

**EXPERIMENTAL VALIDATION OF THE DISCRETE  
FRAMEWORK**

**By: Paula Pugliese, Ph.D. and Jorge G. Zornberg, Ph.D., P.E.**

**Geotechnical Research Report  
University of Colorado at Boulder**

**June 2000**

# EXPERIMENTAL VALIDATION OF THE DISCRETE FRAMEWORK

By: Paula Pugliese, Ph.D. and Jorge G. Zornberg, Ph.D., P.E.

## Introduction

The discrete approach was applied in four design projects with the objectives of experimentally establishing the equivalent shear strength of the soil to be reinforced. The experimental results were compared with the shear strength of fiber-reinforced soil predicted using the discrete approach. The four projects are: LA 22 slopes, Cardinal Road Slope Failures, Vanderbilt Stadium, and Las Colinas Slopes (TETCO).

Information regarding to the fiber-reinforcement material is shown in Table 1. Information on the soil characteristics for each of the 4 cases investigated is shown in Table 2.

**Table 1: Fiber-reinforced material properties**

Properties	Units	Values
Length	in	2
Width	in	0.289
Thickness	in	0.0017
$G_f$ (specific gravity of the fibers)		0.91
Fiber Linear Density	deniers	2610
Fiber Tensile Strength ( $\sigma_{f,ult}$ )	psi	40000

**Table 2: Soil and interface properties used for each of the 4 cases investigated**

SOIL DATA	UNIT S	LA-22	CARDINAL	VANDERBILT	TETCO
Dry unit weigh of soil ( $\gamma$ )	pcf	103.67	100.83	104.9	89.7
Friction angle' ( $\phi'$ )	( $^{\circ}$ )	26.2	24.1	35.8	11.2
Cohesion' ( $c'$ )	psi	1.6	1.5	0.8	4.1
$C_{i,c}$		0.8	0.8	0.8	0.8
$C_{i,\theta}$		0.8	0.8	0.8	0.8
$\alpha$		1	1	1	1
Fiber content ( $X_w$ )	%	0.2	0.2	0.2	0.2

## First case study – LA 22 slopes

The following five steps [(a) through (e)] lead to the determination of the equivalent shear strength to be assumed by the designer after applying the discrete approach. The fiber-induced tension is assumed to be parallel to the failure plane. The soil and fiber reinforcement characteristics used in the calculations are those indicated in Tables 1 and 2.

(a) Determination of the volumetric fiber content,  $\chi$ :

$$\chi = (\chi_w \cdot \gamma) / ((1 + \chi_w) \cdot G_f \cdot \gamma_w)$$

$$\Rightarrow \chi = 0.0036425$$

(b) Determination of the equivalent diameter,  $d_f$ :

(b1) Using reported fibers geometry:

$$d_f = ( (4 \cdot A_f) / \pi )^{1/2}$$

$$A_f = 3.1709 \times 10^{-7} \text{ m}^2 \quad \text{or} \quad 4.9149 \times 10^{-4} \text{ in}$$

$$\Rightarrow d_f = 0.0006354 \text{ m} \quad \text{or} \quad d_f = 0.0250157 \text{ in}$$

(b2) Using reported linear density of the fibers. This second calculation is one way to check the value obtained in (b1).

$$d_f = ( (4 \cdot A_f) / \pi )^{1/2}$$

$$A_f = 3.1868 \times 10^{-7} \text{ m}^2 \quad \text{or} \quad 4.93954 \times 10^{-4} \text{ in}$$

$$\Rightarrow d_f = 0.0006372 \text{ m} \quad \text{or} \quad d_f = 0.0250866 \text{ in}$$

(c) Determination of fiber aspect ratio,  $\eta$ :

$$\eta = l_f / d_f$$

$$\Rightarrow \eta = 79.73$$

Where:

$l_f$  is the length of the fiber in (m),

$d_f$  is the equivalent diameter in (m) obtained in (b2).

(d) Determination of  $\sigma_{n,crit}$

$$\sigma_{n,crit} = (\sigma_{f,ult} - \eta \cdot c_{i,c} \cdot c) / (\eta \cdot c_{i,\phi} \cdot \tan\phi)$$

$$\Rightarrow \sigma_{n,crit} = 7323.89 \text{ kPa or } 1041.69 \text{ psi}$$

The critical confining pressure is too high for practical applications. Consequently, only the first portion of the bilinear equivalent shear strength envelope of the fiber-reinforced composite is of interest.

(e) Determination of equivalent shear strength,  $S_{eq}$

The equivalent shear strength for the range of confining pressure of interest is obtained as:

$$S_{eq,1} = c_{eq,1} + (\tan\phi)_{eq,1} \cdot \sigma_n$$

The cohesive component of the equivalent shear strength is estimated by:

$$c_{eq,1} = (1 + \alpha \cdot \eta \cdot \chi \cdot c_{i,c}) c$$

$$\Rightarrow c_{eq,1} = 13.86 \text{ kPa or } 1.97 \text{ psi}$$

The frictional component of the equivalent shear strength is estimated by:

$$(\tan\phi)_{eq,1} = (1 + \alpha \cdot \eta \cdot \chi \cdot c_{i,\phi}) \tan\phi$$

$$\Rightarrow (\phi)_{eq,1} = 31.23^\circ$$

Table 3 shows the results of soil parameters obtained from the application of discrete approach, the parameters of the soil without reinforcement and the results obtained in triaxial tests using reinforced soil.

**Table 3: Comparison between experimental and predicted soil parameters**

<b>Results</b>	<b>Predicted parameters using the discrete approach</b>	<b>Without Fibers</b>	<b>Best-fit parameters defined from test results</b>
$\phi^{(o)}$	31.23	26.20	33.3
<b>c' (psi)</b>	1.97	1.60	1.4
<b>c' (kPa)</b>	13.86	11.25	9.84

The predicted and best-fit parameters shown in Table 3 correspond to 2 inch fibers mixed at a dosage of 0.2%. Figure 1 shows the corresponding Mohr circles and Mohr-Coulomb shear strength envelope for the unreinforced soil. Figure 2 shows the experimental results obtained from testing of fiber-reinforced specimens (actual results at three confining pressures) as well as the shear strength envelope predicted using the discrete approach. As can be observed in the figure, there is a very good agreement between analytic and experimental shear strength values.

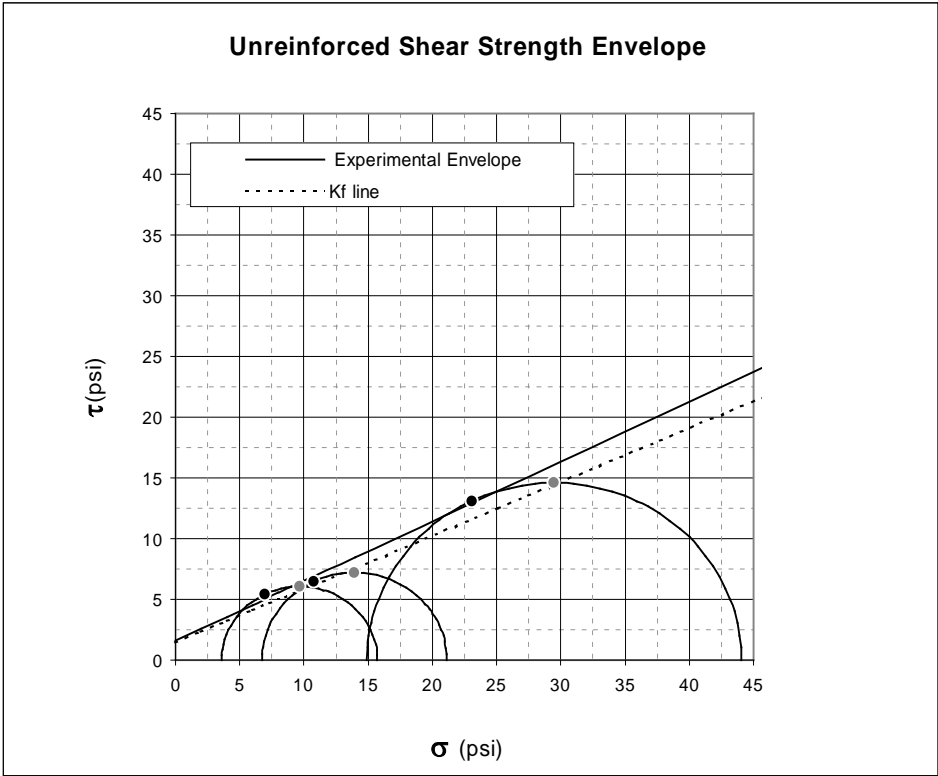


Figure 1: Shear strength results of unreinforced specimens

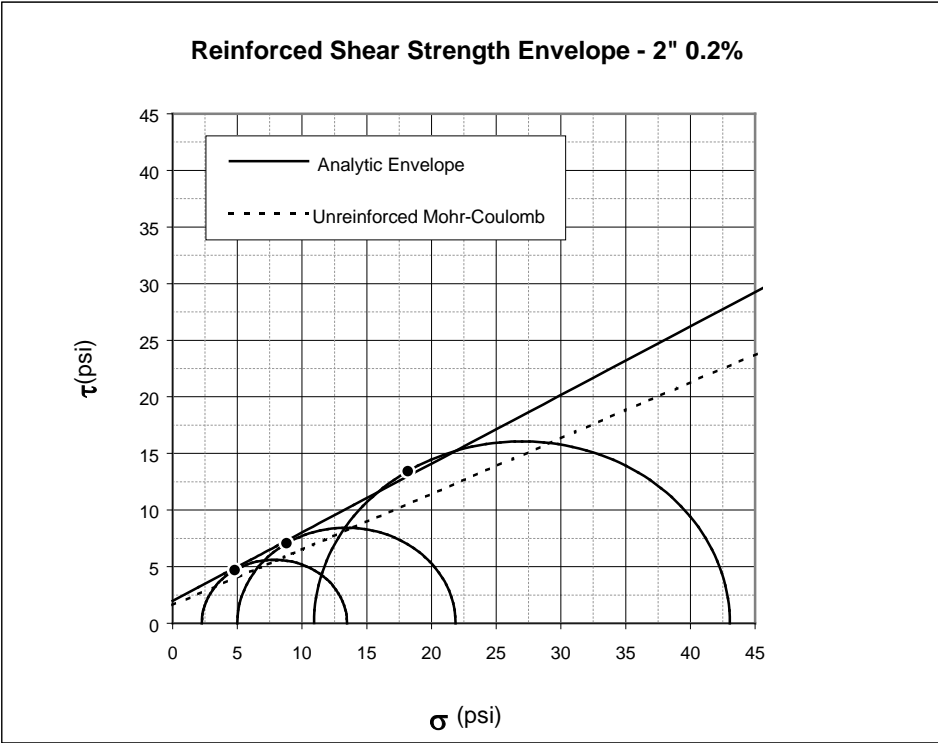


Figure 2: Shear strength results of the fiber reinforced specimens (experimental and predicting using the discrete approach). Fiber length: 2". Fiber content: 0.2%

## Second case study – Cardinal Road Slope Failures

Steps (a) through (e), indicated below, lead to the determination of the equivalent shear strength to be assumed by the designer after applying the discrete approach. The fiber-induced tension is assumed parallel to the failure plane. The soil and fiber reinforcement characteristics used in the calculations are those indicated in Tables 1 and 2.

(a) Determination of the volumetric fiber content,  $\chi$ :

$$\chi = (\chi_w \cdot \gamma) / ((1 + \chi_w) \cdot G_f \cdot \gamma_w)$$

$$\Rightarrow \chi = 0.0035427$$

(b) Determination of the equivalent diameter,  $d_f$ :

(b1) Using reported fibers geometry:

$$d_f = ( (4 \cdot A_f) / \pi )^{1/2}$$

$$A_f = 3.1709 \times 10^{-7} \text{ m}^2 \quad \text{or} \quad 4.9149 \times 10^{-4} \text{ in}$$

$$\Rightarrow d_f = 0.0006354 \text{ m} \quad \text{or} \quad d_f = 0.0250157 \text{ in}$$

(b2) Using reported linear density of the fibers. This second calculation is one way to check the value obtained in (b1)

$$d_f = ( (4 \cdot A_f) / \pi )^{1/2}$$

$$A_f = 3.1868 \times 10^{-7} \text{ m}^2 \quad \text{or} \quad 4.93954 \times 10^{-4} \text{ in}$$

$$\Rightarrow d_f = 0.0006372 \text{ m} \quad \text{or} \quad d_f = 0.0250866 \text{ in}$$

(c) Determination of fiber aspect ratio,  $\eta$ :

$$\eta = l_f / d_f$$

$$\Rightarrow \eta = 79.73$$

Where:

$l_f$  is the length of the fiber in (m),  
 $d_f$  is the equivalent diameter in (m) obtained in (b2).

(d) Determination of  $\sigma_{n,crit}$

$$\sigma_{n,crit} = (\sigma_{f,ult} - \eta \cdot c_{i,c} \cdot c) / (\eta \cdot c_{i,\phi} \cdot \tan\phi)$$

$$\Rightarrow \sigma_{n,crit} = 7325.06 \text{ kPa or } 1041.85 \text{ psi}$$

The critical confining pressure is too high for practical applications. Consequently, only the first portion of the bilinear equivalent shear strength envelope of the fiber-reinforced composite is of interest.

(e) Determination of equivalent shear strength,  $S_{eq}$

The equivalent shear strength for the range of confining pressure of interest is obtained as:

$$S_{eq,1} = c_{eq,1} + (\tan\phi)_{eq,1} \cdot \sigma_n$$

The cohesive component of the equivalent shear strength is estimated by:

$$c_{eq,1} = (1 + \alpha \cdot \eta \cdot \chi \cdot c_{i,c}) c$$

$$\Rightarrow c_{eq,1} = 12.93 \text{ kPa or } 1.84 \text{ psi}$$

The frictional component of the equivalent shear strength is estimated by:

$$(\tan\phi)_{eq,1} = (1 + \alpha \cdot \eta \cdot \chi \cdot c_{i,\phi}) \tan\phi$$

$$\Rightarrow (\phi)_{eq,1} = 28.74^\circ$$

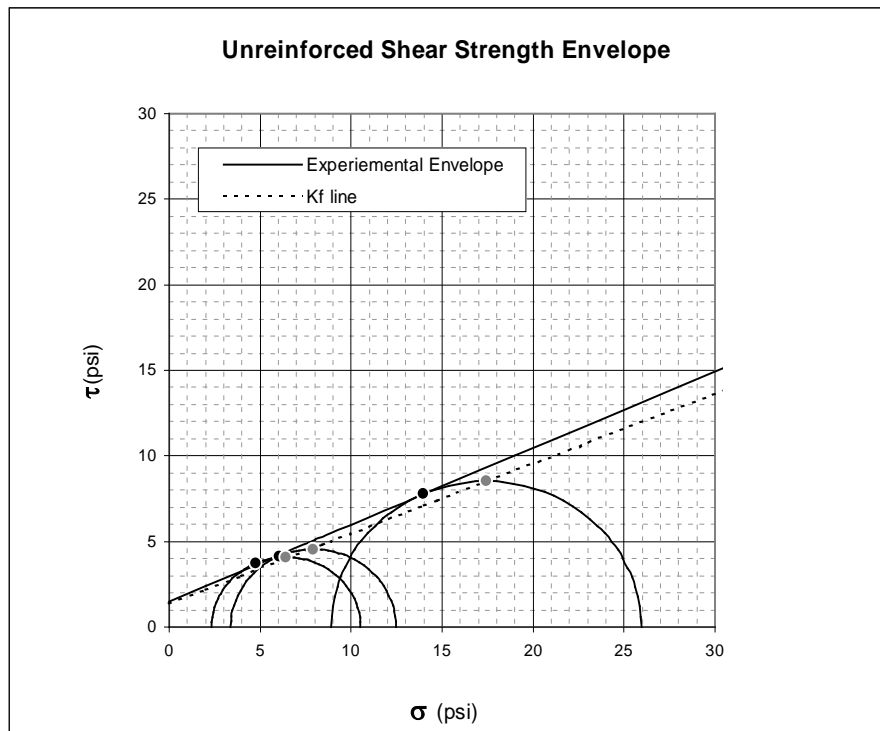
Table 4 shows the results of soil parameters obtained from the application of discrete approach, the parameters of the soil without reinforcement and the results obtained in triaxial tests using reinforced soil.



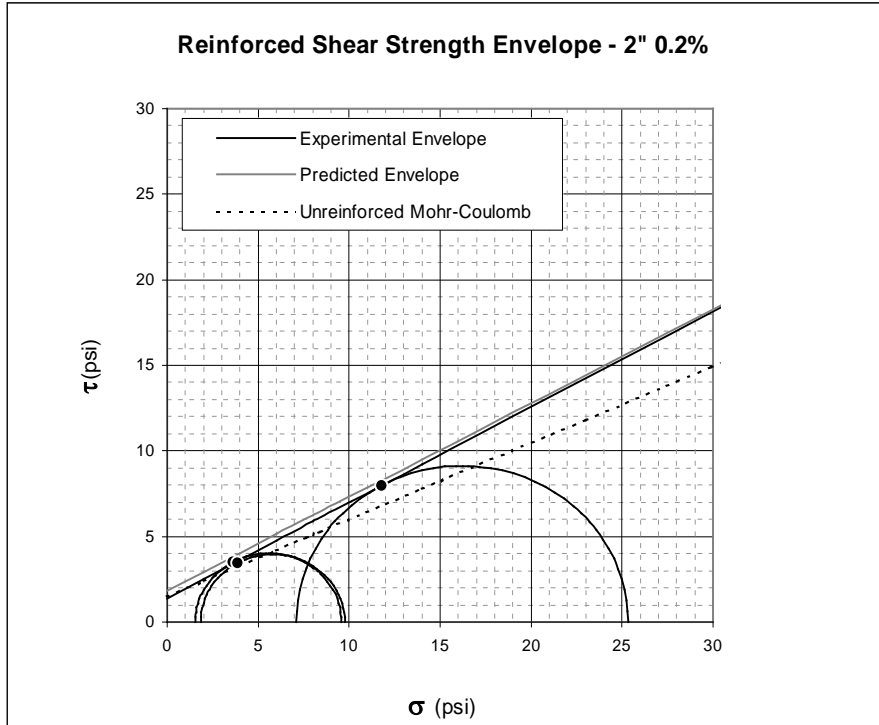
**Table 4: Comparison between experimental and predicted soil parameters**

Results	Predicted parameters using the discrete approach	Without Fibers	Best-fit parameters defined from test results
$\phi^{(o)}$	28.74	24.10	29.2
<b>C' (psi)</b>	1.84	1.50	1.4
<b>C' (kPa)</b>	12.93	10.54	9.84

The predicted and best-fit parameters shown in Table 4 correspond to 2 inch fibers mixed at a dosage of 0.2%. Figure 3 shows the corresponding Mohr circles and Mohr-Coulomb shear strength envelope for the unreinforced soil. Figure 4 shows the experimental results obtained from testing of fiber-reinforced specimens (actual results at three confining pressures) as well as the shear strength envelope predicted using the discrete approach. As can be observed in the figure, there is a very good agreement between analytic and experimental shear strength values.



**Figure 3: Shear strength results of unreinforced specimens**



**Figure 4: Shear strength results of the fiber reinforced specimens (experimental and predicting using the discrete approach). Fiber length: 2". Fiber content: 0.2%**

### **Third case study – Vanderbilt Stadium**

Steps (a) through (e), indicated below, lead to the determination of the equivalent shear strength to be assumed by the designer after applying the discrete approach. The fiber-induced tension is assumed parallel to the failure plane. The soil and fiber reinforcement characteristics used in the calculations are those indicated in Tables 1 and 2.

(a) Determination of the volumetric fiber content,  $\chi$ :

$$\chi = (\chi_w \cdot \gamma) / ((1 + \chi_w) \cdot G_f \cdot \gamma_w)$$

$$\Rightarrow \chi = 0.0036857$$

(b) Determination of the equivalent diameter,  $d_f$ :

(b1) Using reported fibers geometry:

$$d_f = ((4 \cdot A_f) / \pi)^{1/2}$$

$$A_f = 3.1709 \times 10^{-7} \text{ m}^2 \quad \text{or} \quad 4.9149 \times 10^{-4} \text{ in}$$

$$\Rightarrow d_f = 0.0006354 \text{ m} \quad \text{or} \quad d_f = 0.0250157 \text{ in}$$

(b2) Using reported linear density of the fibers. This second calculation is one way to check the value obtained in (b1)

$$d_f = ((4 \cdot A_f) / \pi)^{1/2}$$

$$A_f = 3.1868 \times 10^{-7} \text{ m}^2 \quad \text{or} \quad 4.93954 \times 10^{-4} \text{ in}$$

$$\Rightarrow d_f = 0.0006372 \text{ m} \quad \text{or} \quad d_f = 0.0250866 \text{ in}$$

(c) Determination of fiber aspect ratio,  $\eta$ :

$$\eta = l_f / d_f$$

$$\Rightarrow \eta = 79.73$$

Where:

$l_f$  is the length of the fiber in (m),  
 $d_f$  is the equivalent diameter in (m) obtained in (b2).

(d) Determination of  $\sigma_{n,crit}$

$$\sigma_{n,crit} = (\sigma_{f,ult} - \eta \cdot c_{i,c} \cdot c) / (\eta \cdot c_{i,\phi} \cdot \tan\phi)$$

$$\Rightarrow \sigma_{n,crit} = 7333.26 \text{ kPa or } 1043.02 \text{ psi}$$

The critical confining pressure is too high for practical applications. Consequently, only the first portion of the bilinear equivalent shear strength envelope of the fiber-reinforced composite is of interest.

(e) Determination of equivalent shear strength,  $S_{eq}$

The equivalent shear strength for the range of confining pressure of interest is obtained as:

$$S_{eq,1} = c_{eq,1} + (\tan\phi)_{eq,1} \cdot \sigma_n$$

The cohesive component of the equivalent shear strength is estimated by:

$$c_{eq,1} = (1 + \alpha \cdot \eta \cdot \chi \cdot c_{i,c}) c$$

$$\Rightarrow c_{eq,1} = 6.95 \text{ kPa or } 0.99 \text{ psi}$$

The frictional component of the equivalent shear strength is estimated by:

$$(\tan\phi)_{eq,1} = (1 + \alpha \cdot \eta \cdot \chi \cdot c_{i,\phi}) \tan\phi$$

$$\Rightarrow (\phi)_{eq,1} = 41.7^\circ$$

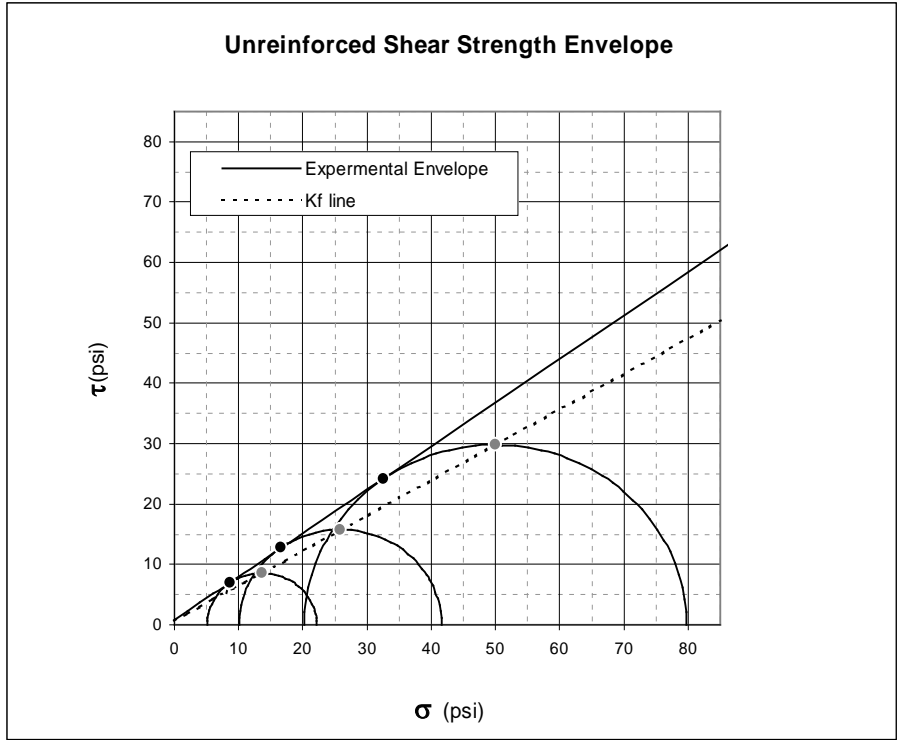
The predicted and best-fit parameters shown in Table 5 correspond to 2 inch fibers mixed at a dosage of 0.2%. Figure 5 shows the corresponding Mohr circles and Mohr-Coulomb

shear strength envelope for the unreinforced soil. Figure 6 shows the experimental results obtained from testing of fiber-reinforced specimens (actual results at three confining pressures) as well as the shear strength envelope predicted using the discrete approach. As can be observed in the figure, there is a very good agreement between analytic and experimental shear strength values.

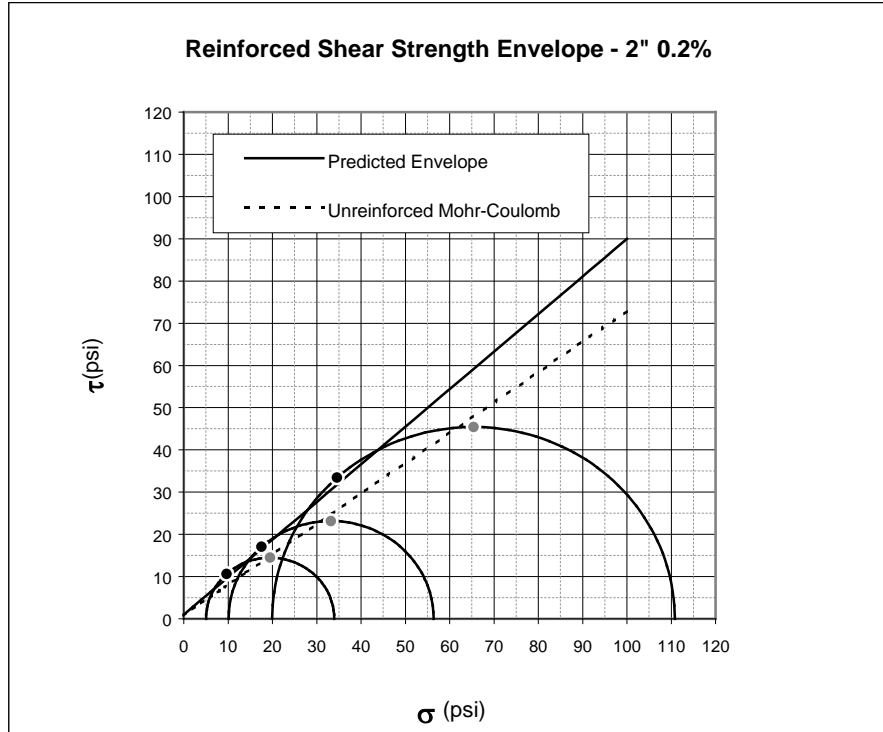
**Table 5: Comparison between experimental and predicted soil parameters**

<b>Results</b>	<b>Predicted parameters using the discrete approach</b>	<b>Without Fibers</b>	<b>Best-fit parameters defined from test results</b>
$\phi^{(o)}$	41.7	35.8	42.7
<b>C' (psi)</b>	0.99	0.8	1.4
<b>C' (kPa)</b>	6.96	5.62	9.84

In the following figures, Figure 5 and 6, it can be seen the Mohr circles, the Mohr-Coulomb Envelop of the soil with and without reinforcement while making a comparison with the results obtained with the discrete approach.



**Figure 5: Shear strength results of unreinforced specimens**



**Figure 6: Shear strength results of the fiber reinforced specimens (experimental and predicting using the discrete approach). Fiber length: 2". Fiber content: 0.2%**

**Fourth case study – Las Colinas Slopes (TETCO)**

Steps (a) through (e), indicated below, lead to the determination of the equivalent shear strength to be assumed by the designer after applying the discrete approach. The fiber-induced tension is assumed parallel to the failure plane. The soil and fiber reinforcement characteristics used in the calculations are those indicated in Tables 1 and 2.

(a) Determination of the volumetric fiber content,  $\chi$ :

$$\chi = (\chi_w \cdot \gamma) / ((1 + \chi_w) \cdot G_f \cdot \gamma_w)$$

$$\Rightarrow \chi = 0.0031366$$

(b) Determination of the equivalent diameter,  $d_f$ :

(b1) Using reported fibers geometry:

$$d_f = \left( (4 \cdot A_f) / \pi \right)^{1/2}$$

$$A_f = 3.1709 \times 10^{-7} \text{ m}^2 \quad \text{or} \quad 4.9149 \times 10^{-4} \text{ in}$$

$$\Rightarrow d_f = 0.0006354 \text{ m} \quad \text{or} \quad d_f = 0.0250157 \text{ in}$$

(b2) Using reported linear density of the fibers. This second calculation is one way to check the value obtained in (b1)

$$d_f = \left( (4 \cdot A_f) / \pi \right)^{1/2}$$

$$A_f = 3.1868 \times 10^{-7} \text{ m}^2 \quad \text{or} \quad 4.93954 \times 10^{-4} \text{ in}$$

$$\Rightarrow d_f = 0.0006372 \text{ m} \quad \text{or} \quad d_f = 0.0250866 \text{ in}$$

(c) Determination of fiber aspect ratio,  $\eta$ :

$$\eta = l_f / d_f$$

$$\Rightarrow \eta = 79.73$$

Where:

$l_f$  is the length of the fiber in (m),

$d_f$  is the equivalent diameter in (m) obtained in (b2).

(d) Determination of  $\sigma_{n,crit}$

$$\sigma_{n,crit} = \left( \sigma_{f,ult} - \eta \cdot c_{i,c} \cdot c \right) / \left( \eta \cdot c_{i,\phi} \cdot \tan\phi \right)$$

$$\Rightarrow \sigma_{n,crit} = 7294.62 \text{ kPa} \quad \text{or} \quad 1037.52 \text{ psi}$$

The critical confining pressure is too high for practical applications. Consequently, only the first portion of the bilinear equivalent shear strength envelope of the fiber-reinforced composite is of interest.



(e) Determination of equivalent shear strength,  $S_{eq}$

The equivalent shear strength for the range of confining pressure of interest is obtained from:

$$S_{eq,l} = c_{eq,1} + (\tan\phi)_{eq,1} \cdot \sigma_n$$

The cohesive component of the equivalent shear strength is estimated by:

$$c_{eq,1} = (1 + \alpha \cdot \eta \cdot \chi \cdot c_{i,c}) c$$

$$\Rightarrow c_{eq,1} = 34.59 \text{ kPa or } 4.92 \text{ psi}$$

The frictional component of the equivalent shear strength is estimated by:

$$(\tan\phi)_{eq,1} = (1 + \alpha \cdot \eta \cdot \chi \cdot c_{i,\phi}) \tan\phi$$

$$\Rightarrow (\phi)_{eq,1} = 13.37^\circ$$

The predicted and best-fit parameters shown in Table 6 correspond to 2 inch fibers mixed at a dosage of 0.2%. Figure 7 shows the corresponding Mohr circles and Mohr-Coulomb shear strength envelope for the unreinforced soil. Figure 8 shows the experimental results obtained from testing of fiber-reinforced specimens (actual results at three confining pressures) as well as the shear strength envelope predicted using the discrete approach. As can be observed in the figure, there is a very good agreement between analytic and experimental shear strength values.

**Table 6: Comparison between experimental and predicted soil parameters**

Results	Predicted parameters using the discrete approach	Without Fibers	Best-fit parameters defined from test results
$\phi^{(o)}$	13.37	11.20	15.5
<b>C' (psi)</b>	4.92	4.1	4.1
<b>C' (kPa)</b>	34.54	28.82	28.82

In the following figures, Figure 7 and 8, it can be seen the Mohr circles, the Mohr-Coulomb Envelop of the soil with and without reinforcement while making a comparison with the results obtained with the discrete approach.

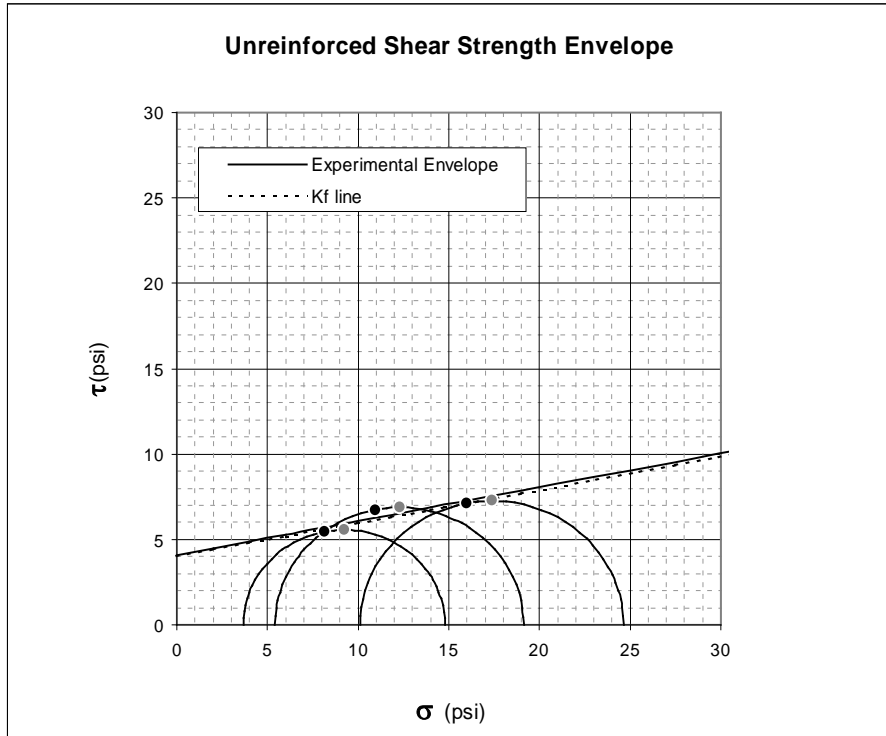
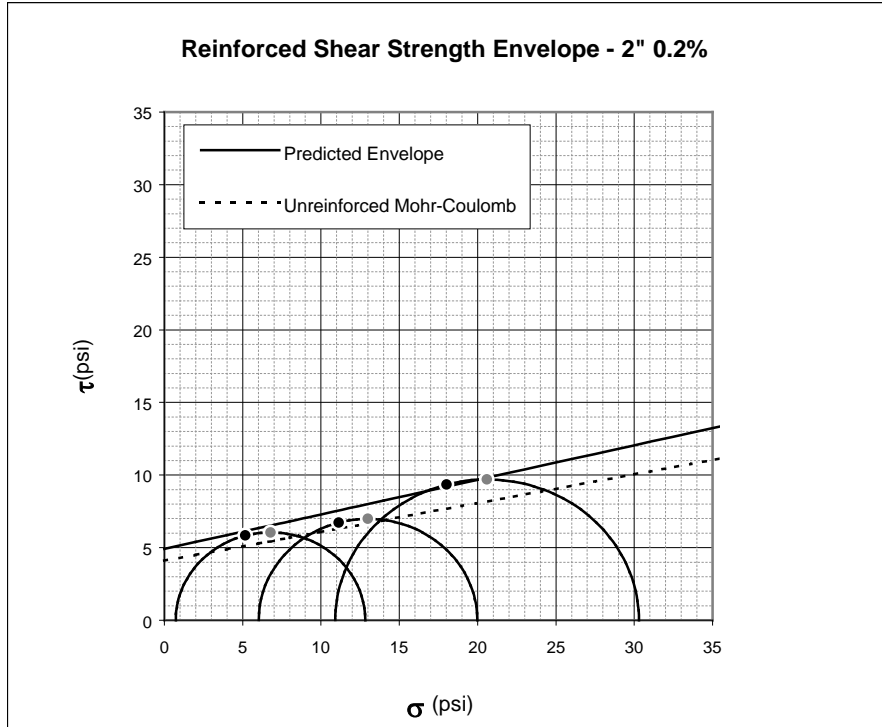


Figure 7: Shear strength results of unreinforced specimens



**Figure 8: Shear strength results of the fiber reinforced specimens (experimental and predicting using the discrete approach). Fiber length: 2". Fiber content: 0.2%**

### **Final Remarks**

Very good agreement was obtained for the case of four additional soils between experimental shear strength values obtained on fiber-reinforced soil specimens and the analytical shear strength envelopes obtained using the discrete approach. This provides significant additional evidence on the suitability of the discrete method proposed in this investigation.