly related to the applied stress. In contrast, RF_{CD}, RF_{PC}, RF_{CC}, and RF_{BC} result from physico-chemical mechanisms and, as such, they are not directly related to the applied stress.
- The physico-chemical mechanisms do not occur during typical hydraulic transmissivity tests that are performed with pure water. In contrast, the mechanical mechanisms may occur during the hydraulic transmissivity test, which affects the magnitude of RF_{IMCO}, RF_{IMIN}, RF_{CR}, and RF_{IN}, as discussed below.

It is important to note that the four reduction factors that result from mechanical mechanisms depend on the conditions under which the hydraulic transmissivity is measured. These conditions include: the stress applied to the specimen of transmissive material (e.g. the transmissive core or the geocomposite) during the hydraulic transmissivity test, the time during which the stress is applied before the flow rate (from which the hydraulic transmissivity is derived) is measured (the "seating time"), and the nature and behavior of the materials in contact with the transmissive material during the hydraulic transmissivity test. From this viewpoint, the following comments can be made:

- RF_{IMCO} can be eliminated (i.e. RF_{IMCO} = 1.0) if the hydraulic transmissivity is measured after a stress equal to, or greater than, the stress in the soil is applied to the specimen of transmissive material subjected to the hydraulic transmissivity test.

- RF_{IMIN} can be eliminated (i.e. RF_{IMIN} = 1.0) if the hydraulic transmissivity test simulates the boundary conditions created by the presence of materials adjacent to the transmissive material.
- RF_{CR} and RF_{IN} can be decreased if the hydraulic transmissivity is measured after the stress has been applied for a certain period of time (seating time), because part of the creep of the transmissive core and part of the delayed intrusion would have occurred before the hydraulic transmissivity is measured.

The extreme theoretical case would be the case where the hydraulic transmissivity is measured on the transmissive core placed between two smooth plates, under no load, with pure water (so none of the physico-bio-chemical mechanisms can take place), and during a period of time that is so short that none of the time-dependent mechanisms can develop. In this extreme theoretical case, all of the eight reduction factors defined above would have their maximum value. This extreme theoretical case may not exist in reality. A typical hydraulic transmissivity test is between: (i) the ideal case where all mechanisms are perfectly simulated and, consequently, all reduction factors would be equal to 1.0; and (ii) the extreme theoretical case where all of the eight reduction factors would have their maximum value. Two typical cases of laboratory test conditions can be considered and are described below.

In the first typical case of test conditions, the transmissive core is placed between two rigid plates and a load equal to or greater than the design load is sustained for a certain period of time (the seating time). In this case, the instantaneous compression takes place before the hydraulic transmissivity is measured. Therefore, RF_{IMCO} = 1.0. Also, some creep occurs during the seating time. As a result, the value of RF_{CR} is less than in the theoretical case where the hydraulic transmissivity would be measured at time zero. Equation 12 then becomes:

$$\theta_{allow} = \frac{\theta_{measured}}{\prod(RF)} = \frac{\theta_{measured}}{RF_{IMIN} \times RF_{CR} \times RF_{IN} \times RF_{CD} \times RF_{PC} \times RF_{CC} \times RF_{BC}} \tag{13}$$

Seating times of 100 or 300 hours are often recommended in the United States (Holtz et al. 1997). During such seating times, a significant amount of creep takes place. As a result, RF_{CR} is significantly less than it would be if the seating time were short.

If a geocomposite (i.e. transmissive core plus one or two geotextiles) is placed between the two rigid plates (instead of only the transmissive core), then, in addition to creep, some time-dependent intrusion of geotextile into the transmissive core channels occurs during the seating time. As a result, the longer the seating time, the smaller the value of RF_{IN} .

In the second typical case of test conditions, the boundary conditions created by the presence of adjacent materials are simulated. To that end, the geocomposite is placed between two materials (soil or geosynthetic) that are identical to, or that simulate, the materials that are adjacent to the considered geocomposite in the field, and the sustained load is equal to or greater than the design load. Therefore, RF_{IMCO} = 1.0 and RF_{IMIN} = 1.0. Also, some creep and some time-dependent intrusion of geotextile into the transmissive core channels occur during the seating time. As a result, the values of RF_{CR} and RF_{IN} are less than in the theoretical case where the hydraulic transmissivity would be measured at time zero. Equation 12 then becomes:

$$\theta_{LTIS} = \frac{\theta_{measured}}{\prod(RF)} = \frac{\theta_{measured}}{RF_{CR} \times RF_{IN} \times RF_{CD} \times RF_{PC} \times RF_{CC} \times RF_{BC}} \tag{14}$$

The determination of RF_{CR} , RF_{IN} , RF_{CD} , RF_{PC} , RF_{CC} , and RF_{BC} requires long-duration tests. Due to lack of time, such tests cannot be performed for the design of a specific project. Therefore, values obtained from Table 1, from the literature, or from the geosynthetic manufacturer should be used. Table 1 provides guidance regarding the values of the reduction factors for geonets and geocomposites having geonets as the transmissive core (which are the most frequently used geosynthetic liquid collection layers in landfills in the United States). However, the design engineer is cautioned that the values of the reduction factors may vary significantly depending on the type of geocomposite and the exposure conditions (stress, chemical composition of the soil and liquid). Also, as pointed out above, the values of some of the time-dependent reduction factors (e.g. RF_{CR} and RF_{IN}) may significantly vary depending on the conditions under which the hydraulic transmissivity is measured. The values given in Table 1 correspond to the case where the seating time exceeds 100 hours and the boundary conditions due to adjacent materials are simulated in the hydraulic transmissivity test. It is also possible to calculate RF_{CR} from the results of thickness measurements during compressive creep tests (tests that are easy to conduct) without the need for performing long-term hydraulic transmissivity tests (tests that are impractical and expensive), using the method developed by Giroud et al. (2000b).

Table 1. Guidance for the selection of some of the reduction factors on the flow capacity of geonets and geocomposites having a geonet transmissive core.

Examples of application	Normal stress	Liquid	RF_{IN}	RF_{CR}	RF_{CC}	RF_{BC}
Landfill cover drainage layer Low retaining wall drainage	Low	Water	1.0 to 1.2	1.1 to 1.4	1.0 to 1.2	1.2 to 1.5
Embankment, Dams, Landslide repair High retaining wall drainage	High	Water	1.0 to 1.2	1.4 to 2.0	1.0 to 1.2	1.2 to 1.5
Landfill leachate collection layer Landfill leakage collection and detection layer Leachate pond leakage collection and detection layer	High	Leachate	1.0 to 1.2	1.4 to 2.0	1.5 to 2.0	1.5 to 2.0

Notes: The reduction factors, RF_{IN} , RF_{CR} , RF_{CC} , and RF_{BC} are defined in Section 1.7.2. Table 1 was developed for the present paper, using some reduction factor values from Koerner (1998). Design engineers are cautioned that the values of the reduction factors may significantly vary depending on the type of geocomposite and the exposure conditions (stress, chemical composition of the soil and liquid). Also, as discussed in Section 1.7.2, RF_{IN} and RF_{CR} depend on the testing conditions under which the hydraulic transmissivity is measured. The reduction factor values given in Table 1 correspond to the case where the seating time exceeds 100 hours and the boundary conditions due to adjacent materials are simulated in the hydraulic transmissivity test. Finally, due to lack of relevant data, no guidance is provided for RF_{CD} and RF_{PC} . Additional information on reduction factors may be found in a paper by Zanzinger and Gartung (1999).

Also, it should be noted that RF_{CR} , RF_{CD} , RF_{CC} , and RF_{BC} (and, to a lesser degree, RF_{IN} and RF_{PC}) correspond to time-dependent mechanisms. Therefore, the values of RF_{CR} , RF_{CD} , RF_{CC} , and RF_{BC} (and, to a lesser degree, RF_{IN} and RF_{PC}) selected by the design engineer depend on the design life of the liquid collection layer. In cases where the liquid supply rate varies with time, the design engineer may consider several time periods. For example, in the case of landfills with no leachate recirculation, three phases may be considered: (i) construction and pre-operational phase; (ii) operational phase; and (iii) post-closure phase. As time elapses, the leachate collection system will typically experience a reduction in the rate of leachate that needs to be collected, but may concurrently experience a reduction of its flow capacity due to time-dependent mechanisms such as creep and clogging.

The above discussion is for geocomposites, in particular, for geocomposites whose transmissive core is a geonet (which are the most frequently used geosynthetic liquid collection layers in landfills in the United States). In the case where the geosynthetic liquid collection layer is a thick needle-punched nonwoven geotextile, the mechanisms described above exist with the exception of geotextile intrusion into the transmissive core since, in this case, the geotextile itself is the transmissive medium. In this case, the reduction factors presented above exist, but no guidance is proposed herein regarding their values.

It should be noted that the various reduction factors may not be completely independent. For example, chemical degradation may affect creep resistance (i.e. may increase RF_{CR}), and, as shown by Palmeira and Gardoni (2000), the presence of soil particles in a needle-punched nonwoven geotextile (i.e. particulate clogging) may reduce the geotextile's compressibility (i.e. it may reduce RF_{IMCO} and RF_{CR} while increasing RF_{PC}).

Finally, it should be noted that, contrary to the case of geosynthetics used for soil reinforcement, no reduction factor for installation damage is included in Equations 12 to 14. Indeed, it is generally considered that damage caused by installation is not likely to affect the hydraulic transmissivity of the geosynthetics typically used in liquid collection layers. However, design engineers may use a reduction factor for installation damage in all cases where this may appear appropriate. Also, it is possible to find in the technical literature "survivability criteria" that evaluate the ability of geotextiles, used alone or as part of geocomposites, to resist damage during installation.

1.7.3 *Flow Capacity Reduction in the Case of Granular Liquid Collection Layers*

When a granular liquid collection layer is used, the mechanisms of thickness reduction are negligible because granular materials do not exhibit any significant instantaneous thickness reduction (compression) nor time-dependent thickness reduction (creep), and the reduction in flow capacity due to geotextile intrusion is negligible because the geotextile thickness is negligible with respect to the thickness of the granular layers. Furthermore, chemical degradation that could affect the thickness and the hydraulic conductivity of a granular liquid collection layer can be avoided by proper selection of the granular material. Therefore, in the case of a granular liquid collection layer, the only relevant reduction affecting the flow capacity is the reduction of hydraulic conductivity due to clogging. As a result, the reduction in flow capacity results from a reduction in hydraulic conductivity, which can be expressed as follows:

$$k_{LTIS} = \frac{k_{measured}}{\prod(RF)} = \frac{k_{measured}}{RF_{PC} \times RF_{CC} \times RF_{BC}} \tag{15}$$

where: k_{LTIS} = long-term-in-soil hydraulic conductivity of the granular material, i.e. hydraulic conductivity of the granular material located in the soil and subjected to conditions that can cause the development of clogging during the design life of the liquid collection layer; and $\boldsymbol{k_{measured}}$ = hydraulic conductivity of a specimen of granular material representative of the granular material as installed, measured in a hydraulic conductivity test performed with water during a short period of time so that clogging does not develop.

1.7.4 *Factor of Safety*

In addition to the reduction factors described in Sections 1.7.2 and 1.7.3, a factor of safety, *FS*, is used in all calculations to take into account possible uncertainties, such as the fact that the measurement of hydraulic characteristics (i.e. hydraulic conductivity and hydraulic transmissivity) is generally delicate and prone to errors. Values such as 2 or 3, or sometimes greater values, are typically recommended for the factor of safety.

In the equations provided in the present paper, there are two ways of using a factor of safety. The factor of safety can be applied to the maximum liquid thickness, FS_T , or to the relevant hydraulic characteristic, FS_H , i.e. the hydraulic transmissivity in the case of a geosynthetic liquid collection layer or the hydraulic conductivity in the case of a granular liquid collection layer. The two ways (factor of safety on the maximum liquid thickness and factor of safety on the hydraulic characteristic) will be compared.

It is important to note that FS_T and FS_H are not partial factors of safety to be used simultaneously. They are two ways of expressing the factor of safety of the liquid collection layer.

1.7.5 *Use of Reduction Factors and Factor of Safety*

As indicated in Section 1.3, there are two design approaches: the thickness approach (described in Section 3) that consists of calculating the maximum liquid thickness, and the hydraulic characteristic approach (described in Section 4) that consists of calculating the required hydraulic conductivity of the liquid collection layer material or the hydraulic transmissivity of the liquid collection layer. Use of the reduction factors in these two approaches is described in Section 3 for the thickness approach and in Section 4 for the hydraulic characteristic approach.

1.8 Design Options

The flow capacity of a liquid collection layer depends on two sets of characteristics: the intrinsic characteristics of the liquid collection layer and the characteristics of the slope on which the liquid collection layer is installed. The intrinsic characteristics are the thickness of the liquid collection layer and the hydraulic conductivity of the liquid collection layer material (or the hydraulic transmissivity of the liquid collection layer, which is the product of the thickness and hydraulic conductivity). The characteristics of the slope on which the liquid collection layer is installed are the slope angle and the slope length.

If design calculations show that the considered liquid collection layer material does not provide adequate flow capacity, the design engineer has the following options: (i) a liquid collection layer with a greater thickness (but this option is inappropriate if, in the original design, the liquid thickness was equal to a regulatory maximum liquid thickness); (ii) a different drainage material with a greater hydraulic conductivity (or greater hydraulic transmissivity); and (iii) a liquid collection layer with a shorter length and/or a steeper slope. The last option is the only one available if the liquid collection layer and its material may not be changed. However, slope steepness may be limited by stability and/or by waste storage capacity considerations.

2 AVAILABLE EQUATIONS AND DISCUSSIONS

2.1 Overview

As indicated in Section 1.5, the liquid flowing in a liquid collection layer has a maximum thickness at a certain location along the slope on which the liquid collection layer is constructed. Equations given in Section 2 provide the maximum liquid thickness as a function of the following parameters: the hydraulic conductivity of the liquid collection layer material; the length and slope of the liquid collection layer; and the rate of liquid supply. Some solutions provide an accurate value of the maximum liquid thickness, some provide an approximate value. The validity of the equations that provide approximate values is assessed. Section 2 only presents the equations and discusses their accuracy. The use of the equations will be presented in Sections 3 and 4.

2.2 Solution Based on Simplified Assumptions

As indicated in Section 1.5, the hydraulic gradient increases from the top to the toe of the slope on which the liquid collection layer is constructed. Giroud (1985) used an average value of the hydraulic gradient and developed the following equation:

$$t_{max} = \frac{\sqrt{\tan^2\beta + 4\, q_h / k} - \tan\beta}{2\cos\beta} L \tag{16}$$

The demonstration of Equation 16 can be found in Appendix A.

Equation 16 can be written as follows:

$$t_{max} = \frac{\sqrt{1 + 4\lambda} - 1}{2\cos\beta / \tan\beta} L \tag{17}$$

where λ is the characteristic parameter (dimensionless) defined by Equation 9.

It is important to note that, as shown in Appendix A, Equations 16 and 17 tend toward well known equations for the two limit cases where λ tends toward zero and toward infinity. This confirms the validity of Equations 16 and 17. Equation 16 (or 17, which is equivalent) has been used as "Giroud's equation" for the design of numerous leachate collection layers and leakage collection layers in many landfills in the United States. Giroud's equation has progressively replaced the use, in the United States, of "Moore's equations", which had been presented in documents published by the US Environmental Protection Agency (Moore 1980, 1983; USEPA 1989), but for which no derivation was ever published or otherwise made available, to the best of the authors' knowledge. In the early 1980s, the senior author suspected the validity of Moore's equations because they did not tend toward the well-known limits when λ tends toward zero or toward infinity. This prompted the development of equations by the senior author. In the remainder of the present paper, Equation 16 (or 17) will be referred to as the "original Giroud's equation".

2.3 Governing Differential Equation

2.3.1 *Establishment of the Governing Differential Equation*

The differential equation that governs the flow of liquid in a liquid collection layer receiving a uniform liquid supply can be established as follows. Based on the principle of mass conservation, the flow rate at abscissa x is:

$$Q = q_h B x \tag{18}$$

where: Q = flow rate; B = width of the liquid collection layer in the direction perpendicular to the direction of the flow; and x = distance, measured horizontally, between the top of the liquid collection layer slope and the point where the liquid thickness is evaluated (Figure 5).

Darcy's equation can be written as follows:

$$Q = k B t i \tag{19}$$

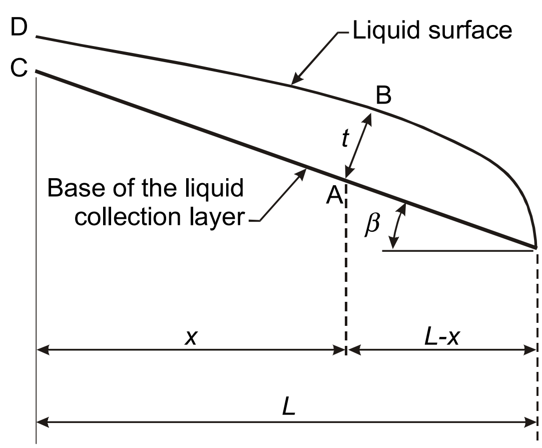

Figure 5. Definition of parameters used in the establishment of the governing differential equation.

where i is the hydraulic gradient.

Combining Equations 18 and 19 gives:

$$q_h \, x \; = \; k \, t \, i \tag{20}$$

The hydraulic gradient, i, is derived as follows from the hydraulic head:

$$i = -\frac{\mathrm{d}h}{\mathrm{d}x / \cos\beta} \tag{21}$$

where h is the hydraulic head, which is given by the following equation:

$$h \; = \; \left(L - x\right) \tan\beta \; + \; h_{AB} \tag{22}$$

where h_{AB} is the hydraulic head that corresponds to the liquid thickness AB (Figure 5). In Equation 22, the term $(L–x)$ tanβ represents the elevation of Point A.

Combining Equation 4 with $h = h_{AB}$ and Equations 21 and 22 gives:

$$i \; = \; \sin\beta - \cos^2\beta \; \frac{\mathrm{d}t}{\mathrm{d}x} \tag{23}$$

It should be noted that Equation 4 is valid when the liquid surface is parallel to the slope. As shown in Figure 4, this is true only at the location of the maximum liquid thickness. Since the ultimate goal is to calculate the maximum liquid thickness, the approximation made when Equation 4 is used should be acceptable. Further discussion on approximations associated with the evaluation of the hydraulic head may be found in Section 2.6.

Combining Equations 20 and 23 gives:

$$q_h \, x \; = \; k \; t \left(\sin\beta - \cos^2\beta \; \frac{\mathrm{d}t}{\mathrm{d}x} \right) \tag{24}$$

Equation 24 is the differential equation governing the flow of liquid in a liquid collection layer exposed to a uniform liquid supply. In the present paper, this equation is referred to as the "governing equation". Equation 24 can also be written as follows:

$$\frac{q_h \, x}{k} \; = \; t \; \sin\beta \; - \cos^2\beta \; t \, \frac{\mathrm{d}t}{\mathrm{d}x} \tag{25}$$

Combining Equations 9 and 25 gives the following expression for the governing differential equation:

$$\lambda \, x \; = \; \frac{t \; \cos^2\beta}{\sin\beta} \; - \; \frac{\cos^4\beta}{\sin^2\beta} \; t \; \frac{\mathrm{d}t}{\mathrm{d}x} \tag{26}$$

2.3.2 *Limit Cases*

The governing differential equation, Equation 24 (or Equations 25 and 26, which are equivalent), becomes simpler in two extreme cases. In each of these two cases, the governing differential equation can easily be solved, as shown below.

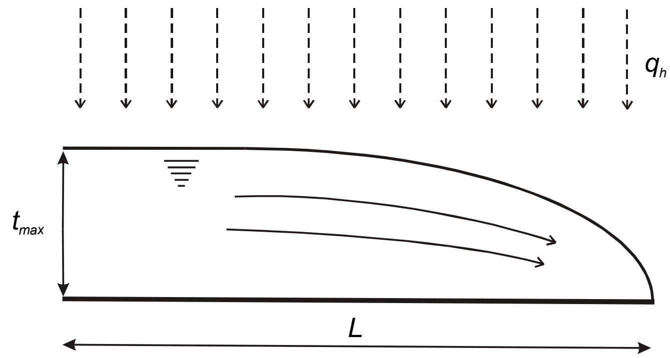

Figure 6. Profile of the liquid surface in the limit case where β = 0.

Case Where β = 0. In this case (Figure 6), Equation 25 becomes:

$$\frac{q_h}{k}\, x \; \mathrm{d}x \; = -t \; \mathrm{d}t \tag{27}$$

Integration of this differential equation gives:

$$t^2 = -\frac{q_h}{k} x^2 + C \tag{28}$$

where C is a constant with respect to the variable x. The value of the constant, C, can be determined by considering that $t = 0$ for $x = L$, hence:

$$C = \frac{q_h}{k} L^2 \tag{29}$$

Combining Equations 28 and 29 gives:

$$t = \sqrt{\frac{q_h}{k}\left(L^2 - x^2\right)} \tag{30}$$

The maximum liquid thickness occurs for $x = 0$, hence:

$$t_{max} = \sqrt{\frac{q_h}{k}} \; L \tag{31}$$

Equation 31 is well known. It is used to design horizontal liquid collection layers, such as drainage layers under road pavements. It should be noted that, based on Equation 9, λ tends toward infinity when β tends toward zero.

Case Where λ is Very Small. The characteristic parameter λ is very small if q_h is very small and/or k and β are very large. If λ is very small, Equation 26 shows that t must be small. Consequently, $t\mathrm{d}t$ is negligible compared to t, and Equation 26 becomes:

$$\lambda x \approx \frac{t \cos^2\beta}{\sin\beta} \tag{32}$$

hence:

$$t \approx \frac{\lambda\ x \sin\beta}{\cos^2\beta} = \frac{\lambda\ x \tan\beta}{\cos\beta} \tag{33}$$

Combining Equations 9 and 33 gives:

$$t = \frac{q_h\ x}{k\ \sin\beta} \tag{34}$$

Equation 34 shows that, when λ is very small, the liquid thickness, t, varies linearly with the abscissa, x. Therefore, the maximum value of t occurs for $x = L$, hence:

$$t_{max} \approx \frac{\lambda\ L \sin\beta}{\cos^2\beta} = \frac{\lambda\ L \tan\beta}{\cos\beta} = \frac{q_h L}{k\ \sin\beta} \tag{35}$$

Equation 35 is well known because it can easily be obtained using Darcy's equation, as shown in Section 2.8. Further discussion of Equation 35 can be found in Sections 2.8 and 2.11.

2.3.3 *Numerical Solution*

In 1992, the governing differential equation was solved numerically (Giroud et al. 1992). To solve the differential equation, the boundary condition used to represent free drainage at the toe of the liquid collection layer (in accordance with an assumption presented in Section 1.4.1) was zero liquid thickness and infinite hydraulic gradient. The validity of this boundary condition is discussed in Appendix B. The numerical solution was deemed accurate because it was consistent with values obtained analytically for the two limit cases, i.e. when λ tends toward infinity (Equation 31) and toward zero (Equation 35).

2.4 Empirical Solution Based on Numerical Results

It was noted by Giroud et al. (1992) that the values of the maximum liquid thickness calculated using Equation 16 (or Equation 17, which is equivalent) were very close to the accurate values obtained by numerically solving the governing differential equation. The difference between the values calculated using Equation 16 or 17 and the accurate values is shown in Table 2. It appears in Table 2 that, for typical slopes, the difference between the values calculated using Equation 16 or 17 and the accurate values is less than 13%, the values calculated using Equation 16 or 17 being greater than the accurate values obtained numerically. A dimensionless modifying factor, j, was then added to Equation 16 to improve its accuracy, hence:

$$t_{max} = j\ \frac{\sqrt{\tan^2\beta + 4\,q_h/k} - \tan\beta}{2\cos\beta} L \tag{36}$$

Table 2. Comparison between values of t_{max}/L obtained numerically (Giroud et al. 1992) and calculated using Giroud's equations and McEnroe's equations given in the present paper.

$\tan\beta$	q_h / k	Numerical solution (Giroud et al. 1992)	Original Giroud's equation Eq. 16 or 17	Modified Giroud's equation Eq. 40 or 41	McEnroe's equations for $i_{toe} = \cos\beta$ Eq. 49 to 51	McEnroe's equations for $t_{toe} = 0$ Eq. 52 to 54	Transformed McEnroe's equations for $t_{toe} = 0$ Eq. 66 to 68
0.02	1×10^{-9}	5.00×10^{-8}	5.00×10^{-8}	5.00×10^{-8}	5.00×10^{-8} (1)	5.00×10^{-8} (1)	5.00×10^{-8} (1)
	1×10^{-8}	5.00×10^{-7}	5.00×10^{-7}	5.00×10^{-7}	5.00×10^{-7} (1)	5.00×10^{-7}	5.00×10^{-7} (1)
	1×10^{-7}	4.99×10^{-6}	5.00×10^{-6}	4.99×10^{-6}	4.99×10^{-6}	4.99×10^{-6}	4.99×10^{-6}
	1×10^{-6}	4.94×10^{-5}	4.99×10^{-5}	4.93×10^{-5}	4.94×10^{-5}	4.94×10^{-5}	4.94×10^{-5}
	1×10^{-5}	4.65×10^{-4}	4.88×10^{-4}	4.61×10^{-4}	4.65×10^{-4}	4.65×10^{-4}	4.65×10^{-4}
	1×10^{-4}	3.68×10^{-3}	4.14×10^{-3}	3.68×10^{-3}	3.68×10^{-3}	3.68×10^{-3}	3.68×10^{-3}
	1×10^{-3}	2.09×10^{-2}	2.32×10^{-2}	2.08×10^{-2}	2.09×10^{-2}	2.09×10^{-2}	2.09×10^{-2}
	1×10^{-2}	8.64×10^{-2}	9.05×10^{-2}	8.65×10^{-2}	8.67×10^{-2}	8.63×10^{-2}	8.63×10^{-2}
	1×10^{-1}	3.03×10^{-1}	3.06×10^{-1}	3.04×10^{-1}	3.16×10^{-1}	3.01×10^{-1}	3.01×10^{-1}
1/3	1×10^{-9}	3.16×10^{-9}	3.16×10^{-9}	3.16×10^{-9}	3.16×10^{-9} (1)(2)	3.16×10^{-9} (1)(2)	3.16×10^{-9} (1)(2)
	1×10^{-8}	3.16×10^{-8}	3.16×10^{-8}	3.16×10^{-8}	3.16×10^{-8} (1)	3.16×10^{-8} (1)	3.16×10^{-8} (1)
	1×10^{-7}	3.16×10^{-7}	3.16×10^{-7}	3.16×10^{-7}	3.16×10^{-7} (1)	3.16×10^{-7} (1)	3.16×10^{-7} (1)
	1×10^{-6}	3.16×10^{-6}	3.16×10^{-6}	3.16×10^{-6}	3.16×10^{-6}	3.16×10^{-6}	3.16×10^{-6} (1)
	1×10^{-5}	3.16×10^{-5}	3.16×10^{-5}	3.16×10^{-5}	3.16×10^{-5}	3.16×10^{-5}	3.16×10^{-5}
	1×10^{-4}	3.15×10^{-4}	3.16×10^{-4}	3.14×10^{-4}	3.14×10^{-4}	3.14×10^{-4}	3.15×10^{-4}
	1×10^{-3}	3.06×10^{-3}	3.13×10^{-3}	3.03×10^{-3}	3.05×10^{-3}	3.05×10^{-3}	3.06×10^{-3}
	1×10^{-2}	2.69×10^{-2}	2.92×10^{-2}	2.65×10^{-2}	2.66×10^{-2}	2.64×10^{-2}	2.67×10^{-2}
	1×10^{-1}	1.87×10^{-1}	2.01×10^{-1}	1.77×10^{-1}	1.85×10^{-1}	1.73×10^{-1}	1.78×10^{-1}
0.5	1×10^{-9}	2.24×10^{-9}	2.24×10^{-9}	2.24×10^{-9}	2.24×10^{-9} (1)	2.24×10^{-9} (1)	2.24×10^{-9} (1)
	1×10^{-8}	2.24×10^{-8}	2.24×10^{-8}	2.24×10^{-8}	2.24×10^{-8} (1)	2.24×10^{-8} (1)	2.24×10^{-8} (1)
	1×10^{-7}	2.24×10^{-7}	2.24×10^{-7}	2.24×10^{-7}	2.24×10^{-7} (1)	2.24×10^{-7} (1)	2.24×10^{-7} (1)
	1×10^{-6}	2.24×10^{-6}	2.24×10^{-6}	2.24×10^{-6}	2.24×10^{-6} (1)	2.24×10^{-6} (1)	2.24×10^{-6} (1)
	1×10^{-5}	2.24×10^{-5}	2.24×10^{-5}	2.24×10^{-5}	2.23×10^{-5}	2.23×10^{-5}	2.24×10^{-5}
	1×10^{-4}	2.23×10^{-4}	2.24×10^{-4}	2.23×10^{-4}	2.23×10^{-4}	2.23×10^{-4}	2.23×10^{-4}
	1×10^{-3}	2.20×10^{-3}	2.23×10^{-3}	2.19×10^{-3}	2.19×10^{-3}	2.19×10^{-3}	2.20×10^{-3}
	1×10^{-2}	2.04×10^{-2}	2.15×10^{-2}	2.01×10^{-2}	2.01×10^{-2}	1.99×10^{-2}	2.02×10^{-2}
	1×10^{-1}	1.61×10^{-1}	1.71×10^{-1}	1.51×10^{-1}	1.57×10^{-1}	1.44×10^{-1}	1.51×10^{-1}
1.0	1×10^{-9}	1.41×10^{-9}	1.41×10^{-9}	1.41×10^{-9}	1.41×10^{-9} (1)	1.41×10^{-9} (1)(2)	1.41×10^{-9} (1)(2)
	1×10^{-8}	1.41×10^{-8}	1.41×10^{-8}	1.41×10^{-8}	1.41×10^{-8} (1)	1.41×10^{-8} (1)(2)	1.41×10^{-8} (1)(2)
	1×10^{-7}	1.41×10^{-7}	1.41×10^{-7}	1.41×10^{-7}	1.41×10^{-7} (1)	1.41×10^{-7} (1)	1.41×10^{-7} (1)
	1×10^{-6}	1.41×10^{-6}	1.41×10^{-6}	1.41×10^{-6}	1.41×10^{-6} (1)	1.41×10^{-6} (1)	1.41×10^{-6} (1)
	1×10^{-5}	1.41×10^{-5}	1.41×10^{-5}	1.41×10^{-5}	1.41×10^{-5}	1.41×10^{-5}	1.41×10^{-5}
	1×10^{-4}	1.41×10^{-4}	1.41×10^{-4}	1.41×10^{-4}	1.41×10^{-4}	1.41×10^{-4}	1.41×10^{-4}
	1×10^{-3}	1.41×10^{-3}	1.41×10^{-3}	1.40×10^{-3}	1.41×10^{-3}	1.40×10^{-3}	1.40×10^{-3}
	1×10^{-2}	1.38×10^{-2}	1.40×10^{-2}	1.35×10^{-2}	1.41×10^{-2}	1.33×10^{-2}	1.36×10^{-2}
	1×10^{-1}	1.28×10^{-1}	1.30×10^{-1}	1.17×10^{-1}	1.41×10^{-1}	1.08×10^{-1}	1.18×10^{-1}

Notes: (1) A Hewlett Packard HP 15 C calculator was unable to perform these calculations, although it uses 10 digits (see the discussion near the end of Example 1). The calculation had to be done using Microsoft Excel, which uses 15 digits. (2) Microsoft Excel was unable to perform this calculation, although it uses 15 digits (see the discussion near the end of Example 1). For example, the value 3.16×10^{-9} was obtained using Equation 74 as follows: $(q_h / k)/\sin\beta = 1\times10^{-9} / \sin[\tan^{-1}(1/3)] = 3.16\times10^{-9}$. Ironically, in this particular case, the "approximate" Equation 74 gives the accurate value whereas McEnroe's Equations 49, 52, and 66, even when used with Microsoft Excel, do not.

Similarly, Equation 17 becomes:

$$t_{max} = j \, \frac{\sqrt{1+4\lambda}-1}{2\cos\beta / \tan\beta} L \tag{37}$$

The derivation of the modifying factor, j, is provided in Appendix C. The expression of the modifying factor, j, is:

$$j \;=\; 1 - 0.12\exp\left[-\left[\log\left(\frac{8(q_h/k)}{5\,\tan^2\beta}\right)^{5/8}\right]^2\right] \tag{38}$$

Combining Equations 9 and 38 gives:

$$j \;=\; 1 - 0.12\exp\left[-\left[\log\left(\frac{8\lambda}{5}\right)^{5/8}\right]^2\right] \tag{39}$$

The values of the modifying factor, j, range between 0.88 and 1.00 (Figure 7). Combining Equations 36 and 38 gives:

$$t_{max} = \frac{\sqrt{\tan^2\beta + 4\,q_h/k} - \tan\beta}{2\cos\beta}\left\{1 - 0.12\exp\left[-\left[\log\left(\frac{8(q_h/k)}{5\,\tan^2\beta}\right)^{5/8}\right]^2\right]\right\} L \tag{40}$$

Combining Equations 37 and 39 gives:

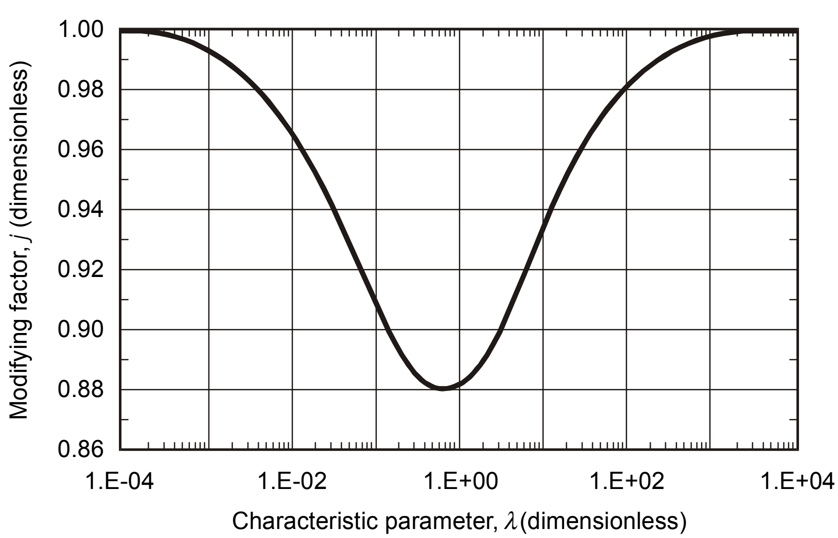

Figure 7. Value of the modifying factor, j, as a function of the characteristic parameter, λ.

$$t_{max} = \frac{\sqrt{1+4\lambda}-1}{2\cos\beta / \tan\beta}\left\{1-0.12\exp\left[-\left[\log\left(\frac{8\lambda}{5}\right)^{5/8}\right]^2\right]\right\}L \tag{41}$$

Table 2 shows that values of the maximum liquid thickness calculated using Equations 40 (or 41, which is equivalent) are within 1% of the accurate results obtained by numerically solving the governing differential equation for q_h/k less than 1×10^{-1}, which is the case in virtually all practical applications. Equation 40 (or Equation 41) has been used as the "modified Giroud's equation" in a number of applications. Since the modified Giroud's equation was developed based on the results of the numerical solution of the governing differential equation, it is legitimate to consider that the modified Giroud's equation corresponds to boundary conditions identical to those used to numerically solve the governing differential equation, i.e. zero liquid thickness at the toe of the liquid collection layer ($t_{toe} = 0$), which implies an infinite hydraulic gradient at the toe of the liquid collection layer ($i_{toe} = \infty$).

Equations 36 to 41 can be used with any set of coherent units. The relevant basic SI units are: t_{max} (m), q_h (m/s), k (m/s), L (m), and β (°); λ and j are dimensionless.

2.5 Analytical Solution

McEnroe (1993) used the following differential equation for the flow in a liquid collection layer:

$$q_h\ x = k\,D\ \cos^2\beta\left(\tan\beta - \frac{dD}{dx}\right) \tag{42}$$

Equation 42 is different from Equation 25. In Equation 42, the liquid depth, D, is used, whereas, in Equation 25, the liquid thickness, t, is used. A detailed comparison between Equations 25 and 42 is presented in Section 2.6.

McEnroe (1993) analytically solved Equation 42, which was a major step forward in the design of liquid collection layers. The analytical solution consists of a set of three equations giving the maximum depth of liquid. Each of the three equations corresponds to a value or a range of values of the dimensionless parameter R defined by Equation 10. The equations presented below were obtained by multiplying by $\cos\beta$ the equations originally presented by McEnroe in order to obtain the liquid thickness from the liquid depth, in accordance with Equation 3, hence:

- for $R < 0.25$

$$t_{max} = L\ \sin\beta\left[R - \frac{t_{toe}}{L\sin\beta} + \left(\frac{t_{toe}}{L\sin\beta}\right)^2\right]^{1/2}\left[\frac{\left(1-A^*-2R\right)\left(1+A^*-\dfrac{2\,t_{toe}}{L\sin\beta}\right)}{\left(1+A^*-2R\right)\left(1-A^*-\dfrac{2\,t_{toe}}{L\sin\beta}\right)}\right]^{1/\left(2A^*\right)} \tag{43}$$

- for $R = 0.25$

$$t_{max} = L\,\sin\beta\,\frac{R\left(1-\dfrac{2\,t_{toe}}{L\sin\beta}\right)}{1-2R}\exp\left[\frac{2\left(\dfrac{t_{toe}}{L\sin\beta}-R\right)}{\left(1-\dfrac{2\,t_{toe}}{L\sin\beta}\right)(1-2R)}\right]=\left(\frac{L\sin\beta}{2}-t_{toe}\right)\exp\left[\frac{4\,t_{toe}-L\sin\beta}{L\sin\beta-2\,t_{toe}}\right] \tag{44}$$

- for $R > 0.25$

$$t_{max} = L\,\sin\beta\left[R-\frac{t_{toe}}{L\sin\beta}+\left(\frac{t_{toe}}{L\sin\beta}\right)^2\right]^{1/2}\exp\left[\frac{1}{B^*}\tan^{-1}\left(\frac{\dfrac{2\,t_{toe}}{L\sin\beta}-1}{B^*}\right)-\frac{1}{B^*}\tan^{-1}\left(\frac{2R-1}{B^*}\right)\right] \tag{45}$$

where t_{toe} is the liquid thickness at the toe of the liquid collection layer slope, R is defined by Equation 10, and A^* and B^* are dimensionless parameters defined by:

$$A^* = \sqrt{1-4R} \qquad B^* = \sqrt{4R-1} \tag{46}$$

Two expressions of t_{max} are given for $R = 0.25$ (Equation 44): the first expression is that given by McEnroe (1993) and the second expression was simplified by the authors of the present paper using $R = 0.25$. Equations 43 to 45 depend on the value of the liquid thickness at the toe of the liquid collection layer slope, t_{toe} . To select the value of t_{toe} that represents free drainage at the toe, McEnroe (1993) made an assumption on the hydraulic head at the toe of the liquid collection layer slope. This assumption is discussed in Appendix B where it is shown that the liquid thickness at the toe that results from this assumption is:

$$t_{toe} = \frac{q_h\;L}{k\;\cos\beta} \tag{47}$$

and the hydraulic gradient at the toe that results from this assumption is:

$$i_{toe} = \cos\beta \tag{48}$$

Combining Equations 43 to 45 with Equation 47 gives the following equations, which were proposed by McEnroe for the case where there is free drainage at the toe:

- for $R < 0.25$

$$t_{max} = L\,\sin\beta\left[R-R\tan\beta+(R\tan\beta)^2\right]^{1/2}\left[\frac{\left(1-A^*-2R\right)\left(1+A^*-2R\tan\beta\right)}{\left(1+A^*-2R\right)\left(1-A^*-2R\tan\beta\right)}\right]^{1/\left(2A^*\right)} \tag{49}$$

- for $R = 0.25$

$$t_{max} = L\ \sin\beta\ \frac{R(1-2R\tan\beta)}{1-2R}\ \exp\left[\frac{2R(\tan\beta-1)}{(1-2R\tan\beta)(1-2R)}\right] = \frac{L\sin\beta}{2}\left(1-\frac{\tan\beta}{2}\right)\exp\left(\frac{\tan\beta-1}{1-\frac{\tan\beta}{2}}\right) \tag{50}$$

- for $R > 0.25$

$$t_{max} = L\ \sin\beta\left[R - R\tan\beta + (R\tan\beta)^2\right]^{1/2} \exp\left[\frac{1}{B^*}\tan^{-1}\left(\frac{2R\tan\beta-1}{B^*}\right) - \frac{1}{B^*}\tan^{-1}\left(\frac{2R-1}{B^*}\right)\right] \tag{51}$$

Equations 49 to 51 are known as McEnroe's equations. Herein, Equations 49 to 51 will be referred to as "McEnroe's equations for $i_{toe} = \cos\beta$", when necessary for clarity. Two expressions of t_{max} are given for $R = 0.25$ (Equation 50): the first expression is that given by McEnroe (1993) and the second expression was simplified by the authors of the present paper using $R = 0.25$.

It is interesting to derive, from Equations 43 to 45, equations for the case where the condition for free drainage at the toe of the liquid collection layer slope is expressed by $t_{toe} = 0$, the boundary condition used by Giroud et al. (1992). The resulting equations are:

- for $R < 0.25$

$$t_{max} = L\ \sin\beta\sqrt{R}\left[\frac{(1-A^*-2R)(1+A^*)}{(1+A^*-2R)(1-A^*)}\right]^{1/(2A^*)} \tag{52}$$

- for $R = 0.25$

$$t_{max} = \frac{L\ \sin\beta}{2}\ \exp(-1)\ =\ 0.18394\ L\ \sin\beta \tag{53}$$

- for $R > 0.25$

$$t_{max} = L\ \sin\beta\ \sqrt{R}\ \ \exp\left[\frac{1}{B^*}\tan^{-1}\left(\frac{-1}{B^*}\right) - \frac{1}{B^*}\tan^{-1}\left(\frac{2R-1}{B^*}\right)\right] \tag{54}$$

Equations 52 to 54 will be referred to as "McEnroe's equations for $t_{toe} = 0$". These equations are simpler than Equations 49 to 51.

It will be shown in Section 2.7 that McEnroe's equations are not easy to use for the numerical calculations typically involved in the design of practical applications. However, being an analytical solution, McEnroe's equations should be regarded as the reference against which other solutions are to be evaluated.

2.6 Transformed Analytical Solution

To compare the two differential equations mentioned in preceding sections, Equations 25 and 42, the authors of the present paper combined Equations 3 and 42 to obtain the following differential equation where the variable is t instead of D:

$$\frac{q_h\ x}{k} = t\ \sin\beta - t\frac{\mathrm{d}t}{\mathrm{d}x} \tag{55}$$

Inspection of Equation 25, the governing differential equation provided in the present paper, and Equation 55, which is equivalent to the differential equation used by McEnroe (1993), reveals a difference between these two equations. The difference is $\cos^2\beta$ in the last term of the equation (which indicates that the two differential equations are very close if β is small). The difference between the two equations results from different approximations made in the evaluation of the hydraulic head in the development of Equations 25 and 42. Approximations are needed because the hydraulic head varies along the liquid collection layer slope. The approximation made in the present paper consists of using the hydraulic head for the case of flow parallel to the slope, whereas the flow is parallel to the slope only at the location of the maximum thickness. The approximation made by McEnroe (1993) consists of assuming that the hydraulic head is equal to the liquid depth, which is only correct when β is small. The authors of the present paper believe that it is preferable to use the hydraulic head for flow parallel to the slope because the liquid surface, as an average, is parallel to the slope. Furthermore, the discussion presented in Section 2.9 tends to show that Equation 25 is a better governing differential equation than Equation 42 because it leads to normalized solutions that do not directly depend on β.

Combining Equations 10 and 55 gives:

$$R\,x = \frac{t}{\sin\beta} - \frac{t}{\sin^2\beta}\frac{\mathrm{d}t}{\mathrm{d}x} \tag{56}$$

Equation 26 can be written as follows:

$$\lambda\,x = \frac{t\ \cos^2\beta}{\sin\beta} - \frac{t\cos^2\beta}{\sin^2\beta}\,\frac{\mathrm{d}\left(t\ \cos^2\beta\right)}{\mathrm{d}x} \tag{57}$$

Comparing Equations 56 and 57 shows that Equation 56 (equivalent to Equation 42, the differential equation used by McEnroe) becomes identical to Equation 57 (equivalent to Equation 25, the differential equation used in the present paper) if R is replaced by λ and t is replaced by $t\cos^2\beta$, which can be summarized as follows:

$$\begin{aligned} R &\to \lambda \\ t &\to t\cos^2\beta \end{aligned} \tag{58}$$

This simple transformation makes is possible to convert the analytical solutions obtained by McEnroe for Equation 42 into analytical solutions for Equation 25, the governing equation provided in the present paper. Thus, the following equations were derived from Equations 43 to 45:

- for $\lambda < 0.25$

$$t_{max} = L\frac{\tan\beta}{\cos\beta}\left[\lambda - \frac{t_{toe}\cos\beta}{L\tan\beta} + \left(\frac{t_{toe}\cos\beta}{L\tan\beta}\right)^2\right]^{1/2}\left[\frac{(1-A'-2\lambda)\left(1+A'-\dfrac{2\,t_{toe}\cos\beta}{L\tan\beta}\right)}{(1+A'-2\lambda)\left(1-A'-\dfrac{2\,t_{toe}\cos\beta}{L\tan\beta}\right)}\right]^{1/(2A')} \quad (59)$$

- for $\lambda = 0.25$

$$t_{max} = \left(\frac{L\tan\beta}{2\cos\beta} - t_{toe}\right)\exp\left(\frac{4t_{toe} - L\tan\beta/\cos\beta}{L\tan\beta/\cos\beta - 2t_{toe}}\right) \quad (60)$$

- for $\lambda > 0.25$

$$t_{max} = L\,\frac{\tan\beta}{\cos\beta}\left[\lambda - \frac{t_{toe}\cos\beta}{L\tan\beta} + \left(\frac{t_{toe}\cos\beta}{L\tan\beta}\right)^2\right]^{1/2}\exp\left[\frac{1}{B'}\tan^{-1}\left(\frac{\dfrac{2\,t_{toe}\cos\beta}{L\tan\beta} - 1}{B'}\right) - \frac{1}{B'}\tan^{-1}\left(\frac{2\lambda-1}{B'}\right)\right] \quad (61)$$

where A' and B' are dimensionless parameters defined by:

$$A' = \sqrt{1-4\lambda} \qquad B' = \sqrt{4\lambda - 1} \quad (62)$$

Using the transformation defined by Equation 58 on Equations 49 to 51 gives the following equations for the case where free drainage at the toe is represented by Equations 47 and 48:

- for $\lambda < 0.25$

$$t_{max} = L\,\frac{\tan\beta}{\cos\beta}\left[\lambda - \lambda\tan\beta + (\lambda\tan\beta)^2\right]^{1/2}\left[\frac{(1-A'-2\lambda)(1+A'-2\lambda\tan\beta)}{(1+A'-2\lambda)(1-A'-2\lambda\tan\beta)}\right]^{1/(2A')} \quad (63)$$

- for $\lambda = 0.25$

$$t_{max} = \frac{L\,\tan\beta}{2\,\cos\beta}\left(1 - \frac{\tan\beta}{2}\right)\exp\left(\frac{\tan\beta - 1}{1 - \dfrac{\tan\beta}{2}}\right) \quad (64)$$

- for $\lambda > 0.25$

$$t_{max} = L\,\frac{\tan\beta}{\cos\beta}\left[\lambda - \lambda\tan\beta + (\lambda\tan\beta)^2\right]^{1/2}\exp\left[\frac{1}{B'}\tan^{-1}\left(\frac{2\lambda\tan\beta - 1}{B'}\right) - \frac{1}{B'}\tan^{-1}\left(\frac{2\lambda-1}{B'}\right)\right] \quad (65)$$

Equations 63 to 65 will be referred to as the "transformed McEnroe's equations for $i_{toe} = \cos\beta$ ", or simply as "transformed McEnroe's equations".

Finally, the transformed McEnroe's equations for the case where the condition for free drainage at the toe of the liquid collection layer slope is expressed by $t_{toe} = 0$, the boundary condition used by Giroud et al. (1992), can be derived from Equations 52 to 54 using the transformation defined by Equation 58. The transformed equations are:

- for $\lambda < 0.25$

$$t_{max} = L\,\frac{\tan\beta}{\cos\beta}\sqrt{\lambda}\left[\frac{(1-A'-2\lambda)(1+A')}{(1+A'-2\lambda)(1-A')}\right]^{1/(2A')} \tag{66}$$

- for $\lambda = 0.25$

$$t_{max} = \frac{L\,\tan\beta}{2\cos\beta}\exp(-1) = 0.18394\,\frac{L\,\tan\beta}{\cos\beta} \tag{67}$$

- for $\lambda > 0.25$

$$t_{max} = L\,\frac{\tan\beta}{\cos\beta}\sqrt{\lambda}\;\exp\left[\frac{1}{B'}\tan^{-1}\left(\frac{-1}{B'}\right)-\frac{1}{B'}\tan^{-1}\left(\frac{2\lambda-1}{B'}\right)\right] \tag{68}$$

Equations 66 to 68 will be referred to as the "transformed McEnroe's equations for $t_{toe} = 0$". These equations are simpler than Equations 63 to 65.

As an ultimate example of the transformation defined by Equation 58, the modified Giroud's equation (Equation 37) can be transformed backward, which gives:

$$t_{max} = j\,\frac{\sqrt{\sin^2\beta + 4q_h/k} - \sin\beta}{2}L = j\,\frac{\sqrt{1+4R}-1}{2}\sin\beta\;L \tag{69}$$

where the dimensionless factor j is defined by Equation 39 with R instead of λ.

Equation 69 is not used in the present paper. However, it would be an appropriate equation for calculating the maximum liquid thickness if it were shown that Equation 42 is a better governing differential equation than Equation 25. This discussion is mostly of academic interest because, as will be shown in Section 2.7, the two differential equations lead to numerical results that are extremely close. Furthermore, the discussion presented in Section 2.9 tends to show that Equation 25 is a better governing differential equation than Equation 42 because it leads to normalized solutions that do not directly depend on β.

2.7 Comparison of the Available Equations

Table 2 compares, for four liquid collection layer slopes ($\tan\beta$ = 0.02, 1/3, 0.5, and 1.0), values of the maximum liquid thickness obtained as follows: (i) by numerical solution of the governing differential equation for the case where $t_{toe} = 0$ (Giroud et al. 1992); (ii) calculated using the original Giroud's equation (Equation 16 or 17); (iii) calculated using the modified Giroud's equation (Equation 40 or 41); (iv) calculated using McEnroe's equations for $i_{toe} = \cos\beta$ (Equations 49 to 51), i.e. the equations proposed by McEnroe for free drainage at the toe; (v) calculated using McEnroe's equations for $t_{toe} = 0$ (Equations 52 to 54), i.e. the equations derived by the authors of the present paper, from the general equations developed by McEnroe, to represent free drainage at the toe of the liquid collection layer slope better than by the McEnroe's equations for $i_{toe} = \cos\beta$; and (vi) calculated using the transformed McEnroe's equations for $t_{toe} = 0$ (Equations 66 to 68). The following comments can be made based on inspection of Table 2:

- The difference between the results obtained using the McEnroe's equations with the two considered boundary conditions is generally very small. For $q_h/k \leq 1 \times 10^{-2}$, the difference is less than 1% except for $\tan\beta = 1.0$ where the difference is 9%. For $q_h/k = 1 \times 10^{-1}$, the difference is less than 7% for $\tan\beta \leq 0.5$ and is 23% for $\tan\beta = 1.0$.
- For $q_h/k \leq 1 \times 10^{-2}$, there is an excellent agreement between the numerical solution (Giroud et al. 1992) and the values calculated using McEnroe's equations, for both boundary conditions. There are some discrepancies for $q_h/k = 1 \times 10^{-1}$. The discrepancies become very small when the transformed McEnroe's equations are used.
- There is very little difference between the values calculated using McEnroe's equations for $i_{toe} = \cos\beta$ and the modified Giroud's equation: the difference is less than 1%, except for $q_h/k = 1 \times 10^{-1}$ where the difference is 5% for $\tan\beta = 1/3$ and 17% for $\tan\beta = 1.0$. There is virtually no difference between the values calculated using McEnroe's equations for $t_{toe} = 0$ and the modified Giroud's equation (which also corresponds to $t_{toe} = 0$, as pointed out in Section 2.3) for $q_h/k \leq 1 \times 10^{-2}$. The difference disappears almost completely, even for $q_h/k = 1 \times 10^{-1}$, when the transformed McEnroe's equations are used. Considering that this boundary condition is the most relevant of the two considered types of boundary conditions (as discussed in Appendix B), it may be concluded that the modified Giroud's equation is equivalent to McEnroe's equations for the usual range of parameter values.
- The difference between the values calculated using the original Giroud's equation and accurate solutions (i.e. McEnroe's equations and the modified Giroud's equation) is always small and is, at most, 13% for the usual range of parameter values.

In conclusion, from the viewpoint of the calculated numerical values: (i) there is virtually no difference between the two considered differential equations; (ii) there is virtually no difference between the two considered types of boundary conditions at the toe of the liquid collection layer slope; (iii) the modified Giroud's equation is as accurate as McEnroe's equations; and (iv) the original Giroud's equation provides an approximation that is sufficient in most applications.

From a practical standpoint, Giroud's equations have the following advantages, compared to McEnroe's equations:

- When the original Giroud's equation or the modified Giroud's equation are used, only one equation is needed to cover the entire range of parameters, whereas three McEnroe's equations are necessary.
- Giroud's equations are simpler than McEnroe's equations, while being almost as accurate. The modified Giroud's equation is virtually as accurate as McEnroe's equations.
- Numerical calculations with the McEnroe's equation for $R < 0.25$ require extremely high precision (15 digits or more in some cases), which is very impractical for engineering calculations. This problem does not exist with Giroud's equations. This important point is illustrated by Example 1, below. See also the notes below Table 2.

- The original Giroud's equation (i.e. Equation 16 or 17) can be solved for all parameters: k, q_h , or even β.

Because it is simple and accurate, the modified Giroud's equation (i.e. Equation 40 or 41) will be used extensively in the remainder of the present paper. Also, because it is accurate and easily programmable, the modified Giroud's equation is the recommended equation for parametric studies where precision is required. Finally, the original Giroud's equation is recommended for usual design calculations because it is very simple, while being sufficiently accurate for all values of the parameters.

The purpose of Example 1, which follows, is to illustrate the comparison between various equations. For the sake of simplicity, and to focus the attention on the comparisons between equations, the reduction factors presented in Section 1.7 are not used in Example 1. Therefore, Example 1 should not be considered as a "design example". Design examples illustrating the use of reduction factors and factor of safety are presented in Sections 3 and 4.

Example 1. A liquid collection layer used in a landfill cover has a length (measured horizontally) of 30 m and a 2% slope. The rate of liquid supply is 100 mm per day. A drainage geocomposite having a core thickness of 9 mm is used. Its hydraulic transmissivity, measured under hydraulic gradients consistent with a slope of 2%, is 3.6×10^{-3} m^2/s. Calculate the maximum liquid thickness.

The liquid supply rate is calculated using Equation 8 as follows:

$$q_h = \frac{0.1}{86{,}400} = 1.157 \times 10^{-6} \ \text{m/s}$$

The hydraulic conductivity of the geosynthetic is derived from its hydraulic transmissivity and its thickness using Equation 2 as follows:

$$k = \frac{3.6 \times 10^{-3}}{9 \times 10^{-3}} = 0.4 \ \text{m/s}$$

An approximate value of the maximum liquid thickness can be obtained by using the original Giroud's equation (i.e. Equation 16) as follows:

$$t_{max} = \left[\frac{\sqrt{(0.02)^2 + (4)\left(1.157 \times 10^{-6}\right)/0.4} - 0.02}{2\cos\left(\tan^{-1} 0.02\right)} \right](30) = 4.31 \times 10^{-3}\,\text{m} = 4.31\,\text{mm}$$

An accurate value of the maximum liquid thickness can be obtained by using the modified Giroud's equation (i.e. Equation 36), which is equivalent to multiplying the value calculated above using Equation 16 by the dimensionless modifying factor, j, calculated using Equation 38 as follows:

$$j = 1 - 0.12 \exp\left\{ -\left[\log\left[\frac{(8)\left(1.157 \times 10^{-6}\right)/(0.4)}{(5)(0.02)^2} \right]^{5/8} \right]^2 \right\} = 0.972$$

hence:

$$t_{max} = 4.31 \times 0.972 = 4.19 \text{ mm}$$

Alternatively, the dimensionless characteristic parameter, λ, can be calculated using Equation 9 as follows:

$$\lambda = \frac{1.157 \times 10^{-6}}{(0.4)(0.02)^2} = 7.231 \times 10^{-3}$$

Then, the maximum liquid thickness can be calculated using Equation 17 as follows:

$$t_{max} = \left[\frac{\sqrt{1 + (4)(7.231 \times 10^{-3})} - 1}{2 \cos(\tan^{-1} 0.02)} \right] (0.02)(30) = 4.31 \times 10^{-3} \text{m} = 4.31 \text{mm}$$

Then, the dimensionless modifying factor, j, can be calculated using Equation 39 as follows:

$$j = 1 - 0.12 \exp \left\{ - \left[\log \left[\frac{(8)(7.231 \times 10^{-3})}{5} \right]^{5/8} \right]^2 \right\} = 0.972$$

Another way to calculate an accurate value of the maximum liquid thickness consists of using McEnroe's equations. First, McEnroe's equations will be used as proposed by McEnroe, i.e. as expressed by Equations 49 to 51, that is for the case where the boundary condition at the toe of the liquid collection layer slope is defined by $i_{toe} = \cos\beta$. The use of McEnroe's equations requires two steps. First, the dimensionless parameter R must be calculated to determine which one of the three McEnroe's equations should be used.

The dimensionless parameter R is calculated using Equation 10 as follows:

$$R = \frac{1.157 \times 10^{-6}}{(0.4) \sin^2(\tan^{-1} 0.02)} = 7.234143 \times 10^{-3}$$

Since R is less than ¼, Equation 49 is to be used. To use this equation, it is necessary to first calculate the dimensionless parameter A^* using Equation 46 as follows:

$$A^* = \sqrt{1 - (4)(7.234143 \times 10^{-3})} = 0.9854255$$

It should be noted that R and A^* are calculated with great precision. The reason for that will be discussed later. Then, Equation 49 can be used as follows:

$$t_{max} = (30)\sin\left(\tan^{-1} 0.02\right)\left\{7.234143\times10^{-3} - \left(7.234143\times10^{-3}\right)(0.02) + \left[\left(7.234143\times10^{-3}\right)(0.02)\right]^2\right\}^{1/2}$$

$$\left\{\frac{\left[1-0.9854255-(2)\left(7.234143\times10^{-3}\right)\right]\left[1+0.9854255-(2)\left(7.234143\times10^{-3}\right)(0.02)\right]}{\left[1+0.9854255-(2)\left(7.234143\times10^{-3}\right)\right]\left[1-0.9854255-(2)\left(7.234143\times10^{-3}\right)(0.02)\right]}\right\}^{\frac{1}{2\times0.9854255}}$$

hence:

$$t_{max} = 0.00422 \text{ m} = 4.22 \text{ mm}$$

It should be noted that the two equations that are supposed to give an accurate value of the maximum liquid thickness indeed give virtually the same result (4.19 and 4.22 mm). However, as seen above, the modified Giroud's equation (i.e. Equation 36) is much simpler and easier to use for numerical calculations than McEnroe's equation. It is important to note that the value of t_{max} calculated using McEnroe's equation for $R < 0.25$ (Equation 49) may vary significantly according to the precision of the calculations because one of its terms is extremely sensitive to the number of decimal places in the input parameters. For example, using seven digits for the dimensionless parameters R and A^*, leads to the correct value of 4.22 mm, whereas using four digits leads to a calculated value of 4.70 mm, i.e. an inaccuracy of 11%. This is due to the following term whose value calculated with four-digit input parameters is:

$$1-0.9854-(2)\left(7.234\times10^{-3}\right) = 1.320\times10^{-4}$$

whereas the value of the same term calculated with seven-digit input parameters is:

$$1-0.9854255-(2)\left(7.234143\times10^{-3}\right) = 1.062140\times10^{-4}$$

This difference results in the 11% inaccuracy on the final result when only four digits are used. Clearly, it is impractical to have to use seven digits for ordinary engineering calculations and it is potentially unsafe to use an equation that is so sensitive to the precision of the input parameters. There are even cases where the use of McEnroe's equation for $R < 0.25$ (Equation 49) requires a number of digits that exceeds the capacity of electronic calculators (e.g. Hewlett Packard 15 C) or software (e.g. Microsoft Excel) routinely used for engineering calculations, as illustrated by the difficulty encountered in the preparation of Table 2 where, for some entries, the 10 digits used by Hewlett Packard 15 C or the 15 digits used by Microsoft Excel were not sufficient. This problem does not exist with Giroud's equations. In fact, Giroud's equations can be used with rounded values of the input parameters for quick calculations, as illustrated by the following example using the original Giroud's equation (i.e. Equation 16):

$$t_{max} = \left[\frac{\sqrt{(0.02)^2 + (4)\left(1.2\times10^{-6}\right)/0.4} - 0.02}{(2)(1)}\right](30) = 4.5\times10^{-3}\,\text{m} = 4.5\,\text{mm}$$

which is a good approximation considering that the more accurate calculation using the same equation with non-rounded values of the parameters gave 4.31 mm.

Finally, McEnroe's equations for the case where the boundary conditions are defined by $t_{toe} = 0$, i.e. Equations 52 to 54, are used. Since R is less than 0.25, Equation 52 must be used, hence:

$$t_{max} = (30)\sin\left(\tan^{-1} 0.02\right)\sqrt{7.234143\times10^{-3}}$$

$$\left\{\frac{\left[1-0.9854255-(2)\left(7.234143\times10^{-3}\right)\right]\left[1+0.9854255\right]}{\left[1+0.9854255-(2)\left(7.234143\times10^{-3}\right)\right]\left[1-0.9854255\right]}\right\}^{\frac{1}{2\times0.9854255}}$$

hence:

$$t_{max} = 0.00422 \text{ m} = 4.22 \text{ mm}$$

It is interesting to note that the same result as with Equation 49 is obtained, which confirms the comments made in Section 2.5 regarding the fact that there is virtually no difference between the two considered types of boundary conditions at the toe of the liquid collection layer slope.

In conclusion, Example 1 shows that using Giroud's equations is as accurate and much simpler than using McEnroe's equations.

END OF EXAMPLE 1

2.8 Limit Case

The limit case where q_h is small and β and k are large is considered. If q_h is small (i.e. if there is little liquid to convey) and β and k are large (i.e. if the liquid collection layer has a high flow capacity), the thickness of liquid in the liquid collection layer is very small (Figure 4c). As a result, the slope of the liquid surface in the liquid collection layer is virtually identical to the slope of the liner underlying the liquid collection layer. The hydraulic gradient is then approximately constant along the slope and equal to the ratio between the height and the length of the slope, which can be expressed as follows (Figure 1):

$$i = \frac{L\tan\beta}{L/\cos\beta} = \sin\beta \tag{70}$$

In this case, the maximum liquid thickness occurs approximately at the toe of the slope (Figure 4c). To calculate the maximum liquid thickness, Darcy's equation can be used at the toe of the slope. Darcy's equation is:

$$Q = k\,i\,A \tag{71}$$

where A is the cross-sectional area of the flow. The cross-sectional area of the flow at the toe of the slope is:

$$A = B\,t_{lim} \tag{72}$$

where t_{lim} is the maximum thickness of the liquid in the limit case defined at the beginning of this Section. Based on the principle of mass conservation, and considering steady-state flow conditions, the flow rate in the liquid collection layer at the toe of the slope (where t_{lim} occurs) is equal to the rate of liquid supplied to the entire liquid collection layer, hence:

$$Q = q_h L B \tag{73}$$

Combining Equations 9 and 70 to 73 gives the following equation:

$$t_{lim} = \frac{q_h L}{k \sin\beta} = \frac{\lambda L \tan\beta}{\cos\beta} \tag{74}$$

Equation 74 is identical to Equation 35 obtained from the differential equation. Equation 74 could have been derived from Equation 41 as follows. If q_h is small and β and k are large, λ is small. In this case, a limited series calculation gives the following limit for t_{max} when λ tends toward zero:

$$t_{max} \rightarrow t_{lim} = \frac{(1+2\lambda)-1}{2\cos\beta / \tan\beta}\left\{1 - 0.12 \text{ e}^{-\infty}\right\}L = \frac{\lambda \tan\beta}{\cos\beta}\{1\}L = \frac{\lambda\tan\beta}{\cos\beta}L \tag{75}$$

which is identical to Equation 74.

The case where t_{max} is exactly equal to t_{lim} cannot be represented in Figure 4 because, in this case, $\lambda = 0$ and Equation 74 shows that $t_{lim} = 0$. Therefore, only the case where $t_{max} \approx t_{lim}$ can be represented (Figure 4c). In this case, the maximum thickness occurs near the toe, but not exactly at the toe.

2.9 Normalized Solutions

The maximum liquid thickness expressed by the equations presented in Sections 2.2, 2.3, and 2.4 can be normalized using, as a reference value, the limit liquid thickness defined by Equation 74.

The maximum liquid thickness expressed by the original Giroud's equation can be normalized as follows by combining Equations 17 and 74:

$$\frac{t_{max}}{t_{lim}} = \frac{\sqrt{1+4\lambda}-1}{2\lambda} \tag{76}$$

Similarly, the maximum liquid thickness expressed by the modified Giroud's equation can be normalized as follows by combining Equations 36 and 74:

$$\frac{t_{max}}{t_{lim}} = j\,\frac{\sqrt{1+4\lambda}-1}{2\lambda} \tag{77}$$

Replacing j by its value as a function of λ given by Equation 38 gives:

$$\frac{t_{max}}{t_{lim}} = \frac{\sqrt{1+4\lambda}-1}{2\lambda}\left\{1-0.12\exp\left[-\left[\log\left(\frac{8\lambda}{5}\right)^{5/8}\right]^{2}\right]\right\} \tag{78}$$

It should be noted that the normalized maximum liquid thickness values expressed by Equations 76 and 78 are a function of only one parameter, the dimensionless characteristic parameter, λ, expressed by Equation 9. Figure 8 shows the normalized maximum liquid thickness obtained using the modified Giroud's equation. An expedient procedure for calculating an accurate value of the maximum liquid thickness would involve calculating λ using Equation 9, calculating t_{lim} using Equation 74, and obtaining the normalized value, t_{max}/t_{lim} , using Figure 8.

The normalized maximum liquid thickness, t_{max}/t_{lim}, can also be obtained using the maximum liquid thickness values expressed by McEnroe's equations (Section 2.5). Below, only the McEnroe's equations for the boundary conditions defined by $t_{toe} = 0$ are used. Combining Equations 52 and 74 gives the following normalized maximum liquid thickness for the case of $R < 0.25$ (i.e. $\lambda < 0.25\cos^2\beta$):

$$\frac{t_{max}}{t_{lim}} = \frac{\cos\beta}{\sqrt{\lambda}}\left[\frac{\left(\cos^2\beta-\sqrt{\cos^2\beta-4\lambda}-2\lambda\right)\left(\cos^2\beta+\sqrt{\cos^2\beta-4\lambda}\right)}{\left(\cos^2\beta+\sqrt{\cos^2\beta-4\lambda}-2\lambda\right)\left(\cos^2\beta-\sqrt{\cos^2\beta-4\lambda}\right)}\right]^{\frac{1/2}{\sqrt{1-\frac{4\lambda}{\cos^2\beta}}}} \tag{79}$$

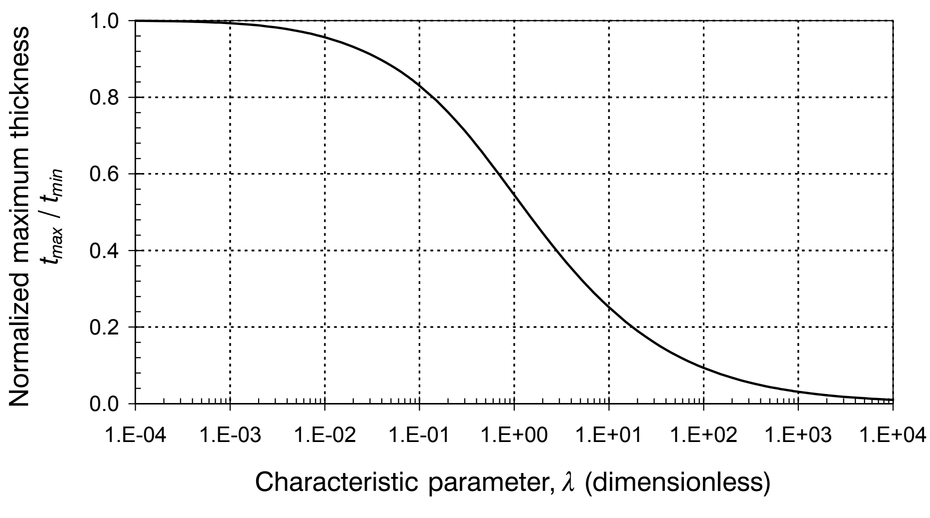

Figure 8. Normalized maximum thickness, t_{max} / t_{lim} , as a function of the characteristic parameter, λ.

Notes: t_{max} in Figure 8 was defined using the modified Giroud's Equation. As indicated in Section 2.10, t_{max} / t_{lim} = x_m / L. Therefore, Figure 8 also provides the location of the maximum liquid thickness (Figure 11).

Combining Equations 53 and 74 gives the following normalized maximum liquid thickness for the case of $R = 0.25$ (i.e. $\lambda = 0.25 \cos^2\beta$):

$$\frac{t_{max}}{t_{lim}} = 2 \ \exp(-1) = 0.73576 \tag{80}$$

Combining Equations 54 and 74 gives the following normalized maximum liquid thickness for the case of $R > 0.25$ (i.e. $\lambda > 0.25 \cos^2\beta$):

$$\frac{t_{max}}{t_{lim}} = \frac{\cos\beta}{\sqrt{\lambda}} \exp\left\{\frac{\cos\beta}{\sqrt{4\lambda - \cos^2\beta}}\left[\tan^{-1}\left(\frac{-\cos\beta}{\sqrt{4\lambda-\cos^2\beta}}\right) - \tan^{-1}\left(\frac{2\lambda - \cos^2\beta}{\cos\beta\ \sqrt{4\lambda - \cos^2\beta}}\right)\right]\right\} \tag{81}$$

As already mentioned in Section 2.7, three McEnroe's equations are necessary while only one Giroud's equation is valid for the entire range of values of λ. Also, a comparison of Equations 79 to 81, on one hand, and Equations 76 or 78, on the other hand, shows that the normalized McEnroe's equations are more complex than Giroud's equations and depend on two parameters, λ and β, whereas the normalized Giroud's equations depend on only λ.

Figures 9 and 10 show comparisons between the normalized maximum liquid thickness obtained using the original Giroud's equation (Equation 17), the modified Giroud's equation (Equation 41), and McEnroe's equations for $t_{toe} = 0$ (Equations 52 to 54). The curves for the normalized Giroud's equations depend only on λ, whereas the curves for the normalized McEnroe's equations depend on both λ and β.

Figure 9 shows the normalized maximum liquid thickness obtained using McEnroe's equations for $\tan\beta = 0.05$, the original Giroud's equation, and the modified Giroud's equation. Figure 9 illustrates the fact that three McEnroe's equations are needed to generate the entire curve, whereas one Giroud's equation suffices. For the slope used in Figure 9 ($\tan\beta = 0.05$), there is an excellent agreement between McEnroe's equations and the modified Giroud's equation for the entire range of λ values. Figure 9 also shows that the original Giroud's equation gives a good approximation (sufficient in most practical cases) for the entire range of λ values.

Figure 10 illustrates the influence of β on results obtained with McEnroe's equations (Equations 52 to 54). Inspection of Figure 10 reveals an excellent agreement between McEnroe's equations and the modified Giroud's equation for the entire range of λ values when $\tan\beta = 1/3$. It may, therefore, be concluded from Figures 9 and 10 that there is an excellent agreement between McEnroe's equations and the modified Giroud's equation for the entire range of λ values when $0 \le \tan\beta \le 1/3$. Figure 10 shows some discrepancy between McEnroe's equations and the modified Giroud's equation for large values of λ when $\tan\beta > 1/3$. However, it should be noted that, when $\tan\beta > 1/3$, the characteristic parameter, λ, is usually small (Equation 9) because q_h/k is generally less than 1×10^{-1} (and is often much less than 1×10^{-1}); for example, when $\tan\beta = 1.0$, λ is generally less than 1×10^{-1} (and is often much less than 1×10^{-1}). For such small values of λ, Figure 10 shows a very good agreement between McEnroe's equations and both the original Giroud's equation and the modified Giroud's equation for all values of the slope angle, β. In conclusion, there is a very good agreement between McEnroe's equations

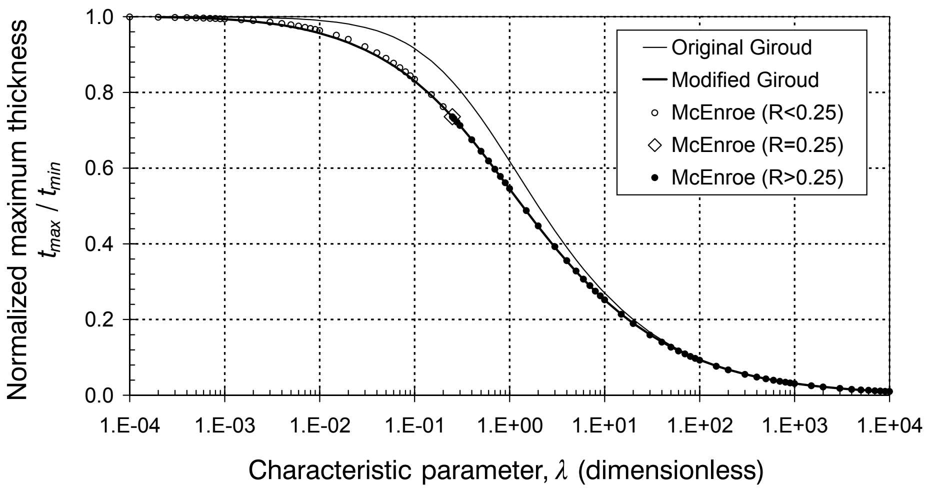

Figure 9. Comparison of values of normalized maximum thickness defined using McEnroe's equations for tanβ = 0.05, the modified Giroud's equation, and the original Giroud's equation.

Notes: Equations 79 to 81 were used for the McEnroe's curve, Equation 78 for the modified Giroud's curve, and Equation 76 for the original Giroud's curve.

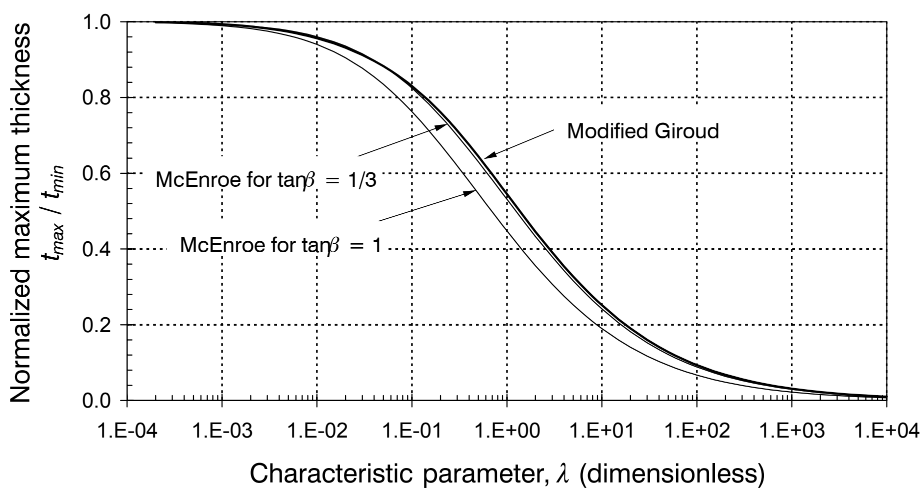

Figure 10. Comparison of values of normalized t_{max} defined using McEnroe's equations for tanβ = 1/3 and 1, and the modified Giroud's equation.

Notes: Equations 79 to 81 were used for the McEnroe's curve and Equation 78 for the modified Giroud's curve.

and the modified Giroud's equation for the entire range of usual values of the parameters, which confirms the comments made in Section 2.5 based on Table 2.

Finally, it is important to note that normalization of the transformed McEnroe's equations (Equations 63 to 65 and 66 to 68) leads to a set of normalized equations (not shown here) that depend only on λ. These normalized transformed equations are, therefore, significantly simpler than Equations 79 to 81, which depend on λ and β. Since the transformed McEnroe's equations are solutions of Equation 25, it is concluded that Equation 25 leads to solutions that are significantly simpler than the solutions derived from Equation 42. Clearly, Equation 25 can be considered the preferred governing differential equation for flow in liquid collection layers. Furthermore, an excellent agreement is found between numerical values of t_{max} obtained with the modified Giroud's equation and the transformed McEnroe's equations.

2.10 Location of the Maximum Liquid Thickness

The location of the maximum liquid thickness can be determined as follows. At any abscissa, x (Figure 11), Darcy's equation can be written as follows:

$$Q = k\,i\,A = k\,i\,B\,t \qquad (82)$$

where: Q = flow rate in the liquid collection layer; k = hydraulic conductivity of the liquid collection layer material; i = hydraulic gradient; A = cross-sectional area of the flow; B = width of the liquid collection layer in the direction perpendicular to the flow direction; and t = thickness of the liquid in the liquid collection layer.

Based on mass conservation, the flow rate through a cross section located at a horizontal distance x from the top of the slope is given by:

$$Q_t = q_h\,x\,B \qquad (83)$$

Combining Equations 82 and 83 gives:

$$t = \frac{q_h\,x}{k\,i} \qquad (84)$$

The hydraulic gradient at the location of the maximum liquid thickness is sinβ since the liquid surface is parallel to the slope at the location where the maximum liquid thickness takes place. Therefore, Equation 84 gives:

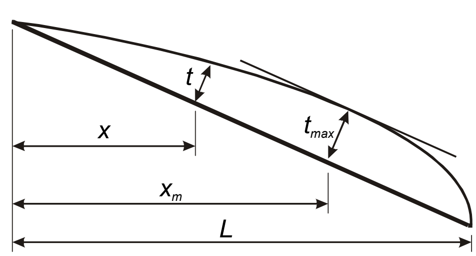

Figure 11. Location of the maximum liquid thickness.

$$t_{max} = \frac{q_h \, x_m}{k \, \sin\beta} \tag{85}$$

where x_m is the horizontal distance between the top of the slope and the location of the maximum liquid thickness (Figure 11).

Equation 85 gives:

$$x_m = \frac{k \, \sin\beta \; t_{max}}{q_h} \tag{86}$$

Combining Equations 74 and 86 gives:

$$\frac{x_m}{L} = \frac{t_{max}}{t_{lim}} \tag{87}$$

It is interesting to note that the location of the maximum liquid thickness, expressed as the x_m /L ratio, is equal to the normalized maximum liquid thickness. Therefore, x_m /L is given by Equation 77 or 78, and by Figure 8, as a function of the characteristic parameter, λ. Identifying the location of the maximum liquid thickness is not necessary in most design situations. However, it is useful in the design of some special cases, such as the design of liquid collection systems comprising two sections of different slope inclinations (Giroud et al. 2000c).

2.11 Simple Equation for Calculating an Approximate Value of the Liquid Thickness

2.11.1 *Presentation of the Simple Equation*

As indicated in Section 2.7, a good approximation of the maximum liquid thickness is given by Equation 16 or 17. However, a simpler equation is provided by Equation 74. Although Equation 74 was developed for certain values of the parameters (see the beginning of Section 2.8), it is often used to calculate an approximate value of the maximum thickness of liquid in the liquid collection layer for a wide range of values of the parameters (q_h , β, and k) that govern the flow of liquid in the liquid collection layer.

Equation 74 has two advantages compared to other equations: (i) it is simpler and can even be memorized easily; and (ii) it can readily be transformed to calculate any of the parameters, or any group of the parameters, as a function of the other parameters. For example, it can be used to calculate the required hydraulic conductivity of the liquid collection layer material to ensure that the flow is not confined (Section 4.3).

Since Equation 74 is often used for a wide range of values of the parameters, it is important to check if this practice is legitimate and if the approximation thus obtained is acceptable.

2.11.2 *Evaluation of the Approximation*

The approximation made when using Equation 74 is evaluated by comparing t_{lim} expressed by Equation 74 to t_{max} expressed by Equation 40 (or 41, which is equivalent) since Equation 40 (or 41) gives an accurate value of the maximum liquid thickness, as discussed in Section 2.7. The t_{max}/t_{lim} ratio expressed by Equation 78 is represented in Figure 8.

Equation 78 shows that t_{max}/t_{lim} depends on only one parameter, the characteristic parameter, λ (Equation 9). Numerical values of the t_{max}/t_{lim} ratio are also presented in Table 3. Since the ratio is always less than 1.0, Figure 8 and Table 3 show that it is always conservative to use t_{lim}, calculated using Equation 74, instead of t_{max}, the value rigorously calculated using Equation 40 (or 41). Figure 8 and Table 3 show that, if the value of λ is small, t_{lim} is an acceptable approximation of the maximum liquid thickness. For example, the inaccuracy of the approximation is less than 5% if $\lambda < 0.01$ and less than 1% if $\lambda < 0.001$.

2.11.3 *Discussion of the Approximation*

Table 3 is sufficient to evaluate the approximation made when Equation 74 is used because t_{max}/t_{lim} depends on only one parameter, λ. However, since design engineers are more familiar with the parameters q_h/k and β, the discussion presented below will be conducted using the following equation obtained by combining Equations 9 and 78:

$$\frac{t_{max}}{t_{lim}} = \frac{\tan\beta\left(\sqrt{\tan^2\beta + 4\,q_h/k} - \tan\beta\right)}{2q_h/k}\left\{1 - 0.12\exp\left[-\left[\log\left(\frac{8\left(q_h/k\right)}{5\tan^2\beta}\right)^{5/8}\right]^2\right]\right\} \quad (88)$$

Table 3. Ratio between the rigorously calculated value, t_{max}, and the approximate value, t_{lim}, of the maximum thickness of liquid in a liquid collection layer as a function of the dimensionless parameter λ defined by Equation 9.

λ	t_{max}/t_{lim}	Approximation
0.0000	1.000	0
0.0007	0.995	≤ 0.5%
0.0015	0.990	≤ 1%
0.0035	0.980	≤ 2%
0.0059	0.970	≤ 3%
0.0088	0.960	≤ 4%
0.0123	0.950	≤ 5%
0.0378	0.900	≤ 10%
0.0789	0.850	≤ 15%
0.1000	0.830	≤ 17%
0.1383	0.800	≤ 20%
0.3310	0.700	≤ 30%
0.6845	0.600	≤ 40%
1.3647	0.500	≤ 50%

Note: The values of t_{max}/t_{lim} tabulated above were calculated using Equation 78.

Numerical values of t_{max}/t_{lim} calculated using Equation 88 are given in Table 4 as a function of q_h/k and β. It appears in Table 4 that: (i) for values of q_h/k equal to or smaller than 1×10^{-5}, t_{lim} is an excellent approximation of t_{max}, regardless of the slope of the liquid collection layer; and (ii) for small values of the slope angle, β, associated with large values of q_h/k, t_{lim} is much greater than t_{max}, which makes it overly conservative to use Equation 74 to calculate an approximate value of t_{max}. In the case of leachate collection layers and leakage detection and collection layers used in landfills, q_h is usually less than 1×10^{-7} m/s and the ratio q_h/k is usually less than 1×10^{-6} when geosynthetic drainage layers such as geonets and geocomposites are used since their hydraulic conductivity is usually greater than 1×10^{-1} m/s. Therefore, based on Table 4, when geosynthetic drainage layers such as geonets and geocomposites are used, t_{lim} provided by Equation 74 is generally an excellent approximation of t_{max}. The same conclusion applies to leachate collection layers and leakage detection and collection layers constructed with clean gravel having a hydraulic conductivity greater than 1×10^{-1} m/s. In the case of leachate collection layers and leakage detection and collection layers constructed with sand having a hydraulic conductivity of 1×10^{-4} to 1×10^{-2} m/s, the ratio q_h/k is less than 1×10^{-3} and, as seen in Table 4, t_{lim} provided by Equation 74 is an excellent approximation of t_{max} on the landfill side slopes (e.g. $\tan\beta \geq 0.25$), but not on the landfill base (e.g. $\tan\beta = 2$ to 5%).

2.11.4 *Simple Rule for the Validity of t_{lim} as an Approximation for t_{max}*

It appears in Table 3 that, if λ is less than 0.1 (a simple value that is easy to remember), the difference between t_{lim} and t_{max} is less than 17%. This approximation is acceptable in most design situations since it is small and conservative. Equation 74 with $\lambda < 0.1$ becomes:

Table 4. Ratio between the rigorously calculated value, t_{max}, and the approximate value, t_{lim}, of the maximum thickness of liquid in a liquid collection layer as a function of the slope angle, β, and the q_h/k ratio (where q_h is the rate of liquid supply and k is the hydraulic conductivity of the liquid collection layer material).

q_h/k	Slope, $\tan\beta$					
	2%	3%	5%	1/4	1/3	1/2
1×10^{-8}	1.00	1.00	1.00	1.00	1.00	1.00
1×10^{-7}	1.00	1.00	1.00	1.00	1.00	1.00
1×10^{-6}	0.98	0.99	1.00	1.00	1.00	1.00
1×10^{-5}	0.92	0.95	0.98	1.00	1.00	1.00
1×10^{-4}	0.73	0.82	0.90	0.99	0.99	1.00
1×10^{-3}	0.42	0.53	0.68	0.94	0.96	0.98
1×10^{-2}	0.17	0.24	0.35	0.79	0.84	0.90
1×10^{-1}	0.06	0.09	0.14	0.48	0.56	0.68

Note: The values of t_{max}/t_{lim} tabulated above were calculated using Equation 88. The 1.00 values correspond to an error less than, or equal to, 0.5% when the approximate value of the liquid thickness is calculated using Equation 74. These values are consistent with the values $\lambda = 0.0007$ in Table 3.

$$t_{lim} < \frac{0.1 L \tan\beta}{\cos\beta} \tag{89}$$

Therefore, t_{lim} is an acceptable approximation of t_{max} if it meets the condition expressed by Equation 89. Since $\cos\beta$ is always less than 1:

$$0.1\, L \tan\beta < \frac{0.1\, L \tan\beta}{\cos\beta} \tag{90}$$

Therefore, the condition expressed by Equation 89 is met if the condition expressed by the following equation is met:

$$t_{lim} < 0.1\ L\ \tan\beta \tag{91}$$

hence:

$$t_{lim} < 0.1 H \tag{92}$$

where H is the height of the liquid collection layer slope (Figure 1):

$$H = L\ \tan\beta \tag{93}$$

The condition expressed by Equation 92 is overly conservative if β is greater than 45° because, then, $\cos\beta$ is much smaller than 1.0 and it is overly conservative to replace Equation 89 by Equation 92.

The condition expressed by Equation 92 is very important. It indicates that, if t_{lim} is less than one tenth of the height of the liquid collection layer slope, then t_{lim} can be used as an acceptable approximation of t_{max} because it is less than 17% above the rigorously calculated value. Therefore, the following recommendation can be made to design engineers: to calculate the maximum liquid thickness in a liquid collection layer, use Equation 74; if the value t_{lim} calculated using Equation 74 is less than one tenth of the height of the liquid collection layer slope, then t_{lim} is an acceptable approximation of t_{max} (i.e. $t_{max} < t_{lim} < 1.17\, t_{max}$), and there is generally no need to perform a more rigorous calculation; and if the value t_{lim} calculated using Equation 74 is greater than one tenth of the height of the liquid collection layer slope, then a more rigorous calculation should be done using Equation 16 or 17, or an even more rigorous calculation using Equation 40 or 41.

It was indicated above, based on Table 4, that the use of t_{lim} as an approximation for t_{max} is virtually always valid when typical geosynthetic drainage materials are used (provided, of course, that the maximum liquid thickness is less than the thickness of the liquid collection layer). The same conclusion can be derived from Equation 92, as follows. Typical geosynthetic drainage materials have a thickness of the order of 10 mm. Therefore, a typical geosynthetic liquid collection layer meets the condition expressed by Equation 92 if the difference in elevation between the top and the toe of its slope is greater than 100 mm, which is virtually always the case, even at the base of a landfill.

Example 2. For the case described in Example 1, calculate an approximate value of the maximum liquid thickness.

As indicated in Example 1, the liquid supply rate is 1.157×10^{-6} m/s and the hydraulic conductivity of the geosynthetic, measured under hydraulic gradients consistent with a slope of 2%, is 0.4 m/s.

An approximate value of the maximum liquid thickness, t_{max}, is provided by t_{lim}, calculated using Equation 74 as follows:

$$t_{lim} = \frac{\left(1.157 \times 10^{-6}\right)(30)}{(0.4)\,\sin\left(\tan^{-1} 0.02\right)} \;=\; 4.34 \times 10^{-3}\,\text{m} \;=\; 4.34\,\text{mm}$$

The accurately calculated value is 4.22 mm, as indicated in Example 1. Therefore, the value of 4.34 mm calculated using Equation 74 is a good approximation. Another good approximation (4.31 mm) was obtained in Example 1 by using the original Giroud's equation (Equation 16). However, there is a major difference between these two equations: (i) the original Giroud's equation always (i.e. for any value of λ) gives a good approximation since the difference between the values given by this equation and the accurate values is always less than 13%, as pointed out in Section 2.3; and (ii) Equation 74 gives a good approximation only if the condition expressed by Equation 92 is satisfied, which is the case in Example 2. Indeed, 4.2 mm is much smaller than 60 mm, which is one tenth of the height of the liquid collection layer given by Equation 93 as follows:

$$H \;=\; (30)(0.02) \;=\; 0.60 \text{ m} = 600 \text{ mm}$$

The approximation made by using Equation 74 would not be good if the condition expressed by Equation 92 were not met.

END OF EXAMPLE 2

2.12 Other Solutions for the Hydraulic Design of Liquid Collection Layers

The focus of Section 2 was on the presentation and evaluation of a set of equations, both simple and accurate, for the design of liquid collection layers, as will be described in Sections 3 and 4. However, in addition to the "simple" equation (Equation 74), the original Giroud equation (Equation 16), the modified Giroud equation (Equation 40), and McEnroe's equations (Equations 49 to 51), other solutions have been proposed to this problem. These other solutions include the original Moore's equation (Moore 1980), the revised Moore's equation (1983), the equation developed by Lesaffre (1987), the equations proposed by McEnroe (1989b), and the equations proposed by Masada (1998). Appendix D presents a brief overview and discussion of some of these other solutions.

3 HYDRAULIC DESIGN BY DETERMINATION OF LIQUID THICKNESS

3.1 Scope of Section 3

In a number of design situations, a given liquid collection layer is considered. In such cases, the thickness of the liquid collection layer is known. For example: (i) if a granular material (sand, gravel) is used, the thickness is typically of the order of 0.2 to 0.6 m; and (ii) if a geosynthetic drainage material is used, the thickness to consider is the thickness of the geotextile (if the geosynthetic drainage material is a transmissive geotextile, such as a needle-punched nonwoven geotextile) or the thickness of the transmissive core (if the geosynthetic drainage material is a geocomposite, which is the typical case when a geosynthetic is used). It is important to note that, in the case where a geosynthetic liquid collection layer is used, the thickness of the liquid collection layer depends on the applied load.

In those cases where the liquid collection layer is given, the hydraulic design of the liquid collection layer consists of checking that the maximum liquid thickness is less than an allowable liquid thickness, t_{allow}, which is the lesser of the thickness of the liquid collection layer and a maximum thickness prescribed by regulation, if any (Section 1.6). This is the approach referred to as the "thickness approach" in Section 1.3. The thickness approach is summarized by the following equation:

$$t_{max} < t_{allow} \leq t_{LTIS} \tag{94}$$

where t_{LTIS} is the long-term-in-soil thickness of the transmissive core of the geosynthetic. Section 3 is devoted to the thickness approach. The general methodology for the thickness approach is presented in Section 3.2, and the implementation of the methodology is presented in Section 3.3 for geosynthetic liquid collection layers and in Section 3.4 for granular liquid collection layers. Design examples are provided in Section 3.5, and conclusions of Section 3 are presented in Section 3.6.

3.2 General Methodology for the Thickness Approach

The maximum liquid thickness during the design life of the considered liquid collection layer is calculated using one of the equations presented in Section 2. These equations give the maximum liquid thickness as a function of the hydraulic conductivity of the liquid collection layer material. There are two different ways of using these equations, depending on the type of factor of safety that is used, i.e. factor of safety on the maximum liquid thickness or factor of safety on the relevant hydraulic characteristic of the liquid collection layer (Section 1.7.4).

If the factor of safety is applied to the maximum liquid thickness, the long-term-in-soil hydraulic conductivity of the liquid collection layer material, k_{LTIS}, must be used in the equations to obtain the maximum liquid thickness during the design life of the considered liquid collection layer, hence the symbol $t_{max}(k_{LTIS})$, i.e. t_{max} as a function of k_{LTIS}. Then the factor of safety of the liquid collection layer is calculated as the ratio between the allowable liquid thickness and the maximum liquid thickness thus calculated, hence:

$$FS_T = \frac{t_{allow}}{t_{max}\left(k_{LTIS}\right)} \tag{95}$$

If the factor of safety is applied to the relevant hydraulic characteristic (hydraulic conductivity or hydraulic transmissivity) of the liquid collection layer, the factored long-term-in-soil hydraulic conductivity of the liquid collection layer material, k_{LTISFS}, must be used in the equations to obtain the maximum liquid thickness during the design life of the considered liquid collection layer, hence the symbol $t_{max}(k_{LTISFS})$. The factored long-term-in-soil hydraulic conductivity is:

$$k_{LTISFS} = \frac{k_{LTIS}}{FS_H} \tag{96}$$

Then, the ratio between the allowable liquid thickness and the maximum liquid thickness thus calculated is equal to 1.0 since the factor of safety, FS_H , is included in the term k_{LTISFS} , hence:

$$\frac{t_{allow}}{t_{max}(k_{LTISFS})} = 1.0 \tag{97}$$

Values of FS_H and FS_T are compared in Appendix E. It appears that, in most practical cases, FS_H is slightly greater than FS_T . However, FS_T can be larger than FS_H for large values of λ and β.

The implementation of the general methodology presented in Section 3.2 is discussed in Section 3.3 for the case of geosynthetic liquid collection layers and in Section 3.4 for the case of granular liquid collection layers.

3.3 The Thickness Approach for Geosynthetic Liquid Collection Layers

3.3.1 *The Problem With Geosynthetic Liquid Collection Layers*

As indicated in Section 1.6, the allowable liquid thickness in the case of geosynthetic liquid collection layers is virtually always the thickness of the liquid collection layer, i.e. the thickness of the transmissive core if the geosynthetic is a geocomposite. This thickness decreases under load and with time from the virgin thickness to the long-term-in-soil thickness. Therefore, in the case of geosynthetic liquid collection layers, the allowable liquid thickness is the long-term-in-soil thickness of the transmissive core of the geosynthetic, t_{LTIS} . As a result, in the case of a geosynthetic liquid collection layer, Equation 95 becomes:

$$FS_T = \frac{t_{LTIS}}{t_{max}(k_{LTIS})} \tag{98}$$

and Equation 97 becomes:

$$t_{max}(k_{LTISFS}) = t_{LTIS} \tag{99}$$

The problem that results from the fact that the thickness of the geosynthetic varies with time is compounded by the fact that t_{LTIS} , k_{LTIS} , and k_{LTISFS} are generally unknown because only the hydraulic transmissivity is typically reported for the geosynthetics used in liquid collection layers. This problem is solved as shown in Sections 3.3.2 and 3.3.3. In Section 3.3.2, a simple solution using an approximate value of t_{max} will be used. In Section 3.3.3, a more complex solution using an accurate value of t_{max} will be used. The two solutions will be compared in Section 3.3.4.

3.3.2 *Solution Using an Approximate Value of the Maximum Liquid Thickness*

A first solution to the problem mentioned in Section 3.3.1 consists of using Equation 74 to calculate an approximate value, t_{lim}, of the maximum liquid thickness, t_{max}. As indicated in Section 2.11, in most cases with geosynthetic liquid collection layers, Equation 74 provides a good approximation of the maximum liquid thickness (i.e. t_{lim} is a good approximation of t_{max}).

In accordance with Section 1.3, two ways of applying the factor of safety are considered: factor of safety applied to the maximum liquid thickness, FS_T, and factor of safety applied to the relevant hydraulic characteristic of the liquid collection layer, FS_H. These two ways are addressed below.

Factor of Safety on Maximum Liquid Thickness. To determine FS_T using Equation 98, it is necessary to calculate $t_{max}(k_{LTIS})$. Using Equation 74 to calculate $t_{max}(k_{LTIS})$ gives:

$$t_{max}\left(k_{LTIS}\right) = \frac{q_h \; L}{k_{LTIS} \; \sin\beta} \tag{100}$$

Combining Equations 98 and 100 gives:

$$FS_T = k_{LTIS} \; t_{LTIS} \; \frac{\sin\beta}{q_h \; L} \tag{101}$$

From Equation 1:

$$\theta_{LTIS} = k_{LTIS} \; t_{LTIS} \tag{102}$$

Combining Equations 101 and 102 gives:

$$FS_T = \theta_{LTIS} \; \frac{\sin\beta}{q_h \; L} \tag{103}$$

Combining Equations 12 and 103 gives:

$$FS_T = \frac{\theta_{measured}}{\prod\left(RF\right)} \; \frac{\sin\beta}{q_h \; L} \tag{104}$$

hence, from Equation 12:

$$FS_T = \frac{\theta_{measured}}{RF_{IMCO} \times RF_{IMIN} \times RF_{CR} \times RF_{IN} \times RF_{CD} \times RF_{PC} \times RF_{CC} \times RF_{BC}} \; \frac{\sin\beta}{q_h \; L} \tag{105}$$

Clearly, it appears that, when $t_{max}(k_{LTIS})$ is calculated using Equation 74 (i.e. when t_{lim} is used as an approximation for t_{max}), the problem mentioned in Section 3.3.1 is solved easily. The use of Equations 104 and 105 is illustrated by Example 3.

Factor of Safety on Hydraulic Characteristic. To determine FS_H using Equations 96 and 99, it is necessary to calculate $t_{max}(k_{LTISFS})$. Using Equation 74 to calculate $t_{max}(k_{LTISFS})$ gives:

$$t_{max}(k_{LTISFS}) = \frac{q_h \; L}{k_{LTISFS} \; \sin\beta} \qquad (106)$$

Combining Equations 99 and 106 gives:

$$t_{LTIS} \; k_{LTISFS} = \frac{q_h \; L}{\sin\beta} \qquad (107)$$

Combining Equations 96 and 107 gives:

$$\frac{t_{LTIS} \; k_{LTIS}}{FS_H} = \frac{q_h \; L}{\sin\beta} \qquad (108)$$

Combining Equations 102 and 108 gives:

$$FS_H = \theta_{LTIS} \; \frac{\sin\beta}{q_h \; L} \qquad (109)$$

Combining Equations 12 and 109 gives:

$$FS_H = \frac{\theta_{measured}}{\prod(RF)} \; \frac{\sin\beta}{q_h \; L} \qquad (110)$$

hence, from Equation 12:

$$FS_H = \frac{\theta_{measured}}{RF_{IMCO} \times RF_{IMIN} \times RF_{CR} \times RF_{IN} \times RF_{CD} \times RF_{PC} \times RF_{CC} \times RF_{BC}} \; \frac{\sin\beta}{q_h \; L} \qquad (111)$$

The expression obtained for FS_H (Equation 110 or 111) is the same as the expression obtained for FS_T (Equation 104 or 105). This is due to the fact that, when Equation 74 is used to calculate t_{max} (i.e. when t_{lim} is used as an approximate value for t_{max}), t_{max} is proportional to k. Therefore, the factor of safety is the same whether it is applied to t or to k. It will be shown in Section 3.3.3 that this is not the case when Equation 40 (or 41) is used.

3.3.3 *Solution Using an Accurate Value of the Maximum Liquid Thickness*

Instead of using the approximate Equation 74, the maximum liquid thickness can be calculated with more precision using the original Giroud's equation (Equation 16 or 17) or, with even more precision, using the modified Giroud's equation (Equation 36, 37, 40, or 41). The use of Giroud's equations is described below.

In accordance with Section 1.3, two ways of applying the factor of safety are addressed: factor of safety applied to the maximum liquid thickness, FS_T, and factor of safety applied to the relevant hydraulic characteristic of the liquid collection layer, FS_H.

Factor of Safety on Maximum Liquid Thickness. To determine FS_T using Equation 98, it is necessary to calculate $t_{max}(k_{LTIS})$. To use Equations 16, 17, 40, or 41 to calculate $t_{max}(k_{LTIS})$ it is necessary to have the value of the long-term-in-soil hydraulic conductivity of the geosynthetic, k_{LTIS} . Since the long-term-in-soil hydraulic conductivity is usually unknown, as indicated in Section 3.3.1, the following method is proposed.

As indicated in Section 1.7.2, the decrease of hydraulic transmissivity from θ_{virgin} (hydraulic transmissivity of the virgin geosynthetic) to θ_{LTIS} is due in part to a decrease of thickness and in part to a decrease of hydraulic conductivity. It is assumed here that the decrease of hydraulic transmissivity is entirely due to a decrease of hydraulic conductivity. Therefore, Equation 98 becomes:

$$FS_T = \frac{t_{virgin}}{t_{max}\left(k_{LTIS}\right)} \qquad (112)$$

where t_{virgin} is the thickness of the liquid collection layer under no compressive stress.

Combining Equations 1 and 12 gives:

$$k_{LTIS} = \frac{\theta_{measured} / t_{virgin}}{\prod\left(RF\right)} \qquad (113)$$

hence, from Equation 12:

$$k_{LTIS} = \frac{\theta_{measured} / t_{virgin}}{RF_{IMCO} \times RF_{IMIN} \times RF_{CR} \times RF_{IN} \times RF_{CD} \times RF_{PC} \times RF_{CC} \times RF_{BC}} \qquad (114)$$

The methodology for obtaining FS_T for a geosynthetic liquid collection layer using an accurate calculation of t_{max} the factor of safety with equation 16 or 19 can be summarized as follows: (i) calculate k_{LTIS} using Equation 113 or 114; (ii) calculate t_{max} using Equation 16 (or 17, which is equivalent) or 36 (or 37, 40, or 41, which are equivalent) with k_{LTIS}; and (iii) calculate the factor of safety, FS_T , using Equation 112. The methodology is illustrated by Example 3.

Factor of Safety on Hydraulic Characteristic. To determine FS_H using Equation 96, it is necessary to calculate $t_{max}(k_{LTISFS})$. To use Equations 16, 17, 36, 37, 40, and 41 to calculate $t_{max}(k_{LTISFS})$ it is necessary to have the value of the factored long-term-in-soil hydraulic conductivity of the geosynthetic, k_{LTISFS} . Since the factored long-term-in-soil hydraulic conductivity is usually unknown, as indicated in Section 3.3.1, the following method is proposed.

As indicated in Section 1.7.2, the decrease of hydraulic transmissivity from θ_{virgin} to θ_{LTIS} is due in part to a decrease of thickness and in part to a decrease of hydraulic conductivity. It assumed here that the decrease of hydraulic transmissivity is entirely due to a decrease of hydraulic conductivity. Therefore, Equation 99 becomes:

$$t_{max}\left(k_{LTISFS}\right) = t_{virgin} \qquad (115)$$

Combining Equations 1 and 12 gives:

$$k_{LTISFS} = \frac{\theta_{measured} / t_{virgin}}{FS_H \times \prod\left(RF\right)} \qquad (116)$$

hence, from Equation 12:

$$k_{LTISFS} = \frac{\theta_{measured} / t_{virgin}}{FS_H \times RF_{IMCO} \times RF_{IMIN} \times RF_{CR} \times RF_{IN} \times RF_{CD} \times RF_{PC} \times RF_{CC} \times RF_{BC}} \quad (117)$$

The methodology for obtaining FS_H for a geosynthetic liquid collection layer using an accurate calculation of t_{max} can be summarized as follows: (i) select a tentative value for FS_H ; (ii) calculate k_{LTISFS} using Equation 116 or 117; (iii) calculate t_{max} using Equation 16 (or 17, which is equivalent) or 36 (or 37, 40, or 41, which are equivalent) with k_{LTISFS}; and (iv) try several values of FS_H until Equation 115 is satisfied. The methodology is illustrated by Example 3.

3.3.4 *Comparison of the Two Solutions*

The solution which consists of calculating the factor of safety based on an accurate calculation of the maximum liquid thickness (Section 3.3.3) has two drawbacks: (i) the calculation of FS_H is complex; and (ii) the values obtained for FS_T and FS_H are different. Furthermore, the factor of safety estimated using an accurate calculation of the maximum liquid thickness (Section 3.3.3) is not significantly more accurate than the factor of safety estimated using an approximate calculation of the maximum liquid thickness (Section 3.3.2). Indeed, it has been shown in Section 2.11 that Equation 74 (which is used for the approximate calculation of t_{max} in Section 3.3.2) provides a good approximation of the maximum liquid thickness in the case of geosynthetic liquid collection layers. Also, as shown in Section 2.9, the approximation that results from the use of Equation 74 is conservative, which adds to the merit of the approximate method presented in Section 3.3.2. Therefore, the use of the method described in Section 3.3.2 is recommended for the case of geosynthetic liquid collection layers. This method gives the same value for FS_T and FS_H (Equations 104 or 105 for FS_T , and 110 or 111 for FS_H). As indicated in Section 1.7.4, it is important to note that FS_T and FS_H are not partial factors of safety to be used simultaneously. They are two ways of expressing the factor of safety of the liquid collection layer.

3.4 The Thickness Approach for Granular Liquid Collection Layers

3.4.1 *General Methodology for Granular Liquid Collection Layers*

When a granular liquid collection layer is used, the mechanisms of thickness reduction are negligible as indicated in Section 1.7.3. Therefore, the thickness of the liquid collection layer is constant. As a result, the allowable liquid thickness is constant since, as indicated in Section 1.6, the allowable liquid thickness is the lesser of the thickness of the liquid collection layer and a maximum thickness prescribed by regulation, if any. Equation 95 can then be used to calculate FS_T and Equation 97 to calculate FS_H . Two cases can be considered for the calculation of t_{max} to be used in Equations 95 and 97: a simple solution using an approximate value of t_{max} (Section 3.4.2) and a more complex solution using an accurate value of t_{max} (Section 3.4.3). The two solutions will be compared in Section 3.4.4.

3.4.2 *Solution Using an Approximate Value of the Maximum Liquid Thickness*

As indicated in Section 2.9, under certain conditions, Equation 74 provides a good approximation of the maximum liquid thickness. A method to determine the factor of safety of the liquid collection layer based on maximum liquid thickness calculated using Equation 74 is presented below.

In accordance with Section 1.3, two ways of applying the factor of safety are addressed: factor of safety applied to the maximum liquid thickness, FS_T, and factor of safety applied to the relevant hydraulic characteristic of the liquid collection layer, FS_H.

Factor of Safety on Maximum Liquid Thickness. To determine FS_T using Equation 95, it is necessary to calculate $t_{max}(k_{LTIS})$. Using Equation 74 to calculate $t_{max}(k_{LTIS})$ gives Equation 100. Combining Equations 95 and 100 gives:

$$FS_T = k_{LTIS} \, t_{allow} \frac{\sin\beta}{q_h \, L} \tag{118}$$

Combining Equations 15 and 118 gives:

$$FS_T = \frac{k_{measured}}{\prod(RF)} \frac{t_{allow} \sin\beta}{q_h \, L} \tag{119}$$

hence, from Equation 15:

$$FS_T = \frac{k_{measured}}{RF_{PC} \times RF_{CC} \times RF_{BC}} \frac{t_{allow} \sin\beta}{q_h \, L} \tag{120}$$

It should be noted that Equations 119 and 120 are valid only if the conditions for the validity of Equation 74 are met (Section 2.9).

Factor of Safety on Hydraulic Characteristic. To determine FS_H using Equation 96, it is necessary to calculate $t_{max}(k_{LTISFS})$. Using Equation 74 to calculate $t_{max}(k_{LTISFS})$ gives Equation 106. Combining Equations 97 and 106 gives:

$$t_{allow} \, k_{LTISFS} = \frac{q_h \, L}{\sin\beta} \tag{121}$$

Combining Equations 96 and 121 gives:

$$FS_H = k_{LTIS} \, t_{allow} \frac{\sin\beta}{q_h \, L} \tag{122}$$

Combining Equations 15 and 122 gives:

$$FS_H = \frac{k_{measured}}{\prod(RF)} \frac{t_{allow} \sin\beta}{q_h \, L} \tag{123}$$

hence, from Equation 15:

$$FS_H = \frac{k_{measured}}{RF_{PC} \times RF_{CC} \times RF_{BC}} \; \frac{t_{allow} \sin \beta}{q_h \; L} \tag{124}$$

The expression obtained for FS_H (Equation 123 or 124) is the same as the expression obtained for FS_T (Equation 119 or 120). This is due to the fact that, when Equation 74 is used to calculate t_{max} (i.e. when t_{lim} is used as an approximate value for t_{max}), t_{max} is proportional to k. Therefore, the factor of safety is the same whether it is applied to t or to k. This is very convenient. However, Equation 74 is often not valid for the case of granular liquid collection layers, as shown in Section 2.11. Therefore, it is generally necessary, in the case of granular liquid collection layers, to use an equation more accurate than Equation 74 to calculate t_{max} . This is shown in Section 3.4.3, where it appears that different values are obtained for FS_T and FS_H .

3.4.3 *Solution Using an Accurate Value of the Maximum Liquid Thickness*

As indicated in Section 3.4.2, in the case of a granular liquid collection layer, it is often necessary to calculate the maximum liquid thickness more accurately than by using Equation 74. As indicated in Section 2.5, an accurate calculation of the maximum liquid thickness can be done using the original Giroud's equation (Equation 16 or 17) or the modified Giroud's equation (Equation 40 or 41).

In accordance with Section 1.3, two ways of applying the factor of safety are considered: factor of safety applied to the maximum liquid thickness, FS_T , and factor of safety applied to the relevant hydraulic characteristic of the liquid collection layer, FS_H . These two ways are addressed below.

Factor of Safety on Maximum Liquid Thickness. To determine FS_T using Equation 95, it is necessary to calculate $t_{max}(k_{LTIS})$. To use Equations 16, 17, 40, or 41 to calculate $t_{max}(k_{LTIS})$ it is necessary to have the value of the long-term-in-soil hydraulic conductivity of the granular liquid collection layer material, k_{LTIS} . This is given by Equation 15.

The methodology for obtaining FS_T for a granular liquid collection layer using an accurate calculation of the maximum liquid thickness is the following: (i) select the allowable liquid thickness in accordance with Section 1.6; (ii) calculate k_{LTIS} using Equation 15; (iii) calculate t_{max} using Equation 16 (or 17, which is equivalent) or 40 (or 41, which is equivalent) with k_{LTIS}; and (iv) calculate FS_T using Equation 95. The methodology is illustrated by Example 4.

Factor of Safety on Hydraulic Characteristic. To determine FS_H using Equations 96 and 97, it is necessary to calculate $t_{max}(k_{LTISFS})$. To use Equations 16, 17, 40, or 41 to calculate $t_{max}(k_{LTISFS})$ it is necessary to first calculate the value of the factored long-term-in-soil hydraulic conductivity of the granular material, k_{LTISFS} , using the following equation derived from Equation 15:

$$k_{LTISFS} = \frac{k_{measured}}{FS_H \times \prod(RF)} = \frac{k_{measured}}{FS_H \times RF_{PC} \times RF_{CC} \times RF_{BC}} \tag{125}$$

The methodology for obtaining FS_H for a granular liquid collection layer using an accurate calculation of the maximum liquid thickness is the following: (i) select the allowable liquid thickness in accordance with Section 1.6; (ii) select a tentative value for FS_H ; (iii) calculate k_{LTISFS} using Equation 125; (iv) calculate t_{max} using Equation 16 (or 17, which is equivalent) or 40 (or 41, which is equivalent) with k_{LTISFS} ; and (v) try several values of FS_H until Equation 97 is satisfied. The methodology is illustrated by Example 4.

3.4.4 *Comparison of the Two Solutions*

The solution which consists of calculating the factor of safety of a granular liquid collection layer based on an accurate value of the maximum liquid thickness (Section 3.4.3) has two drawbacks: (i) the calculation of FS_H is complex; and (ii) the values obtained for FS_T and FS_H are different. However, it is generally necessary to use this solution in the case of granular liquid collection layers. This is because the approximate method presented in Section 3.4.2 is often not applicable since Equation 74 on which it is based is often not valid in the case of granular liquid collection layers (Section 2.9). This is a major difference between the approaches used for geosynthetic and granular liquid collection layers.

As indicated in Section 1.7.4, it is important to note that FS_T and FS_H are not partial factors of safety to be used simultaneously. They are two ways of expressing the factor of safety of the liquid collection layer.

3.5 Design Examples

Example 3. A liquid collection layer is designed for a landfill cover. The rate of liquid supply is 100 mm per day. A drainage geocomposite having a core thickness of 9 mm under no load has been selected. A hydraulic transmissivity test was performed on this geocomposite (including the geotextile filters) under stresses and hydraulic gradients consistent with those expected in the field. The stresses were applied for 100 hours before the hydraulic transmissivity was measured. The transmissivity value thus measured was 3.6×10^{-3} m^2/s The following geometric characteristics of the liquid collection layer are tentatively considered: a length (measured horizontally) of 30 m and a 2% slope. Check that the factor of safety is greater than 2.5, or redesign.

The liquid supply rate is calculated using Equation 8 as follows:

$$q_h = \frac{0.1}{86{,}400} = 1.157 \times 10^{-6} \text{ m/s}$$

The following values are selected for the reduction factors:

- RF_{IMCO} = 1.0 and RF_{IMIN} = 1.0 because the hydraulic transmissivity is measured after the load application (Section 1.7.2).
- RF_{CR} = 1.1, RF_{IN} = 1.2, RF_{CC} = 1.2, RF_{BC} = 1.5, based on Table 1.
- RF_{PC} = 1.0, assuming that the geotextile filter has been properly selected, and RF_{CD} = 1.0, assuming that the geocomposite will not degrade during the design life of the landfill cover, considering that the landfill cover will not be exposed to chemicals.

Alternative 1. A first alternative consists of using the approximate method described in Section 3.3.2. Equations 105 and 111 are used as follows:

$$FS_T = FS_H = \left(\frac{3.6\times10^{-3}}{1.0\times1.0\times1.1\times1.2\times1.0\times1.0\times1.2\times1.5}\right)\frac{\sin\left(\tan^{-1}0.02\right)}{\left(1.157\times10^{-6}\right)(30)} = 0.87$$

Since the factor of safety is less than the target value of 2.5, the liquid collection layer must be redesigned. Based on the discussion in Section 1.8, the only solution is to change the geometry of the liquid collection layer. The following new values are considered for the geometry of the liquid collection layer: a slope of 3% and a length (measured horizontally) of 15 m. Equations 105 an 111 can then be written as follows:

$$FS_T = FS_H = \left(\frac{3.6\times10^{-3}}{1.0\times1.0\times1.1\times1.2\times1.0\times1.0\times1.2\times1.5}\right)\frac{\sin\left(\tan^{-1}0.03\right)}{\left(1.157\times10^{-6}\right)(15)} = 2.62$$

This factor of safety is greater than 2.5 and, therefore, the new geometry (slope 3% and length 15 m) is acceptable.

Alternative 2. A second alternative consists of using the solution described in Section 3.3.3. In this case, different values are obtained for FS_T and FS_H .

First, FS_T is calculated. The three steps described in Section 3.3.3, after Equation 114, are followed.

The first step consists of calculating k_{LTIS} using Equation 114 as follows:

$$k_{LTIS} = \frac{\left(3.6\times10^{-3}\right)/\left(9\times10^{-3}\right)}{1.0\times1.0\times1.1\times1.2\times1.0\times1.0\times1.2\times1.5} = 0.1684 \text{ m/s}$$

The second step consists of calculating t_{max} using Equation 17 with the value of k_{LTIS} calculated above. To that end, λ is calculated using Equation 9 as follows:

$$\lambda = \frac{1.157\times10^{-6}}{(0.1684)(0.02)^2} = 0.0172$$

At this point, after having calculated the value of λ, the engineer may quickly assess the accuracy of the method used for Alternative 1, which consists of using t_{lim} as an approximate value for t_{max} . This can be done by using Figure 8, which shows that, for $\lambda = 0.0172$, $t_{max} / t_{lim} = 0.94$. This indicates that t_{lim} is a close approximation of t_{max} . Consequently, it is anticipated that a more rigorous estimate of t_{max} will not yield a factor of safety significantly different from the one calculated using the Alternative 1 method ($FS_T = FS_H = 0.87$ for the case of the 2% slope). Indeed, the values calculated below are 0.89 and 0.93.

Then, $t_{max}(k_{LTIS})$ is calculated using Equation 17 as follows:

$$t_{max}\left(k_{LTIS}\right) = \frac{\sqrt{1 + (4)(0.0172)} - 1}{(2)\ \cos\left(\tan^{-1}0.02\right)/0.02}(30) = 1.015\times10^{-2}\text{ m} = 10.15 \text{ mm}$$

The third step consists of calculating FS_T using Equation 112 as follows:

$$FS_T = \frac{9}{10.15} = 0.89$$

Instead of Equation 17, Equation 37 can be used for more precision. Equation 37 is equal to Equation 17 multiplied by the dimensionless factor j. The value of j is calculated using Equation 39 as follows:

$$j = 1 - 0.12 \exp\left\{ -\left[\log\left(\frac{(8)(0.0172)}{5} \right)^{5/8} \right]^2 \right\} = 0.954$$

Then, t_{max} is calculated using Equation 37 as follows:

$$t_{max}(k_{LTIS}) = (0.954)(10.15) = 9.68 \text{ mm}$$

Finally, FS_T is calculated using Equation 112 as follows:

$$FS_T = \frac{9}{9.68} = 0.93$$

Since the factor of safety (0.89 or 0.93) is less than the target value of 2.5, the liquid collection layer must be redesigned. Based on the discussion in Section 1.8, the only solution is to change the geometry of the liquid collection layer. The following new values are considered for the geometry of the liquid collection layer: a slope of 3% and a length (measured horizontally) of 15 m. The new value of λ is calculated using Equation 9 as follows:

$$\lambda = \frac{1.157\times10^{-6}}{(0.1684)(0.03)^2} = 7.634\times10^{-3}$$

At this point, after having calculated the value of λ, the engineer may quickly assess the accuracy of the method used for Alternative 1, which consists of using t_{lim} as an approximate value for t_{max}. This can be done by using Figure 8, which shows that, for $\lambda = 0.0076$, $t_{max} / t_{lim} = 0.99$. This indicates that t_{lim} is a very close approximation of t_{max}. Consequently, it is anticipated that a more rigorous estimate of t_{max} will not yield a factor of safety significantly different from the one calculated using the Alternative 1 method ($FS_T = FS_H = 2.62$ for the case of the 3% slope). Indeed, the values calculated below using Alternative 2 are 2.64 and 2.72 for FS_T, and 2.67 and 2.82 for FS_H.

Then, $t_{max}(k_{LTIS})$ is calculated using Equation 17 as follows:

$$t_{max}(k_{LTIS}) = \frac{\sqrt{1 + (4)(7.634\times10^{-3})} - 1}{(2)\ \cos(\tan^{-1} 0.03)/0.03}(15) = 3.41\times10^{-3}\text{ m} = 3.41\text{ mm}$$

The third step consists of calculating FS_T using Equation 112 as follows:

$$FS_T = \frac{9}{3.41} = 2.64$$

Instead of Equation 17, Equation 37 can be used for more precision. Equation 37 is equal to Equation 17 multiplied by the dimensionless factor j. The value of j is calculated using Equation 39 as follows:

$$j = 1 - 0.12\exp\left\{-\left[\log\left(\frac{(8)\left(7.634\times10^{-3}\right)}{5}\right)^{5/8}\right]^2\right\} = 0.971$$

Then, t_{max} is calculated using Equation 37 as follows:

$$t_{max}\left(k_{LTIS}\right) = (0.971)(3.41) = 3.31 \text{ mm}$$

Finally, FS_T is calculated using Equation 112 as follows:

$$FS_T = \frac{9}{3.31} = 2.72$$

Then, the method described in Section 3.3.3 is used for calculating FS_H. The four steps described at the end of Section 3.3.3 are followed. At this point, one could skip the calculation of FS_H for $\tan\beta = 0.02$ and $L = 30$ and immediately calculate FS_H for $\tan\beta = 0.03$ and $L = 15$, knowing that the liquid collection layer has to be redesigned. However, FS_H is calculated herein for both set of values of $\tan\beta$ and L to give a complete example.

The first step consists of selecting a tentative value for FS_H. The value of 2.5 is selected.

The second step consists of calculating k_{LTISFS} using Equation 117 as follows:

$$k_{LTISFS} = \frac{\left(3.6\times10^{-3}\right)/\left(9\times10^{-3}\right)}{2.5\times1.0\times1.0\times1.1\times1.2\times1.0\times1.0\times1.2\times1.5} = 6.734\times10^{-2} \text{ m/s}$$

The third step consists of calculating t_{max} using Equation 17 with the value of k_{LTISFS} calculated above. To that end, λ is calculated using Equation 9 as follows:

$$\lambda = \frac{1.157\times10^{-6}}{\left(6.734\times10^{-2}\right)(0.02)^2} = 0.0430$$

Then, $t_{max}(k_{LTISFS})$ is calculated using Equation 17 as follows:

$$t_{max}\left(k_{LTISFS}\right) = \frac{\sqrt{1 + (4)(0.0430)} - 1}{(2)\ \cos\left(\tan^{-1} 0.02\right)/0.02}(30) = 2.478\times10^{-2} \text{ m} = 24.78 \text{ mm}$$

The fourth step consists of comparing $t_{max}(k_{LTISFS})$ with t_{virgin} in accordance with Equation 115. It appears that $t_{max}(k_{LTISFS})$ is much greater than t_{virgin} (which is 9 mm). Clearly, the factor of safety is much less than the target value of 2.5. The liquid collection layer must be redesigned. The following new values are considered for the geometry of the liquid collection layer: a slope of 3% and a length (measured horizontally) of 15 m. A new value of λ is calculated using Equation 9 as follows:

$$\lambda = \frac{1.157\times10^{-6}}{\left(6.734\times10^{-2}\right)\left(0.03\right)^2} = 0.0191$$

Then, $t_{max}(k_{LTISFS})$ is calculated using Equation 17 as follows:

$$t_{max}\left(k_{LTISFS}\right) = \frac{\sqrt{1+\left(4\right)\left(0.0191\right)}-1}{\left(2\right)\cos\left(\tan^{-1}0.03\right)/0.03}\left(15\right) = 8.44\times10^{-3}\,\text{m} = 8.44\text{ mm}$$

Finally, $t_{max}(k_{LTISFS})$ is compared with t_{virgin} in accordance with Equation 115. It appears that $t_{max}(k_{LTISFS})$, i.e. 8.4 mm, is close to and slightly less than t_{virgin} (which is 9 mm). Therefore, the target factor of safety of 2.5 is slightly exceeded. Then several iterations (not shown herein) were performed to determine that the value of FS_H for this example, which is 2.67.

The above calculations could be redone using Equation 36 (or Equations 37, 40, or 41, which are equivalent) for more precision. The lengthy calculations, including iterations, are not shown herein. A value of 2.82 is obtained for FS_H .

Comparison of the Two Alternatives. The calculations presented in Example 3 can be summarized as follows:

- The method based on an approximate calculation of the maximum liquid thickness (Section 3.3.2) gives $FS_T = FS_H = 2.62$ and the calculations involved are simple and short.
- The method based on an accurate calculation of the maximum liquid thickness (Section 3.3.3) using the original Giroud's equation gives $FS_T = 2.64$ and $FS_H = 2.67$. The calculations involved are simple and moderately long for FS_T , but are very long (including iterations) for FS_H . (Iterations can, however, be done rapidly using available computer programs.)
- The method based on an accurate calculation of the maximum liquid thickness (Section 3.3.3) using the modified Giroud's equation (which is more precise than the original Giroud's equation) gives $FS_T = 2.72$ and $FS_H = 2.82$. The calculations involved are simple and rather long for FS_T and are extremely long (including iterations) for FS_H . (Iterations can, however, be done rapidly using available computer programs.)
- The factors of safety obtained using the approximate Equation 74 are more conservative (i.e. lower) than those obtained using more accurate approaches.

Example 3 confirms that the approximate method based on Equation 74 (i.e. the method that consists of using t_{lim} as an approximate value of t_{max}) and described in Section 3.3.2 gives an excellent approximation of the factor of safety in the case of geosynthetic liquid collection layers. Example 3 confirms that the extra precision obtained with the method based on an accurate calculation of the maximum liquid thickness and described in Section 3.3.3 is not worth the effort required to use this method. Therefore, confirming the recommendation made in Section 3.3.4, the method described in Section 3.3.2 is recommended for the case of geosynthetic liquid collection layers.

END OF EXAMPLE 3

Example 4. A granular liquid collection layer is designed for a landfill cover. The rate of liquid supply is 100 mm per day. The thickness of the liquid collection layer is 0.45 m and the hydraulic conductivity of the granular material is 1.6×10^{-2} m/s, as measured in a laboratory test. The following geometric characteristics of the liquid collection layer are tentatively considered: a length (measured horizontally) of 30 m and a 2% slope. The maximum liquid thickness prescribed by the applicable regulation is 0.30 m. Check that the factor of safety is greater than 2.0, or redesign.

The liquid supply rate is calculated using Equation 8 as follows:

$$q_h = \frac{0.1}{86{,}400} = 1.157 \times 10^{-6} \text{ m/s}$$

The following values are selected for the reduction factors:

- RF_{CC} = 1.2 and RF_{BC} = 1.5, using Table 1 for guidance, even though Table 1 was developed for geonets.
- RF_{PC} = 1.0, assuming that an adequate filter has been selected.

The allowable liquid thickness is t_{allow} = 0.3 m, which is the lesser of the prescribed maximum liquid thickness and the thickness of the liquid collection layer.

A first alternative consists of using the approximate method described in Section 3.4.2, even though it is realized that this approximate method is often inaccurate for granular liquid collection layers. Equations 120 and 124 are used as follows:

$$FS_T = FS_H = \left(\frac{1.6 \times 10^{-2}}{1.0 \times 1.2 \times 1.5} \right) \frac{(0.3)\ \sin\left(\tan^{-1} 0.02\right)}{\left(1.157 \times 10^{-6}\right)(30)} = 1.54$$

If the calculated factor of safety of 1.54 is correct, the liquid collection layer should be redesigned since 1.54 is less than the target value of 2.0. Knowing that the value of 1.54 was obtained using the approximate method, its validity must be checked before making the decision of redesigning the liquid collection layer. A factor of safety of 1.54 means that the calculated liquid thickness is 0.3/1.54 = 0.195 m. This is greater than one tenth of the height of the liquid collection layer, which is 0.06 m, based on the height of the liquid collection layer, which is calculated using Equation 93 as follows:

$$H = (30)(0.02) = 0.60 \text{ m}$$

Therefore, the condition of validity of Equation 74 expressed by Equation 92 is not met. Consequently, to determine whether or not the liquid collection layer must be redesigned, the factor of safety must be calculated using the more accurate method described in Section 3.4.3. In this case, different values are obtained for FS_T and FS_H .

First, FS_T is calculated. The three steps described in Section 3.4.3 for the calculation of FS_T are followed.

The first step consists of selecting the allowable liquid thickness. A value of 0.3 m was selected at the beginning of Example 4.

The second step consists of calculating k_{LTIS} using Equation 15 as follows:

$$k_{LTIS} = \frac{1.6\times10^{-2}}{1.0\times1.2\times1.5} = 8.889\times10^{-3} \text{ m/s}$$

The third step consists of calculating t_{max} using Equation 17 with the value of k_{LTIS} calculated above. To that end, λ is calculated using Equation 9 as follows:

$$\lambda = \frac{1.157\times10^{-6}}{\left(8.889\times10^{-3}\right)\left(0.02\right)^2} = 0.3254$$

For this value of λ, Figure 8 shows that t_{max} / t_{lim} = 0.70, which indicates that t_{lim} is not a close approximation of t_{max} . This confirms the need of using a more rigorous method to estimate the maximum liquid thickness and the factor of safety.

Then, $t_{max}(k_{LTIS})$ is calculated using the original Giroud's equation (Equation 17) as follows:

$$t_{max}\left(k_{LTIS}\right) = \frac{\sqrt{1 + \left(4\right)\left(0.3254\right)} - 1}{\left(2\right)\ \cos\left(\tan^{-1} 0.02\right)/\,0.02}\left(30\right) = 0.155 \text{ m}$$

The fourth step consists of calculating FS_T using Equation 95 as follows:

$$FS_T = \frac{0.3}{0.155} = 1.94$$

If the calculated factor of safety of 1.94 is correct, the liquid collection layer should be redesigned since 1.94 is less than the target value of 2.0. However, before making the decision of redesigning the liquid collection layer, another calculation can be made using the modified Giroud's equation, which is more precise than the original Giroud's equation that was used above.

The modified Giroud's equation (Equation 37) is equal to Equation 17 multiplied by the dimensionless factor j. The value of j is calculated using Equation 39 as follows:

$$j = 1 - 0.12\exp\left\{-\left[\log\left(\frac{\left(8\right)\left(0.3254\right)}{5}\right)^{5/8}\right]^2\right\} = 0.884$$

Then, t_{max} is calculated using Equation 37 as follows:

$$t_{max}\left(k_{LTIS}\right) = \left(0.884\right)\left(0.155\right) = 0.137 \text{ m}$$

It should be noted that the accurately calculated maximum liquid thickness (0.137 m) is 70% of the previously calculated value (0.195 m), which confirms a comment made above.

Then FS_T is calculated using Equation 95 as follows:

$$FS_T = \frac{0.3}{0.137} = 2.19$$

This is the correct value of FS_T because the modified Giroud's equation gives an accurate value of the maximum liquid thickness. Since FS_T is greater than the target value of 2.0, the considered granular liquid collection layer is satisfactory from the viewpoint of FS_T . Then, FS_H is calculated using the methodology presented at the end of Section 3.4.3.

The first step consists of selecting the allowable thickness. The value selected is 0.3 m, as indicated above.

The second step consists of selecting a tentative value for FS_H . For the considered value of λ (0.3254), no guidance is provided in Appendix E regarding the FS_H / FS_T ratio. The value of 2.5 is tentatively selected.

The third step consists of calculating k_{LTISFS} using Equation 125 as follows:

$$k_{LTISFS} = \frac{1.6\times10^{-2}}{2.5\times1.0\times1.2\times1.5} = 3.556\times10^{-3} \text{ m/s}$$

The fourth step consists of calculating t_{max} using Equation 41 with the value of k_{LTISFS} calculated above. To that end, λ is calculated using Equation 9 as follows:

$$\lambda = \frac{1.157\times10^{-6}}{\left(3.556\times10^{-3}\right)\left(0.02\right)^2} = 0.8135$$

Then, $t_{max}(k_{LTISFS})$ is calculated using Equation 41 as follows:

$$t_{max}\left(k_{LTISFS}\right) = \left\{1-0.12\exp\left[-\left[\log\left(\frac{8\times0.8135}{5}\right)^{5/8}\right]^2\right]\right\}\frac{\sqrt{1+(4)(0.8135)}-1}{(2)\cos\left(\tan^{-1}0.02\right)/0.02} \quad (30)$$

hence:

$$t_{max}\left(k_{LTISFS}\right) = 0.281\,\text{m}$$

The fourth step consists of comparing $t_{max}(k_{LTISFS})$ with t_{allow} in accordance with Equation 97. It appears that $t_{max}(k_{LTISFS})$ is less than t_{allow} (which is 0.3 m). This means that FS_H is greater than 2.5. Iterations show that FS_H = 2.73.

Example 4 shows that:

- Based on the calculated value for the maximum liquid thickness (0.137 m), the target factor of safety of 2.0 is met.
- The value of the maximum liquid thickness calculated using the original Giroud's equation (0.155 m) is a good approximation, but it leads to believe that the target factor of safety of 2.0 is not met.

- The value of the maximum liquid thickness calculated using t_{lim} (0.195 m) is incorrect because Equation 74 was used outside the range of values of the parameters for which it is valid.

Clearly, Example 4 shows that it was worth using Giroud's equations, in particular the modified Giroud's equation, in the case of granular liquid collection layers.

END OF EXAMPLE 4

3.6 Conclusions of Section 3

The discussions presented in Section 3 and Example 3 show that, in the case of geosynthetic liquid collection layers, the use of t_{lim} as an approximate value for t_{max} (i.e. the use of Equation 74) is sufficiently accurate, and the use of Giroud's equations (Equations 16, 17, 40, and 41) provides an additional precision that is insignificant, while requiring significantly more effort from the design engineer. In contrast, the discussions presented in Section 3 and Example 4 show that, in the case of granular liquid collection layers, Equation 74 is often overconservative and the use of Giroud's equations (Equations 16 or 17, and 40 or 41) is recommended.

When t_{lim} is used as an approximation of t_{max} , the numerical value of the factor of safety is the same whether the factor of safety is applied to the maximum liquid thickness, FS_T , or to the hydraulic characteristics of the liquid collection layer, FS_H . In contrast, when Giroud's equations are used, different values are obtained for FS_T and FS_H .

4 HYDRAULIC DESIGN BY DETERMINATION OF REQUIRED CHARACTERISTICS OF THE LIQUID COLLECTION LAYER

4.1 Scope of Section 4

In Section 3, it was assumed that the liquid collection layer material had been previously selected and the task of the design engineer was to check that the maximum liquid thickness is less than an allowable value. In Section 4, the situation that exists at a preliminary design stage is considered. The liquid collection layer material has not been selected yet, and the task of the design engineer is to specify the material to be used to construct the liquid collection layer. To that end, the design engineer must determine the hydraulic characteristics that the liquid collection layer should have to provide the required flow capacity. This is the approach referred to as the "hydraulic characteristic approach" in Section 1.3.

Section 4 is devoted to the hydraulic characteristic approach. The general methodology for implementing this approach is presented in Section 4.2. Essentially, Section 4.2 shows how to derive equations for the required hydraulic transmissivity of the liquid collection layer from the equations presented in Section 2. Then, Sections 4.3 and 4.4 present equations developed using the methodology presented in Section 4.2, Section 4.3 presents equations that provide the required hydraulic transmissivity of geosynthetic liquid collection layers, and Section 4.4 presents equations that provide the required hydraulic conductivity of granular liquid collection layers. Design examples are provided in Section 4.5, and conclusions of Section 4 are presented in Section 4.6.

4.2 General Methodology

4.2.1 *Principle*

A liquid collection layer has the required flow capacity if the maximum liquid thickness calculated using the long-term-in-soil hydraulic conductivity, k_{LTIS} , is less than the allowable thickness, t_{allow} (which is the lesser of the thickness of the liquid collection layer and the maximum liquid thickness prescribed by the applicable regulations), affected by a factor of safety (typically equal to 2.0 or more). Two ways of applying the factor of safety are described in Section 3.2: a factor of safety applied to the maximum liquid thickness, FS_T , and a factor of safety applied to the relevant hydraulic characteristic of the liquid collection layer, FS_H . The equations (Equations 95 to 97), presented in Section 3.2, that define these two factors of safety will be used in Section 4.

4.2.2 *Development of Equations*

To express the required value of the hydraulic conductivity based on Equations 95 to 97, it is necessary to use an equation that gives the hydraulic conductivity, k, of the liquid collection layer material as a function of the maximum liquid thickness, t_{max} . The equations presented in Section 2 give the opposite: they give t_{max} as a function of k. Therefore, to be used in Section 4, the equations presented in Section 2 must first be solved for k. Some of the equations presented in Section 2 (i.e. Equations 16 (or 17, which is equivalent) and 74) can be solved in a way that gives an explicit value of k as a function of t_{max} . In contrast, Equation 40 (or 41, which is equivalent) cannot be solved in a way that gives an explicit value of k as a function of t_{max} . When this equation is used, iterations are needed to obtain the required hydraulic conductivity. Finally, McEnroe's equations (Equations 49 to 54) appear to be unsuitable to analytically solve for k.

Solving Equation 74 for the hydraulic conductivity is straightforward. It gives:

$$k = \frac{q_h \ L}{t_{max} \ \sin\beta} \tag{126}$$

The solutions presented in Section 4 based on Equation 126 will only be valid if the use of Equation 74 for calculating t_{max} is legitimate. Equation 74 gives t_{lim} , which is an approximate value of t_{max} only when the dimensionless parameter λ is small (Section 2.9.2 and Figure 6). The condition of validity of Equation 74 is also expressed by Equation 92 (Section 2.9.4). Therefore, solutions presented in Section 4 based on Equation 126 will only be valid for certain values of the parameters. In contrast, Equation 17 (the "original Giroud' equation") always provides a good approximation of t_{max} and can be considered as an accurate equation for the purpose of Section 4.

Equation 17 can be written as follows:

$$\frac{2\, t_{max} \cos\beta}{L \tan\beta} = \sqrt{1 + 4\lambda} - 1 \tag{127}$$

hence:

$$\left(\sqrt{1+4\lambda}\right)^2 = 1+4\lambda = \left(\frac{2\ t_{max}\cos\beta}{L\ \tan\beta}+1\right)^2 \tag{128}$$

hence:

$$\lambda = \frac{t_{max}\cos\beta}{L\tan\beta} + \left(\frac{t_{max}\cos\beta}{L\tan\beta}\right)^2 \tag{129}$$

Combining Equations 9 and 129 gives:

$$\frac{q_h}{k} = \frac{t_{max}\sin\beta}{L} + \left(\frac{t_{max}\cos\beta}{L}\right)^2 \tag{130}$$

hence, the hydraulic conductivity as a function of t_{max} :

$$k = \frac{q_h}{\dfrac{t_{max}\sin\beta}{L} + \left(\dfrac{t_{max}\cos\beta}{L}\right)^2} \tag{131}$$

Equation 131 was derived from Equation 17. The same derivation from Equation 36 gives:

$$k = \frac{q_h}{\dfrac{t_{max}\sin\beta}{j\ L} + \left(\dfrac{t_{max}\cos\beta}{j\ L}\right)^2} \tag{132}$$

Equation 132 does not give an explicit value of k because j depends on k (Equation 38). Therefore, iterations are required when Equation 132 and equations derived from Equation 132 are used. Equations 126, 131, and 132 are the three equations that serve as a basis for developing the equations that will give the required hydraulic characteristics of the liquid collection layer.

As indicated in Section 4.3.1, there are two ways of applying the factor of safety. These two ways are discussed below.

If the factor of safety is applied to the maximum liquid thickness, the governing equation is Equation 95 and, to use Equation 95, one need to express k_{LTIS} as a function of t_{max} . Equation 131 gives:

$$k_{LTIS} = \frac{q_h}{\dfrac{t_{max}(k_{LTIS})\ \sin\beta}{L} + \left(\dfrac{t_{max}(k_{LTIS})\ \cos\beta}{L}\right)^2} \tag{133}$$

If the factor of safety is applied to hydraulic characteristics, the governing equations are Equations 96 and 97. To use Equation 97, one must express k_{LTISFS} as a function of t_{max} . Equation 131 gives:

$$k_{LTISFS} = \frac{q_h}{\dfrac{t_{max}(k_{LTISFS})\ \sin\beta}{L} + \left(\dfrac{t_{max}(k_{LTISFS})\ \cos\beta}{L}\right)^2} \tag{134}$$

Similarly, Equation 132 gives:

$$k_{LTIS} = \frac{q_h}{\dfrac{t_{max}(k_{LTIS})\ \sin\beta}{j\ L} + \left(\dfrac{t_{max}(k_{LTIS})\ \cos\beta}{j\ L}\right)^2} \tag{135}$$

$$k_{LTISFS} = \frac{q_h}{\dfrac{t_{max}(k_{LTISFS})\ \sin\beta}{j\ L} + \left(\dfrac{t_{max}(k_{LTISFS})\ \cos\beta}{j\ L}\right)^2} \tag{136}$$

Equations 133 to 136 will be used for both the geosynthetic and the granular liquid collection layers, in Sections 4.3 and 4.4, respectively.

4.2.3 *Validity of the Equations*

Equations 132, 135, and 136 are accurate, but iterations are required to use them. Equations 131, 133, and 134 are always valid for design purposes. As pointed out in Section 2.3, the maximum error that can be made when Equation 17 (from which Equations 131, 133, and 134 are derived) is used is 13%.

In contrast, Equation 126 is valid for a certain range of the values of the parameters, as indicated in Section 4.2.2. Comparison of Equations 126 and 131 shows that Equation 126 is a good approximation of Equation 131 if the second term of the denominator of Equation 131 is small compared to the first term, e.g. if the ratio between the two terms is 10 or more:

$$\left(\frac{t_{max}\ \cos\beta}{L}\right)^2 \le 0.1\,\frac{t_{max}\ \sin\beta}{L} \tag{137}$$

hence:

$$t_{max} \le 0.1\ L \tan\beta / \cos\beta \tag{138}$$

This is consistent with the discussion of the validity of Equation 74 presented in Section 2.9.4, which confirms that the validity of Equation 126 is linked to the validity of Equation 74.

4.3 Required Hydraulic Characteristics of a Geosynthetic Liquid Collection Layer

4.3.1 *General Methodology For Geosynthetic Liquid Collection Layers*

As indicated in Sections 1.6 and 3.3.1, the allowable liquid thickness in the case of geosynthetic liquid collection layers is virtually always the thickness of the liquid collection layer, i.e. the thickness of the transmissive core when the liquid collection layer is a geocomposite. This thickness decreases with time due to creep. Therefore, in the case of geosynthetic liquid collection layers, the allowable thickness is the minimum thickness that the liquid collection layer will have during its design life, i.e. it is the long-term-in-soil thickness of the transmissive core of the geosynthetic, t_{LTIS} . As a result, in the case of a geosynthetic liquid collection layer, the basic equations that define the factors of safety, Equations 95 and 97, become Equations 98 and 99, respectively. Two types of solutions are presented below: solution based on an approximate value of the maximum liquid thickness (i.e. solutions based on Equation 126) and solution based on an accurate value of the maximum liquid thickness (i.e. solutions based on Equations 131 and 132).

4.3.2 *Solution Using an Approximate Value of the Maximum Liquid Thickness*

The use of Equation 126 leads to the same considerations as in Section 3.3.2. Therefore, the equations presented in Section 3.3.2 can be solved for θ to express the required hydraulic transmissivity. Thus, Equations 104 and 110 give:

$$\theta_{measured-req} = FS \ \prod(RF) \ \frac{q_h \ L}{\sin\beta} \tag{139}$$

where $\theta_{measured\text{-}req}$ indicates that the required hydraulic transmissivity of the geosynthetic must be evaluated in a hydraulic transmissivity test performed under the conditions that correspond to the values of the reduction factors used in Equation 139, and $\Pi(RF)$ is, according to Equations 105 and 111:

$$\prod(RF) = RF_{IMCO} \times RF_{IMIN} \times RF_{CR} \times RF_{IN} \times RF_{CD} \times RF_{PC} \times RF_{CC} \times RF_{BC} \tag{140}$$

It is important to note that, based on the equations presented in Section 3.3.2, the factor of safety, FS, is the same whether it is applied to the maximum liquid thickness (FS_T) or the hydraulic characteristics of the liquid collection layer (FS_H).

The use of Equations 139 and 140 is illustrated by Example 5.

4.3.3 *Solution Using an Accurate Value of the Maximum Liquid Thickness*

Instead of using the approximate solution presented in Section 4.3.2, one may want to use the more precise solution based on Equation 131 or Equation 132. This is described below.

In accordance with Section 1.3, two ways of applying the factor of safety are considered: factor of safety applied to the maximum liquid thickness, FS_T , and factor of safety applied to the relevant hydraulic characteristic of the liquid collection layer, FS_H . These two ways are addressed below.

Factor of Safety Applied to Maximum Liquid Thickness. For the long-term-in-soil situation, Equation 131 gives Equation 133.

As indicated in Section 4.3.1, in the case of a geosynthetic liquid collection layer, the allowable liquid thickness, t_{allow} , is virtually always the thickness of the liquid collection layer, and the governing equation, Equation 95, becomes Equation 98. Combining Equations 98 and 133 gives:

$$k_{LTIS} = \frac{q_h}{\dfrac{t_{LTIS}}{L} \dfrac{\sin\beta}{FS_T} + \left(\dfrac{t_{LTIS}}{L} \dfrac{\cos\beta}{FS_T}\right)^2} \tag{141}$$

Combining Equations 102 and 141 gives:

$$\theta_{LTIS} = k_{LTIS}\ t_{LTIS} = \frac{FS_T\ q_h\ L}{\sin\beta + \left(\dfrac{t_{LTIS}}{L\ FS_T}\right)\cos^2\beta} \tag{142}$$

Combining Equations 12 and 142, and using the notation $\theta_{measured\text{-}req}$ for the required hydraulic transmissivity, gives:

$$\theta_{measured-req} = \frac{FS_T\ \prod(RF)\quad q_h\ L}{\sin\beta + \left(\dfrac{t_{LTIS}}{L\ FS_T}\right)\cos^2\beta} \tag{143}$$

where $\Pi(RF)$ is given by Equation 140.

In Equation 143, t_{LITS} is unknown. To be conservative, it can be replaced by a known value that is smaller. Since the mechanisms that are quantified by the reduction factors affect partly the thickness and partly the hydraulic conductivity:

$$t_{LTIS} > \frac{t_{virgin}}{\prod(RF)} \tag{144}$$

Therefore, it is conservative to combine Equations 143 and 144, which gives:

$$\theta_{measured-req} = \frac{FS_T\ \prod(RF)\quad q_h\ L}{\sin\beta + \left(\dfrac{t_{virgin}/L}{FS_T\ \prod(RF)}\right)\cos^2\beta} \tag{145}$$

Since t_{virgin} is small compared to L, the second term in the denominator is small compared to the first term. Therefore, the values of $\theta_{measured\text{-}req}$ calculated using Equation 145 are close to the values calculated using Equation 139. It is concluded that the extra effort associated with the use of Equation 145, compared with Equation 139, is not justified in the case of geosynthetic liquid collection layers. This will be illustrated by Example 5. It would be even less justified to derive an equation from Equation 135. The use of this equation would be time-consuming due to the iterations and it would not result in more precision.

Factor of Safety Applied to Hydraulic Characteristic. For the long-term-in soil situation, with the factor of safety, FS_H , included in the hydraulic conductivity, Equation 131 gives Equation 134. Combining Equations 96, 99, and 134 gives:

$$\frac{k_{LTIS}}{FS_H} = \frac{q_h}{\frac{t_{LTIS}\ \sin\beta}{L} + \left(\frac{t_{LTIS}\ \cos\beta}{L}\right)^2} \tag{146}$$

Combining Equations 102 and 146 gives:

$$\theta_{LTIS} = k_{LTIS}\ t_{LTIS} = \frac{FS_H\ q_h\ L}{\sin\beta + \frac{t_{LTIS}}{L}\cos^2\beta} \tag{147}$$

Combining Equations 12 and 147 gives:

$$\theta_{measured-req} = \frac{FS_H\ \prod(RF)\ q_h\ L}{\sin\beta + \frac{t_{LTIS}}{L}\cos^2\beta} \tag{148}$$

Combining Equations 144 and 148 gives:

$$\theta_{measured-req} = \frac{FS_H\ \prod(RF)\ q_h\ L}{\sin\beta + \frac{t_{virgin}/L}{\prod(RF)}\cos^2\beta} \tag{149}$$

Equation 149 is similar to Equation 145. The only difference is the presence of the factor of safety in the denominator of Equation 145. As a result, for similar values of FS_T and FS_H , a slightly greater value of $\theta_{measured\text{-}req}$ is obtained with Equation 145 rather than with Equation 149.

Since t_{virgin} is small compared to L, the second term in the denominator is small compared to the first term. Therefore, the values of $\theta_{measured\text{-}req}$ calculated using Equation 149 are close to the values calculated using Equation 139. It is concluded that the extra effort associated with the use of Equation 149, compared with Equation 139, is not justified in the case of geosynthetic liquid collection layers. It would be even less justified to derive an equation from Equation 136. The use of this equation would be time-consuming due to the iterations and it would not result in more precision.

4.3.4 *Discussion*

The analysis presented in Section 4.3 shows, and Example 5 will confirm, that, in the case of geosynthetic liquid collection layers, the required hydraulic transmissivity should be calculated using Equation 139, which was derived from Equation 74. Using the more precise Equations 145 and 149 adds insignificant precision and requires significantly more effort. It should also be noted that the approximation provided by Equation 139 is not only excellent, but it is also conservative.

4.4 Required Hydraulic Characteristics of a Granular Liquid Collection Layer

4.4.1 *General Methodology for Granular Liquid Collection Layers*

When a granular liquid collection layer is used, the mechanisms of thickness reduction are negligible as indicated in Section 1.7.3. Therefore, the thickness of the liquid collection layer is constant. As a result, the allowable liquid thickness is constant since, as indicated in Section 1.6, the allowable liquid thickness is the lesser of the thickness of the liquid collection layer and a maximum liquid thickness prescribed by regulation, if any. Therefore, the case of a granular liquid collection layer is simpler than the case of a geosynthetic liquid collection layer.

The governing equations are Equation 95, for the case where the factor of safety is applied to the maximum liquid thickness, and Equations 96 and 97, for the case where the factor of safety is applied to the hydraulic characteristics of the liquid collection layer. Two types of solutions are presented below: solution based on an approximate value of the maximum liquid thickness (i.e. solution based on Equation 126) and solutions using an accurate value of the maximum liquid thickness (i.e. solutions based on Equation 131 or 132). These two cases are discussed in Sections 4.4.2 and 4.4.3, respectively.

4.4.2 *Solution Using an Approximate Value of the Maximum Liquid Thickness*

The use of Equation 126 for a granular liquid collection layer leads to the same considerations as in Section 3.4.2. The equations presented in Section 3.4.2 can be solved for θ to express the required hydraulic conductivity. Thus, Equations 119 and 123 give:

$$k_{measured-req} = FS \ \prod(RF) \ \frac{q_h \ L}{t_{allow} \ \sin\beta} \tag{150}$$

where $k_{measured\text{-}req}$ indicates that the required hydraulic conductivity of the geosynthetic must be evaluated in a hydraulic conductivity test performed under the conditions that correspond to the values of the reduction factors used in Equation 150, and $\Pi(RF)$ is, according to Equation 120 or 124:

$$\prod(RF) = RF_{PC} \times RF_{CC} \times RF_{BC} \tag{151}$$

The use of Equation 150 is illustrated by Example 6. It should be noted that Equation 150 is valid only when t_{lim} is a good approximation of t_{max} (i.e. when the use of Equation 74 for calculating t_{max} is legitimate). The conditions for the validity of Equation 74 (and, therefore, Equation 150) are discussed in Section 2.11, where it is shown that Equation 74 is often not valid for the case of granular liquid collection layers. Therefore, it is generally necessary, in the case of granular liquid collection layers, to use an equation more accurate than Equation 74 to calculate t_{max}. This is addressed in Section 4.4.3.

4.4.3 *Solution Using an Accurate Value of the Maximum Liquid Thickness*

As indicated in Section 4.4.2, it is often necessary in the case of granular liquid collection layers to use solutions derived from Equations 131 and 132, because equations derived from Equation 126 (such as Equation 150) are often not valid in the case of granular liquid collection layers. Accordingly, Section 4.4.3 presents solutions derived from Equations 131 and 132.

In accordance with Section 1.3, two ways of applying the factor of safety are considered: factor of safety applied to the maximum liquid thickness, FS_T, and factor of safety applied to the relevant hydraulic characteristic of the liquid collection layer, FS_H. These two ways are addressed below.

Factor of Safety Applied to Maximum Liquid Thickness. For the long-term-in-soil conditions, Equation 131 gives Equation 133. Combining Equations 95 and 133 gives:

$$k_{LTIS} = \frac{q_h}{\dfrac{t_{allow}\ \sin\beta}{FS_T\ L} + \left(\dfrac{t_{allow}\ \cos\beta}{FS_T\ L}\right)^2} \tag{152}$$

Combining Equations 12 and 152, and using the notation $k_{measured\text{-}req}$ for the required hydraulic conductivity, gives:

$$k_{measured-req} = \frac{\prod(RF)\ q_h}{\dfrac{t_{allow}\ \sin\beta}{FS_T\ L} + \left(\dfrac{t_{allow}\ \cos\beta}{FS_T\ L}\right)^2} \tag{153}$$

Similarly, combining Equations 12, 95, and 135 gives:

$$k_{measured-req} = \frac{\prod(RF)\ q_h}{\dfrac{t_{allow}\ \sin\beta}{j\ FS_T\ L} + \left(\dfrac{t_{allow}\ \cos\beta}{j\ FS_T\ L}\right)^2} \tag{154}$$

Equation 153 is explicit, whereas Equation 154 requires iterations. The use of Equations 153 and 154 will be illustrated by Example 6.

Factor of Safety Applied to Hydraulic Characteristic. For the long-term-in-soil conditions, and considering a factor of safety, FS_H, applied to the hydraulic conductivity, Equation 131 becomes Equation 134. Combining Equations 96, 97, and 134 gives:

$$\frac{k_{LTIS}}{FS_H} = \frac{q_h}{\dfrac{t_{allow}\ \sin\beta}{L} + \left(\dfrac{t_{allow}\ \cos\beta}{L}\right)^2} \tag{155}$$

Combining Equations 12 and 155 gives:

$$k_{measured-req} = \frac{FS_H\ \prod(RF)\ q_h}{\dfrac{t_{allow}\sin\beta}{L} + \left(\dfrac{t_{allow}\cos\beta}{L}\right)^2} \tag{156}$$

Similarly, combining Equations 12, 96, and 136 gives:

$$k_{measured-req} = \frac{FS_H \ \prod\left(RF\right) q_h}{\dfrac{t_{allow} \sin\beta}{j\ L} + \left(\dfrac{t_{allow} \cos\beta}{j\ L}\right)^2} \tag{157}$$

Equation 156 is explicit, whereas Equation 157 requires iterations. The use of Equations 156 and 157 will be illustrated by Example 6.

4.4.4 *Discussion*

The analysis presented in Section 4.4 shows, and Example 6 will confirm, that, in the case of granular liquid collection layers, the required hydraulic conductivity should generally not be calculated using Equation 147, which was derived from Equation 74. In the case of granular liquid collection layers, it is generally necessary to use the more accurate Equations 153 and 156, or even 154 and 157, to calculate the required hydraulic conductivity.

4.5 Design Examples

Example 5. A liquid collection layer has a length (measured horizontally) of 30 m and a 2% slope. The rate of liquid supply is 100 mm per day. Calculate the required hydraulic transmissivity for a geocomposite liquid collection layer.

First, the liquid supply rate is calculated using Equation 8 as follows:

$$q_h = \frac{0.1}{86,400} = 1.157 \times 10^{-6} \text{ m/s}$$

The following values are selected for the reduction factors:

- $RF_{IMCO} = 1.0$ and $RF_{IMIN} = 1.0$ assuming that the hydraulic transmissivity is measured after the load application (Section 1.7.2).
- $RF_{CR} = 1.1$, $RF_{IN} = 1.2$, $RF_{CC} = 1.2$, and $RF_{BC} = 1.5$, based on Table 1.
- $RF_{PC} = 1.0$, assuming that the geotextile filter has been properly selected, and $RF_{CD} = 1.0$, assuming that the geocomposite will not degrade during the design life of the landfill cover, considering that the landfill cover will not be exposed to chemicals.

Also, a factor of safety of 2.5 is selected.

A first alternative consists of using the solution described in Section 4.3.2. Equation 140 gives $\Pi(RF)$ as follows:

$$\prod\left(RF\right) = 1.0 \times 1.0 \times 1.1 \times 1.2 \times 1.0 \times 1.0 \times 1.2 \times 1.5 = 2.376$$

Then, Equation 139 gives:

$$\theta_{measured-req} = \left(2.5 \times 2.376\right) \frac{\left(1.157 \times 10^{-6}\right)\left(30\right)}{\sin\left(\tan^{-1} 0.02\right)} = 1.03 \times 10^{-2} \text{ m}^2\text{/s} \approx 1.0 \times 10^{-2} \text{ m}^2\text{/s}$$

Based on this result, the selected geocomposite should have a hydraulic transmissivity of at least 1.0×10^{-2} m^2/s. This hydraulic transmissivity should be measured in a hydraulic transmissivity test performed under a stress equal to the maximum stress expected in the actual liquid collection layer and with a seating time of at least 100 hours, since these are the testing conditions assumed to develop Table 1 that was used to select the values of the reduction factors. Furthermore, the hydraulic transmissivity test should be performed with a hydraulic gradient of 0.02 or more, since the hydraulic transmissivity tends to decrease with increasing hydraulic gradient.

A second alternative consists of using the solution described in Section 4.3.3. The solution described in Section 4.3.3 depends on the thickness of the liquid collection layer. Since no thickness is given in Example 5, a thickness of 9 mm is assumed. In the case of the solution described in Section 4.3.3, two different values are obtained for the required hydraulic transmissivity depending on how the factor of safety is applied.

If the factor of safety is applied to the maximum liquid thickness, Equation 145 gives:

$$\theta_{measured-req} = \frac{(2.5)(2.376)\left(1.157\times10^{-6}\right)(30)}{\sin\left(\tan^{-1}0.02\right) + \dfrac{\left(9\times10^{-3}\right)/30}{(2.5)(2.376)}\cos^2\left(\tan^{-1}0.02\right)} = 1.03\times10^{-2}\ \text{m}^2/\text{s}$$

If the factor of safety is applied to the hydraulic transmissivity, Equation 149 gives:

$$\theta_{measured-req} = \frac{(2.5)(2.376)\left(1.157\times10^{-6}\right)(30)}{\sin\left(\tan^{-1}0.02\right) + \dfrac{\left(9\times10^{-3}\right)/30}{(2.376)}\cos^2\left(\tan^{-1}0.02\right)} = 1.02\times10^{-2}\ \text{m}^2/\text{s}$$

The very small difference between 1.02 and 1.03 confirms that Equation 139 gives a good approximation of the required hydraulic transmissivity in the case of geosynthetic liquid collection layers. This is consistent with the comment made at the end of Section 4.4.2.

END OF EXAMPLE 5

Example 6. A liquid collection layer has a length (measured horizontally) of 30 m and a 2% slope. A granular liquid collection layer with a thickness of 0.45 m is considered at the design stage. The rate of liquid supply is 100 mm per day. The maximum liquid thickness value prescribed by the applicable regulation is 0.3 m. Calculate the required hydraulic conductivity of the granular liquid collection layer.

First, the liquid supply rate is calculated using Equation 8 as follows:

$$q_h = \frac{0.1}{86{,}400} = 1.157 \times 10^{-6}\ \text{m/s}$$

The following values are selected for the reduction factors:

- RF_{CC} = 1.2 and RF_{BC} = 1.5, using Table 1 for guidance, even though Table 1 was developed for geonets.
- RF_{PC} = 1.0, assuming that an adequate filter has been selected.

Also, a factor of safety of 2.5 is selected.

Then, it is necessary to know the allowable thickness. As indicated in Section 3.2.3, it is the lesser of: (i) a maximum liquid thickness prescribed by regulations; and (ii) the thickness of the liquid collection layer. Therefore, the allowable thickness is 0.3 m.

A first approximation of the required hydraulic conductivity can be calculated using Equation 150. This equation is valid only if the condition expressed by Equation 92 is met. The height of the liquid collection layer is given by Equation 93 as follows:

$$H = (30)(0.02) = 0.60 \text{ m}$$

The allowable liquid thickness is greater than one tenth of the height of the liquid collection layer. As a result, Equation 150 is not valid in the considered case. Therefore, the more accurate Equations 153, 154, 156, and 157 must be used.

Equation 153, which corresponds to the case where the factor of safety is applied to the maximum liquid thickness gives:

$$k_{measured-req} = \frac{(1.0 \times 1.2 \times 1.5)(1.157 \times 10^{-6})}{\dfrac{(0.3)\sin(\tan^{-1} 0.02)}{(2.5)(30)} + \left[\dfrac{(0.3)\cos(\tan^{-1} 0.02)}{(2.5)(30)}\right]^2} = 2.16988 \times 10^{-2} \text{ m/s}$$

hence:

$$k_{measured-req} = 2.17 \times 10^{-2} \text{ m/s}$$

Equation 156, which corresponds to the case where the factor of safety is applied to the hydraulic conductivity of the liquid collection layer gives:

$$k_{measured-req} = \frac{(2.5)(1.0 \times 1.2 \times 1.5)(1.157 \times 10^{-6})}{\dfrac{(0.3)\sin(\tan^{-1} 0.02)}{30} + \left[\dfrac{(0.3)\cos(\tan^{-1} 0.02)}{30}\right]^2} = 1.73596 \times 10^{-2} \text{ m/s}$$

hence:

$$k_{measured-req} = 1.74 \times 10^{-2} \text{ m/s}$$

Equations 153 and 156 are explicit. More precision can be obtained from Equations 154 and 157 that require iterations. The values obtained using Equations 153 and 156 are used to start the iteration process.

To use Equation 154, it is necessary to first calculate λ to calculate j. Equation 9 gives:

$$\lambda = \frac{1.157 \times 10^{-6}}{\left(2.16988 \times 10^{-2}\right)(0.02)^2} = 0.13302$$

Then Equation 39 gives:

$$j = 1 - 0.12 \exp\left\{-\left[\log\left(\frac{(8)(0.13302)}{5}\right)^{5/8}\right]^2\right\} = 0.899356$$

Then, the required hydraulic conductivity is obtained using Equation 156 as follows:

$$k_{measured-req} = \frac{(1.0 \times 1.2 \times 1.5)\left(1.157 \times 10^{-6}\right)}{\dfrac{(0.3)\sin\left(\tan^{-1} 0.02\right)}{(0.899356)(2.5)(30)} + \left[\dfrac{(0.3)\cos\left(\tan^{-1} 0.02\right)}{(0.899356)(2.5)(30)}\right]^2} = 1.91577 \times 10^{-2} \text{ m/s}$$

This is only a first iteration. Then, λ must be recalculated using Equation 9 as follows:

$$\lambda = \frac{1.157 \times 10^{-6}}{\left(1.91577 \times 10^{-2}\right)(0.02)^2} = 0.15098$$

Then Equation 39 gives:

$$j = 1 - 0.12 \exp\left\{-\left[\log\left(\frac{(8)(0.15098)}{5}\right)^{5/8}\right]^2\right\} = 0.896578$$

Then, the required hydraulic conductivity is obtained using Equation 156 as follows:

$$k_{measured-req} = \frac{(1.0 \times 1.2 \times 1.5)\left(1.157 \times 10^{-6}\right)}{\dfrac{(0.3)\sin\left(\tan^{-1} 0.02\right)}{(0.896578)(2.5)(30)} + \left[\dfrac{(0.3)\cos\left(\tan^{-1} 0.02\right)}{(0.896578)(2.5)(30)}\right]^2} = 1.9088 \times 10^{-2} \text{ m/s}$$

It appears that 1.9088 and 1.91577 are very close. Therefore, there is no need for additional iterations, hence:

$$k_{measured-req} = 1.91 \times 10^{-2} \text{ m/s}$$

The above value of the required hydraulic conductivity is for a 2.5 safety factor applied to the liquid collection layer thickness. Then, the case of a 2.5 safety factor applied to the liquid collection layer hydraulic conductivity must be considered. To that end, Equation 157 should be used. To use Equation 157, it is necessary to first calculate λ to calculate j. Equation 9 gives:

$$\lambda = \frac{1.157\times10^{-6}}{\left(1.73596\times10^{-2}\right)\left(0.02\right)^2} = 0.16662$$

Then Equation 39 gives:

$$j = 1 - 0.12\exp\left\{-\left[\log\left(\frac{(8)(0.16662)}{5}\right)^{5/8}\right]^2\right\} = 0.89450$$

Then, the required hydraulic conductivity is obtained using Equation 157 as follows:

$$k_{measured-req} = \frac{(2.5)(1.0\times1.2\times1.5)\left(1.157\times10^{-6}\right)}{\dfrac{(0.3)\sin\left(\tan^{-1}0.02\right)}{(0.89450)30} + \left[\dfrac{(0.3)\cos\left(\tan^{-1}0.02\right)}{(0.89450)30}\right]^2} = 1.4941\times10^{-2}\ \text{m/s}$$

This is only a first iteration. Then, λ must be recalculated using Equation 9 as follows:

$$\lambda = \frac{1.157\times10^{-6}}{\left(1.4941\times10^{-2}\right)\left(0.02\right)^2} = 0.19359$$

Then Equation 39 gives:

$$j = 1 - 0.12\exp\left\{-\left[\log\left(\frac{(8)(0.19359)}{5}\right)^{5/8}\right]^2\right\} = 0.89155$$

Then, the required hydraulic conductivity is obtained using Equation 157 as follows:

$$k_{measured-req} = \frac{(2.5)(1.0\times1.2\times1.5)\left(1.157\times10^{-6}\right)}{\dfrac{(0.3)\sin\left(\tan^{-1}0.02\right)}{(0.89155)30} + \left[\dfrac{(0.3)\cos\left(\tan^{-1}0.02\right)}{(0.89155)30}\right]^2} = 1.4874\times10^{-2}\ \text{m/s}$$

It appears that 1.4941 and 1.4874 are very close. Therefore, there is no need for additional iterations, hence:

$$k_{measured-req} = 1.49\times10^{-2}\ \text{m/s}$$

In conclusion, the required hydraulic conductivity with a factor of safety of 2.5 is 1.91 m/s (i.e. the higher of the two values obtained using FS_T and FS_H). This example shows that it is worth using Equations 154 and 157, i.e. the equations derived from the modified Giroud's equation, even though these equations require iterations, because they result in a more accurate value of the required hydraulic conductivity.

As indicated above, Equation 150 is not valid in the case considered. If Equation 150 had been used, it would have given:

$$k_{measured-req} = 2.5\times1.0\times1.2\times1.5\,\frac{\left(1.157\times10^{-6}\right)(30)}{(0.3)\sin\left(\tan^{-1}0.02\right)} = 2.60\times10^{-2}\ \text{m/s}$$

It appears that Equation 150 overestimates the required hydraulic conductivity.

END OF EXAMPLE 6

4.6 Conclusion of Section 4

Based on the discussions presented in Section 4 and Example 5, it appears that the approximate solution (Equation 139) derived from Equation 74 (i.e. t_{lim} used as an approximate value for t_{max}) provides a good approximation of the required hydraulic transmissivity of the liquid collection layer in all cases where a geosynthetic liquid collection layer is used. Use of the more accurate equations (i.e. equations derived from Giroud's equations) requires significantly more effort, but does not provide significant improvement in accuracy.

In contrast, based on the discussions presented in Section 4 and Example 6, it appears that, in the case of granular liquid collection layers, the use of equations derived from Equation 74 is not accurate and leads to an overestimate of the required hydraulic conductivity. Therefore, in the case of granular liquid collection layers it is recommended to use Equations 153, 154, 156, and 157 (i.e. equations derived from Giroud's equations) to calculate the required hydraulic conductivity of the liquid collection layer material. When Equation 74 is used, the numerical value of the factor of safety is the same whether the factor of safety is applied to the maximum liquid thickness, FS_T, or to the hydraulic characteristics of the liquid collection layer, FS_H. In contrast, when Giroud's equations are used, different values are obtained for FS_T and FS_H.

5 CONCLUSIONS

The basic requirement for the hydraulic design of liquid collection layers is that the maximum thickness of liquid in the liquid collection layer be less than an allowable thickness. Accordingly, equations are presented for calculating the maximum liquid thickness in liquid collection layers. Also, equations are presented for determining the required hydraulic conductivity of the liquid collection layer material and the required hydraulic transmissivity of the liquid collection layer to ensure that the maximum liquid thickness is less than the allowable thickness.

The following conclusions can be drawn from the present paper:

- The differential equation governing the thickness of liquid in a liquid collection layer has been solved numerically by Giroud et al. (1992) and analytically by McEnroe (1993) for the maximum liquid thickness. The development of the analytical solution by McEnroe is a major step forward in the design of liquid collection layers. The numerical values of the maximum liquid thickness provided by both the numerical and analytical solutions are very close. It may, therefore, be concluded that both solutions are correct.

- The analytical solution by McEnroe consists of a set of three equations (known as "McEnroe's equations", Equations 49 to 51). The use of McEnroe's equations requires lengthy calculations. Furthermore, one of McEnroe's equations (i.e. the equation for $R < 0.25$) is extremely sensitive to the precision of the input parameters. For some values of the parameters, calculations must be performed with more than 10 digits (and even, in some cases, more than 15) to avoid significant inaccuracies. This makes it potentially unsafe to use McEnroe's equations with pocket calculators routinely used for engineering calculations. However, being the correct analytical solution to the governing differential equation, McEnroe's equations should be regarded as the reference against which other solutions are to be evaluated.
- For practical applications, an accurate value of the maximum thickness of liquid in a liquid collection layer can be calculated using a semi-empirical equation known as the "modified Giroud's equation" (Equation 40). This equation is simple to use and is not overly sensitive to the precision of the input parameters. The difference between values of the maximum liquid thickness calculated using the modified Giroud's equation and the reference values calculated using McEnroe's equations is very small; it is less than 1% for most usual values of the parameters. Therefore, use of the modified Giroud's equation is the recommended approach for the design of liquid collection layers when accurate determination of the maximum liquid thickness is needed. Differently than in McEnroe's solution, which requires a set of three equations to cover the entire range of parameter values, the modified Giroud's equation is a single equation that covers the entire range of parameter values.
- A good approximation of the maximum thickness of liquid in a liquid collection layer is provided by a simple equation known as the "original Giroud's equation" (Equation 16). The difference between values of the maximum liquid thickness calculated using the original Giroud's equation and the reference values calculated using McEnroe's equations is less than 13% for the entire range of usual values of the parameters. This degree of precision is sufficient in most applications. Furthermore, the use of the original Giroud's equation is conservative (i.e. the values of the maximum liquid thickness calculated using the original Giroud's equation are higher than the reference values calculated using McEnroe's equations).
- An advantage of both the original and the modified Giroud's equations is that they are particularly suitable to perform parametric studies.
- A very simple equation (Equation 74) provides an approximate value of the liquid thickness in a liquid collection layer that is valid only within a certain range of parameter values. A discussion of the influence of the parameters (Section 2.11) shows that, if the approximate value of the maximum liquid thickness calculated using Equation 74 is less than one tenth of the height of the liquid collection layer slope, this approximate value is an acceptable approximation because it is less than 17% above the rigorously calculated value. As a result, Equation 74 is valid in virtually all cases where a typical geosynthetic drainage layer is used, provided that the maximum liquid thickness is less than the thickness of the geosynthetic drainage layer.
- The various equations proposed for calculating the maximum liquid thickness can be normalized in relation to the liquid thickness that corresponds to the limit case in which the dimensionless parameter λ tends toward zero, t_{lim} . The normalized solu-

tions are helpful for comparing of the accuracy of different solutions and for defining the location of the maximum liquid thickness along the liquid collection layer.

- The governing differential equation used by McEnroe (Equation 42) is slightly different from the governing differential equation used in the present paper (Equation 25). The difference is due to different approximations regarding the hydraulic head. These approximations are needed because the hydraulic head varies along the liquid collection layer slope. A discussion presented in Section 2.9 shows that Equation 25 is preferable because it leads to normalized solutions that do not directly depend on β.
- Giroud's equations (Equation 16 and 40) and the simple equation (Equation 74) can be solved for the hydraulic conductivity and can, therefore, be used to obtain equations that give the required value of the hydraulic conductivity of the liquid collection layer material and the hydraulic transmissivity of the liquid collection layer.
- The use of the McEnroe's equations appears to be unsuitable for the determination of the characteristics of the liquid collection material needed to ensure that the liquid thickness is less than the allowable thickness.
- From a practical standpoint, the equations provided in the present paper are used with reduction factors and a factor of safety. The reduction factors account for the decrease in hydraulic properties (characteristics) of the liquid collection layer in soil, under stress, and with time. The factor of safety can be applied in two ways: it can be applied to the maximum liquid thickness, FS_T , or to the hydraulic characteristics of the liquid collection layer, FS_H. Detailed guidance on the use of reduction factors and factor of safety is provided in Sections 3 and 4.
- Equations other than McEnroe's and Giroud's have been published and are discussed in Appendix D. Most of these equations lead to incorrect values of the liquid thickness. This is, in particular, the case of Moore's equations, which are still widely used in the United States, even though they may lead to unconservative designs.

In conclusion, it appears that all the equations required for the hydraulic design of liquid collection layers are available. The present paper provides an overview of the available equations (Section 2) and provides guidance for the use of these equations (Sections 3 or 4). In particular, the present paper provides detailed guidance for the use of reduction factors to quantify the decrease in flow capacity of liquid collection layers due to a variety of mechanisms, such as thickness reduction caused by the applied stresses and hydraulic conductivity reduction caused by clogging. Also, the present paper provides guidance for the use of safety factors. Two design approaches are presented: the "thickness approach" (Section 3) and the "hydraulic characteristic approach" (Section 4). With the thickness approach, a given liquid collection material is considered and the design engineer checks that the maximum liquid thickness is less than the allowable thickness; with the hydraulic characteristic approach, the design engineer determines the required hydraulic characteristics of the liquid collection layer. The two approaches are equivalent, and a careful design engineer will use both.

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NOTATIONS

Basic SI units are given in parentheses.

A = cross-sectional area of flow (m^2)

A' = parameter defined by Equation 62 (dimensionless)

A^* = parameter defined by Equation 46 (dimensionless)

A_h = horizontal area through which flow rate Q_h is measured (m^2)

B = width of liquid collection layer in direction perpendicular to direction of flow (m)

B' = parameter defined by Equation 62 (dimensionless)

B^* = parameter defined by Equation 46 (dimensionless)

C = constant used in integration (m^2)

D = depth (measured vertically) of liquid in liquid collection layer (m)

D_L = depth of liquid collected in horizontal pan during time Δt^* (m)

FS	=	factor of safety (dimensionless)
FS_H	=	factor of safety applied to hydraulic characteristics of liquid collection layer (dimensionless)
FS_T	=	factor of safety applied to maximum liquid thickness (dimensionless)
g	=	acceleration due to gravity (m/s^2)
H	=	height (measured vertically) of liquid collection layer (m)
h	=	hydraulic head (m)
h_A	=	hydraulic head at Point A (m)
h_{AB}	=	hydraulic head corresponding to liquid thickness AB (m)
h_B	=	hydraulic head at Point B (m)
h_{max}	=	hydraulic head related to the maximum liquid thickness (m)
i	=	hydraulic gradient (dimensionless)
i_{equiv}	=	equivalent hydraulic gradient (dimensionless)
i_{toe}	=	hydraulic gradient at toe of liquid collection layer slope (dimensionless)
j	=	modifying factor applied to original Giroud's equation to obtain modified Giroud's equation (dimensionless)
k	=	hydraulic conductivity of liquid collection layer material (m/s)
k_{LTIS}	=	long-term-in-soil hydraulic conductivity, i.e. hydraulic conductivity of liquid collection layer material subjected to conditions that can cause development of chemical and/or biological clogging during design life of liquid collection layer (m/s)
k_{LTISFS}	=	factored long-term-in-soil hydraulic conductivity, i.e. in-soil-long-term hydraulic conductivity divided by overall factor of safety (m/s)
$k_{measured}$	=	hydraulic conductivity of specimen of granular material, representative of granular material as installed, measured in hydraulic conductivity test performed with water during short periods of time so that clogging does not develop (m/s)
$k_{measured\text{-}req}$	=	required value of hydraulic conductivity of liquid collection layer material (m/s)
L	=	length (measured horizontally) of rectangular liquid collection layer in direction of flow (m)
p	=	liquid pressure (Pa)
p_B	=	liquid pressure at Point B (Pa)
Q	=	rate of flow in liquid collection layer (m^3/s)
Q_h	=	rate of liquid flow through horizontal area A_h (m^3/s)
Q_t	=	rate of flow through cross section located at horizontal distance x from top of slope (m^3/s)
q_h	=	rate of liquid supply expressed per unit surface area measured horizontally (m/s)

R	=	parameter defined by Equation 9 (dimensionless)
RF	=	reduction factor (dimensionless)
RF_{BC}	=	reduction factor for flow capacity decrease due to biological clogging of liquid collection layer material (dimensionless)
RF_{CC}	=	reduction factor for flow capacity decrease due to chemical clogging of liquid collection layer material (dimensionless)
RF_{CD}	=	reduction factor for chemical degradation, i.e. decrease of hydraulic transmissivity due to chemical degradation of polymeric compound(s) used to make geocomposite (dimensionless)
RF_{CR}	=	reduction factor for flow capacity decrease due to creep of transmissive core of geocomposite or other geosynthetic liquid collection layer (dimensionless)
RF_{IMCO}	=	reduction factor for immediate compression, i.e. decrease of hydraulic transmissivity due to compression of transmissive core following immediate application of stress (dimensionless)
RF_{IMIN}	=	reduction factor for immediate intrusion, i.e. decrease of hydraulic transmissivity due to geotextile intrusion into transmissive core following immediate application of stress (dimensionless)
RF_{IN}	=	reduction factor for delayed intrusion, i.e. decrease of hydraulic transmissivity over time due to geotextile intrusion into transmissive core resulting from time-dependent deformation of geotextile (dimensionless)
RF_{PC}	=	reduction factor for particulate clogging, i.e. decrease of hydraulic transmissivity due to clogging by particles migrating into transmissive core (dimensionless)
r	=	parameter defined by Equation D-5 (dimensionless)
t	=	thickness (measured perpendicular to slope) of liquid in liquid collection layer (m)
t_{LCL}	=	thickness of liquid collection layer (m)
t_{LTIS}	=	long-term-in-soil thickness of transmissive core of geosynthetic (m)
t_{allow}	=	allowable thickness of liquid in liquid collection layer, i.e. lesser of: (i) maximum liquid thickness prescribed by regulations; and (ii) thickness of liquid collection layer (m)
t_{lim}	=	maximum thickness of liquid in liquid collection layer in special case where q_h is small and k and β are large (m)
t_{max}	=	maximum thickness of liquid in liquid collection layer (m)
$t_{max}(k_{LTIS})$	=	maximum thickness of liquid in liquid collection layer as a function of k_{LTIS} (m)
$t_{max}(k_{LTISFS})$	=	maximum thickness of liquid in liquid collection layer as a function of k_{LTISFS} (m)
t_{toe}	=	liquid thickness at toe of liquid collection layer slope (m)

t_{virgin}	=	thickness of liquid collection layer under no compressive stress (m)
v	=	liquid velocity (m/s)
X	=	distance measured along slope between top of slope and considered cross section of liquid collection layer (m)
x	=	horizontal distance between top of slope and considered cross section of liquid collection layer (m)
x_m	=	horizontal distance between top of slope and location of maximum liquid thickness (m)
z	=	elevation (m)
z_A	=	elevation of Point A (m)
z_B	=	elevation of Point B (m)
β	=	slope angle (°)
Δt^*	=	period of time (s)
λ	=	parameter (sometimes called characteristic parameter) defined by Equation 10 (dimensionless)
ρ	=	liquid density (kg/m^3)
θ	=	hydraulic transmissivity of liquid collection layer (m^2/s)
θ_{LTIS}	=	long-term-in-soil hydraulic transmissivity, i.e. hydraulic transmissivity of geosynthetic in soil under maximum stress anticipated during design life of liquid collection layer and subjected to time-dependent mechanisms (such as creep, chemical clogging, and biological clogging) during design life of liquid collection layer (m^2/s)
$\theta_{measured}$	=	value of hydraulic transmissivity of liquid collection layer measured in laboratory test (m^2/s)
$\theta_{measured\text{-}req}$	=	required value of hydraulic transmissivity of liquid collection layer (m^2/s)
θ_{virgin}	=	hydraulic transmissivity of virgin geosynthetic, i.e. geosynthetic subjected to hydraulic transmissivity test under no stress (or, at a maximum, small stress needed to keep geosynthetic flat during hydraulic transmissivity test) and performed with water during short period of time so that clogging does not develop (m^2/s)

APPENDIX A

DERIVATION OF THE ORIGINAL GIROUD'S EQUATION

A.1 Flow Rate

Darcy's equation is written as follows:

$$Q = k\,i\,A = k\,i\,B\,t \tag{A-1}$$

where: Q = flow rate in the liquid collection layer in a given cross section perpendicular to the flow; k = hydraulic conductivity of the liquid collection layer material; i = hydraulic gradient at the considered cross section; A = cross-sectional area of the flow in the considered cross section; B = width of the liquid collection layer; and t = thickness of the liquid in the liquid collection layer at the considered cross section.

Based on mass conservation, the flow rate through a cross section located at a horizontal distance x from the top of the slope (Figure 5) is given by:

$$Q = q_h\,x\,B \tag{A-2}$$

where: q_h = liquid supply rate per unit area measured horizontally; x = horizontal distance from the top of the slope to the considered cross section; and B = width of the liquid collection layer.

A.2 Calculation of the Liquid Thickness

Combining Equations A-1 and A-2 gives:

$$t = \frac{q_h\,x}{k\;i} \tag{A-3}$$

The hydraulic gradient at the location of the maximum liquid thickness is $\sin\beta$ since the liquid surface is parallel to the slope at the location where the maximum liquid thickness takes place. Therefore Equation A-3 gives:

$$t_{max} = \frac{q_h\,x_m}{k\,\sin\beta} \tag{A-4}$$

where: t_{max} = maximum liquid thickness; β = liquid collection layer slope angle; and x_m = horizontal distance between the top of the slope and the location of the maximum liquid thickness.

The distance, x_m , from the top of the slope to the location where the maximum liquid thickness occurs is unknown. If it is replaced by L in Equation A-4, in order to perform a simple calculation, the numerator of Equation A-4 is increased and the denominator should be increased to compensate for that. This can be achieved by using a value of the hydraulic gradient greater than $\sin\beta$, hence:

$$t_{max} = \frac{q_h\,L}{k\;i_{equiv}} \tag{A-5}$$

where i_{equiv} is the equivalent hydraulic gradient used for the calculation of the maximum liquid thickness, t_{max} , using Equation A-5. As indicated above, this equivalent hydraulic gradient must be greater than $\sin\beta$.

A.3 Selection of an Equivalent Hydraulic Gradient

As a guidance for the selection of the equivalent hydraulic gradient, the well-known case where $\beta = 0$ was considered. In this case (Figure 6 in the main text of the present paper), the maximum liquid thickness occurs for $x_m = 0$ and is given by the following classical equation (Giroud and Houlihan 1995):

$$\frac{t_{max}}{L} = \sqrt{\frac{q_h}{k}} \tag{A-6}$$

The derivation of Equation A-6 may be found in Section 2.3.2. Equation A-6 can be written:

$$\frac{q_h}{k} = \left(\frac{t_{max}}{L}\right)^2 = \left(\frac{t_{max}}{L}\right)\left(\frac{t_{max}}{L}\right) \tag{A-7}$$

hence:

$$t_{max} = \frac{q_h\, L}{k\,\left(t_{max} / L\right)} \tag{A-8}$$

Comparing Equations A-5 and A-8 shows that Equation A-6 could have been derived from Equation A-5 using the following value for the equivalent hydraulic gradient, i_{equiv} :

$$i_{equiv} = \frac{t_{max}}{L} \tag{A-9}$$

The hydraulic gradient must be defined based on the hydraulic head, not the liquid thickness. In the case shown in Figure 6, $\beta = 0$; therefore, Equation 4 (in the main text of the present paper) gives:

$$h_{max} = t_{max} \tag{A-10}$$

where h_{max} is the hydraulic head related to the maximum liquid thickness.

Combining Equations A-9 and A-10 shows that, in reality, in Equation A-6, the hydraulic gradient is:

$$i_{equiv} = \frac{h_{max}}{L} \tag{A-11}$$

The foregoing analysis shows that it was possible to establish Equation A-6 by using an equivalent hydraulic gradient equal to the hydraulic head associated with the maximum thickness divided by the horizontal projection of the length of the liquid collection layer. This approach, which gave Equation A-11 for the special case where $\beta = 0$, gives the following equation for the general case where $\beta \neq 0$:

$$i_{equiv} = \sin\beta + \frac{h_{max}}{L / \cos\beta} \tag{A-12}$$

Equation 4 (in the main text of the present paper) gives:

$$h_{max} = t_{max} \cos\beta \tag{A-13}$$

Combining Equations A-12 and A-13 gives:

$$i_{equiv} = \sin\beta + \frac{t_{max}}{L} \cos^2\beta \tag{A-14}$$

Combining Equations A-5 and A-14 gives:

$$\frac{q_h}{k} = \frac{t_{max}}{L}\left(\sin\beta + \frac{t_{max}}{L} \cos^2\beta\right) \tag{A-15}$$

hence:

$$\left(\frac{t_{max}}{L}\right)^2 \cos^2\beta + \frac{t_{max}}{L} \sin\beta - \frac{q_h}{k} = 0 \tag{A-16}$$

Solving this quadratic equation gives:

$$\frac{t_{max}}{L} = \frac{\sqrt{\sin^2\beta + \dfrac{4q_h}{k}\cos^2\beta} - \sin\beta}{2\cos^2\beta} = \frac{\sqrt{\tan^2\beta + \dfrac{4q_h}{k}} - \tan\beta}{2\cos\beta} \tag{A-17}$$

Equation A-17 is identical to Equation 16 in the main text of the present paper.

A.4 Limit Cases

To confirm the validity of Equation A-17, it is important to check that it tends toward the well known equations for the two limit cases: when $\beta = 0$, and when q_h/k is very small.

When $\beta = 0$, Equation A-17 gives:

$$\frac{t_{max}}{L} = \frac{\sqrt{\dfrac{4q_h}{k}}}{2} = \sqrt{\frac{q_h}{k}} \tag{A-18}$$

Equation A-18 is the well-known equation for the case where $\beta = 0$ (see Equation 31 in the main text of the present paper, and Equation A-6). Therefore, Equation A-17 tends toward the appropriate limit for $\beta = 0$.

Equation A-17 can be written as follows:

$$\frac{t_{max}}{L} = \frac{\sqrt{1 + \dfrac{q_h}{k \tan^2\beta}} - 1}{2\cos\beta / \tan\beta} \tag{A-19}$$

When q_h/k is very small, the following classical relationship can be used:

$$\sqrt{1 + \frac{q_h}{k \ \tan^2\beta}} \approx 1 + \frac{q_h}{2 \ k \ \tan^2\beta} \tag{A-20}$$

Combining Equations A-19 and A-20 gives:

$$\frac{t_{max}}{L} \approx \frac{\dfrac{q_h}{2 \ k \ \tan^2\beta}}{\dfrac{2 \ \cos\beta}{\tan\beta}} \approx \frac{q_h}{k \ \sin\beta} \tag{A-21}$$

Equation A-21 is the well-known equation for the case where q_h/k is very small (Section 2.8). Therefore, Equation A-17 tends toward the appropriate limit when q_h/k is very small.

The fact that Equation A-17 tends toward the well-known equations for the two limit cases confirms the validity of Equation A-17.

APPENDIX B

BOUNDARY CONDITIONS AT THE TOE OF THE LIQUID COLLECTION LAYER SLOPE

B.1 Hydraulic Gradient

The hydraulic gradient is defined by:

$$i = -\frac{\mathrm{d}h}{\mathrm{d}x / \cos\beta} \tag{B–1}$$

where: x = horizontal distance from the top of the slope to the considered cross section; β = liquid collection layer slope angle; and h = hydraulic head defined by:

$$h = z + \frac{p}{\rho g} + \frac{v^2}{2g} \tag{B–2}$$

where: z = elevation; p = pressure; ρ = liquid density; g = acceleration due to gravity; and v = liquid velocity. The term v^2 is negligible in the case of laminar flow, hence:

$$h = z + \frac{p}{\rho g} \tag{B–3}$$

In Figure 2 (in the main text of the present paper), the hydraulic head is the same in A and B since AB is an equipotential line. Therefore:

$$h = h_A = h_B = z_B + \frac{p_B}{\rho g} \tag{B–4}$$

where: h_A = hydraulic head at Point A; h_B = hydraulic head at Point B; z_B = elevation at Point B; and h_B = pressure at Point B.

If A is at the horizontal distance x from the top of the liquid collection layer slope, the elevation of B can be calculated as follows based on Figures 2 and 5:

$$z_B = (L - x)\ \tan\beta\ +\ t\ \cos\beta \tag{B–5}$$

where t is the liquid thickness.

Combining Equations B-4 and B-5 gives:

$$h = (L - x)\ \tan\beta\ +\ t\ \cos\beta + \frac{p_B}{\rho g} \tag{B–6}$$

The pressure in B, p_B , is the atmospheric pressure, which does not vary as a function of x. Therefore, combining Equations B-1 and B-6 gives:

$$i = \sin\beta\ -\ \cos^2\beta\ \frac{\mathrm{d}t}{\mathrm{d}x} \tag{B–7}$$

B.2 Darcy's Equation at the Toe of the Liquid Collection Layer

Darcy's equation is given by Equation A-1. At the toe of the liquid collection layer slope, the flow rate per unit width in the liquid collection layer is:

$$\frac{Q}{B} = q_h L \tag{B-8}$$

Combining Equations A-1 and B-8 gives:

$$t_{toe} = \frac{q_h L}{k\, i_{toe}} \tag{B-9}$$

where t_{toe} is the liquid thickness at the toe of the liquid collection layer.

B.3 Liquid Thickness at the Toe of the Liquid Collection Layer

Since there is free drainage at the toe of the liquid collection layer slope, it is logical to assume that the liquid surface is vertical at the toe of the slope (Figure B-1). This assumption is consistent with the shape of the liquid surface in the well-known case where the slope of the liquid collection layer is zero (Figure 6). Based on Figure B-1, the derivative of t with respect to X has the following value at the toe of the liquid collection layer slope:

$$\frac{\mathrm{d}t}{\mathrm{d}X} = -\frac{1}{\tan\beta} \tag{B-10}$$

where X is the abscissa along the slope, hence:

$$x = X \cos\beta \tag{B-11}$$

Combining Equations B-10 and B-11 gives:

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \left(\frac{1}{\cos\beta}\right)\frac{\mathrm{d}t}{\mathrm{d}X} \tag{B-12}$$

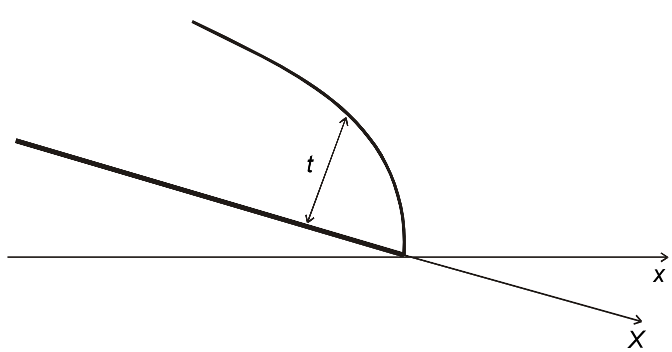

Figure B-1. Assumed profile of the liquid surface at the toe of the liquid collection layer slope.

Combining Equations B-10 and B-12 gives:

$$\frac{\mathrm{d}t}{\mathrm{d}x} = -\left(\frac{1}{\cos\beta}\right)\left(\frac{1}{\tan\beta}\right) = -\frac{1}{\sin\beta} \tag{B–13}$$

Combining Equations B-7 and B-13 gives the hydraulic gradient at the toe of the liquid collection layer slope as follows:

$$i_{toe} = \sin\beta - \left(\frac{\cos^2\beta}{-\sin\beta}\right) = \frac{\sin^2\beta + \cos^2\beta}{\sin\beta} = \frac{1}{\sin\beta} \tag{B–14}$$

Combining B-9 and B-14 gives the thickness of liquid at the toe of the liquid collection layer slope as follows:

$$t_{toe} = \frac{q_h \; L \; \sin\beta}{k} \tag{B–15}$$

In the special case where $\beta = 0$, $i_{toe} = \infty$ according to Equation B-14 and $t_{toe} = 0$ according to Equation B-15. This is consistent with Figure 6. It is important to note that vertical flow corresponds to a hydraulic gradient of one only if the liquid collection layer is vertical. If the liquid collection layer is horizontal, vertical flow corresponds to an infinite gradient. If the liquid collection layer slope is β, vertical flow corresponds to a hydraulic gradient of $1/\sin\beta$, as shown above.

B.4 Boundary Conditions at Toe Used for the Development of Equations

To solve the governing differential equation numerically, Giroud et al. (1992) assumed $t_{toe} = 0$ for all values of β, which implies $i_{toe} = \infty$ for all values of β. Since the modified Giroud's equation was developed based on the results of the numerical solution of the governing differential equation (Section 2.3), it is legitimate to consider that the modified Giroud's equation corresponds to the same boundary conditions, i.e. $t_{toe} = 0$ and $i_{toe} = \infty$. The same boundary conditions were used by the authors of the present paper to derive Equations 52 to 54, i.e. the McEnroe's equations for $t_{toe} = 0$.

To develop his equations for free drainage at the toe of the liquid collection layer slope, McEnroe (1993) used the boundary conditions defined by:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = -1 \tag{B–16}$$

The rationale presented by McEnroe (1993) for this assumption shows that it is plausible, but does not demonstrate that it is the best possible assumption to represent free drainage conditions at the toe. Combining Equations B-1 and B-16 gives:

$$i_{toe} = \cos\beta \tag{B–17}$$

It should be noted that the assumption made by McEnroe does not correspond to a hydraulic gradient of one at the toe of the slope (contrary to what is sometimes stated by users of McEnroe's equations).

Combining Equations B-9 and B-17 gives:

$$t_{toe} = \frac{q_h\ L}{\cos\beta} \tag{B–18}$$

A summary of the boundary conditions used by different authors is presented in Table B-1. The values of t_{toe} resulting from the analysis presented in Section B.3 (based on vertical liquid surface assumption at the toe) and from the assumptions made by Giroud et al. (1992) and McEnroe (1993) are compared in Table B-2. Inspection of Table B-2 shows that the value of t_{toe} used by Giroud et al. (1992) is closer than the value used by McEnroe (1993) to the value of t_{toe} obtained for vertical flow at the toe, which may be considered as the most logical assumption.

B.5 Conclusions

The purpose of Appendix B was to clarify the question of the boundary conditions at the toe of a liquid collection layer slope. It appears that different boundary conditions were used by Giroud et al. (1992) and McEnroe (1993), and it appears that these boundary conditions may not be as logical as the boundary conditions that consist of assuming that the liquid flow is vertical at the toe of the liquid collection layer slope. Fortunately, the comparisons presented in Section 2.5 show that the boundary conditions used by Giroud et al. (1992) and McEnroe (1993) have a negligible influence on the calculated value of the maximum liquid thickness.

Table B-1. Summary of boundary conditions at the toe of a liquid collection layer.

Reference	Assumption	i_{toe}	t_{toe}
The present paper	Vertical liquid surface at toe	$\frac{1}{\sin\beta}$	$\frac{q_h L\ \sin\beta}{k}$
Giroud et al. (1992)	Zero liquid thickness at toe	∞	0
McEnroe (1993)	$\frac{dh}{dx} = -1$ at toe	$\cos\beta$	$\frac{q_h L}{k\cos\beta}$

Table B-2. Values of t_{toe} for different boundary conditions at the toe of a liquid collection layer.

β	$\tan\beta$	The present paper $\frac{t_{toe}}{q_h L/k} = \sin\beta$	Giroud et al. (1992) $t_{toe} = 0$	Giroud et al. (1992) $\frac{t_{toe}}{q_h L/k} = \frac{1}{\cos\beta}$
0°	0	0	0	1.00
1.1°	0.02	0.02	0	1.00
5.7°	0.10	0.10	0	1.00
18.4°	0.33	0.32	0	1.05
26.6°	0.50	0.45	0	1.12
45.0°	1.00	0.71	0	1.41

APPENDIX C

DERIVATION OF THE MODIFYING FACTOR

As indicated in Section 2.3.3 in the main text of the present paper, the governing differential equation for the liquid thickness was solved numerically (Giroud et al. 1992). The senior author then compared values calculated using the original Giroud's equation (i.e. Equation 16 in the main text of the present paper) with the accurate values of the maximum liquid thickness obtained numerically. It appeared that the difference between the values calculated using the original Giroud's equation and the values obtained numerically depended only on the dimensionless parameter λ defined by Equation 9. The senior author noted that, when the difference was plotted as a function of logλ, it was represented by an inverted bell-shaped curve (Figure 7) with values between 1.00 and 0.88. Therefore, the senior author proposed the following equation for the dimensionless factor j:

$$j = 1 - 0.12 \ e^{-y^2} \tag{C-1}$$

where y is a function of logλ.

After trial and errors, the senior author found that the following equation ensures that, when Equation A-17 is multiplied by the dimensionless factor j, one obtains values of the maximum liquid thickness very close to the accurate values obtained numerically:

$$j = 1 - 0.12 \exp\left[-\left[\log\left(\frac{8(q_h / k)}{5 \tan^2\beta} \right)^{5/8} \right]^2 \right] \tag{C-2}$$

APPENDIX D

REVIEW OF OTHER REPORTED SOLUTIONS FOR THE MAXIMUM LIQUID THICKNESS IN LIQUID COLLECTION LAYERS

D.1 Presentation of the Equations

As indicated in Section 2.12, solutions other than those discussed in the present paper have been proposed in the technical literature for the maximum liquid thickness in liquid collection layers. A brief review is presented below. Different authors have reported their solutions as the maximum head, maximum depth, or maximum thickness of the liquid. For consistency, all equations below are reported as maximum liquid thickness, t_{max}.

Moore (1980) proposed an equation ("the first Moore's equation") for which, to the best of the authors' knowledge, no derivation was published. The equation is:

$$t_{max} = \cos\beta \sqrt{\frac{q_h}{k}} \left(\frac{k \ \tan^2\beta}{q_h} + 1 - \frac{k \ \tan\beta}{q_h} \sqrt{\frac{q_h}{k} + \tan^2\beta} \right) L \qquad \text{(D-1)}$$

Giroud and Houlihan (1995) compared the first Moore's equation (Equation D-1) to the accurate numerical solution (Giroud et al. 1992) and showed that the first Moore's equation significantly overestimates the leachate thickness in most practical cases (e.g. overestimation by a ratio of 1.0 to 1500, i.e. by 0 to 150,000%).

Moore (1983) proposed another equation ("the second Moore's equation") for which, to the best of the authors' knowledge, no derivation was published. The equation is:

$$t_{max} = \cos\beta \left(\sqrt{\frac{q_h}{k} + \tan^2\beta} \ - \ \tan\beta \right) L \qquad \text{(D-2)}$$

Giroud and Houlihan (1995) compared the second Moore's equation (Equation D-2) to the accurate numerical solution (Giroud et al. 1992) and showed that the second Moore's equation significantly underestimates the leachate thickness in most practical cases, which leads to unconservative designs (e.g. underestimation by 55%).

Lesaffre (1987) proposed the following equation:

$$t_{max} = \frac{L}{\cos\beta \left[\frac{4k}{q_h} + \left(\frac{k}{q_h} - 1 \right)^2 \tan^2\beta \right]^{1/2}} \qquad \text{(D-3)}$$

Lesaffre equation (Equation D-3) was developed only for the case illustrated by Figure 4b, and gives accurate numerical values for that case. This equation should not be used for the case illustrated by Figure 4a.

Masada (1998) developed a solution by making a number of simplifying assumptions. The solution proposed by Masada (1998) for the conditions illustrated by Figure 4a is:

$$t_{max} = \frac{\cos\beta}{r - \tan^2\beta}\left(-r\ \tan\beta + \sqrt{r}\ \tan^2\beta - \sqrt{r\ \tan^4\beta - 2r^2\ \tan^2\beta + r^3}\right)L \quad \text{(D-4)}$$

where:

$$r = \frac{q_h}{k\ \cos^2\beta} \quad \text{(D-5)}$$

The solution proposed by Masada (1998) for the conditions illustrated by Figure 4b is:

$$t_{max} = \frac{2}{3}\frac{q_h}{k\ \sin\beta}L \quad \text{(D-6)}$$

Since the equations proposed by Masada have been published only recently, no evaluation of these equations has been published yet. The evaluation of Masada's equations by the authors of the present paper is provided in Section D.2.

D.2 Discussion of Masada's Equations

D.2.1 *Relationship Between r and λ*

It should be noted that:

$$r = \lambda\ (\tan\beta\ /\cos\beta\ )^2 \quad \text{(D-7)}$$

where r is defined by Equation D-5 and λ is defined by Equation 9 in the main text of the present paper. Therefore:

- Equation D-4, which is valid for $\lambda > 0.25$ (Figure 4a), is valid for $r > 0.25\ (\tan\beta\ /\cos\beta)^2$; and
- Equation D-6, which is valid for $\lambda < 0.25$ (Figure 4b), is valid for $r < 0.25\ (\tan\beta\ /\cos\beta)^2$.

D.2.2 *Discussion of Equation D-4*

It should be noted that Equation D-4 could have been simplified. Indeed, Equation D-4 is equal to:

$$t_{max} = \frac{L\sqrt{r}\cos\beta}{r - \tan^2\beta}\left[-\sqrt{r}\ \tan\beta + \tan^2\beta - \sqrt{\left(\tan^2\beta - r\right)^2}\ \right] \quad \text{(D-8)}$$

This leads to two cases.

First Case. If $r > \tan^2\beta$ (which is equivalent to $\lambda > \cos^2\beta$):

$$\sqrt{\left(\tan^2\beta - r\right)^2} = r - \tan^2\beta \quad \text{(D-9)}$$

Combining Equations D-8 and D-9 gives:

$$t_{max} = \frac{L\sqrt{r}\cos\beta}{r - \tan^2\beta}\left(-\sqrt{r}\ \tan\beta + 2\tan^2\beta\ - r\right) \tag{D-10}$$

hence:

$$t_{max} = \frac{L\sqrt{r}\cos\beta\left[-\left(\sqrt{r} - \tan\beta\right)\left(\sqrt{r} + 2\ \tan\beta\right)\right]}{\left(\sqrt{r} - \tan\beta\right)\left(\sqrt{r} + \tan\beta\right)} \tag{D-11}$$

hence:

$$t_{max} = -\frac{L\ \sqrt{r}\ \ \cos\beta\ \left(\sqrt{r} + 2\ \tan\beta\right)}{\sqrt{r}\ +\ \tan\beta} = -\ L\ \sqrt{r}\ \ \cos\beta\ \left(1 + \frac{\tan\beta}{\sqrt{r}\ +\ \tan\beta}\right) \tag{D-12}$$

It appears that Equation D-12 gives negative values of t_{max} . Therefore, Equation D-4 proposed by Masada (1998) does not provide a solution for $\lambda > \cos^2\beta$.

Second Case. If $r < \tan^2\beta$ (which is equivalent to $\lambda < \cos^2\beta$):

$$\sqrt{\left(\tan^2\beta\ - r\right)^2}\ =\ \tan^2\beta\ -\ r \tag{D-13}$$

Combining Equations D-8 and D-13 gives:

$$t_{max} = \frac{L\sqrt{r}\cos\beta}{r - \tan^2\beta}\left(r - \sqrt{r}\ \tan\beta\right) \tag{D-14}$$

hence:

$$t_{max} = \frac{L\ r\ \ \cos\beta\ \left(\sqrt{r} - \tan\beta\right)}{\left(\sqrt{r} - \tan\beta\right)\left(\sqrt{r} + \tan\beta\right)} \tag{D-15}$$

hence:

$$t_{max} = \frac{L\ \ r\ \ \cos\beta}{\sqrt{r}\ +\ \tan\beta} \tag{D-16}$$

Equation D-16 is much simpler than the equation given by Masada (Equation D-4). The above discussion regarding Equation D-4 can be summarized as follows: in the paper by Masada (1998), the limit of validity of Equation D-4 is not indicated and the expression given for D-4 is much more complex than it needs to be. Systematic calculations performed by the authors of the present paper using Equation D-16 (which is equivalent to Masada's Equation D-4) show that this equation can be very inaccurate (by up to 40%) and systematically underestimates t_{max} , which leads to unconservative designs.

D.2.3 *Discussion of Equation D-6*

The equation proposed by Masada (1998) for the case where $\lambda < 0.25$, Equation D-6, appears to be incorrect, because it is Equation 74 of the present paper multiplied by an arbitrary factor 2/3. In particular, Equation D-6 does not provide the accurate solution for the well-known limit case where q_h/k is very small.

D.2.4 *Conclusion on Masada's Equations*

In conclusion, the equations provided by Masada do not appear to provide an accurate solution for the maximum liquid thickness in a liquid collection layer.

D.3 Conclusion of Appendix D

The equations discussed in Appendix D do not provide a valid solution for the maximum liquid thickness in a liquid collection layer. One equation (Lesaffre's equation) is correct, but was developed for another field of application, the drainage of slopes, and cannot be used for the liquid collection layers discussed in the present paper. Comparisons of the other equations with accurate solutions show significant inaccuracies, which often lead to major unconservatism.

It should be emphasized that the solutions discussed in Appendix D were obtained for steady-state conditions. Insight into the evaluation of the liquid thickness in liquid collection layers under unsteady-state conditions is provided by Wong (1977), Lentz (1981), Demetracopoulos et al. (1984), McEnroe (1989a), and Giroud and Houlihan (1995).

APPENDIX E

COMPARISON BETWEEN FS_T AND FS_H

The analysis presented in Appendix E is based on the original Giroud's equation (Equation 16 or 17). Therefore, the FS_H/FS_T ratio expressed by Equation E-4 is valid only when the original Giroud's equation is used to calculate t_{max}. When the modified Giroud's equation is used, the FS_H/FS_T ratio expressed by Equation E-4 gives only an approximate indication of the actual value of FS_H/FS_T.

Combining Equations 17 and 98 from the main text of the present paper gives:

$$FS_T = \frac{2 \cos\beta \ (t_{LTIS} / L)}{\tan\beta \ \left(\sqrt{1+4\lambda} - 1\right)} \tag{E-1}$$

Equation 146 from the main text of the present paper gives:

$$FS_H = \frac{\dfrac{t_{LTIS} \ \sin\beta}{L} + \left(\dfrac{t_{LTIS} \ \cos\beta}{L}\right)^2}{q_h / k} \tag{E-2}$$

Combining Equation 9 from the main text of the present paper and Equation E-2 gives:

$$FS_H = \frac{\dfrac{t_{LTIS} \ \sin\beta}{L} + \left(\dfrac{t_{LTIS} \ \cos\beta}{L}\right)^2}{\lambda \ \tan^2\beta} \tag{E-3}$$

Combining Equations E-1 and E-3 gives:

$$\frac{FS_H}{FS_T} = \left[\frac{\sqrt{1+4\lambda} - 1}{2 \ \lambda}\right]\left[1 + \left(\frac{t_{LTIS}}{L}\right)\left(\frac{\cos\beta}{\tan\beta}\right)\right] \tag{E-4}$$

The term in the first set of brackets is smaller than one. However, it is close to one if λ is small, e.g. smaller than 0.25. The term in the second set of brackets is always greater than one. As a result, FS_H/FS_T can be smaller or greater than one depending on the values of the parameters. Some numerical calculations seem to provide the following approximate indications:

- in the case of a geosynthetic liquid collection layer (Figure E-1), $FS_H/FS_T \approx 1$ if $\lambda \leq 0.01$, which is the only practical case; and
- in the case of a granular liquid collection layer (Figure E-2), $FS_H/FS_T > 1$ if $\lambda < 0.1$, $FS_H/FS_T < 1$ if $\lambda > 1$, and FS_H/FS_T may be smaller or greater than 1 if $0.1 \leq \lambda \leq 1$.

It should be remembered that, based on the comment made at the beginning of Appendix E, the above conclusions on the value of the FS_H/FS_T ratio are valid only when the original Giroud's equation is used to calculate t_{max}. When the modified Giroud's equation is used, the FS_H/FS_T ratio expressed by Equation E-4 gives only an approximate indication of the actual value of FS_H/FS_T.

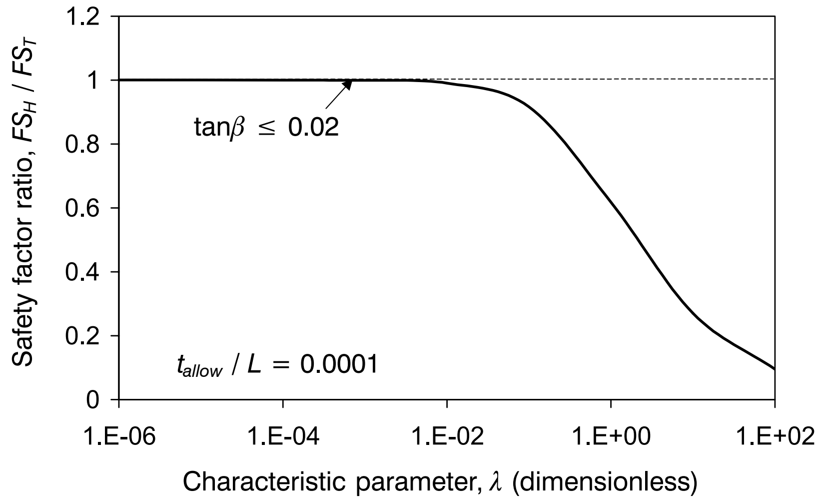

Figure E-1. Safety factor ratio, FS_H/FS_T, as a function of the characteristic parameter, λ, for typical geosynthetic liquid collection layers.

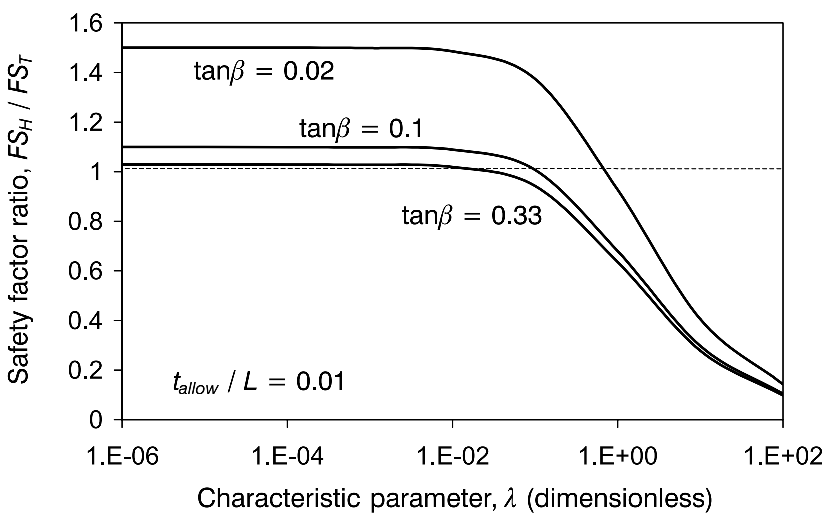

Figure E-2. Safety factor ratio, FS_H/FS_T, as a function of the characteristic parameter, λ, for typical granular liquid collection layers.